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REPORT OF A THESIS ADVISOR

Master's thesis title: "Relativistic stochastic processes, their description and use in financial markets"
Candidate: Rastislav Blaho
Institution: Department of Physics, Czech Technical University in Prague, Faculty of Nuclear Sciences and Physical Engineering, Břehová 7, 115 19 Czech Republic
Assessment: A (výborně)

It is a pleasure to report on Master's Thesis of Rastislav Blaho. Mr. Blaho worked under my supervision during the academic years 2019-2021. In that time he focused his attention on issues related to relativistic stochastic processes with a particular emphasize on financial markets.

In his work was Mr. Blaho inspired not only by classical works of P. Hänggi and J. Dunkel (relativistic Brownian motion), H. Risken (Fokker–Planck equations), J.C. Hull (derivative markets) but also by a number of original recent papers. He approached the subject of his Thesis with a good knowledge of prerequisite mathematical concepts.

The Thesis itself is well structured and basic concepts are easy to grasp. In Introduction some information about the organization of the Thesis is provided. Chapters 1 is logically divided into two parts. First part discusses Hänggi et al. approach to a relativistic generalization of the Ornstein–Uhlenbeck stochastic equation, which currently represents a canonical version. Here Mr. Blaho provides quite detailed discussion of various approaches that allow to obtain generalized position distribution, which is of interest in Chapter 3 in connection with option pricing. Apart from the conventional material Mr. Blaho also provided original derivation of the generalized position distribution in terms of multi-variable Fokker–Planck equation. In the second part of Chapter 1 Mr. Blaho puts emphasize directly on Fokker–Planck equations rather than ensuing stochastic equations. He discussed possible ways how to formulate this equation in manifestly covariant way and he went some way towards this goal by utilizing Hänggi–Klimontovic post-point discretization, which allowed him to find a relativistic backward Kolmogorov equation. In Chapter 2 is provided a

brief review of a mathematical and conceptual background of the Black–Scholes option pricing model. Beside a standard textbook material, Mr. Blaho also discussed various salient issues, such as binomial tree option pricing model, Ito diffusion and Feynman–Kac formula, which I have found very illuminating. Chapter 3 is devoted to relativistic generalization of the Black–Scholes option pricing model. This chapter forms a basic interpretational frame for discussing the role of relativity in financial theory of derivatives. This part belongs to the most original part of the Thesis. Apart from Hänggi et al. approach Mr. Blaho also employed the connection between Schrödinger and Fokker–Planck equation, which is often used in the so-called quantum finance theory. This allowed him to obtain the original Fock–Goldstein relativistic generalization of the diffusion equation in terms of the telegrapher's equation. This finding is very interesting, since it is based on a very different premises and methodology than Fock–Goldstein derivation. It is expected that this issue will be explored in more detail during Mr. Blahos's prospective doctoral study. Thesis closes with Conclusion where a brief summary of results is presented. Finally, there are four appendices, which give necessary background on such issues as: a) special relativity, b) stochastic calculus, c) Fokker–Plack equation and d) non-Gaussian increment distribution.

All in all, Master Thesis of Rastislav Blaho has in my opinion quite high quality. It offers an interesting and in many respects original selection and discussion of topics that are not only important in their own right, but also, e.g., in modern theory of financial markets, such as derivative markets or nanotrading. At the same time I should say that the Thesis is unfortunately riddled with a large number of typos both in text and formulas. This was caused by the fact that most of the text was finalized during a very short period before submission and there was basically no possibility for me to check the final text. Finally, I should add that theory of relativistic stochastic processes provides a novel paradigm within the steadily growing field of Quantitative Finance. I am sure that expertise gained by Mr. Blaho will be beneficial to him in the years to come. For the originality of the work I propose the mark A "výborně".

Petr Jizba