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Summary of Dissertation Thesis

Higher Order Neural Unit Adaptive Control and Stability Analysis for Industrial System Applications

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Title: *Higher Order Neural Unit Adaptive Control and Stability Analysis for Industrial System Applications*

Abstract

Given the push in our modern digitalized industry for advanced, yet comprehensible methods for process identification and control, computational intelligence methods are readily ongoing in study. Higher-order neural units (HONUs) have proven to be such computationally efficient and comprehensible nonlinear polynomial models for application as standalone process models or as a nonlinear control loop where one recurrent HONU is a plant model and another HONU is as a nonlinear state feedback (neuro)controller (via MRAC scheme). Alternative approaches as the widely used Lyapunov function, can be used for design of the control law or prove of stability for existing control laws in state space for a given equilibrium and a given input.

However, in practical engineering applications such methods although proving stability about an equilibrium point may still result in bad performance or damage if they are also not proven to be bounded-input-bounded-output/state (BIBO/BIBS) stable with respect to the control inputs. Therefore, the main contribution of this dissertation is to introduce two novel real-time BIBO/BIBS based stability evaluation methods for HONUs and for their closed control loops. The proposed methods being derived from the core polynomial architectures of HONUs themselves provides a straightforward and comprehensible framework for stability monitoring that can be applied to other forms of recurrent polynomial neural networks. New results are presented from the rail automation field as well as several non-linear dynamics system examples. Further directions are also highlighted for sliding mode design via HONUs and multi-layered HONU feedback control presented as a framework for low to moderately nonlinear systems.

Keywords

model reference adaptive control; discrete-time nonlinear dynamic systems; polynomial neural networks; point-wise state-space representation; stability

Název práce: Adaptivní řízení s polynomiálními neuronovými architekturami a analýza stability pro průmyslové aplikace

Anotace

Metody výpočetní inteligence pro identifikaci a řízení procesů jsou v této práci studovány vzhledem k rozvoji moderního digitalizovaného průmyslu a s ohledem na potřebu pokročilých avšak pro praxi srozumitelných algoritmů. Neuronové jednotky vyšších stupňů (HONU) se ukázaly jako výpočetně efektivní a přitom srozumitelné nelineární polynomiální modely pro řízení samotných soustav nebo pro optimalizaci již existujících regulačních smyček, kde jedna jednotka HONU představuje na datech založený model soustavy a druhá HONU je implementována jako nelineární stavový (neuro) regulátor (řízení typu MRAC).

Mezi dnes široce rozšířené přístupy nelineárního řízení patří metody pomocí Lyapunovovi funkce pro návrh regulátoru, který v principu garantuje stabilitu ve stavovém prostoru pro daný rovnovážný bod a vstup systému, avšak v principu nepředepisuje požadovanou dynamiku chování (jako např. řízení typu MRAC). V praktických inženýrských aplikacích také může stále dojít ke špatnému řízení a tudíž i poškození, pokud se také neprokáže, že regulační smyčka je stabilní ve smyslu omezený-vstup omezený-výstup/stav (BIBO / BIBS). Hlavním přínosem této disertační práce je proto zavedení dvou nových metod vyhodnocení stability nelineárních adaptivních smyček s HONU založených na BIBO / BIBS. Navrhované metody jsou odvozeny na základě polynomiálních neuronových architektur HONU a poskytují přímý a komplexní rámec pro sledování stability v reálném čase, který lze použít i na jiné formy rekurentních polynomiálních neuronových sítí (lineárních v parametrech). Jsou prezentovány nové výsledky adaptivního řízení v oblasti kolejových vozidel a na několika dalších příkladech nelineárních dynamických systémů. Další směry jsou také naznačeny pro návrh řízení typu „sliding mode control“ s HONU a zpětnovazebního řízení se sítěmi HONU pro mírně až středně nelineární systémy, kde lineární způsoby řízení nedosahují požadovaných vlastností.

Klíčová slova

adaptivní řízení s referenčním modelem; diskrétní nelineární dynamické systémy; polynomiální neuronové sítě; bodová stavová reprezentace; stabilita

Contents

1. INTRODUCTION

With respect to the last decade, many powerful and quite novel approaches in the field of computational intelligence have brought to the fore in our industry. In accordance to the AI Maturity Model by Gartner (Aug 2018) less than 3% of the worlds large scale engineering companies are readily employing computational intelligence for real-time process identification and control. The more analyzable the computational algorithm, the higher chance there is for its adoption by industry because a rigorous stability analysis helps to guarantee and avoid the unexpected financial loss resulting from interrupted production or damages. Also, advanced algorithms require highly skilled and educated specialists to be available within engineering teams to maintain the algorithms and to prevent the algorithms from misuse and from causing problems (including getting unstable). Then, employee fluctuation is another natural risk factor for applying too complicated algorithms into practice.

Polynomial Neural Networks (PNN) and Higher Order Neural Networks (HONN) represent a family of relatively well analyzable neural architectures. These directions of polynomial neural computation significantly originate from works [1]; furthermore, the usefulness of HONNs for adaptive control via Lyapunov function approach was successfully demonstrated also in last decade, e.g. in [1], [2] (to mention at least a few most significant out of much more). The comprehensibility of the polynomial architectures relates to the fact that polynomials are linear in parameters, while the quality of nonlinear approximation is adjustable via the polynomial order. This also makes polynomial neural architectures attractive for recently hot topics such as ridge regression and extreme learning machines [3], or information theoretic learning approaches [4].

In this dissertation, a focus on Higher Order Neural Units (HONUs) [5]–[7] that can be applied individually or as building units of more complicated polynomial structures is considered. And generally, a focus on discrete-time systems that involves recurrent HONUs as well as feedforward ones (e.g., for prediction, novelty detection, and control where dynamical models are obtained from data only). For stability analysis of nonlinear dynamical systems including HONUs, we incline to the area of recurrent neural networks (RNNs) as tremendous effort has been carried out on stability in there. The algorithms for stability of RNNs have been massively researched as reviewed in [8] and references therein and further below. Nevertheless, review paper [8] naturally concludes that we have no universally best stability evaluation algorithm for RNNs as for nonlinear dynamical systems; a proper stability evaluation is always case dependent considering different aspects of each particular purpose. Earlier methods for stability analysis of RNNs utilized linearization analysis and then Lyapunov function based techniques for which the stability is related to control law design. A traditional approach as exampled in [9] and [10] is to construct the adaptive control law via a suitable Lyapunov function candidate, such to ensure a global rule to constrain the process inputs. Another technique in [11] is the global robust exponential stability of an equilibrium point for delayed neural networks developed via homeomorphism and Lyapunov function based techniques. In [12] via a delay partitioning technique, a novel stability approach justified via the Lyapunov-Krasovskii criteria is investigated to warrant the global stability of static neural networks with time delays.

In practical control applications, although stability of the equilibrium point may be justified as exampled in [11], it is often necessary and more practical to ensure boundedness of the resulting process states with respect to bounded control inputs. As a result, a rather readily researched field in the realm of nonlinear discrete-time systems is bounded-input-bounded- state (BIBS) stability [13], [14] with a more universal definition being the condition of input-to-state stability (ISS) [15]–[18]. Several studies of ISS for RNNs with time-varying delays may include [19], where two algebraic criteria backbone from a Lyapunov functional and they are derived for the verification of ISS for a class of time-delayed RNNs.

The previous work [7], has focused on the stability of the real-time adaptation of HONUs. Newly, this dissertation focuses on a stability framework for standalone HONU models or HONU-MRAC control loops as whole, even applied as a constant parameter loop. Being motivated with some analogies from linear and nonlinear concepts, two novel (BIBO and BIBS based) stability analysis techniques (extended from [20]) for discrete-time recurrent HONUs are proposed. The first derived technique is denoted as DHS (Dynamic HONU Stability), which is BIBO-based approach that monitors eigenvalues of a linearized state-space model to determine the local stability of state point neighborhood. The second derived technique is denoted DDHS (Decomposed DHS), and it is a BIBS-based (ISS-based) approach that results from a newly introduced nonlinear state-space decomposition via polynomial architecture of HONUs. DDHS is rather different from the ISS based forms presented in [19], [21], [22], because it derives from the same nonlinear architecture at a given state and for given inputs. DDHS then monitors stability of every state transition from initial conditions; thus, DDHS evaluates stability of polynomial time-variant systems for given inputs without being limited to specifically applied control laws or training methods. Both DHS and DDHS stability evaluation techniques are derived for single recurrent HONUs as well as for dynamical closed loop with two HONUs. Sections 6 and 7 illustrate the performance of both methods on several nonlinear system examples as well as real-time rail automation examples to highlight its applicability in real industrial process identification and control.

2. CURRENT STATUS OF RESEARCH

In this dissertation, three main areas of adaptive control were researched and classified as follows:

1. *Control input adjustment methods*: Such approach relies on the justified stability of the control law via consideration of the negative derivate in each time point of the Lyapunov control function. Another popular form is Model Predictive Control (MPC) [23]–[25]. Other advanced forms include adaptive backstepping design [1] and a quite trending technique of sliding mode control [26], [27]
2. *Heuristic tuning based methods*: A popular form of such control is Adaptive Dynamic Programming (ADP) [28], [29] which may be divided into the basic structures of heuristic dynamic programming (HDP) as exampled in the works [30]-[31] and Dual heuristic programming (DHP). In principle, the approach is focussed on minimisation of an error measure over time. A penalisation or reward is calculated to compute the output of the critic network which is an estimate of the cost function J .
3. *Parameter adjustment-based control*: More conventional forms include control are adaptive PID controller tuning or Model Reference Adaptive Control (MRAC). Till this day more emphasis is pushed towards variations with use of computational intelligence methods as such neural-networks and fuzzy based methods. Parameter adjustment forms may also be combined with input adjustment forms, an example of such approach is the Model-Reference Sliding Mode Control as reviewed in [32].

Following comprehensive review of the fields 1)-3), the main motivation of this dissertation is to advance field 3) in the realm of model reference based higher order neural unit (HONU-MRAC) based control. Primarily, to bridge an open gap till this date of practical application and theories of stability analysis for such forms of adaptive nonlinear polynomial models and their use for control.

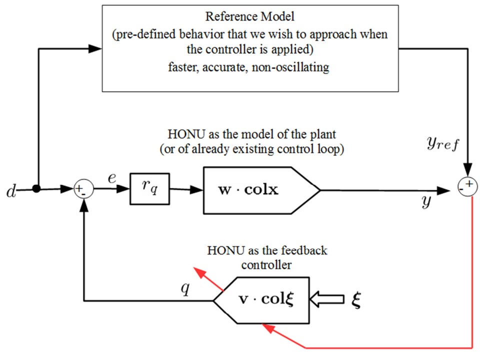

Figure 1- Higher order neural unit model reference based control scheme (HONU-MRAC), where one HONU is identified as a plant and one standalone HONU controller is extended in feedback (possibility of multiple HONU controllers in feedback also presented in [33]).

The origin of HONUs derive from earlier works of M.Gupta [5] where the Higher Order Neural Network (HONN) was presented and in I. Bukovsky and N. Homma [34], where a quadratic neural unit (QNU) as a advancement on the SONN by [35] was analysed as a powerful medium for plant modelling and control. The most up to date architecture of a HONU-MRAC control loop is illustrate in Figure 1 where one HONU is used for plant identification and another single HONU in feedback as a adaptive controller. Featuring training via fast real time algorithms as such Levenberg-Marquardt and conjugate gradient decent algorithm [33] or earlier in [36] (I.Bukovsky, S.Redlapalli) where HONUs are computationally faster in achieving adequate convergence in square error whilst achieving desirable control performance for both nonlinear unknown systems as well as linear systems of SISO structure as compared to MLP based architectures. In the work [37] by L. Smetana, the superiority of QNU architecture as compared to the conventional PID controller was exhibited, which also highlights a use case in control loop optimization with extension of a HONU in feedback. Though HONUs have shown promising results in field of adaptive identification and control. Study into the very stability of such HONUs as standalone models and HONU-MRAC control loops is limited. In [10] though a stability approach for stable gradient descent learning of HONUs themselves was presented. Studies focussed on justifying both the online adaptive control law itself is stable as well as dynamically the whole control loop including plant, has not readily been presented. Nor the stability of offline tuned neural weight parameters with online application on new data, especially for use in real industrial applications, and is hence the key problem that is addressed in this dissertation work.

3. GOALS OF DISSERTATION

1) *Propose pointwise state-space representation of a HONU*: As a major contribution, this dissertation investigates and derives the transformation of nonlinear HONU architectures to a linearly approximated state-space model in discrete state points. Following this newly derived state-space form the Discrete-time HONU Stability Condition (DHS) is developed as a pointwise state-space based evaluation of BIBO stability and further proven for justifying asymptotic stability in neighbourhood of the evaluated state point via Lyapunov stability theory.
2) *Propose a pointwise state-space representation of a HONU via polynomial decomposition*: Due to an intrinsic relation of HONU architectures to discrete time nonlinear state space models via their in-parameter factorization, another major objective and contribution of this dissertation is to derive a new decomposition approach for modelling HONU models and further their whole HONU-MRAC control loop. Due to this intrinsic relation as extension of works [20], [38] a decomposition of the HONU polynomial equation to sub-polynomial state space form is proposed.
3) *Derive a new ISS based stability condition for BIBS stability assessment of HONU polynomial architectures*: As a result of 2), the concepts of ISS stability are extended to the derived decomposed state space form and a method for justification of BIBS stability, termed as Decomposed Discrete-time HONU Stability Condition (DDHS) and the further resulting DDHS(Strict) condition. An added advantage of this decomposition approach is the preservation of dynamical accuracy for higher order nonlinear polynomial structures (presented in this dissertation for up to 3rd order as a HONU-MRAC control loop).
4) *Experimental analysis to validate the proposed DHS and DDHS approaches*: A final objective and contribution is the deep experimental analysis and comparison of the derived DHS and DDHS approaches. The methods are also analysed with respect to Lyapunov control function stability approach. Both methods are implemented on physical industrial systems with focus to rail automation applications to validate the feasibility of use for our modern industry. Future directions via sliding mode control approach with the presented HONU decomposition and multi-layered HONU feedback control for low to moderately nonlinear systems are also discussed.

4. FUNDAMENTALS OF HIGHER ORDER NEURAL UNITS AND THEIR ADAPTIVE CONTROL

Since recently e.g. [10], HONUs have been appearing in long-vector notation, e.g. for QNU or CNU as follows

$$\tilde{y} = \mathbf{w} \cdot col^{r=2}(\mathbf{x}), \;\; \tilde{y} = \mathbf{w} \cdot col^{r=3}(\mathbf{x}), \tag{1}$$

Further $\mathbf{w}$ is the long-vector of all neural weights (i.e. r-dimensional $\mathbf{W}$ flattens to 1-D vector), for example of QNU it is as follows

$$\mathbf{w} = \left[w_{0,0} \;\; w_{0,1} \;\dots\; w_{i,j} \;\dots\; w_{n_x,n_x} \right] = \left[\left\{ w_{i,j} \right\} \begin{matrix} i=0\dots n_x \\ j=i\dots n_x \end{matrix} \right], \tag{2}$$

where $n_x = n_y + n_u$, which was the length of x that was the vector of neural inputs (including step-delayed feedbacks $\tilde{y}$ and control inputs u) as

$$\mathbf{x}(k) = [1 \;\; \tilde{y}(k-n_y+1) \;\dots\; \tilde{y}(k) \;\; u(k-n_u+1) \;\dots\; u(k)]^T . \tag{3}$$

In [10] $col(\mathbf{x})$ stands for transformation of $\mathbf{x}$ into the long-vector consisting of all (not repeated) polynomial terms up to the order of HONU, so for QNU it yields that

$$col^{r=2}(\mathbf{x}) = \mathbf{colx} = \{ x_i \cdot x_j \; ; \; i=0 \dots n_x, \; j=i \dots n_x \}, \tag{4}$$

and for CNU

$$col^{r=3}(\mathbf{x}) = \mathbf{colx} = \{ x_i \cdot x_j \cdot x_\kappa \; ; i=0 \dots n_x , j=i \dots n_x , \kappa=j \dots n_x \}. \tag{5}$$

In similar analogy, from (3) an additional recurrent HONU in the forward-branch according to Figure 1 where $\tilde{y}$ is the forward-branch HONU output and u is the external (control) input. The feedback law is as follows

$$u(k) = d(k) - p(k) \cdot \tilde{q}(k), \tag{6}$$

where $\tilde{q}(k)$ is output of the feedback-branch HONU defined in (8), $d(k)$ can be desired value, and $p(k)$ is an adaptable proportional gain. Optionally, in order to reflect either too high or too small static gain of controlled plant. a mutation of control law (6) can be

$$u(k) = p(k) \cdot \big(d(k) - q(k) \big). \tag{7}$$

The feedback-branch HONU output $\tilde{q}$ is then defined as follows

$$\tilde{q}(k) = \mathbf{v}(k) \cdot col^{\gamma} \left(\xi(k) \right), \tag{8}$$

where $\mathbf{v}$ is the long vector of weights, $\boldsymbol{\xi}$ is customizable input vector involving step delays of $\tilde{y}$ and u with augmenting unit $\xi_0 = 1$, and γ is customable nonlinear polynomial order of the feedback-branch HONU. For a list of fundamental weight update laws for general or of HONUs refer to the appendix section, where the weight updates themselves are performed as $\mathbf{w}(k) = \mathbf{w}(k-1) + \Delta\mathbf{w}$.

5. MAIN PRINCIPLES OF STABILITY ANALYSIS FOR NON-LINEAR SYSTEMS

Considering the general class of discrete-time nonlinear dynamic system defined as

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}(k)\cdot\mathbf{x}(k) + \mathbf{B}(k)\cdot\mathbf{u}(k) \\ y(k) &= \mathbf{C}\cdot\mathbf{x}(k)\ ; k \geq 0, \end{aligned} \tag{9}$$

where, both $\mathbf{A}(k)$ and $\mathbf{B}(k)$ are both continuous in $\mathbf{x}$ and bounded with respect to k. The following definitions may be stated.

***Definition 1*: For a discrete-time state-space model, two equally justified statements ensure the respective model is asymptotically stable about its equilibrium point [17], [42]:

1) The time-invariant HONU matrix $\mathbf{A}(k)$ is asymptotically stable if and only if the relation (10) holds.
2) Given any matrix $\mathbf{Q} = \mathbf{Q}^T > 0$ there exists a positive definite matrix $\mathbf{P} = \mathbf{P}^T$ such that satisfies the proceeding relation

$$\mathbf{A}(k)^{\ T}\mathbf{P}\mathbf{A}(k) - \mathbf{P} = -\mathbf{Q}. \tag{10}$$

Definition 2 [21, Chapter 2.9]: The time-variant state-space representation of the form (9) is ISS stable provided that

$$\left\|\mathbf{x}(k)\right\| \leq \beta\left(\left\|\mathbf{x}(k_0)\right\|\right) + \gamma\left(\left\|u(k)\right\|_{\infty}\right), \tag{11}$$

where $\beta(\cdot)$ represents a κL-class function which is asymptotically stable such that the function converges to a minimum for $k \to \infty$ and for a zero equilibrium that $\beta(\cdot) \to 0$; further, $\gamma(\cdot)$ represents a κ_{∞} class function which is unbounded and strictly increasing from a zero initial state (i.e. $\gamma = 0$ and for $k > k_0$ that $k \to \infty, \gamma \to \infty$).

Definition 3 *(General BIBS for a Discrete Time System)*: the general solution of a discrete time nonlinear state-space system is defined as

$$\mathbf{x}(k)=\prod_{\kappa=k_0}^{k-1}\mathbf{A}(\kappa)\cdot\mathbf{x}(k_0)+\sum_{\kappa=k_0}^{k-1}\prod_{i=\kappa}^{k-1}\mathbf{A}(i)\cdot\mathbf{B}(\kappa)\cdot\mathbf{u}(\kappa), \tag{12}$$

where via taking we take the norms of both sides and via triangular inequality it yields that

$$\begin{aligned}\left\|\mathbf{x}(k)\right\|&\leq\left\|\prod_{\kappa=k_0}^{k-1}\mathbf{A}(\kappa)\mathbf{x}(k_0)\right\|+\left\|\sum_{\kappa=k_0}^{k-1}\prod_{i=\kappa}^{k-1}\mathbf{A}(i)\cdot\mathbf{B}(\kappa)\mathbf{u}(\kappa)\right\|\leq\\&\leq\left\|\prod_{\kappa=k_0}^{k-1}\mathbf{A}(\kappa)\right\|\cdot\left\|\mathbf{x}(k_0)\right\|+\sum_{\kappa=k_0}^{k-1}\left\|\prod_{i=\kappa}^{k-1}\mathbf{A}(i)\cdot\mathbf{B}(\kappa)\mathbf{u}(\kappa)\right\|,\end{aligned} \tag{13}$$

where, it can be proven for bounded-input-bounded-state (BIBS) stability of the general non-linear dynamic system in (9), (13) must hold.

6. DISCRETE TIME HIGHER ORDER NEURAL UNIT STABILITY METHOD (DHS)

In line with definition 1, the DHS method solves the local bounded-input-bounded-output (BIBO) stability of the pointwise state-space representation of HONUs. In this dissertation, a general form was introduced for in vector form with respect to the state variable $\overline{\mathbf{x}}$ as

$$\begin{aligned}&\overline{\mathbf{x}}(k+1)=\overline{\mathbf{f}}(\overline{\mathbf{x}}(k))+\overline{\mathbf{u}}(k)\\&\tilde{y}(k)=\overline{x}_{n_y}(k).\end{aligned} \tag{14}$$

The DHS method, inspired from classical theories re-expresses the form (14) into an incremental linear approximation via derivation of the Jacobian matrix of partial derivatives to the nonlinear state-space representation yields the simplified pointwise general state-space form

$$\Delta\overline{\mathbf{x}}(k+1)=\overline{\mathbf{A}}(k)\cdot\Delta\overline{\mathbf{x}}(k)+\Delta\overline{\mathbf{u}}(k), \tag{15}$$

where $\overline{\mathbf{A}}(k)$ is the Jacobian matrix $\mathbf{J}_m$ of the recurrent HONU, indicated in (16) and the respective difference of the state variable vector is denoted as $\Delta\overline{\mathbf{x}}(k)$ and state input vector as $\Delta\overline{\mathbf{u}}(k)$ then,

$$\bar{\mathbf{A}}(k)=\frac{\partial\bar{\mathrm{f}}}{\partial\bar{\mathrm{x}}(k)}=\begin{bmatrix} 0 & 1 & 0\ 0 & 0 \\ : & \dots & 1\ 0 & 0 \\ \bar{a}_{n_y,1} & \bar{a}_{n_y,2} & \dots\dots & \bar{a}_{n_y,n_{\bar{x}}} \\ 0 & 0 & \dots\ 1 & 0 \\ 0 & 0 & \dots\ 0 & 0 \end{bmatrix}=\underset{\substack{i=1\dots n_{\bar{x}};\\ j=1\dots n_{\bar{x}}}}{\left[\{\bar{a}_{i,j}\}\right]}, \tag{16}$$

where the matrix $\bar{\mathbf{A}}(k)$ represents a $n_{\bar{x}}\times n_{\bar{x}}$ dimension matrix. For an arbitrary r order of HONU it yields that the matrix coefficients $\bar{\mathbf{A}}(k)$ may be expressed as

$$\bar{a}_{i,j}=\begin{cases} =1 \qquad \text{for } i=2,3,\dots,n_x\wedge i\neq n_y\,;\, j=i+1 \\ =\boldsymbol{\psi}_j col^r(\mathbf{x}(k+1))=w[p]+\dfrac{\partial}{\partial\bar{x}_j}\left(\bar{x}_{n_y}(k)\right) \text{ for } j=1,2,\dots,n_{\bar{x}}\wedge i=n_y \\ =0 \qquad \text{else} \quad , \end{cases}, \tag{17}$$

where $col^r(\mathbf{x}(k+1))=\mathbf{x}(k+1)$ in the sense of a QNU, denotes the original neural weight vector $\mathbf{x}$ defined in (4) for one sample ahead. The term $w[n]$ denotes n^{th} element of $\mathbf{w}$ from the corresponding $\bar{x}_j(k)$ in vector $\mathbf{x}(k+1)$. Therefore the newly introduced weight vector $\boldsymbol{\psi}$ for a QNU may be explicitly computed term-by-term as

$$\boldsymbol{\psi}_j=f(\mathbf{w})=\begin{cases} \sum\limits_{l=0}^{n-1} w_{l,n} \\ \sum\limits_{s=n}^{n_x}\alpha_{n,s}w_{n,s} \quad where \quad \alpha_{n,s}=1 \ \ \forall s\neq n \ \ \& \ \ \alpha_{n,n}=2. \end{cases} \tag{18}$$

Thus, in similar analogy to (15) the linear approximation of the HONU model with extension of a HONU feedback controller may be defined as

$$\Delta\bar{\mathbf{x}}(k+1)=\bar{\mathbf{M}}(k)\cdot\Delta\bar{\mathbf{x}}(k)+\Delta\bar{\mathbf{d}}(k), \tag{19}$$

where $\bar{\mathbf{M}}(k)$ is the Jacobian matrix for extension of a feedback controller.

Theorem 1 (DHS): Recurrent HONUs via the representation (15) or the HONU closed feedback loops via representation (19), are BIBO stable at a given state with actual input at the sample time k if

$$\rho\left(\bar{\mathbf{A}}(k)\right)<1 \text{ or } \rho\left(\bar{\mathbf{M}}(k)\right)<1, \tag{20}$$

where $\rho(.)$ denotes the spectral radius, and $\bar{\mathbf{A}}(k)$ or $\bar{\mathbf{M}}(k)$ are the Jacobians of standalone recurrent HONUs or of the whole closed loops with HONUs respectively. For linear neural units (LNUs, i.e. HONUs of r=1) and for the closed loops composed of LNUs, the Jacobian matrices $\bar{\mathbf{A}}(k)$ and $\bar{\mathbf{M}}(k)$ are time variant only when real-time adaptation is applied. Or similarly $\mathbf{v}(k)=\mathbf{v}(k-1)+\Delta\mathbf{v}(k)$ is on, while for higher nonlinear orders the dynamics is variant because the Jacobians varies in state-space. Figures 2-4 illustrate several experimental analysis results from both simulation and real-time application on a non-linear two-funnel tank and CTU roller rig respectfully.

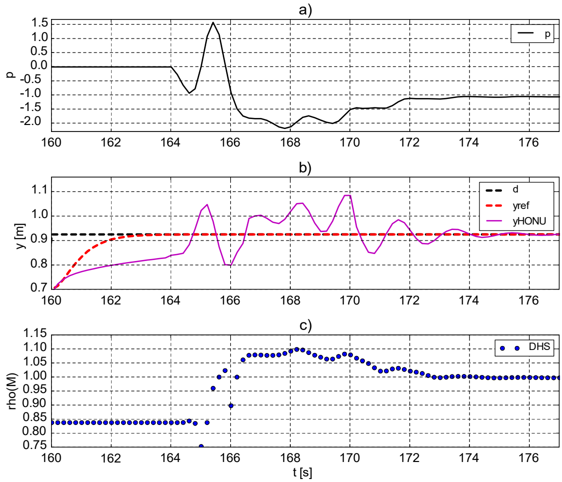

Figure 2- DHS: a) Randomly changed controller gain $p(t)$ in adaptive QNU-QNU control loop of non-linear two-funnel tank (instability onset $p(t=164.6)=-1.18$). b) Control loop output and the reference (desired) output. c) $\rho(t)$ computed via DHS.

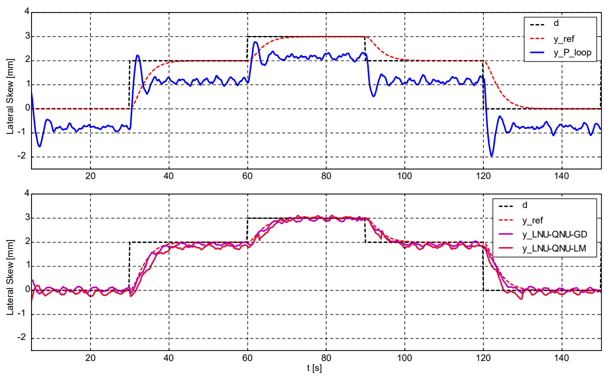

Figure 3 – Real-time HONU-MRAC adaptive control loop via LNU model with QNU controller of constant parameters on CTU roller rig.

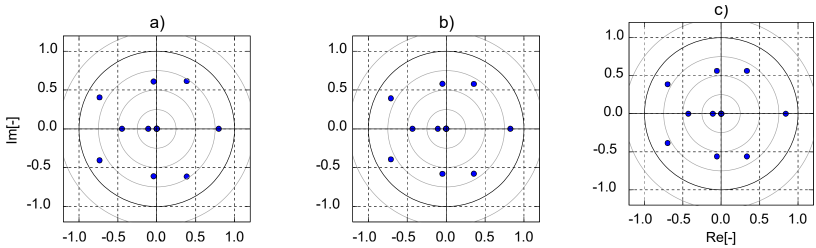

Figure 4 – DHS: Dispersion of GD tuned LNU-QNU closed control loop eigenvalues for real-time control of the roller rig system (Figure 3), evaluated about a) t=120-150 [s] (0 [mm]), b) t=150-180 [s] (2 [mm]) c) t=180-210 [s] (3 [mm]) respectfully.

7. DECOMPOSED DISCRETE-TIME HIGHER ORDER NEURAL UNIT STABILITY METHOD (DDHS)

To develop this approach, let us consider the expanded form of a QNU i.e. HONU, $r=2$ for the input vector (1), where an arbitrary length of previous model outputs n_y and previous process inputs n_u are considered. Then, on observation of the expanded form (21) we may restate the QNU with respect to the principle input vector $\mathbf{x}(k-1)$ as

$$\begin{aligned}\tilde{y}(k)&=\sum_{i=0}^{n_x}\sum_{j=i}^{n_x} w_{i,j}\cdot x_i\cdot x_j\\&=w_{0,0}+\sum_{i=1}^{n_y} x_i\cdot\left(w_{0,i}+\sum_{j=i}^{n_x} w_{i,j}\cdot x_j\right)+\sum_{i=n_y+1}^{n_x} x_i\cdot\left(w_{0,i}+\sum_{j=i}^{n_x} w_{i,j}\cdot x_j\right),\end{aligned}\tag{21}$$

where $n_x=n_y+n_u$. Then, the definition (21) maybe restated as

$$\tilde{y}(k)=\mathbf{w}(k)\cdot col^r\left(\mathbf{x}(k\text{-}1)\right);\mathbf{x}(k-1)=\begin{bmatrix}1\\ \hat{\mathbf{x}}(k-1)\\ \hat{\mathbf{u}}(k-1)\end{bmatrix}.\tag{22}$$

Thus, we may summarize the form (21) as follows

$$\hat{x}_{n_y}(k)=w_{0,0}+\sum_{i=1}^{n_y}\hat{x}_i(k-1)\cdot\hat{a}_i+\sum_{i=1}^{n_u}\hat{u}_i(k-1)\cdot\hat{b}_i,\tag{23}$$

where the coefficients $\hat{a}_i$ and $\hat{b}_i$ maybe respectively defined in sub polynomial form as in the sense of a QNU as

$$\hat{a}_i=\hat{a}_i\left(\hat{\mathbf{x}}(k-1),\hat{\mathbf{u}}(k-1),\mathbf{w}\right)=w_{0,i}+\sum_{j=i}^{n_x} w_{i,j}\cdot x_j(k-1),\tag{24}$$

$$\hat{b}_i = \hat{b}_i\left(\hat{\mathbf{u}}(k-1), \mathbf{w}\right) = w_{0,i} + \sum_{j=i}^{n_x} w_{i,j} \cdot x_j(k-1) \ ; \ i > n_y. \qquad (25)$$

Thus, given the state variable vector definition as the set of step-delayed feedbacks $\tilde{y}$, the original HONU polynomial form may be re-expressed in canonical state-space form as

$$\begin{aligned} &\hat{\mathbf{x}}(k) = \hat{\mathbf{A}}(k\text{-}1) \cdot \hat{\mathbf{x}}(k-1) + \hat{\mathbf{B}}(k-1) \cdot \hat{\mathbf{u}}(k-1) + \hat{\mathbf{w}}_0(k\text{-}1) \\ &\tilde{y}(k) = \hat{\mathbf{C}} \cdot \hat{\mathbf{x}}(k) \, ; \hat{\mathbf{w}}_0 = \left[0 \ldots 0 \ w_{0,0}\right]^T, \end{aligned} \qquad (26)$$

where the matrix of dynamics $\hat{\mathbf{A}}(k\text{-}1)$ and input matrix $\hat{\mathbf{B}}(k-1)$ respectively may be defined as

$$\hat{\mathbf{A}} = \begin{bmatrix} 0 & 1 & 0 \cdots & 0 \\ 0 & 0 & 1 \cdots & \vdots \\ 0 & 0 & \ddots \ \ddots & 0 \\ 0 & 0 & \ddots \ 0 & 1 \\ \hat{a}_{n_y} & \hat{a}_{n_y-1} & \cdots \ \hat{a}_2 & \hat{a}_1 \end{bmatrix}, \hat{\mathbf{B}} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ \hat{b}_{n_u} & \hat{b}_{n_u-1} & \cdots & \hat{b}_1 \end{bmatrix}. \qquad (27)$$

Further, the output matrix $\hat{\mathbf{C}} = \left[0 \cdots 0 \ 1\right]^T$. However, we may further simplify this expression on considering the neural weight bias as an additional input vector $w_{0,0} = w_{0,0}(k)$. Therefore, the augmented input matrix and input vector may be defined as

$$\hat{\mathbf{x}}(k) = \hat{\mathbf{A}}(k\text{-}1) \cdot \hat{\mathbf{x}}(k-1) + \hat{\mathbf{B}}_{\mathbf{a}} \cdot \hat{\mathbf{u}}_{\mathbf{a}}(k-1) \, ; \tilde{y}(k) = \hat{\mathbf{C}} \cdot \hat{\mathbf{x}}(k-1), \qquad (28)$$

where

$$\hat{\mathbf{B}}_{\mathbf{a}} = \begin{bmatrix} & 0 \\ \hat{\mathbf{B}} & \vdots \\ & 0 \\ & 1 \end{bmatrix}, \hat{\mathbf{u}}_{\mathbf{a}}(k\text{-}1) = \left[\hat{\mathbf{u}}(k\text{-}1) \ w_{0,0}(k\text{-}1)\right]^T. \qquad (29)$$

$\hat{\mathbf{A}}$ results in a time-variant matrix of dynamics for polynomial orders $r>1$ and it is also further due to applied learning; thus, the matrix $\hat{\mathbf{A}}$ is also termed as the Local Matrix of Dynamics (LMD). For a closed loop LNU with nonlinear control loop it leads to the following canonical state-space form with matrix of dynamics $\hat{\mathbf{M}}(k\text{-}1)$ and augmented input matrix $\hat{\mathbf{N}}_{\mathbf{a}}(k\text{-}1)$

$$\hat{\mathbf{x}}(k) = \hat{\mathbf{M}}(k\text{-}1) \cdot \hat{\mathbf{x}}(k-1) + \hat{\mathbf{N}}_{\mathbf{a}} \cdot \hat{\mathbf{u}}_{\mathbf{a}}(k-1) \ ; \ \tilde{y}(k) = \hat{\mathbf{C}} \cdot \hat{\mathbf{x}}(k). \qquad (30)$$

Theorem 2: (DDHS) The discrete-time recurrent HONUs, via their decomposed state-space representation (26), are BIBS stable leading from definition 2 & 3 from an initial sample in time k_0 until k provided

$$S(k)=\left\|\hat{\mathbf{x}}(k)\right\|-\left\|\prod_{\kappa=k_0}^{k-1}\hat{\mathbf{A}}(\kappa)\right\|\cdot\left\|\hat{\mathbf{x}}(k_0)\right\|-\sum_{\kappa=k_0}^{k-1}\left\|\prod_{i=\kappa}^{k-1}\hat{\mathbf{A}}(i)\cdot\hat{\mathbf{B}}_{\mathbf{a}}(\kappa)\right\|\cdot\left\|\hat{\mathbf{u}}_{\mathbf{a}}(\kappa)\right\|\leq 0\,. \tag{31}$$

Theorem 3: (DDHS) The discrete-time polynomial loops of HONUs via their decomposed state-space representation (30) are BIBS stable from initial sample time k_0 until k provided

$$S(k)=\left\|\hat{\mathbf{x}}(k)\right\|-\left\|\prod_{\kappa=k_0}^{k-1}\hat{\mathbf{M}}(\kappa)\right\|\cdot\left\|\hat{\mathbf{x}}(k_0)\right\|-\sum_{\kappa=k_0}^{k-1}\left\|\prod_{i=\kappa}^{k-1}\hat{\mathbf{M}}(i)\cdot\hat{\mathbf{N}}_{\mathbf{a}}(\kappa)\right\|\cdot\left\|\hat{\mathbf{u}}_{\mathbf{a}}(\kappa)\right\|\leq 0\,. \tag{32}$$

Theorem 4 (Strict DDHS): Provided the discrete-time recurrent HONUs (or their loops) are BIBS stable according to Theorem 2 (or Theorem 3) at time k_0, then the BIBS stability will be strictly maintained for HONU models if

$$\begin{aligned}\Delta S(k)&=S(k)-S(k-1)=\left\|\hat{\mathbf{x}}(k)\right\|-\left\|\hat{\mathbf{x}}(k-1)\right\|-\left\|\hat{\mathbf{B}}_{\mathbf{a}}(k-1)\cdot\hat{\mathbf{u}}_{\mathbf{a}}(k-1)\right\|\\&+\left(\left\|\hat{\mathbf{A}}(k-1)\right\|-1\right)\cdot\left(\left\|\hat{\mathbf{A}}(k-2)\cdot\hat{\mathbf{x}}(k-2)\right\|\right)+\left\|\hat{\mathbf{B}}_{\mathbf{a}}(k-2)\cdot\hat{\mathbf{u}}_{\mathbf{a}}(k-2)\right\|\leq 0\ \text{ for } \forall k>k_0.\end{aligned} \tag{33}$$

Analogically, extended for a HONU-MRAC control loop. Figures 5-7 illustrate performance of the DDHS method on a non-linear two funnel tank and CTU roller rig as a real-time implementation respectfully.

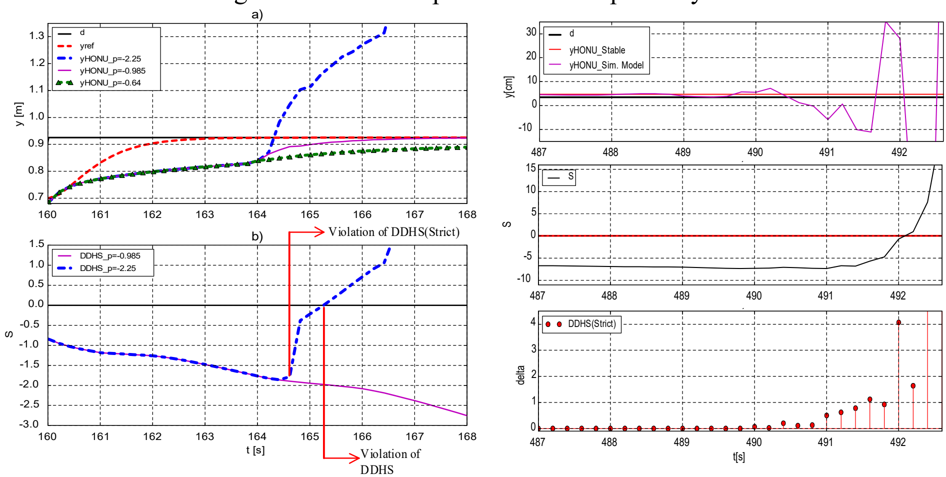

Figure 5 – DDHS (Left): QNU-QNU control loop responses with feedback controller gain on non-linear two funnel tank $p(t>164)=-0.985$ (stable) and $p(t>164)=-2.25$ (unstable). DDHS analysis. (Right): Comparison of DDHS and DDHS(Strict) on non-linear two-tank liquid level system proving unstable LNU-QNU control loop from t>488[s].

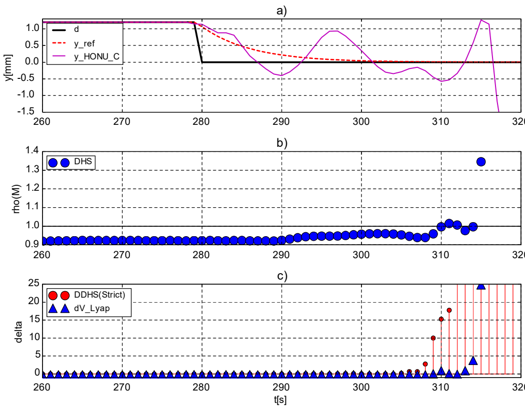

Figure 6: a) LNU-QNU control loop with RLS pre-training and adaptive GD in last epoch for single wheelset active control. b) Analysis of spectral radii through time (BIBO stability) via DHS after learning rate $\mu(t>280)=0.01$. c) Validation and comparison with Lyapunov condition [1] and strict DDHS (33) which reveals clearer condition for ongoing instability of whole LNU-QNU loop.

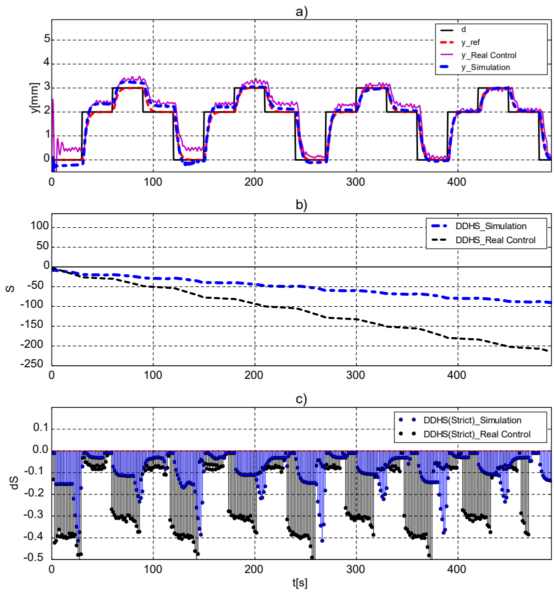

Figure 7 – a) Fully adaptive HONU-MRAC control loop via QNU plant trained via NGD and LNU feedback controller trained via RLS learning algorithm optimises following changed bogie frame stiffness and damping dynamic properties. b) DDHS stability condition (for $k_0=0$). c) Strict DDHS (33) confirms stability of the control loop

8. CONCLUSION

Following from review of the studied computational intelligence based forms of adaptive identification and control, this dissertation has highlighted the comprehensibility and efficient performance of HONUs for real industrial applications. In this dissertation, further fundamentals of HONU-MRAC control were derived, namely an adaptive controller parameter gain and further use of the recursive least squares (RLS) algorithm were presented with successful results on real nonlinear system and rail automation examples. As a major contribution, a new stability framework (DHS/DDHS) was proposed for BIBO/BIBS based stability analysis of nonlinear polynomial neural unit based architectures i.e. HONU and also for a whole HONU-MRAC control loop which can be extended for other polynomial based neural architectures as presented in this dissertation. In Figure 6, a comparison with the presented Lyapunov based criteria in [1] showed superior results, whilst unlike the compared, the DDHS can be applied for constant parameter model and further derived control loop. As a further outcome the ASPI Kit v 1.3 software was released as well as a stability analysis library in the programming language Python software to help engineers and practitioners investigate the potentials of HONU adaptive identification and control for their respective engineering applications.

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Appendix

Sample-by-sample Adaptation	
Gradient Descent (GD)	$\Delta\mathbf{w}=\mu\cdot e(k)\cdot\mathbf{colx}^T$
Normalised GD (NGD)	$\Delta\mathbf{w}=\dfrac{\mu}{\lVert\mathbf{colx}\rVert_2^2+1}\cdot e(k)\cdot\mathbf{colx}^T$
Recursive Least Squares (RLS)	$\Delta\mathbf{w}=e(k)\cdot\mathbf{colx}(k)^T\cdot\mathbf{R}^{-1}(k)$
Batch Adaptation	
LM (Leven. Marq.)	$\Delta\mathbf{w}=(\mathbf{J}^T\cdot\mathbf{J}+\dfrac{1}{\mu}\cdot\mathbf{I})^{-1}\cdot\mathbf{J}^T\cdot\mathbf{e}$
Resilient Backpropagation	$\boldsymbol{\Delta}\mathbf{w}=\boldsymbol{\Delta}\mathbf{w}\left(\Delta\left(\nabla\mathbf{E}\right)\right)$
Conjugate Gradients	$\Delta\mathbf{w}=\alpha\left(\mathbf{r}_\mathbf{e},\mathbf{p},\mathbf{J}\right)\cdot\mathbf{p}\left(\beta(\mathbf{r}_\mathbf{e},\mathbf{J})\right)$

Table 1 – Fundamental Learning Algorithms of HONUS of General Order. Application of RLS and adaptive controller feedback gain for HONU-MRAC as additional results of this dissertation.