THESIS REVIEWER'S REPORT

I. IDENTIFICATION DATA

Thesis title:	**Using Monodromy to Simplify Polynomial Systems**
Author's name:	**Viktor Korotynskiy**
Type of thesis :	master
Faculty/Institute:	Faculty of Electrical Engineering (FEE)
Department:	Department of Cybernetics
Thesis reviewer:	RNDr. Zuzana Kúkelová PhD.
Reviewer's department:	Department of Cybernetic

II. EVALUATION OF INDIVIDUAL CRITERIA

Assignment **extraordinarily challenging**

How demanding was the assigned project?

The topic of Viktor Korotynskiy's master thesis is monodromy, Galois groups, and symmetries in polynomial systems. This is an interesting problem since finding the symmetries of a given polynomial system may significantly simplify the solution of the system. Many problems in computer vision, robotics, and other fields require solving systems of polynomial equations. Therefore, using symmetries may result in the improved efficiency of existing solutions to many computer vision problems or may lead to efficient solutions for unsolved problems. Symmetries are well studied in mathematics but have only recently been used in computer vision. Their obscurity in computer vision applications is likely due to the deep knowledge of numerical algebraic geometry that is required to understand the concept of symmetries

Fulfilment of assignment **fulfilled**

How well does the thesis fulfil the assigned task? Have the primary goals been achieved? Which assigned tasks have been incompletely covered, and which parts of the thesis are overextended? Justify your answer.

The thesis fulfills all three given tasks. The author reviewed Galois theory and relevant parts of algebraic topology related to monodromy of polynomial systems in a deep detail. In the thesis, Viktor Korotynskiy explains complex connections between the monodromy group of a polynomial system with a finite number of solutions, its Galois group and symmetries of this polynomial system. The presented theory is demonstrated on three camera geometry problems, including the well-known 5-point relative pose problem.

Methodology **outstanding**

Comment on the correctness of the approach and/or the solution methods.

The thesis presents several contributions and demonstrates that the author has a deep knowledge and fundamental understanding of advanced concepts from different branches of mathematics. Namely, the thesis reviews complex concepts from group theory, algebraic geometry, algebraic topology and Galois theory. Viktor Korotynskiy shows how to combine these branches in order to understand the concept of symmetries. The knowledge that Viktor presents might be expected from a postdoctoral student in mathematics.

The thesis collects and reviews a large number of results from the theory to understand the symmetries in parametric polynomial systems. The thesis extends the results of the recent work [3], which presents a method for exploiting the structure of the Galois/monodromy group for the simplification of generic polynomial systems. Unlike for generic parametric polynomial systems that do not contain relations among the parameters and for which the situation has been fully characterized in [3], the situation in systems appearing, e.g., in computer vision, is more complex since these systems are structured and contain relations among parameters. To the best of my knowledge, the proposed thesis is the first attempt to develop a complete, rigorous and systematic approach to finding how to use the structure of the Galois/monodromy group for simplification of such parametric polynomial systems.

Technical level **A - excellent.**

Is the thesis technically sound? How well did the student employ expertise in the field of his/her field of study? Does the student explain clearly what he/she has done?

The thesis is very well written, and even though the presented topic requires a non-trivial mathematical language and significant background in different branches of mathematics, most of the definitions and necessary theorems are clearly described in the thesis.
My only concern is that identifying Viktor's Korotynskiy contributions is a bit hard. Contributions are summarized in Section 1.2; however, this section contains only a general summary. In the theoretical chapters 2-8 it is not clear whether all results, theorems, propositions and proofs are known and were collected by the author from different sources or whether some of these propositions and proofs are introduced by the author, e.g., the results presented in Chapter 7. The contribution of Viktor Korotynskiy to the results presented in Chapter 9 is also not clearly specified. This Chapter refers to [9] [23], which is a personal communication with T.Duff and M.Regan; however, it is not clear how Viktor Korotynskiy contributed to it. I believe that Viktor contributed to these results significantly, however, in this case, instead of using "it was discovered [9]/[23]", the author should use "we discovered" and specify his contribution.

Formal and language level, scope of thesis **A - excellent.**

Are formalisms and notations used properly? Is the thesis organized in a logical way? Is the thesis sufficiently extensive? Is the thesis well-presented? Is the language clear and understandable? Is the English satisfactory?

Despite the fact that the thesis contains a huge number of non-trivial mathematical expressions, I found only a few minor errors. The thesis is well structured and is technically excellent. All necessary definitions, concepts and theorems are included in the thesis. English is satisfactory.
The clarity could be improved by summarizing the proposed method for identifying symmetries and simplifying equations at the end of the thesis, e.g., in the form of an algorithm, similar as it was proposed in [3]. Having such an algorithm or a set of instructions, including instructions and commands in GAP that the user should run, together with one simple example of 2-3 equations in 2-3 unknowns, would significantly help, especially non-experts. Even though the thesis contains some simple examples and Chapter 9 provides three minimal problems, the theory is extremely complex. Therefore, a non-expert can easily get lost and would most likely not be able to use the method for her/his own systems of equations. Providing a summarizing algorithm/set of instructions or even some black-box for detecting symmetries, similar to the one that is e.g. contained in the automatic generator for minimal solvers, would be extremely useful. Maybe an automatic "black box" can be future work.

Selection of sources, citation correctness **B - very good.**

Does the thesis make adequate reference to earlier work on the topic? Was the selection of sources adequate? Is the student's original work clearly distinguished from earlier work in the field? Do the bibliographic citations meet the standards?

The references are satisfactory. However, I would appreciate a little bit more detailed description of the state-of-the-art methods in Section 1.2 including a more detailed comparison of the proposed results to the results from [1] and [3]. It would also be useful to add references to books/ papers that cover the theory of each of the considered mathematical branches at the beginning of Chapters 2-8, e.g, adding references to books that the author used as a motivation, and a source of theorems and definitions, and which maybe include a more detailed description. Some sections contain such references to e.g., proofs. However, I would appreciate having such references also at the beginning of each Chapter.

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III. OVERALL EVALUATION, QUESTIONS FOR THE PRESENTATION AND DEFENSE OF THE THESIS, SUGGESTED GRADE

Summarize your opinion on the thesis and explain your final grading. Pose questions that should be answered during the presentation and defense of the student's work.

The thesis is an excellent submission, and it fulfills all its stated goals. The author demonstrates a deep understanding of different fields of mathematics, including group theory, algebraic geometry, algebraic topology and Galois theory, and he mastered advanced methods for finding symmetries of polynomial systems. In summary, the topic of the thesis is of importance to the field; the goals of the thesis were met, and interesting results were achieved. I recommend the thesis for defense and propose the grade of A (excellent)

Minor questions:

1. How have you computed the generators of Bir(f) e.g. in Example 7.23?
2. Was the elimination ideal (9.6) computed using some algebraic software or did you use the well-known constraints for the 5pt problem?
3. Was the symmetry (9.8) computed, and if yes, how, or did you use the known constraints/symmetries from geometry?
4. Have you tried to compute f1 for the PLMP 3100 problem? Do you have at least some preliminary results?

The grade that I award for the thesis is **A - excellent.**

Date: **15.6.2020** Signature: