

**CZECH TECHNICAL UNIVERSITY IN PRAGUE**

**Faculty of Mechanical Engineering  
Department of Process Engineering**



**BACHELOR THESIS**

**Light Propagation in Algae Photobioreactors**

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# BACHELOR'S THESIS ASSIGNMENT

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Bachelor's thesis title in Czech:

**Šíření světelného záření v řasových fotobioreaktorech**

Guidelines:

- Make a literature research concerning CFD methods focused on light propagation in algae photobioreactors.
- Describe basic models available in ANSYS Fluent software.
- Perform CFD simulations with some of the available models in ANSYS Fluent for a given case.
- Summarize the results and propose possible improvements in future work.

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
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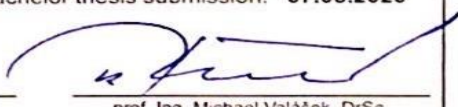
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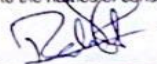
  
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I also declare that this dissertation is my own original work and has not been submitted before to any institution for assessment purpose.

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Prague 12/08/2020

## Annotation Sheet

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**Annotation:** This work aimed at the description and assessment of available numerical methods used for simulations of light propagation in photobioreactors. Different models of radiation were used ANSYS Fluent and their ability to describe the decrease of incident irradiation was investigated. CFD simulations were performed with some of the available models and the results were compared.

**Keywords:** CFD, ANSYS Fluent, microalgae, photobioreactor, radiation, heat flux.

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## **Abstract**

In the work, research concerning CFD methods focused on light propagation in algae photobioreactors using radiation models in ANSYS Fluent software has been carried out. ANSYS Fluent software offers various radiation models which could be used for the simulation of light propagation in photobioreactors. These models require various parameters among others, the absorption coefficient which could describe the effect of light intensity decrease with respect to the distance from illuminated wall, density of algae, and hydrodynamics of the system. All these factors can have substantial impact on the effectivity of such system.

In the ANSYS Fluent software, CFD simulations are performed with some of the available models for the given case with respect to light intensity. The assessment of these models is based on their theoretical ability to account for the light gradients and density of algae culture which is in direct relation with the absorptivity of the culture. Different models offer distinguished outcomes but the aim of this work is to provide the most suitable models, conclude with the results and propose possible improvements in future work.

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## **1. Introduction**

Phototrophic microorganisms like microalgae could provide valuable compounds. For several of these applications such as monocultures which can be sustained for an extended period of time, confined photobioreactors can be used. Because light energy is the growth limiting substrate, light supply plays a key role in photobioreactor design. Since algae are exposed to high-intensity light within the so-called photic zone near the reactor surface and total darkness within the inside, then the light regime inside photobioreactors is said to be complicated. The photosynthesis efficiency of microalgae varies between strains and culture conditions. Typical strategies to accumulate high product concentrations in the micro algal cells are nutrient starvation. However, while most microalgae can obtain high photosynthetic efficiency during growth under nutrient rich conditions and light.

Cultivation of algae in photobioreactors can be done either by utilizing direct sunlight or by employing artificial light such as from lamps, but artificial light can come at a cost and it is significant to reduce the cost of production. The main advantages of microalgae biofilms are a direct harvest of concentrated microalgae paste and they are layered closed and tightly. In order to promote growth, cultivation and better understanding of the process, we will have to develop and design a right radiation model by using Computational Fluids Dynamics (CFD).

CFD is useful tool for optimization of hydrodynamic conditions in photobioreactors which can help to avoid non-optimal light conditions (photo inhibition) and provide better efficiency of such systems.

## **2. The objectives and Motivation**

The work aims at description and assessment of available numerical methods used for simulations of light propagation in algae photo bioreactors. The light is the key factor of algae growth and its permeability decreases with increasing density of algae culture. The light intensity decreases with increasing distance from the illuminated wall in general. It can be affected by the hydrodynamics of the given system which is determined by the geometry and operational parameters. A proper model when describing such system is very important in the design.

Dealing with such an assessment of light propagation in algae photo bioreactors, ANSYS Fluent is one of the many CFD software available in the market which offers number of models to solve light propagation problem. There are many models provided by the software but in our case the main focus will be on the following Radiation models only, namely: DTRM (Discrete Transfer Radiation Model); The P-1 Model; The Rosseland Model; DO (Discrete Ordinates) Radiation Model; S2S (Surface-to-Surface) Radiation Model and MC (Monte Carlo) Radiation Model.

The above mentioned Radiation Models must be carefully evaluated by the necessary parameters for our subject case. Proper overview, limitations, advantages, disadvantages and comparison of these Radiation Models will help us to decide which model is best for the use with algae modelling. Several point of views might be evaluated here, easiness of the solver set-up, and so on.

### 3. Biological fundamentals in Algae production

#### 3.1 Algae in an overview

Algae belong to a large group of diverse organisms which are mainly microscopic. Algae are capable of converting factors like sunlight, CO<sub>2</sub> and water (including nutrients) into biomass by the process of photosynthesis that is similar to other plants.

Generally, algae can be divided according of their life cycle and which comprises of three main groups, namely:

*Autotrophic*: For a full functional growth, inorganic compounds such as light energy, CO<sub>2</sub> and other minerals/salts are required.

*Heterotrophic*: Since heterotrophic algae are non-photosynthetic, external sources of organic compounds such as nutrients and energy sources are required.

*Mixotrophic*: This type of algae are able to use either process for growth, which means that light energy is not a limiting factor for growth as they can utilize carbon substrates to support their growth.

#### 3.2 A glimpse of Algae biomass uses

Algae biomass have many uses like in food industry, in agriculture, as an energy, to create biofuel. In modern world, algae biomass is vastly used for producing biofuel/biodiesel. In general, conversion processes of biomass to biofuels are divided into three groups: chemical, biochemical and thermochemical conversions. Figure 1 illustrates the conversion processes.

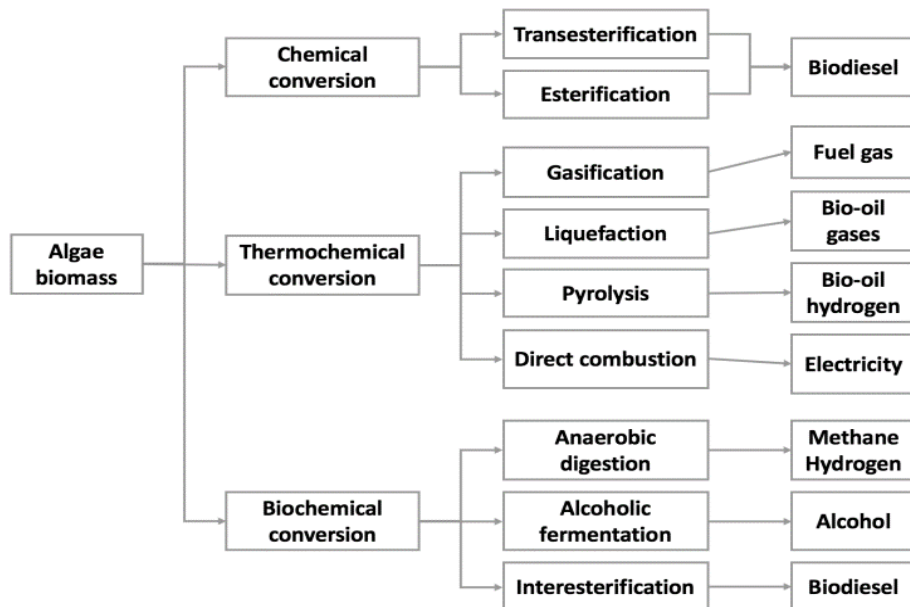


Figure 1: Conversion processes of Algae biomass

[Milano J., Ong H.C., Masjuki H.H., Chong W.T., Lam M.K., Loh P.K. and Vellayan V., 2016].

### 3.3 Algae Cultivation

As mentioned earlier, the main issue associated in mass cultivation of algae for photosynthetic productivity of cell mass and secondary metabolites, concerns with efficient use of strong light. The foremost crucial data concerning in such a productivity is the measurement of the coupling between photosynthesis and irradiance in variety of the steady-state light response curve, which represents the kinetic response of an algal cell to light intensity (Figure 2). [Richmond, 2004]; [Matonoha, Papaček and Štys, 2010].

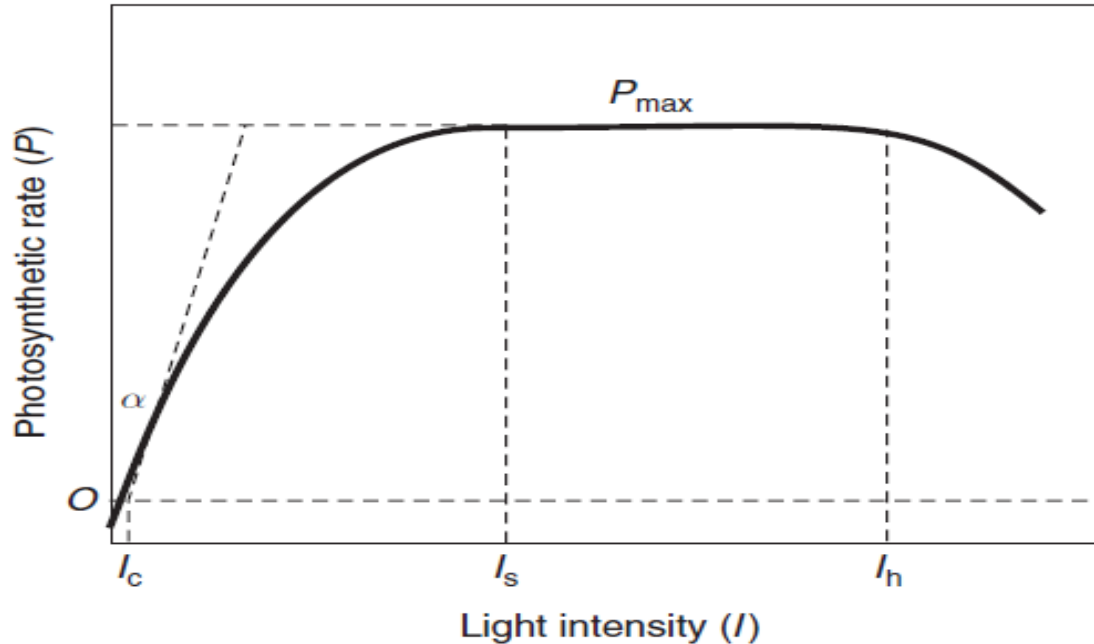


Figure 2: Light response curve of photosynthesis ( $P$ - $I$  curve),  $I_c$  – light compensation point,  $I_s$  – light saturation intensity,  $I_h$  – light intensity value at photo inhibition occurs.

If all growth conditions are optimal (i.e. temperature, amount of nourishments), the intensity of light source is the main factor which controls growth.

In the system for mass algae cultivation another term must be taken under consideration, which is mutual shading [Tamiya, 1957]. The phenomena causes internal shading (Figure 3) (p.14), which results that cells receiving light intermittently rather than continuously. The higher the cell density, the shorter the depth light penetrates the culture. When mutual shading prevails, cells are rather exposed to cycles of light and darkness (L/D cycle, Figure 4) (p.14), which may take from milliseconds to a few seconds to complete. Two basic parameters are associated to the L/D cycle (having the average irradiance fixed):

- Ratio between the light and the dark period in the cycle.
- Frequency of the cycle  $f_c = 1/T_c$

This results, the higher the frequency of L/D cycle, the more efficient light maybe used for photosynthesis.

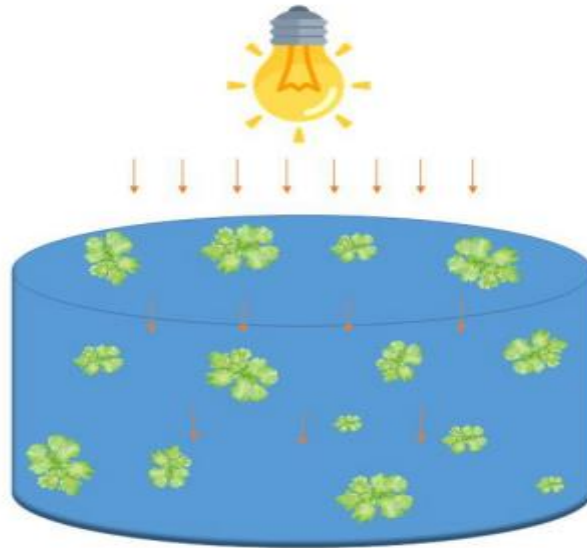


Figure 3: Schematics of mutual shading. The level of irradiance is decreasing with the depth as the photon are absorbed by cells locating in the higher layers of the tank [Tamiya, 1957].

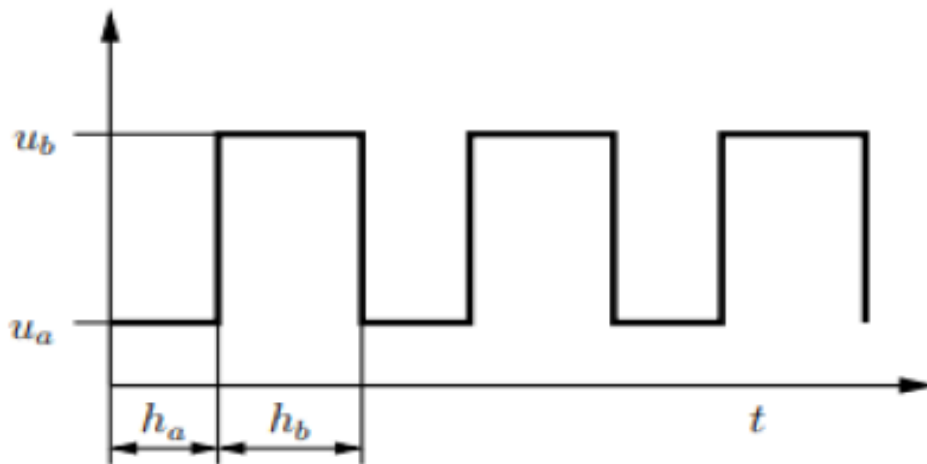


Figure 4: Schematic representation of periodic intermittent input signal  $u(t)$ :  
 $u_o$  – irradiance during period  $h_o$ ,  $u_b$  – irradiance level during period  $h_b$ .  
 Total cycle period  $T_c = h_o + h_b$ . [Tamiya, 1957].

### 3.4 Critical Factors affecting Algae growth

#### 3.41 Light source and Light Intensity

In every algae cultivation set up or system, the light intensity and the light source are critical factors that affects the condition of algae (autotrophic) growth.

Basically, cultivation systems are divided in two groups: indoor and outdoor. For indoor cultivators, artificial light sources are mostly utilized whereas for outdoor cultivators, sunlight is the major light source. But using artificial light sources for indoor cultivators may come with cost of setting up and electricity consumption depending on the type of light used. The characteristics and electricity consumption of different light sources are shown in Table 1 below.

Light source	Characteristics	Electricity consumption	Operational stability
<i>LED (Light-Emitting Diode)</i>	<ul style="list-style-type: none"> <li>• Lower heat generation</li> <li>• Higher stability</li> <li>• Low constructing cost</li> <li>• Lower energy consumption</li> </ul>	Moderate	High
<i>Conventional artificial light sources</i>	<ul style="list-style-type: none"> <li>• Low constructing cost</li> <li>• Higher stability</li> <li>• Higher biomass productivity</li> <li>• Illumination of larger area</li> </ul>	High	High
<i>Optical fiber excited by solar energy</i>	<ul style="list-style-type: none"> <li>• Uniform light distribution</li> <li>• Low cost</li> <li>• Minimum space requirements</li> <li>• Good light path</li> </ul>	Low	Low

*Table 1: Characteristics and electricity consumption of artificial light sources.*

[Chen C.Y. and Chang J.S., 2006].

In addition, the light energy attained by autotrophic microorganisms depends on the irradiance  $I$  (expressed as photon flux density). The daily algal production per irradiated surface can be indicated by the value called Areal productivity  $P_A$ .

Table 2 (p.16) shows an increase of areal productivity with increase in irradiance [Giuseppe O., Salatino P. and Marzocchella A., 2014].

<b>Cultivator</b>	<b>Light Configuration</b>	<b>Irradiance (<math>\mu\text{Em}^{-2}\text{s}^{-1}</math>)</b>	<b>Areal productivity (<math>\text{gm}^{-2}\text{day}^{-1}</math>)</b>
Thin-layer	Outdoor	400	22
Thin-layer	Outdoor	500	33
Thin-layer	Outdoor	540	19
Thin-layer	Outdoor	700	38
Thin-layer with LED	Indoor	141	18
Thin-layer with LED	Indoor	661	57
Thin-layer with LED	Indoor	2100	185
Flat panel	Indoor	17.5	1.3
Flat panel	Indoor	115	2.6
Flat panel	Indoor	980	47
Tubular-conical helical	Indoor	484	31
Tubular-conical helical	Outdoor	580	33
Tubular-static mixer	Indoor	150	7.6
Tubular-static mixer	Outdoor	333	14
Tubular-static mixer	Outdoor	400	10
Tubular-static mixer	Outdoor	420	37
Tubular-static mixer	Outdoor	420	46
Column-rotating	Indoor	1500	103
Internally irradiated	Indoor	205	14
Internally irradiated, mechanically stirred	Indoor	1750	136

*Table 2: Cultivator performance* [Giuseppe O., Salatino P. and Marzocchella A., 2014].

### 3.42 Temperature

Growth of algae culture differs with different temperature. Algae can utilize different temperatures to grow but there are still some optimal range of 15° to 30° C. moreover, some algae species can tolerate temperatures of range 15° to 45° C. Due to this specific reason, it is recommended to use closed-indoor system as it will be easier to control the temperature [Masojidek J., 2014].



## **4. Basics of Computational Fluid Dynamics (CFD)**

### **4.1 Understanding CFD**

Computational Fluid Dynamics (CFD) can be defined as a computer based tool used for analysis of fluid flows using numerical solution methods. Basically, Navier-Stokes equations representing momentum balance are solved numerically, along with other transport equations like energy, mass balance, including additional equations modelling turbulence, radiation etc. Using CFD, we are able to analyze complex problems like fluid-fluid, fluid-solid or fluid-gas interaction.

Application of CFD analyses, commonly in Engineering fields, are usually used in aerodynamics and hydrodynamics, where quantities such as lift and drag or field properties as pressures and velocities are achieved.

### **4.2 Application of CFD**

CFD analysis have a great potential in saving time, designing the process and are therefore cheaper and faster compared to ordinary or traditional testing for data acquisition. During a CFD analysis all desired quantities are often measured at once as compared to tests in real life. CFD analysis cannot fully exclude physical testing procedures, for verification purposes tests should still be performed.

### **4.3 Operation of CFD**

A CFD analysis basically consists of the following three stages:

*Pre-processing:* In this stage the issue articulation is changed into an idealized and discretized computer model. Processes included are mesh generation and application of introductory and boundary conditions.

*Solving:* the genuine computations are performed by the solver, and in this tackling stage computational control is required. There are numerous solvers accessible, varying in efficiency and capability of tackling certain physical phenomena.

*Post-processing:* Finally, the obtained results are visualized and analyzed within the post processing stage. At this stage the analysis can confirm the results and conclusions can be drawn on the obtained results.

## 5. Heat Transfer

### 5.1. Introduction

Heat transfer, moreover alluded to essentially as heat, is the movement of thermal energy from thing to another thing of distinctive temperature. Heat transfer can occur by three main methods: conduction, convection and radiation. The simplest models are physical models which include conduction and/or convection, whereas buoyancy-driven flow or natural convection and radiation models are more complex, which in our case is radiation model. Reckoning on our problem, ANSYS Fluent will solve a variation of the energy equation that takes into consideration the heat transfer methods specified.

### 5.2. Heat Transfer Theory

#### 5.2.1. The energy equation

ANSYS Fluent solves the energy equation in the following form [ANSYS Fluent Theory Guide, release 19.2, 2018]:

$$\frac{\partial}{\partial t}(\rho E) + \nabla \cdot (\vec{v}(\rho E + p)) = \nabla \cdot \left( k_{eff} \nabla T - \sum_j h_j \vec{J}_j + (\bar{\tau}_{eff} \cdot \vec{v}) \right) + S_h \quad (5.11)$$

In equation (5.11),

$$E = h - \frac{p}{\rho} + \frac{v^2}{2} \quad (5.12)$$

Where sensible enthalpy  $h$  is characterized for ideal gas as

$$h = \sum_j Y_j h_j \quad (5.13)$$

and for incompressible flow as

$$h = \sum_i Y_i h_i + \frac{p}{\rho} \quad (5.14)$$

In equation 5.13 and 5.14,  $Y_j$  is the mass fraction of species  $j$  and

$$h_j = \int_{T_{ref}}^T c_{p,j} dT \quad (5.15)$$

The utilized value for  $T_{ref}$  within the sensible enthalpy calculation depends on the solver and model in utilization. For the pressure-based solver  $T_{ref}$  is 298.15 K aside from PDF models in which case  $T_{ref}$  is a client input for the species. For the density-based solver  $T_{ref}$  is 0 K except when the modelling species transport with responses in which case  $T_{ref}$  is a client input for the species.

### 5.2.2. Energy Source Due To Radiation

When one of the radiation model is being utilized,  $S_h$  in equation (5.11) (p.18) also incorporates radiation source terms. For points of interest, follow the Modelling Radiation displaying underneath.

## 6. Radiative Transfer Equation

The radiative transfer equation (RTE) for an absorbing, emitting, and scattering medium at position  $\vec{r}$  in the direction  $\vec{s}$  is [ANSYS Fluent Theory Guide, release 19.2, 2018]:

$$\frac{dI(\vec{r}, \vec{s})}{ds} + (a + \sigma_s)I(\vec{r}, \vec{s}) = an^2 \frac{\sigma T^4}{\pi} + \frac{\sigma_s}{4\pi} \int_0^{4\pi} I(\vec{r}, \vec{s}') \Phi(\vec{s}, \vec{s}') d\Omega' \quad (6.0)$$

Where,

$\vec{r}$  = position vector

$\vec{s}$  = direction vector

$\vec{s}'$  = scattering direction vector

$s$  = path length

$a$  = absorption coefficient

$n$  = refractive index

$\sigma_s$  = scattering coefficient

$\sigma$  = Stefan – Boltzmann constant ( $5.669 \times 10^{-8} \text{ W/m}^2 - \text{K}^4$ )

$I$  = radiation intensity, which depends on position ( $\vec{r}$ ) and direction ( $\vec{s}$ )

$T$  = local temperature

$\Phi$  = phase function

$\Omega'$  = solid angle

$(\sigma + \sigma_s)$  is the optical thickness or opacity of the medium. When considering radiation in semi-transparent media, then the refractive index  $n$  is very important. Figure 5 illustrates the method of radiative heat transfer.

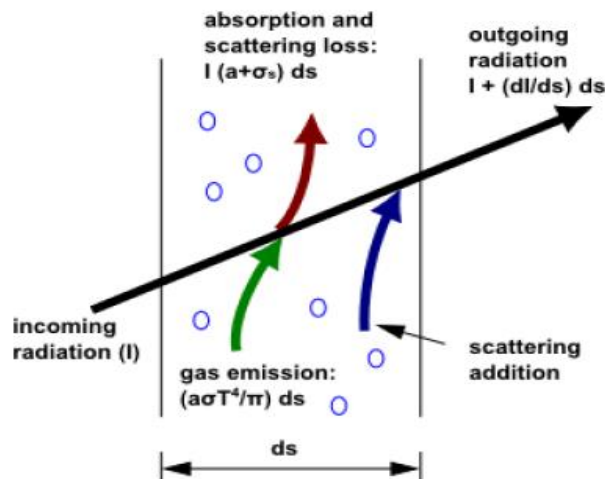


Figure 5: Radiative Heat Transfer. [ANSYS Fluent Theory Guide, release 19.2, 2018]

## 7. Modelling Radiation

### 7.1. Analysis and Drawbacks

ANSYS Fluent gives six radiation models that permit us to incorporate radiation, with or without a partaking medium, in our energy transfer simulations.

Heating or cooling of surfaces due to radiation and/or heat sources or sinks due to radiation within the fluid phase can be included in our model using one of the following radiation models.

- Discrete Transfer Radiation Model (DTRM). [*Carvalho, Farias and Fontes, 1991*]
- P-1 Radiation Model. [*Cheng, 1964*]
- Rosseland Radiation Model. [*Siegal and Howell, 1992*]
- Surface-to-Surface (S2S) Radiation Model. [*Siegal and Howell, 1992*]
- Discrete Ordinates (DO) Radiation Model. [*Chui and Raithby, 1993*]; [*Raithby and Chui, 1990*]
- Monte Carlo (MC) Radiation Model. [*ANSYS Fluent Theory Guide, release 19.2, 2018*]

In expansion to these radiation models, ANSYS Fluent also gives a solar load model that permits you to include the impacts of solar radiation in your simulation.

Other applications suitable for simulation using radiative heat transfer include the following:

- Radiative heat transfer from flames
- Surface-to-surface radiant heating or cooling
- Coupled radiation, convection, and/or conduction heat transfer
- Radiation through windows in HVAC applications, and cabin heat transfer analysis in automotive industry
- Radiation in glass processing, glass fiber drawing, and ceramic processing.

Radiative heat transfer in the simulation ought to be included when the radiative flux,  $Q_{rad} = \sigma (T_{max}^4 - T_{min}^4)$ , is expansive compared to the heat transfer rate because of convection or conduction. Typically this will happen at high temperatures where the fourth-order reliance of the radiative heat flux on temperatures infers that radiation will dominate.

But we can imagine other situations where modelling of radiative transport might be important, like distribution of light in photobioreactors and its dependency on spatial coordinate due to the dense algae culture and increased absorptivity of the liquid medium.

### 7.2. Advantages and Drawbacks of the DTRM

The primary advantages of the DTRM are:

- Relatively straightforward model,
- Accuracy can be increased by increasing the number of rays,
- It applies to a wide extend of optical thickness.

Drawbacks of using the DTRM in ANSYS Fluent:

- DTRM assumes that all surfaces are diffuse. This means the reflection of incident radiation at the surface is isotropic with respect to the solid angle.
- The effect of scattering is not included.
- The implementation assumes gray radiation.
- Solving a problem with a large number of rays is CPU-intensive.
- DTRM is not compatible with non-conformal interfaces or sliding meshes.
- DTRM is not compatible with parallel processing.

Currently, parallel process is a must in CFD simulations of complex cases, therefore the last condition disqualifies the usage of this model except simple cases (possibly 2-D geometries only).

### **7.3. Advantages and Drawbacks of the P-1 Model**

Taking into account the P-1 Model over the DTRM, it has quite a few advantages. For the P-1 model, the RTE [Equation (6.0) (p.19)] is a diffusion equation, which is simple to resolve with lesser CPU demands. The model incorporates the impact of scattering. Applications involving combustion where the optical thickness is large, the P-1 model works sensibly well. In addition to this, the application of P-1 model on certain complicated geometries having curvilinear coordinates can be very easy.

Drawbacks when using the P-1 Model:

- The P-1 model presumes that all surfaces are diffuse. This implies that the reflection of incident radiation at the surface is isotropic with regard to solid angle.
- The usage is restricted to either gray radiation or non-gray radiation employing a gray-band model.
- There could be a loss of accuracy, reckoning on the complexity of the geometry, in the event that the optical thickness is small.
- The P-1 model tends to over-predict radiative fluxes from localized heat sources or sinks.

### **7.4. Advantages and Drawbacks of the Rosseland Model**

The Rosseland model has two points of interest over the P-1 model. Since it does not figure out an additional transport equation for the incident radiation (as the P-1 model does), the Rosseland model is faster than the P-1 model and requires less memory.

The Rosseland Model can have some drawbacks as well when using it. The Rosseland model can be utilized only for optically thick media. It is recommended for use when the optical thickness surpasses 3. Note also that the Rosseland model is not accessible when the density-based solver is being utilized; it is available with the pressure-body solver only.

## 7.5. Advantages and Drawbacks of the DO Model

The DO model extends to the complete range of optical thickness, and allows us to resolve issues extending from surface-to-surface radiation to participating radiation in combustion problems. The solution of radiation occurring at semi-transparent walls, is additionally allowed. For ordinary angular discretization, the computational cost is relatively moderate, and memory requirements are the modest.

The current execution is restricted to either gray radiation or non-gray radiation employing a gray-band model. Solving an issue with a fine angular discretization might be CPU-intensive.

*Note!*

*A gray surface has a spectral emissivity and absorptivity that is not dependent on the wavelength of the radiation, gray radiation refers to the type of radiation one would expect to see from a gray surface.*

*Non-gray radiation is considered as annulus natural convection and the presence of non-gray radiation substantially changes the temperature and flow pattern in the cavity.*

Drawbacks when using the DO Model:

- The DO model is not supported for use with granular (fluid-solid) Eulerian multiphase flows.

## 7.6 Advantages and Drawbacks of the S2S Model

It is recommended to use surface-to-surface (S2S) model when modelling the enclosure radiative transfer without partaking media (for instance, spacecraft heat rejection systems, solar collector systems, radiative space heaters, and automotive under hood cooling systems. In such cases, the strategy for participating radiation may not always be efficient. As compared to the DTRM and the DO radiation model, S2S model encompasses a much quicker time per iteration, in spite of the fact that the view factor calculation itself is CPU-intensive. This increased time for view factor calculation will be particularly decided when the emitting/absorbing surfaces are the polygonal faces of polyhedral cells.

Drawbacks when using S2S Model:

- The S2S model considers that all surfaces are diffuse.
- The utilization/operation expects gray radiation.
- As the number of surface faces increase, there is a quick increment in the capacity and memory prerequisites.
- The S2S model cannot be used to model problems with participating medium (which is important for our case of dense algae culture).
- The S2S model with the hemicube view factor method cannot be used if the model contains symmetry or periodic boundary conditions.
- The S2S model does not support hanging nodes or hanging node adoption on radiating boundary zones.

## 7.7 Advantages and Drawbacks of the MC Model

Problems varying from optically thin ‘transparent’ regions to optically thick ‘diffusion’ regions, like combustion, can be solved using the MC model. Moreover, it is suitable for calculating quasi-exact solution. Whereas it is more precise compared to the other available models, it has a higher computational cost.

Drawbacks when using the Monte Carlo Model:

- For the Monte Carlo Model, all the physical quantities of interest are calculated as surface or volume averages.
- The following notable features/models are supported with the Monte Carlo model:
  - Shell conduction
  - Periodic boundaries
  - Transient
  - Heat exchanger model

The following are currently not supported with the Monte Carlo model:

- 2D cases
- Rotating/moving/overset meshes
- CutCell meshes
- Hanging nodes in the mesh
- Non-conformal interfaces
- Thin walls (example: baffles)
- Solid load model
- DPM
- Multiphase models
- User defined material properties
- Isotropic radiation flux option is not available for the boundary source of radiation on opaque walls.

## 8. Preferred Radiation Models for further analysis

### 8.1. P-1 Radiation Model

#### 8.1.1. P-1 Radiation Model Theory

The P-1 radiation model is the only case of the more common P-N model, which is based on the extension of the radiation intensity  $I$  into an orthogonal arrangement of spherical harmonics [Cheng, 1964]; [Siegal and Howell, 1992]. This segment gives details about the equations utilized in the P-1 Model.

### 8.1.2. The P-1 Model Equations

As mentioned above, the P-1 radiation model is the simplest case of the P-N model. When modelling gray radiation, the subsequent equation is obtained for the radiation flux  $q_r$  if only four terms within the series are used:

$$q_r = -\frac{1}{3(a + \sigma_s) - C\sigma_s} \nabla G \quad (8.11)$$

Where  $a$  is the absorption coefficient,  $\sigma_s$  is the scattering coefficient,  $G$  is the incident radiation, and  $C$  is the linear-anisotropic phase function coefficient, described below. After introducing the parameter

$$\Gamma = -\frac{1}{3(a + \sigma_s) - C\sigma_s} \quad (8.12)$$

Equation (8.11) simplifies to

$$q_r = -\Gamma \nabla G \quad (8.13)$$

The transport equation for  $G$  is

$$\nabla \cdot (\Gamma \nabla G) - aG + 4an^2\sigma T^4 = S_G \quad (8.14)$$

Where  $n$  is the refractive index of the medium,  $\sigma$  is the Stefan-Boltzmann constant and  $S_G$  is a user-defined radiation source. ANSYS Fluent solves this equation to determine the local incident radiation when the P-1 model is active.

Combining Equations (8.13) and (8.14), we get the following equation:

$$-\nabla \cdot q_r = aG - 4an^2\sigma T^4 \quad (8.15)$$

In order to account for heat sources due to radiation, the expression  $-\nabla \cdot q_r$  can be directly inserted into the energy equation.

ANSYS Fluent also allows for the modelling of non-gray radiation using a gray-band model. For non-gray radiation, Equation (8.14) can be written as:

$$\nabla \cdot (\Gamma_\lambda \nabla G_\lambda) - a_\lambda G_\lambda + 4a_\lambda n^2 \sigma T^4 = S_{G\lambda} \quad (8.16)$$

Where  $G_\lambda$  is the spectral incident radiation,  $a_\lambda$  is a spectral absorption coefficient,  $n$  is the refractive index of the medium,  $S_{G\lambda}$  is a user-defined source term, and  $\sigma$  is the Stefan-Boltzmann Constant.  $\Gamma_\lambda$  is defined as:

$$\Gamma_\lambda = \frac{1}{3(a_\lambda + \sigma_{s\lambda}) - C\sigma_{s\lambda}} \quad (8.17)$$

Where  $\sigma_{s\lambda}$  is a spectral scattering coefficient and  $C$  is the linear anisotropic phase function coefficient.



The spectral radiative flux is calculated as

$$q_\lambda = -\Gamma_\lambda \nabla G_\lambda \quad (8.18)$$

The term for radiation source within the energy equation can be obtained as

$$-\nabla q_r = \sum_{ALL\ BANDS} -\nabla q_{r,\lambda} = \sum_{ALL\ BANDS} a_\lambda (G_\lambda - n^2 G_{b\lambda}) \quad (8.19)$$

### 8.1.3. Anisotropic Scattering

The potentiality for modelling anisotropic scattering is already included within the P-1 radiation model. In ANSYS Fluent the modelling of anisotropic scattering is carried out by implying a linear-anisotropic scattering phase function:

$$\Phi(\vec{s}' \cdot \vec{s}) = 1 + C \vec{s}' \cdot \vec{s} \quad (8.20)$$

Where

$\vec{s}$  = unit vector in the direction of scattering

$\vec{s}'$  = unit vector in the direction of the incident radiation

$C$  = linear anisotropic phase function coefficient, which is a property of the fluid

Value of  $C$  ranges from -1 to 1. If the value is positive, then it shows that more radiant energy is scattered forward than backward, and if the value is negative then it means that more radiant energy is scattered backward than forward. A zero value characterizes isotropic scattering (default in ANSYS Fluent).

### 8.1.4. Particulate Effects in the P-1 Model

When the ANSYS Fluent model involves a scattered second phase of particles, we are able to include the impact of particles in the P-1 radiation model. Note that when particles are present, ANSYS Fluent disregards scattering in the gas phase. That is equation (8.21) expects that all scattering is due to particles.

The transport equation for the incident radiation can be written as:

$$\nabla \cdot (\Gamma \nabla G) + 4\pi \left( an^2 \frac{\sigma T^4}{\pi} + E_p \right) - (a + a_p)G = 0 \quad (8.21)$$

Where,  $E_p$  is the equivalent emission of the particles,  $a_p$  is the equivalent absorption coefficient and  $n$  is the refractive index of the medium.

These are defined as follows:

$$E_p = \lim_{V \rightarrow 0} \sum_{n=1}^N \varepsilon_{pn} A_{pn} \frac{\sigma T_{pn}^4}{\pi V} \quad (8.22)$$

and

$$a_p = \lim_{V \rightarrow 0} \sum_{n=1}^N \varepsilon_{pn} \frac{A_{pn}}{V} \quad (8.23)$$

In the equations (8.22) and (8.23),  $\varepsilon_{pn}$  is the emissivity,  $A_{pn}$  is the projected area, and  $T_{pn}$  is the temperature of particle  $n$ . The summation is over  $N$  particles in volume  $V$ .

Then,  $A_{pn}$  of particle  $n$  is defined as:

$$A_{pn} = \frac{\pi d_{pn}^2}{4} \quad (8.24)$$

Where  $d_{pn}$  is the diameter of the  $n^{th}$  particle.

The quantity  $\Gamma$  in equation (8.21) (p.25) is defined as:

$$\Gamma = \frac{1}{3(a + a_p + \sigma_p)} \quad (8.25)$$

The equivalent particle scattering factor can be figured out in the time of particle tracking and is defined as:

$$\sigma_p = \lim_{V \rightarrow 0} \sum_{n=1}^N (1 - f_{pn})(1 - \varepsilon_{pn}) \frac{A_{pn}}{V} \quad (8.26)$$

Here,  $f_{pn}$  is the scattering factor associated with the  $n^{th}$  particle.

Heat sources due to particle radiation are included in the energy equation as follows:

$$-\nabla \cdot q_r = -4\pi \left( an^2 \frac{\sigma T^4}{\pi} + E_p \right) + (a + a_p)G \quad (8.27)$$

### 8.1.5. Boundary Condition Treatment for the P-1 Model at Walls

To induce the boundary condition for the incident radiation equation when modelling gray radiation, the dot product of the outward normal vector  $\vec{n}$  and equation (8.13) (p.24) is computed:

$$q_r \cdot \vec{n} = -\Gamma \nabla G \cdot \vec{n} \quad (8.28)$$

$$q_{r,w} = -\Gamma \frac{\partial G}{\partial n} \quad (8.29)$$

Thus the flux of the incident radiation,  $G$ , at a wall is  $-q_{r,w}$ . The wall radiative heat flux is computed using the subsequent boundary condition:

$$I_w(\vec{r}, \vec{s}) = f_s(\vec{r}, \vec{s}) \quad (8.30)$$

$$f_w(\vec{r}, \vec{s}) = \epsilon_w \frac{n^2 \sigma T_w^4}{\pi} + \rho_w I(\vec{r}, -\vec{s}) \quad (8.31)$$

Where  $\rho_w$  is the wall reflectivity. The Marshak boundary condition is then used to eliminate the angular dependence [Ozisik, 1973]:

$$\int_0^{2\pi} I_w(\vec{r}, \vec{s}) \vec{n} \cdot \vec{s} d\Omega = \int_0^{2\pi} f_w(\vec{r}, \vec{s}) \vec{n} \cdot \vec{s} d\Omega \quad (8.32)$$

Substituting Equation (8.30) and Equation (8.31) into Equation (8.32) and performing the integration yields

$$q_{r,w} = -\frac{4\pi\epsilon_w \frac{n^2 \sigma T_w^4}{\pi} - (1 - \rho_w)G_w}{2(1 + \rho_w)} \quad (8.33)$$

If it assumed that the walls are diffuse gray surfaces, then  $\rho_w = 1 - \epsilon_w$ , and Equation (8.33) becomes

$$q_{r,w} = -\frac{\epsilon_w}{2(2 - \epsilon_w)} (4n^2 \sigma T_w^4 - G_w) \quad (8.34)$$

Equation (8.34) is used to compute  $q_{r,w}$  for the energy equation and for the incident radiation equation boundary conditions.

When using the gray-band model, the flux at the wall can be written as the following (assuming the wall as a diffuse surface):

$$q_{\lambda,w} = -\frac{\epsilon_{\lambda,w}}{2(2 - \epsilon_{\lambda,w})} (4n^2 [F(0 \rightarrow n\lambda_2 T) - F(0 \rightarrow n\lambda_1 T)] \sigma T^4 - G_{\lambda,w}) \quad (8.35)$$

Where  $\epsilon_{\lambda,w}$  is the wall emissivity,  $q_{\lambda,w}$  is the spectral radiative flux at the wall and  $G_{\lambda,w}$  is the spectral radiation at the wall.

### 8.1.6. Boundary Condition Treatment for the P-1 Model at Flow Inlets and Exits

The following radiative heat flux at flow inlets and outlets is computed in the same way as at walls, as described above. One of the assumptions in the ANSYS Fluent is that the emissivity of all flow inlets and outlets is 1.0 (black body absorption) unless we choose to redefine this boundary treatment.

ANSYS Fluent includes a choice that enables us to use different temperatures for radiation and convection at inlets and outlets. This could be valuable where temperature outside the inlet or outlet contrasts significantly from the temperature within the enclosure.

## 8.2. Rosseland Radiation Model

### 8.2.1 Rosseland Radiation Model Theory

The Rosseland or diffusion estimation for radiation is substantial when the medium is optically thick ( $(a + \sigma_s)L \gg 1$ ), and is suggested for use in problems where the optical thickness is greater than 3. It can be determined from the P-1 Model equations, with few approximations. This area gives details about the equations used in the Rosseland model.

### 8.2.2 The Rosseland Model Equations

Similarly to the P-1 model, the radiative heat flux vector in a gray medium can be estimated by Equation (8.13) (p.24):

$$q_r = -\Gamma \nabla G \quad (8.36)$$

Where  $\Gamma$  is given by Equation (8.12) (p.24).

The variation between the Rosseland radiation model and the P-1 model is that the Rosseland model assumes that the intensity is the black-body intensity at the gas temperature. (The P-1 model in fact calculates a transport equation for  $G$ ). Thus  $G = 4\sigma n^2 T^4$ , where  $n$  is the refractive index. Substituting this value for  $G$  into Equation (8.36) gives

$$q_r = -16\sigma \Gamma n^2 T^3 \nabla T \quad (8.37)$$

Since the radiative heat flux has the same form as the Fourier conduction law, it can be written as

$$q = q_c + q_r \quad (8.38)$$

$$q = -(k + k_r) \nabla T \quad (8.39)$$

$$k_r = 16\sigma \Gamma n^2 T^3 \quad (8.40)$$

Where  $k$  is the thermal conductivity and  $k_r$  is the radiative conductivity. Equation (8.38) is used in the energy equation to compute the temperature field.

### 8.2.3 Anisotropic Scattering

The Rosseland model uses the same phase function [Equation (8.20) (p.25)] for anisotropic scattering as described for the P-1 model in Anisotropic Scattering (p.25).

### 8.2.4 Boundary Condition Treatment at Walls

While the diffusion estimation is not substantial close to the walls, it is vital to use a temperature slip boundary condition. The radiative heat flux at the wall boundary,  $q_{r,w}$ , is characterized using the slip coefficient  $\psi$ :

$$q_{r,w} = -\frac{\sigma(T_w^4 - T_g^4)}{\psi} \quad (8.41)$$

Where  $T_w$  is the wall temperature,  $T_g$  is the temperature of the gas at the wall, and the slip coefficient  $\psi$  is approximated by a curve fit to the plot given in [R.Siegal and J.R. Howell, 1992]:

$$\psi = \begin{cases} 1/2 & N_w < 0.01 \\ \frac{2x^3 + 3x^2 - 12x + 7}{54} & 0.01 \leq N_w \leq 10 \\ 0 & N_w > 10 \end{cases} \quad (8.42)$$

Where  $N_w$  is the conduction to radiation parameter at the wall:

$$N_w = \frac{k(a + \sigma_s)}{4\sigma T_w^3} \quad (8.43)$$

and  $x = \log_{10} N_w$

### 8.2.5 Boundary Condition Treatment at Flow Inlets and Exits

No special treatment is required at flow inlets and outlets for the Rosseland model. The radiative heat flux at these boundaries can be determined using Equation (8.38) (p.28).

*Important!*

The **Rosseland** model can be used only with the pressure based solver.

Before starting the simulation for large meshes or complex models, it is recommended to compute view factors outside ANSYS Fluent and then read them into ANSYS Fluent.

## 8.3 Discrete Ordinates (DO) Radiation Model

### 8.3.1 The DO Model Theory

The Radiative Transfer Equation (RTE) for a definite number of discrete solid angles, each related to a vector  $\vec{s}$  fixed within the universal Cartesian system of  $x,y,z$ , can be solved by the DO Model. The DO Model does not perform any ray tracing but it does transform the equation 6.0 (p.19) into a transport equation for radiation intensity in the spectral coordinates  $(x,y,z)$ .

ANSYS Fluent gives two implementations of the DO Model, it can be either coupled (energy) or uncoupled. In coupled ordinates method, the equation of discrete and intensity at every cell will be solved together, assuming the spatial neighbors as known. The coupled method has some advantages like speeding up applications that includes high optical thicknesses or high scattering coefficients.

### 8.3.2 The DO Model Equations

The radiative transfer equation (RTE), according to DO model in the direction  $\vec{s}$  is a field equation. Hence the equation 6.0 (p.19) can be written as:

$$\nabla \cdot (I(\vec{r}, \vec{s})\vec{s}) + (a + \sigma_s)I(\vec{r}, \vec{s}) = an^2 \frac{\sigma T^4}{\pi} + \frac{\sigma_s}{4\pi} \int_0^{4\pi} I(\vec{r}, \vec{s}')\Phi(\vec{s}, \vec{s}')d\Omega' \quad (8.44)$$

### 8.3.3 Energy Coupling and the DO Model

When in a cell there is a coupling between energy and radiation intensities, acceleration of convergence of the finite volume in radiative heat transfer takes place. This method, generally, results in improving the convergence for applications which involves optical thicknesses  $> 10$ .

### 8.3.4 Anisotropic Scattering

Using of the DO model in ANSYS Fluent, different variants of scattering phase function are admitted. The included phase functions are isotropic, linear anisotropic, Delta- Eddington or a user defined phase function.

The Linear phase function is defined in equation 8.20 (p.25), which is then followed by the Delta-Eddington function in the following form:

$$\Phi(\vec{s}, \vec{s}') = 2f\delta(1 - \vec{s} \cdot \vec{s}') + (1 - f)(1 + C\vec{s} \cdot \vec{s}') \quad (8.45)$$

Where,  $f$  is the forward-scattering factor.

Scattering phase function further specifies the user-defined function in the form:

$$\Phi(\vec{s}, \vec{s}') = 2f\delta(1 - \vec{s} \cdot \vec{s}') + (1 - f)\Phi^*(\vec{s} \cdot \vec{s}') \quad (8.46)$$

The user-defined function will specify  $\Phi^*$  and the forward scattering-function.

### 8.3.5 *Particulates Effects in the DO Model*

One of the possibility in the DO model is to include the impact of discrete second phase of particles on radiation.

The particulate phase in the radiative transfer equation can be written as:

$$\nabla \cdot (I\vec{s}) + (a + a_p + \sigma_p)I(\vec{r}, \vec{s}) = an^2 \frac{\sigma T^4}{\pi} + E_p + \frac{\sigma_s}{4\pi} \int_0^{4\pi} I(\vec{r}, \vec{s}')\Phi(\vec{s}, \vec{s}')d\Omega' \quad (8.47)$$

Where  $a_p$  is the equivalent absorption coefficient due to the presence of particles and  $E_p$  is the equivalent emission.

### 8.3.6 *Boundary and Cell Zone Condition Treatment at Walls*

The DO model permits the specification of opaque walls and semi-transparent walls.

Opaque walls are interior to a domain or external to a domain and can be treated according to type of model being used (gray or non-gray). In addition, some amount of the incident radiation is absorbed at the wall surface and partly emitted back, depending on the emissivity.

In the case of semi-transparent walls, incident radiation can pass through the wall and can be transmitted to the adjacent medium (possibly refracted). However, it can be reflected back to the surrounding medium and being absorbed through the wall thickness. The Incident radiant energy which is transmitted through a semi-transparent wall can be transmitted specularly and diffusively.

### 8.3.7 *Boundary Condition Treatment at Periodic Boundaries*

If rotational periodic boundaries are utilized, it is necessary to use pixilation to ensure that the radiant energy is properly transferred between the periodic and shadow faces.

It is recommended to use pixilation between 3x3 and 10x10.

## 9. Using and Setting Up the Radiation Models

For the operational strategy of solving a radiation problem, it is necessary to know how to use and set up every individual radiation model that is being taken into account.

Underneath provided are the necessary steps that are significant for a specific radiation modelling, focusing mainly on the three selected models, namely: P-1 radiation model, the Rosseland radiation model and Discrete Ordinates (DO) radiation model.

(Note: The following points of interest are proposed in ANSYS Fluent User's Guide, release 19.2, 2018).

1. Select a radiation model to enable the radiative heat transfer (**P-1, The Rosseland, and DO**), under **Model** in the **Radiation Model** dialog box.
  - ❖ *Setup* → *Models* → *Radiation* → *Edit...*

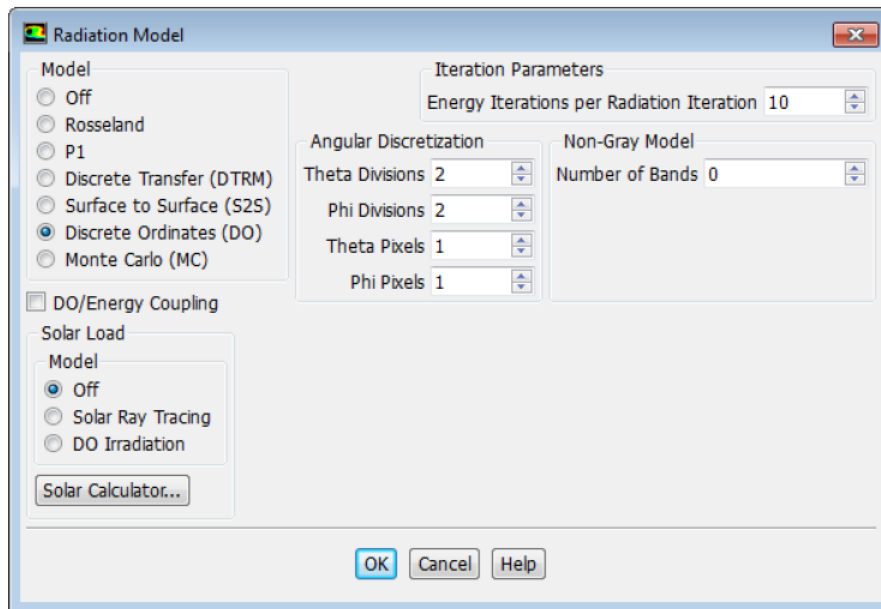


Figure 6: The Radiation Model Dialog Box. [ANSYS Fluent User's Guide, release 19.2, 2018]

### Tips/Hints:

- When radiation model is enabled, ANSYS Fluent will automatically enable the energy equation, reducing one step.
- In an active radiation model, the radiation flux at each iteration will have a solution of the energy equation.
- Select **Off** button in the **Radiation Model** dialog box if you wish to turn off a problem that is being already set up with the radiation model and is already turned on.
- The Radiation Model dialog box extends to show extra parameters when one of the radiation models is enabled.
- The extra parameters will not appear if the Rosseland model is selected.



## 2. Setting up the appropriate radiation criterion

- Define the non-gray radiation parameters in case of modelling non-gray radiation using P-1 model.
- If DO model is being used, then select DO/Energy Coupling if needed, also angular discretization has to be defined if necessary. Besides, the non-gray radiation parameters can be defined too (if necessary).

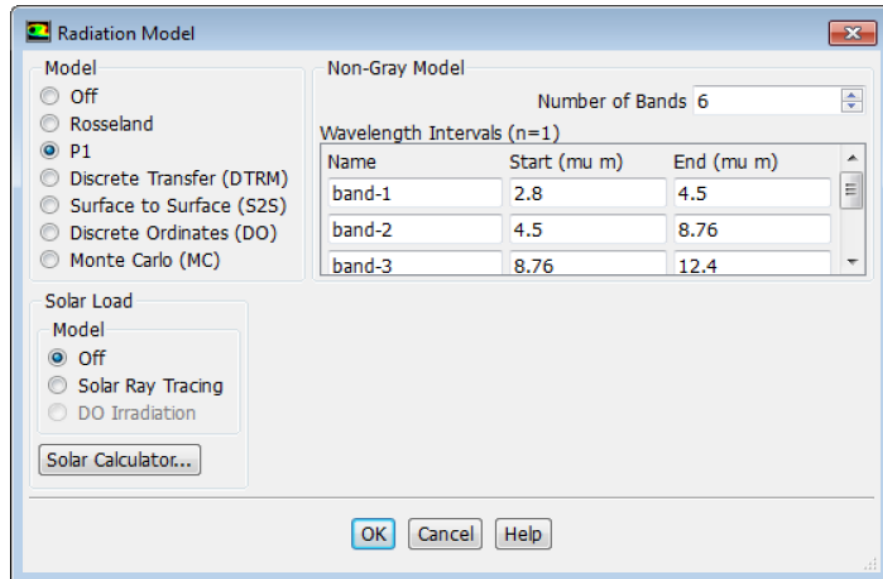


Figure 7: The Radiation Model Dialog Box (Non-Gray P-1 Model).  
[ANSYS Fluent User's Guide, release 19.2, 2018]

## 3. Define the material properties

- It is mandatory to define both the absorption and scattering coefficients of the fluid in the **Create/Edit Materials** dialog box when using the P-1, the Rosseland and DO model in ANSYS Fluent.
- The values entered can either be constant or can be specified by using a user-defined function (UDF).

❖ *Setup* → *Materials*

## 4. Define the boundary conditions

- Setting up of a problem that involves radiation in any ways, it is necessary that additional boundary conditions must be set for inlets, outlets and walls. To set these values, open the boundary condition dialog box by right-clicking the boundary name in the list.
- For DO model, in the **Radiation** tab of the **Wall** dialog box, select **Opaque** or **Semi-transparent Wall** in the **BC** drop down list, according to the prerequisites.

❖ *Setup* → *Boundary Conditions* → *Edit...*

## 5. Set the parameters that control the solution

- In P-1 model, the convergence criterion and under-relaxation factor can be controlled by paying attention to the optical thickness which by default for the P-1 model is  $10^{-6}$ . The convergence criterion for P-1 can be set up in the **Residual Monitors** dialog box.
- In DO model, the frequency can be controlled at the surface intensity which is updated as the continuous phase solution and proceeds through the **Energy Iterations per Radiation Iteration** setting.
  - ❖ *Solution* → *Monitors* → *Residuals* → *Edit...*

## 6. Run the solution

- After successfully setting up the radiation problem, the next step will be to proceed with the calculation. Always keep a note that the P-1 model resolves additional transport equations and reports residuals whereas the Rosseland model does not do any of them as the impact of the Rosseland model on the solution is only via the energy equation.
- In DO model, after every iteration, the DO model reports a normalized-composite residual for all the DO transport equations which is similar to the other transport equations.

## 7. Post process the results

- When the model includes the solution of radiative energy transfer, ANSYS Fluent provides some radiation quantities which can be used further for post processing.
- In the **Radiation...** category (*only for P-1, the Rosseland and the DO models*):
  - Incident Radiation (P-1 and DO models)
  - Incident Radiation (Band n) (non-gray P-1 and non-gray DO models)
  - Absorption Coefficient (P-1, DO and Rosseland models)
  - Scattering Coefficient (P-1, DO and Rosseland models)
  - Refractive Index (P-1, DO and Rosseland models)
  - Radiation Temperature (P-1 and DO models)
- In the **Wall Fluxes...** category (*only for P-1, DO and the Rosseland models*):
  - Radiation Heat Fluxes (all radiation models)
  - Surface Incident Radiation (DO model)
  - Absorbed Radiation Flux (DO model)
  - Reflected Radiation Flux (DO model)
  - Transmitted Radiation Flux (DO model)
- To report Radiative energy transfer through boundaries, **Flux Reports** dialog box can be used
  - ❖ *Results* → *Reports* → *Fluxes* → *Edit...*

## 10. Preliminary Selection of Photobioreactor System and Concept Design

Before moving on to the Modelling, a choice of selecting a photobioreactor and its design is a crucial step in order to perform modelling and further simulations. The photobioreactor system represents either a closed or open system, depending upon the prerequisites and operational circumstances, which in our case will be a closed (indoor) photobioreactor system. Algae in closed systems are cultivated in order to reduce contamination risk from the environment and increase the growth in cells.

However, it is important to consider all the features and other operational parameters necessary for the implementation as well as that affect the system function.

For further information on comparison of the systems, see Table 3 below.

### 10.1 Design Alternatives and Comparison

	<b>Closed Photobioreactor Systems</b>	<b>Open Pond Systems</b>
Contamination Control	Easy	Difficult
Sterility	Achievable	None
Process control	Easy	Difficult
Mixing	Uniform	Very poor
Area/volume ratio	20 - 200 m <sup>-1</sup>	5 - 10 m <sup>-1</sup>
Algal cell density	High	Low
Investment	High	Low
Operation cost	High	low
Light utilization efficiency	High	Poor
Temperature control	Uniform	Difficult
Productivity	Higher	Lower
Evaporation of growth medium	Low	High
Gas transfer control	High	Low
O <sub>2</sub> inhibition	Difficult	uniform

*Table 3: Comparison of Photobioreactor Systems*  
[Mata, Teresa M., Martins A. and Caetano N.S., 2010].

## 10.2 Some Advantages and Drawbacks of Design Alternatives

	<b>Advantages</b>	<b>Drawbacks</b>
Thin-layer cascade	<ul style="list-style-type: none"> <li>• Short light path</li> <li>• High biomass productivity</li> <li>• Easy penetration into algae</li> </ul>	<ul style="list-style-type: none"> <li>• Large area of the land required</li> <li>• Contamination risk</li> <li>• Low temperature control</li> </ul>
Column photobioreactor	<ul style="list-style-type: none"> <li>• High area/volume ratio</li> <li>• High mass transfer</li> <li>• Low energy consumption</li> <li>• Easy to sterilize</li> <li>• compact</li> </ul>	<ul style="list-style-type: none"> <li>• Relatively difficult to scale-up</li> <li>• Small illumination area</li> <li>• High maintenance cost</li> <li>• Expensive</li> </ul>
Internally stirred tank photobioreactor	<ul style="list-style-type: none"> <li>• High mass transfer</li> <li>• Good agitation</li> <li>• Possibilities to scale-up</li> </ul>	<ul style="list-style-type: none"> <li>• High maintenance cost</li> <li>• Higher shear stress on algae</li> </ul>
Photobioreactor with internal optic-fiber illumination	<ul style="list-style-type: none"> <li>• Compact</li> <li>• Large illumination surface area</li> </ul>	<ul style="list-style-type: none"> <li>• Relatively expensive</li> <li>• Sterilization problems</li> <li>• Fouling</li> <li>• Temperature control</li> <li>• Agitation problems</li> </ul>
Couette Taylor photobioreactor	<ul style="list-style-type: none"> <li>• Possibilities to scale-up</li> <li>• Good agitation with low shear stress</li> </ul>	<ul style="list-style-type: none"> <li>• Relatively expensive</li> <li>• High maintenance cost</li> <li>• High energy consumption</li> <li>• Difficult in sterilization</li> </ul>
Tubular photobioreactor	<ul style="list-style-type: none"> <li>• Suitable for outdoor cultures</li> <li>• Easy to implement/operate</li> <li>• Shorter harvest time</li> <li>• High area/volume ratio</li> <li>• High agitation efficiency</li> </ul>	<ul style="list-style-type: none"> <li>• Fouling</li> <li>• Dissolves O<sub>2</sub> and CO<sub>2</sub></li> <li>• High capital cost</li> </ul>
Raceway Pond	<ul style="list-style-type: none"> <li>• Low capital cost</li> <li>• Low operation cost</li> <li>• Easy to clean</li> <li>• Low energy inputs</li> </ul>	<ul style="list-style-type: none"> <li>• Low biomass productivity</li> <li>• Large area of land required</li> <li>• Poor light utilization efficiency</li> <li>• Contamination risk</li> <li>• Low temperature control</li> </ul>
Flat panel photobioreactor	<ul style="list-style-type: none"> <li>• Relatively cheap</li> <li>• Low contamination risk</li> <li>• Easy to clean/sterilize</li> <li>• Large illumination area</li> </ul>	<ul style="list-style-type: none"> <li>• Difficult in temperature control</li> <li>• Difficult to scale-up</li> <li>• Hydrodynamic stress may occur</li> </ul>

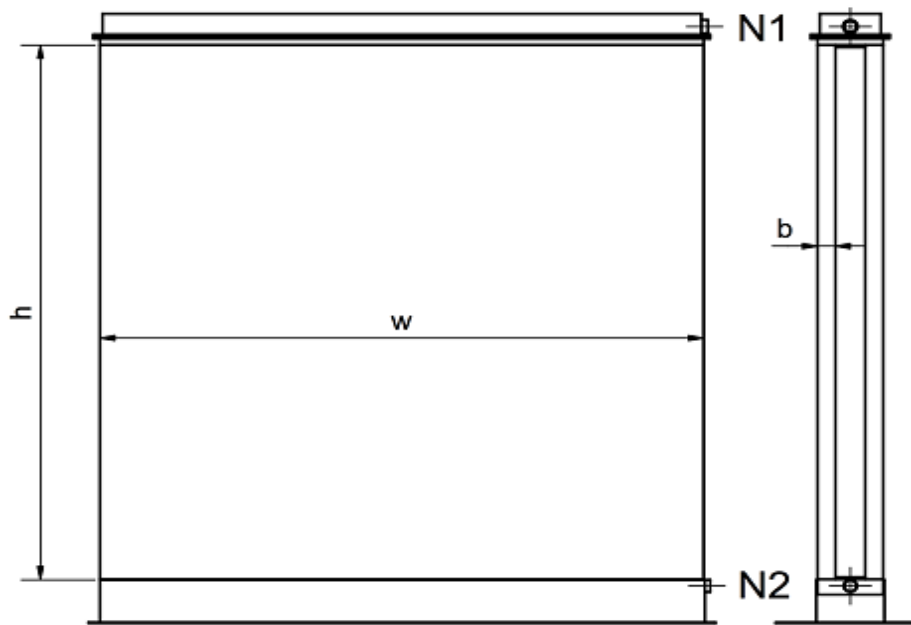
*Table 4: Advantages and Drawbacks of design alternatives  
[Mata, Teresa M., Martins A. and Caetano N.S., 2010].*

### 10.3 Opted Design for CFD Modelling / Simulation

#### *Flat Panel Photobioreactor*

In present day, there exist different variations of design options of flat panel photobioreactors. The aim of the paragraph is to select a more practical design which can be simulated without having any issues. From the previous analytical research, we can say that flat panel photobioreactor can be a suitable candidate as meets most of the prerequisites and environmental demands.

It is also important to design the system mainly for indoor purpose and the construction should be adapted such that it yields a sufficient area irradiated by the main light source as well as to examine its impact on algae production. Necessary parameters such as height, width and volume can be chosen for inputs of design and some other parameters like absorption coefficient, scattering coefficient, temperature and velocity etcetera can be set up in the ANSYS Fluent directly. Therefore, with the selection of right parameters, it will be possible to attain a large irradiated area at a very demand for space.



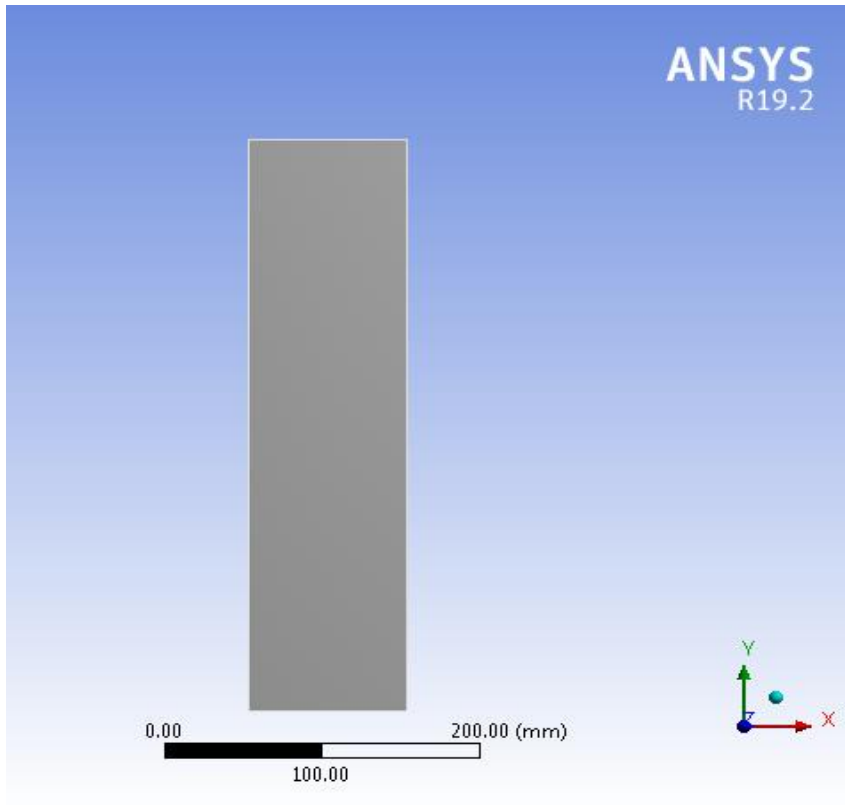
*Figure 8: Illustration for Motivational purpose*  
[Fraunhofer IGB “Flat panel Photobioreactor”, release 2015]

# 11. CFD Simulation

## 11.1 2-D Model

### Flat Panel Model and Fluid Domain creation

The initial step of numerical simulation is to create a model which is more suitable for our case, which is a simple flat panel. For the sake of simplicity, I focused on 2-D geometry only, representing a cross section of the full 3-D geometry. The next step is to create a fluid domain in the interior surface-body of the model. The geometry contemplated will consist of four walls, namely: left wall, right wall, bottom wall and top wall. Since there is no absolute information about the geometrical dimensions, therefore, a geometry of 100mm x 400mm is chosen. The creation of geometry and the following steps were carried out by using the software Workbench 19.2 in ANSYS Fluent R19.2.



*Figure 9: Geometry of the Model*

<b>Bounding Box</b>	
Length X	100. mm
Length Y	400. mm
Length Z	0. mm
<b>Properties</b>	
Volume	1.2e+005 mm <sup>3</sup>
Surface Area(approx.)	40000 mm <sup>2</sup>
Scale Factor Value	1.

*Table 5: Primary Model Dimensions*

Global Coordinate System (Cartesian) is used by the ANSYS Fluent in the creation of the model. The origin of the system  $[x,y,z]$  is all set to 0, whereas the directional vectors are  $[1, 0, 0]$  at x-axis,  $[0, 1, 0]$  at y-axis and  $[0, 0, 1]$  at z-axis.

## 11.2 Mesh

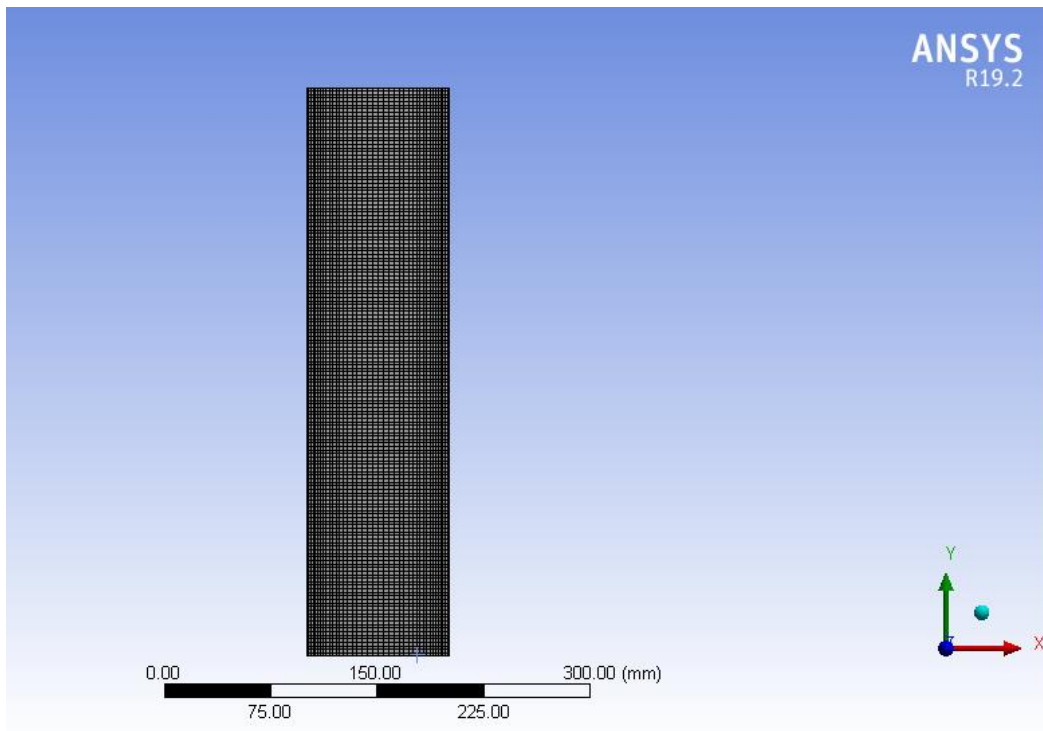
In order to analyze fluid flows, radiation or heat transfer, the flow domains are split into smaller subdomains. This is because the partial differential equations that govern fluid flow and heat transfer are usually not practical to analytical solutions, except for much simpler cases.

Therefore, the governing equations are discretized and solved within each of these subdomains. The subdomains are often called elements or cells, and the collection of all elements or cells is called a mesh or grid. Likewise, quality of mesh and number of elements in mesh model is a crucial factor to get correct results from the simulated model.

Mesh Quality acquired from ANSYS Fluent

Minimum Orthogonal Quality = 1.00000e+00 and Maximum Aspect Ratio = 2.08535e+00.  
To improve orthogonal quality, "Inverse Orthogonal Quality" in Fluent Meshing can be used.  
Where Inverse Orthogonal Quality = 1 - Orthogonal Quality).

In this study, one mesh model is considered to be necessary as the geometry and shape of the model chosen is not very complex. The mesh obtained as a result of simulation can be seen in the figure 10 below.



*Figure 10: Mesh Model*

In ANSYS Fluent, the element order is set to Linear and the element size is 2.5 mm, all by defaults. For mesh generation, quadrilateral method of definition is used.

Some statistical values obtained are the simulation can be seen in the table 6 given below

<b>Sizing</b>	
Growth Rate	Default (1.2)
Defeature Size	Default (1.25e-002 mm)
Curvature Min Size	Default (2.5e-002 mm)
Curvature Normal Angle	Default (18.0°)
Bounding Box Diagonal	412.31 mm
Average Surface Area	40000 mm <sup>2</sup>
Minimum Edge Length	100.0 mm
<b>Inflation</b>	
Inflation Option	Smooth Transition
Transition Ratio	0.272
Maximum Layers	2
Growth Rate	1.2
<b>Advanced</b>	
Number of CPUs for Parallel Part Meshing	Program Controlled
Rigid Body Behavior	Dimensionally Reduced
Triangle Surface Mesher	Program Controlled
Pinch Tolerance	Default (2.25e-002 mm)
<b>Statistics</b>	
Nodes	6601
Elements	6400

*Table 6: Statistical values of mesh model*

### 11.3 Boundary Conditions

For the enclosed model created in Workbench 19.2, taken into account the four sides and including the inner space of the body, we can determine the boundary conditions on each wall individually (depending on the type of radiation model selected).

For the radiation models (P-1 and DO) used, the operating pressure is set to 101325 Pascal with the reference pressure location at the origin [x, y].

In both the Radiation models, the left and the right sides of the enclosure are considered to be the operating walls and the properties are defined specifically, while the top and bottom sides are taken as symmetric walls.



### 11.3.1 Boundary Conditions for P-1 Model

		Left Wall	Right Wall
Momentum	Wall motion	Stationary Wall	Stationary Wall
	Shear condition	No Slip	No Slip
	Roughness height (m)	0	0
	Roughness constant (m)	0.5	0.5
Thermal Conditions	Type	Radiation	Temperature
	External Emissivity	1	-
	External Radiation Temperature (K)	500	300
	Internal Emissivity	1	1
	Wall thickness (m)	0	0
	Heat Generation Rate (W/m <sup>3</sup> )	0	0

Table 7: BC (P-1 Model)

While setting up the radiation model, it is important to keep in note that materials properties are a vital factor in obtaining accurate results. For this purpose, we select random sets of values and alternate them in order get an imitation of exponential curve defined by Beer-Lambert law  $I = I_0 e^{-\mu x}$ . For water-liquid, the refractive index  $n=1$  and the scattering coefficient was set to 0.

Fluid (water-liquid)	
Density (kg/m <sup>3</sup> )	998.2
Specific Heat, C <sub>p</sub> (J/kg-K)	4182
Thermal Conductivity (W/m-K)	0.6
Viscosity (kg/m-s)	0.001003
Absorption Coefficient (1/m)	1, 10, 40, 100, 1000
Solid (aluminum)	
Density (kg/m <sup>3</sup> )	2719
Specific Heat, C <sub>p</sub> (J/kg-K)	871
Thermal Conductivity (W/m-K)	202.4

Table 8: Material properties (P-1 Model)

The absorption coefficient  $a$  is changed periodically in the radiation models (in material properties) for the purpose of better visualization and comparison. [Chosen values for  $a= 1, 10, 40, 100, 1000$ . [1/m]]

### 11.3.2 Boundary Conditions for DO Model (Opaque/ semi-transparent wall)

		Left Wall	Right Wall
Momentum	Wall motion	Stationary Wall	Stationary Wall
	Shear condition	No Slip	No Slip
	Roughness height (m)	0	0
	Roughness constant (m)	0.5	0.5
Thermal Conditions	Type	Radiation	Temperature
	External Emissivity	1	-
	External Radiation Temperature (K)	500	300
	Internal Emissivity	1	1
	Wall thickness (m)	0	0
	Heat Generation Rate (W/m <sup>3</sup> )	0	0
Radiation	BC Type	Opaque	Opaque
		Semi-transparent	Semi-transparent
	Diffuse Fraction	0, 1	0, 1

Table 9: BC (DO Model)

While setting up the Discrete Ordinates (DO) radiation model, there are two possibilities of setting up. One is selecting BC type as “*opaque wall*” and the second is to select BC type as “*semi-transparent wall*”. Similarly as the P-1 model, the top and the bottom walls are set as symmetric.

Fluid (water-liquid)	
Density (kg/m <sup>3</sup> )	998.2
Specific Heat, C <sub>p</sub> (J/kg-K)	4182
Thermal Conductivity (W/m-K)	0.6
Viscosity (kg/m-s)	0.001003
Absorption Coefficient (1/m)	1, 100
Solid (aluminum)	
Density (kg/m <sup>3</sup> )	2719
Specific Heat, C <sub>p</sub> (J/kg-K)	871
Thermal Conductivity (W/m-K)	202.4

Table 10: Material properties (DO Model)

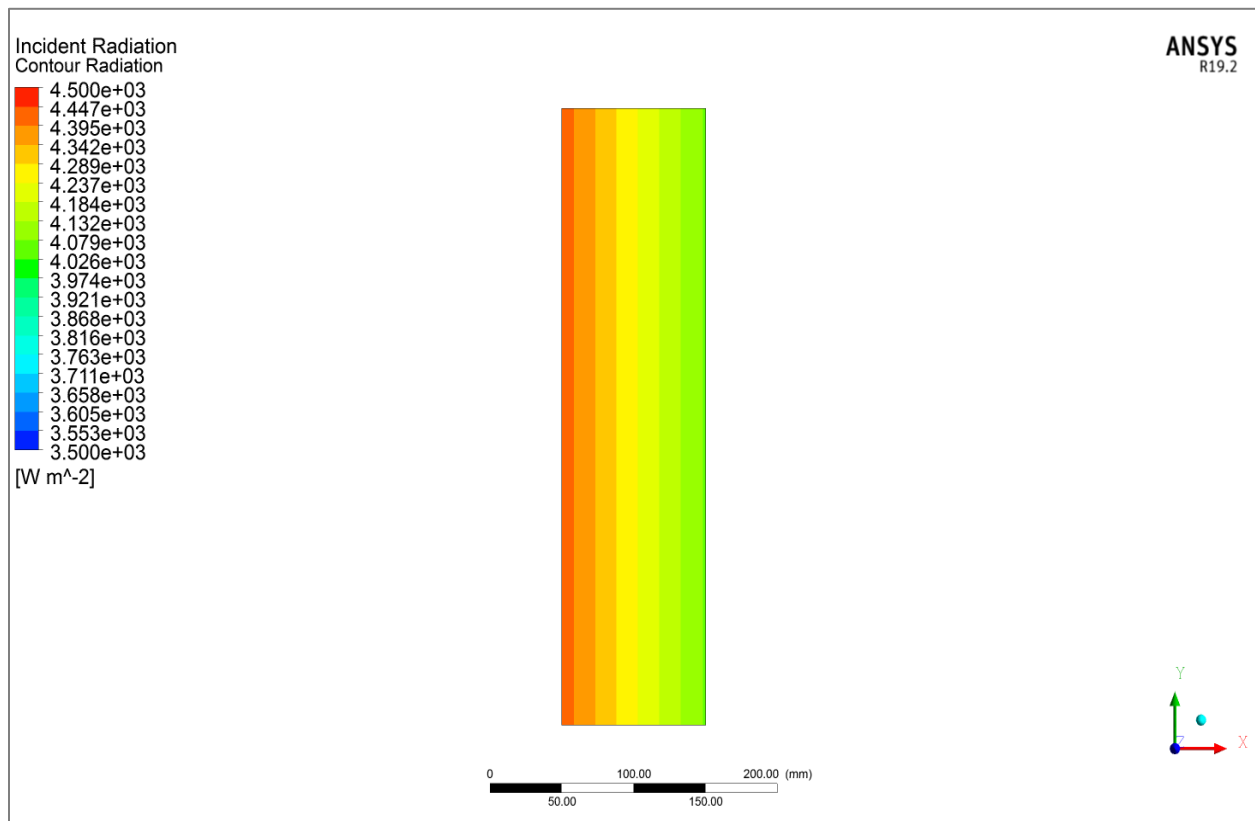
Like for P-1 model, we change the absorption coefficient for both opaque and semi-transparent walls in DO model. [Chosen values for  $a = 1, 100$  [1/m]]

## 11.4 Simulation

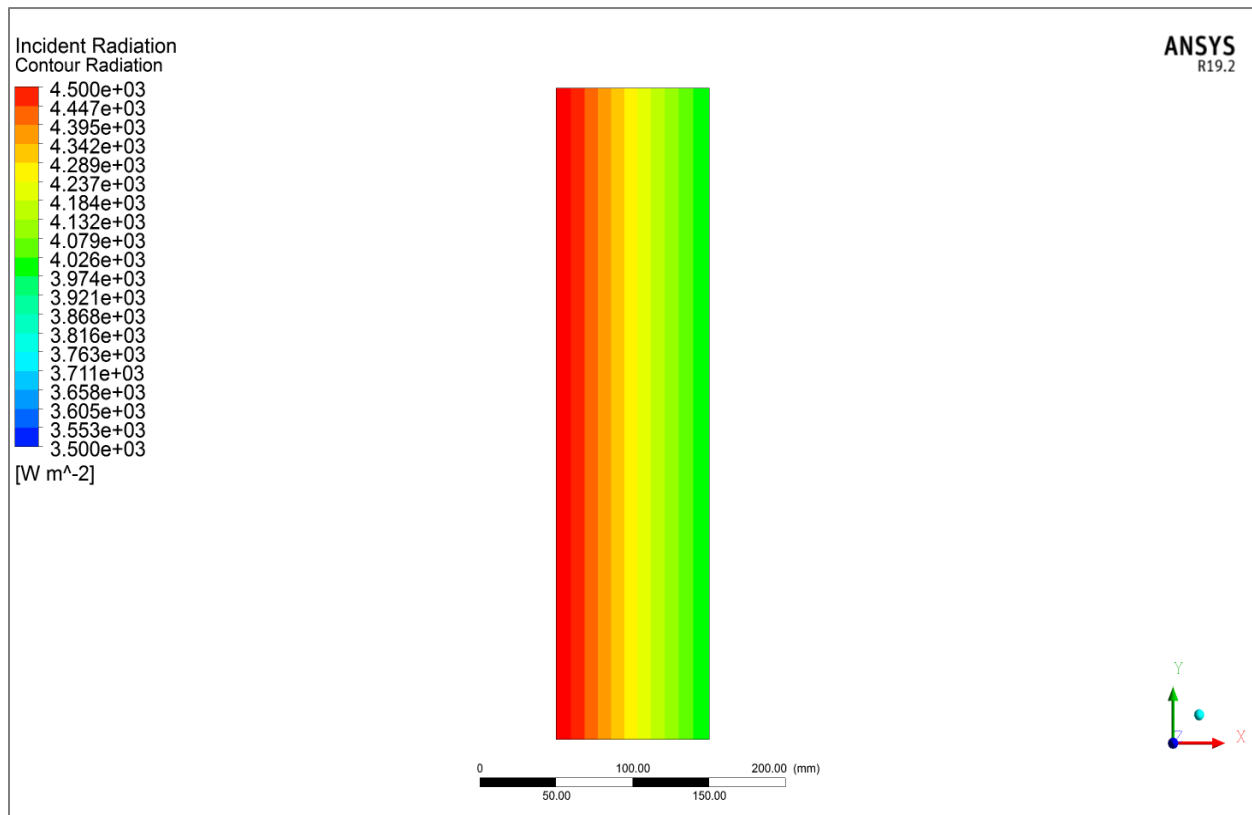
Objective of the simulation is the numerical solution of the model for radiation energy transfer. Two dimensional simulation is done for both the selected radiation models, which are P-1 model and DO model. Each model is simulated with different boundary conditions and most importantly different absorption coefficients are taken into consideration for the performance. Numerical simulation is then carried out using the ANSYS Fluent R19.2 software. The detailed simulation of the study is discussed in the results below.

## 12. Results and Comparison

Simulation results of the mesh model is discussed in this section. For P-1 and DO model, obtained results, variation of boundary conditions (temperature, absorption coefficient, etc.) are further discussed to get a proper proposition of this work.



*Figure 11: Radiation profile for mesh model (All P-1 cases)*



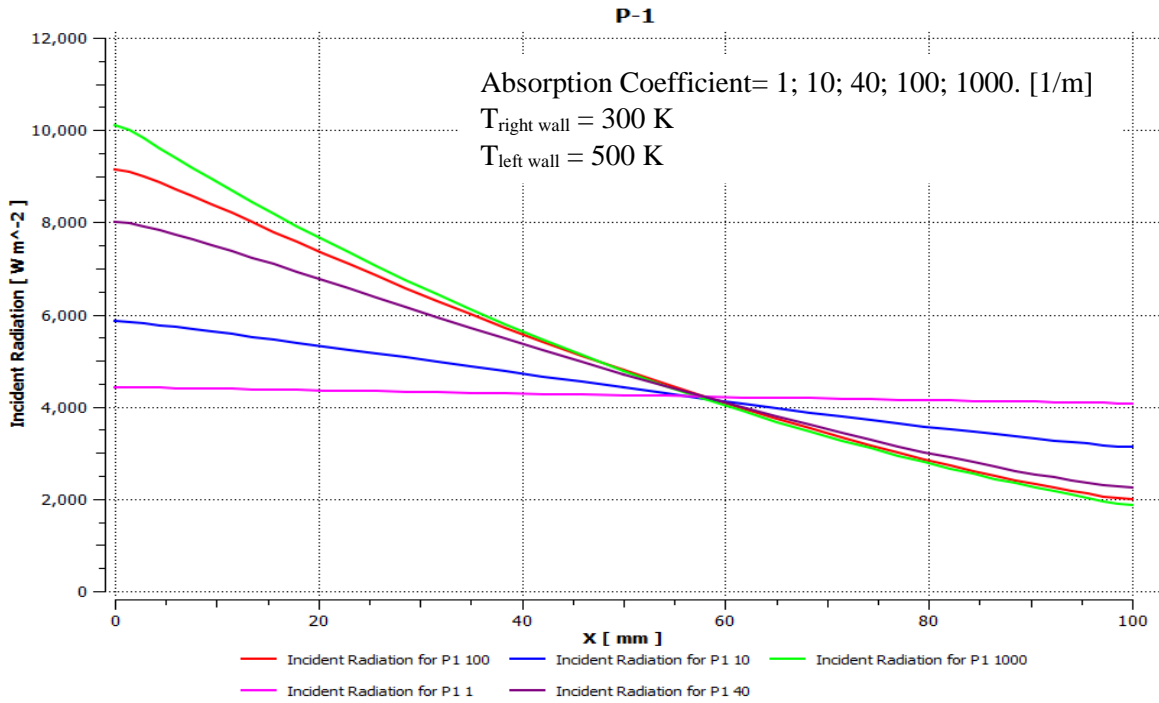
*Figure 12: Radiation profile for mesh model (All DO cases)*

From the Figure 11 (p.43) and Figure 12 above, we can relate the contours of the incident radiation at different level of the enclosed model, from left to right wall and within the inner surface of the enclosed body.

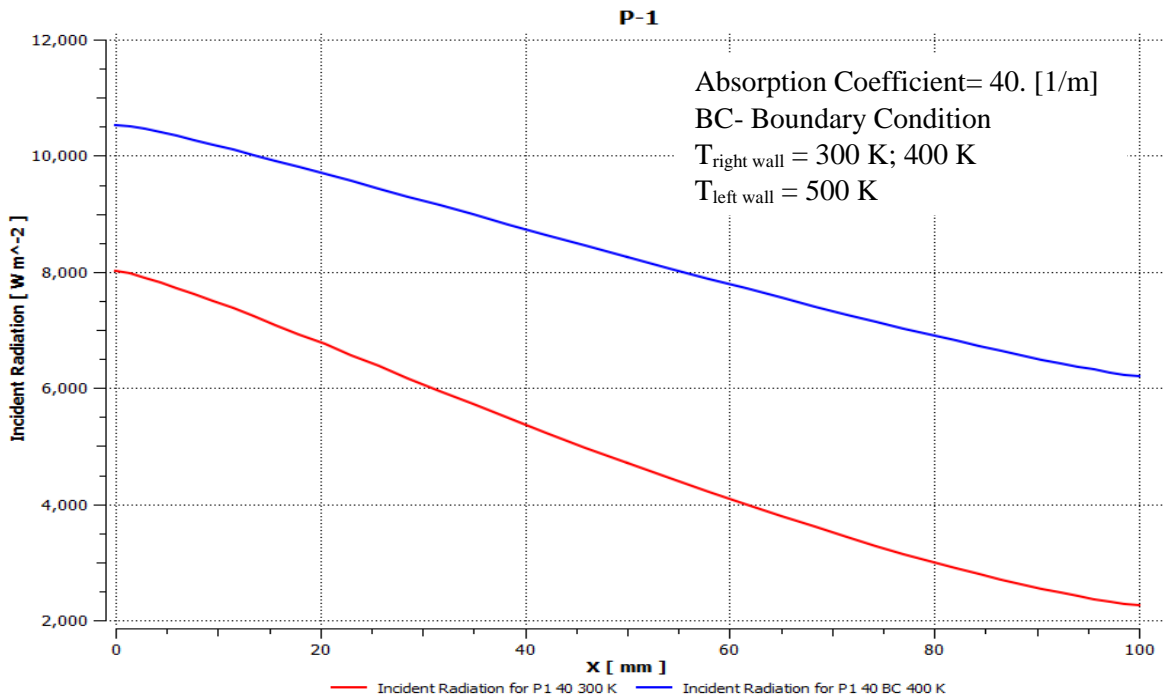
For a clear insight of the results, we can compare the radiation charts (p.45) for both the radiation models at different boundary conditions, obtained as a part of results.

Figure 13 (p.45) illustrates comparison for different absorption coefficients and P-1 model. It is clearly visible here that the larger values of absorption coefficient are used, the larger decrease of the incident radiation flux appears there.

From the following radiation charts, an idea can be estimated whether a model used is most appropriate for the given case or not. While setting up the parameters for P-1 model, for example, slightly changing the temperature on the right wall apprehends a variable curve as seen in Figure 14 (p.45) whereas alternating Diffuse Factor between 1 and 0, in the DO model gives no differences (depending on the absorption coefficient and Type of wall: opaque or semi-transparent)

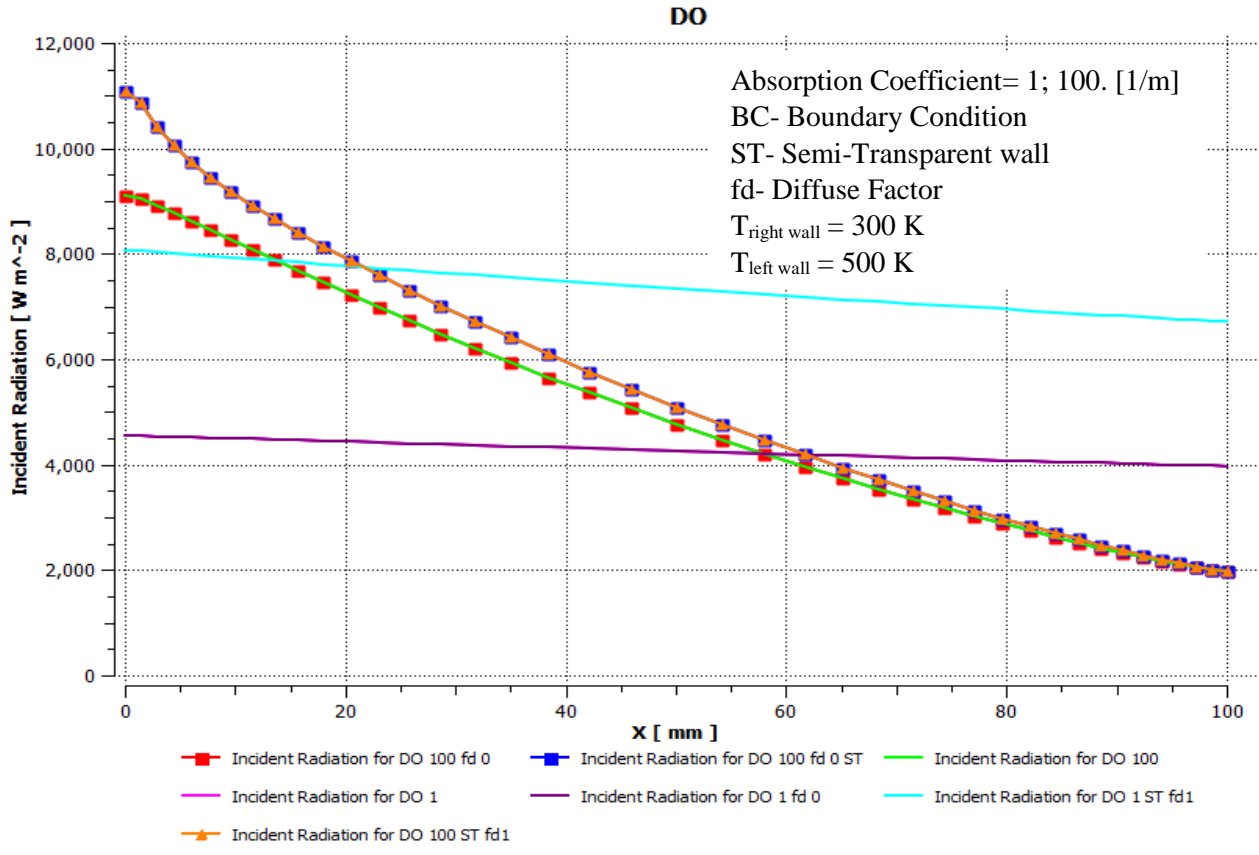


*Figure 13: Incident Radiation Chart 1 (All P-1 cases)*

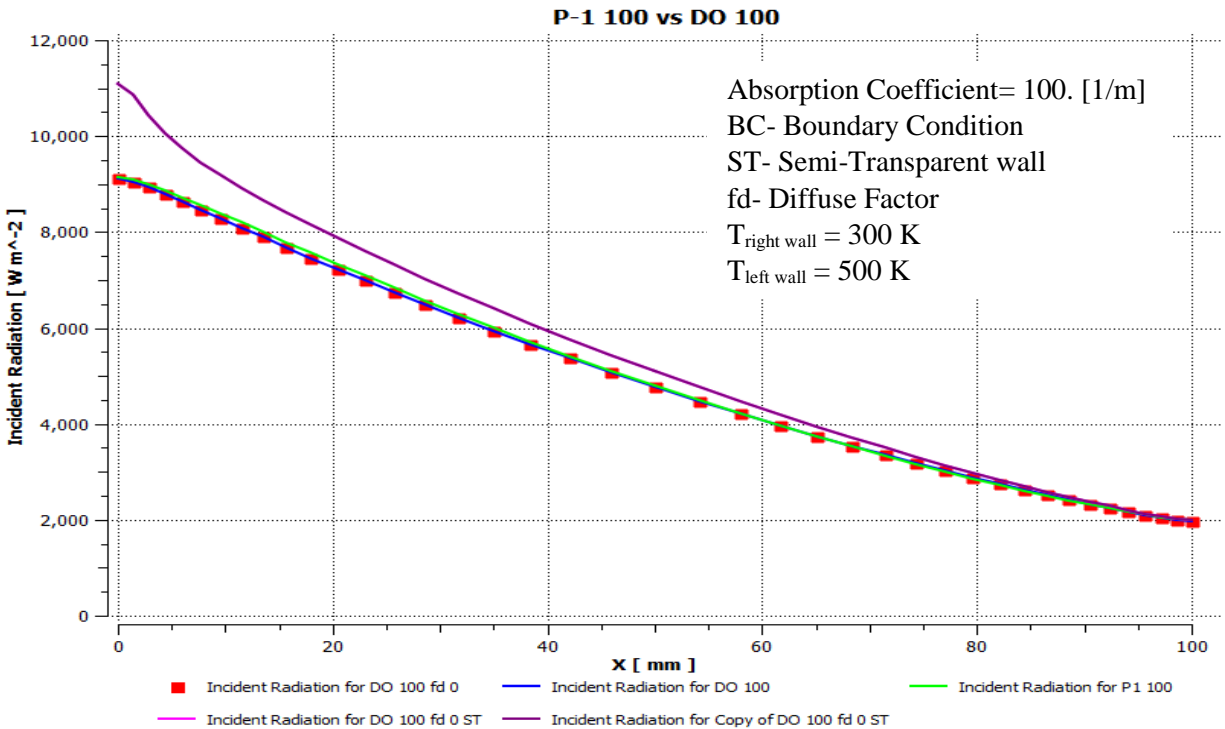


*Figure 14: Incident Radiation Chart 2 (BC)*

Figure 14 represents the incident radiation curves set for the Boundary condition of temperature 300 [K] and 400 [K] at the right wall and 500 [K] at the left wall, with absorption coefficient of 40 [1/m].



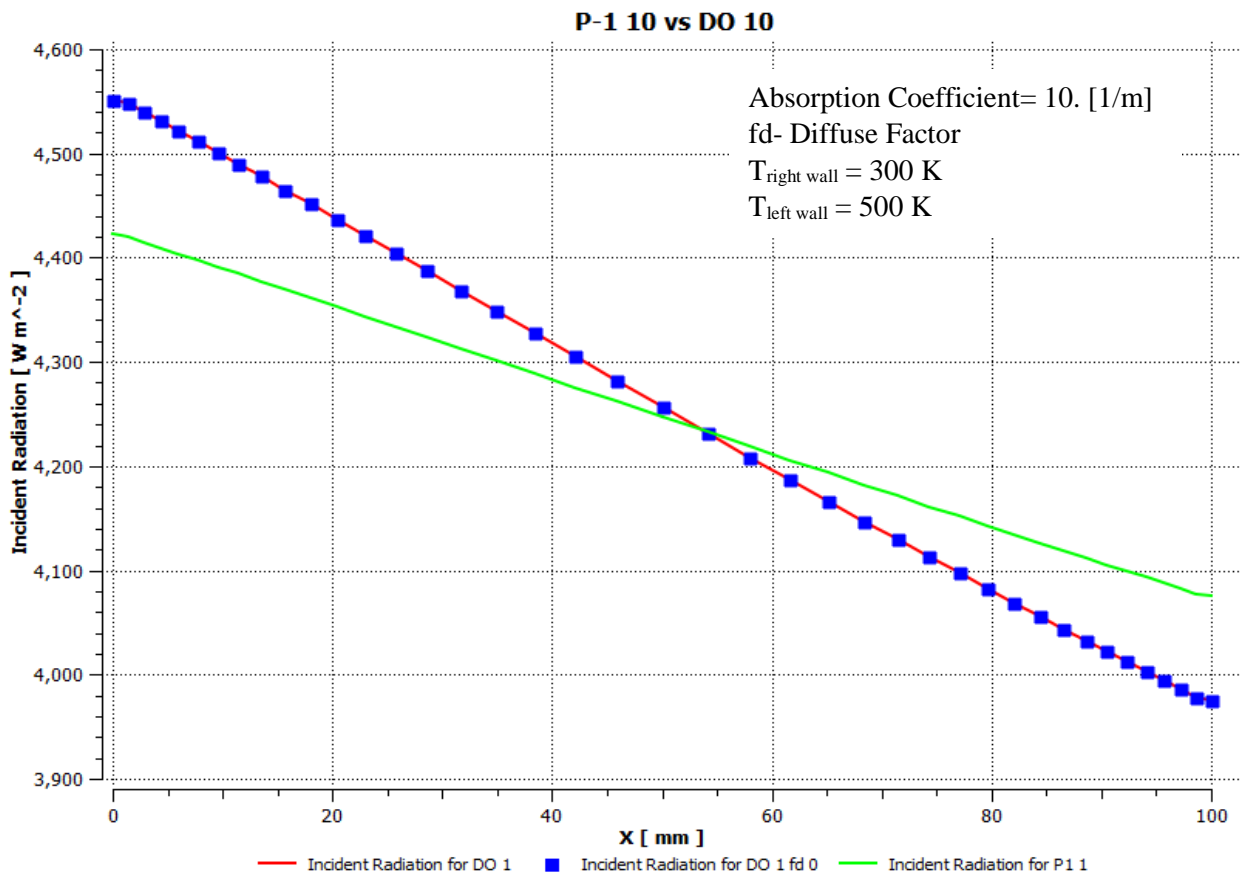
*Figure 15: Incident Radiation Chart 3 (All DO cases)*



*Figure 16: Incident Radiation Chart 4 (P-1 vs DO)*

From the results and radiation charts, it can be said that the P-1 model can be used in our case but only if the optical thickness is larger than 1. For optical thickness, given equation is  $aL > 1$ , for instance, we take  $a=10$  [1/m] and  $L= 0.1\text{m}$  (given), then the equation for optical thickness yields as  $10 \times 0.1 = 1$ . Henceforth, we can prove it by comparing the radiation charts of P-1 and DO model with absorption coefficient  $a=10$  [1/m] in ANSYS Fluent (see Figure 17 below where substantial difference can be seen).

Similarly for the absorption coefficient  $a=100$  [1/m] in both P-1 and DO model and their comparison (see Figure 16), the resulting curves are much closer to each other and shows a smaller difference. Finally it can be said that the P-1 model is more suitable when the optical thickness is larger than 1, 10 etc.



*Figure 17: Incident Radiation Chart 5 (P-1 vs DO)*

Figures 15 and 16 (p.46) also includes cases with different values of diffuse fraction  $fd=0$  and  $1$ , as well as the cases with semi-transparent wall (ST). Some differences are visible there but they are mostly negligible or relatively small.

### **13. Future Proposal related to the present work**

Concerning the future proposal based on the present work, we can say that Monte-Carlo (MC) model exists there as well but in our case it was not used (it cannot be used with 2-D geometry), the final aim is focused on using a radiation model when describing algae cells in a photobioreactor where their growth rate is directly affected by the irradiation in corresponding locations. Because in many cases the algae culture could be quite dense (with respect to light irradiation), it has direct impact on the absorption coefficient of the media. Therefore to be able to describe such situations, we have to solve fluid flow together with radiation where the absorption coefficients depends on coordinates and we could study, for example, the impact of hydrodynamics (mixing) on the overall performance of such system (we usually need to produce the maximum amount of algae biomass).

For future simulation, it would be recommended to achieve radiation curves which start at the same point (from the left wall) and decreases exponentially downwards (towards the right wall). But there could be a problem of how one can achieve it. There is a possibility to use Solar load model in ANSYS Fluent for the description of radiation flux at some wall, but it can be used only in 3-D. In my work, I focused only on 2-D geometry so it was left for the future work. Likewise, it was not possible to use it in parallel in older versions of ANSYS Fluent.

### **14. Conclusion**

The work aims at description and assessment of available numerical methods used for simulations of light propagation in algae photo bioreactors. The light is the key factor of algae growth and its permeability decreases with increasing density of algae culture. The light intensity decreases with increasing distance from the illuminated wall in general. It can be affected by the hydrodynamics of the given system which is determined by the geometry and operational parameters. A proper model when describing such system is very important in the design. This work compares radiation models in ANSYS Fluent which could be used to imitate the light distribution in photobioreactors.

All the radiation models in ANSYS Fluent were evaluated carefully and three of them were selected to compare the outcomes. P-1 model showed relatively good results and could be recommended as a suitable radiation model (but only for optical thickness  $> 1$ ), whereas DO model showed results closer to the P-1 but the complexity of the DO model makes it somehow difficult when it comes to boundary conditions. Rosseland model was chosen as well, but it did not work correctly (as in general, it is recommended for large optical thickness only). Another possible radiation model could be Surface-to-Surface (S2S) radiation model, but it can be used only with a non-participating media, that is the absorption in the media (water in our case) cannot be described with this model. Therefore, simulations were carried out in ANSYS Fluent software with the selected radiation models to obtain the results and make a comparison. Finally, on the basis of my simulations, the P-1 model can be used in cases with larger optical thickness, otherwise DO model should be used. Of course, to be able to describe a real system where density of the algae culture changes with location, it would be necessary to implement a relation between the density and the absorption coefficient in ANSYS Fluent, possibly through UDF (user defined function).



## Nomenclature

$a$	Absorption coefficient [ $\text{cm}^{-1}, \text{m}^{-1}$ ]
$A_{pn}$	Projected area [ $\text{m}^2$ ]
$C_p$	Heat capacity at constant pressure [ $\text{J}/\text{kg}\cdot\text{K}$ ]
$d_{pn}$	Diameter of $n^{\text{th}}$ particle [ $\text{m}$ ]
$E$	Total energy, activation energy [ $\text{J}, \text{kJ}$ ]
$F$	Fraction of radiant energy by black body [ $\text{W}/\text{m}^2\cdot\mu\text{m}$ ]
$f_c$	Frequency of the cycle [ $\text{s}^{-1}$ ]
$h$	Species enthalpy [energy/mass]
$h_j$	Enthalpy of species $j$ [energy/mass]
$I$	Radiation intensity [energy per area surface per unit solid angle]
$k$	Thermal conductivity [ $\text{W}/\text{m}\cdot\text{K}$ ]
$k_{eff}$	Reactivity coefficient [-]
$k_r$	Radiative conductivity [ $\text{W}/\text{m}\cdot\text{K}$ ]
$n$	Refractive index [-]
$N$	Number of particles [-]
$p$	Static pressure [ $\text{Pa}$ ]
$q_r$	Radiation flux [ $\text{W}/\text{m}^2$ ]
$q_\lambda$	Spectral radiative flux [ $\text{W}/\text{m}$ ]
$s$	Path length [ $\text{m}$ ]
$T$	Local temperature [ $\text{K}$ ]
$T_c$	Total cycle period [ $\text{s}^{-1}$ ]
$T_g$	Temperature of gas at wall [ $\text{K}$ ]
$T_{pn}$	Temperature of particle [ $\text{K}$ ]
$T_{ref}$	Reference temperature [ $\text{K}$ ]
$T_w$	Wall temperature [ $\text{K}$ ]
$u$	Velocity magnitude [ $\text{m}/\text{s}$ ]
$v$	Velocity [ $\text{m}/\text{s}$ ]
$V$	Volume [ $\text{m}^3$ ]
$Y_j$	Mass fraction of species $j$ [-]

$\varepsilon_{pn}$	Emissivity [-]
$\varepsilon_{\lambda w}$	Wall emissivity [-]
$\lambda_1, \lambda_2$	Wavelength boundaries of the band [m]
$\rho$	Density [kg/m <sup>3</sup> ]
$\rho_w$	Wall reflectivity [-]
$\sigma$	Stefan-Boltzmann constant [W/m <sup>2</sup> ·K <sup>4</sup> ]
$\sigma_s$	Scattering coefficient [m <sup>-1</sup> ]
$\Phi$	Phase function [°]
$\Omega$	Solid angle [°]
$\Omega'$	Solid angle [°]
$\bar{\tau}_{eff}$	Viscous stress tensor [Pa]
$\vec{n}$	Normal vector [-]
$\vec{r}$	Position vector [-]
$\vec{s}$	Direction vector [-]
$\vec{s}'$	Scattering direction vector [-]
$\vec{v}$	Overall velocity vector [m/s]
$\nabla$	Nabla operator [-]
$\partial$	Partial derivation [-]
$d$	Derivation [-]

## Abbreviations

CFD	Computation Fluid Dynamics
DTRM	Discrete Transfer Radiation Model
DO	Discrete Ordinates
S2S	Surface-to-Surface
MC	Monte Carlo
LED	Light-Emitting Diode
RTE	Radiative Transfer Equation
HVAC	Heating, Ventilation and Air Conditioning
CPU	Central Processing Unit
DPM	Dose Planning Method
UDF	User-Defined Function

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