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FACULTY OF MECHANICAL ENGINEERING

DOCTORAL THESIS STATEMENT

CZECH TECHNICAL UNIVERSITY IN PRAGUE
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DOCTORAL THESIS STATEMENT

Analytical and Computational Methods for Transonic Flow Analysis and Design

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Abstract

The work deals with various solutions of the compressible transonic flows and their applications on academic and practical cases. The main motivation is to remind the classical methods, modify or modernize them with fast computational techniques and show their benefits even in todays modern era of commercial software tools. In order to solve the combination of elliptic and hyperbolic equations describing the near sonic flow, the hodograph transformation based methods are introduced and rheograph solution to the near sonic flow is presented as well as the numerical methods which can solve directly Euler partial differential equations describing general compressible flow. After the validation of individual approaches, classical methods are combined with modern computational abilities to create an academic case of supercritical nozzle, which prove the functionality for the internal aerodynamics. The practical benefits of the analytical solution knowledge are presented on the ERCOFTAC case of transonic blade cascade SE 1050 with specific design and flow pattern with supersonic re-compression issue. The rheograph transformation is used for specific flow analysis and solutions to this problem are proposed.

Anotace

Práce se zabývá různými způsoby řešení transonického proudění stlačitelných tekutin a jejich aplikací na akademické i praktické úlohy. Hlavní motivací je připomenutí clasických metod, jejich modifikace využitím moderních výpočetních postupů a techniky a cílem je ukázat jejich benfity i v denšní době rychlých numerických simulací. Pro potřebu řešení kombinace eliptických a hyperbolických rovnic popisujících proudění blízké rychlosti zvuku se využívají metody založené na hodografické transformaci. Je zde popsána rheografická metoda řešení a rovněž numerické metody řešící přimo Eulerovy rovnice proudění stlačitelné tekutiny. Po validaci jednotlivých přístupů, je navrženo rozšíření rheografické metody použitím moderních výpočetních metod a je tímto způsobem vytvořena akademická úloha superkritické trysky, která mimo jiné dokazuje možnost použití této metody i pro potřeby interní aerodynamiky. Praktické výhody znalosti a využití analytických metod jsou prezentovány na úloze transonické lopatkové mříže SE 1050 z databáze ERCOFTAC. Tato mříž je známá svým specifickým charakterem a přítomností supersonické rekomprese. Rheografická transformace je použita na podrobnou analýzu specifického proudového pole a na jejím základu jsou navrženy opatření pro eliminaci tohoto nežádoucího chování.

Contents

1 Introduction

Among various aerospace vehicle speeds and machinery operating states, the transonic regime has always posed remarkably difficult challenges to systematic design and experimental techniques. The unknown physical limits and by this time unexpected behavior at reaching and exceeding sonic velocities were hard to understand and describe for the first observers. Until only theoretical methods to understand transonic flow phenomena developed in past time shed light into transonic testing techniques by solving the underlying physical relations in various stages of their simplifications. At the same time, first numerical simulations of these physical relations were developed so that prior to experimental investigations a quantitative results were obtained. Accelerated computational technology and numerical codes development then enabled the expansion and introduced this field to the wider community of engineers.

The work makes use of classical concepts for transonic and supersonic flows which have been used for developing design methods in the past century [1], [2]. These methods [3], [4] might be aging today, but can still be capable and compared to presently operational computational methods, they offer valuable extra benefit of understanding the transonic flow phenomena and gas dynamic basic principles often unknown to current engineers. Various applications for different cases in aerospace field were developed, but the extensions of the classical methods can be applied as well on internal aerodynamics [5], [6]. To make these methods still relevant, they can now arise and advance from modern computational and programing abilities. These still theoretically and mathematically valid concepts and cases may be perfectly suitable for validation and application of new approaches, however, the seek for direct practical usage example of any method require application on a industry relevant case. The solution of transonic internal aerodynamics already implies the possible practical application for the turbomachinery blades and cascades [7]. Concerning this, the main idea is to show that all the classic theory, even though up to date renovated, widely used CFD solutions as well as experiments can be used in cooperation and occasionally provide new, more advanced view on flow field analysis and also potentially help with a clever shape design.

1.1 Motivation

As the described problematics reaches the limits of simple mathematical methods, the more attention should be paid to the level of knowledge amongst the researchers. The new modern generation of engineers and even scientists may be skilled in use of modern computational software and hardware tools, but the complexity and abilities of nowadays technology requires less and less of user involvement. No doubt that there are many benefits of such technological progress, but the side effect of the overall knowledge no more being necessary to be present among the research and development society is also apparent due to this evolution. Especially in such complicated and sensitive field of study as the transonic flow regime.

The main philosophical message of the thesis is to remind the basic principles of high speed compressible fluid flow analysis and techniques and show that it is still beneficial to have at least a basic theoretical knowledge among engineering community. To show that the modern numerical computational methods do not have to mean the old practice to be forgotten but may be used in cooperation with theoretical methods and lead to better understanding of the transonic flow basics and smarter, creative engineering.

1.2 Topics and Scheme

Previous thoughts already point out the main theme of the following chapters, however, for the informational impact, some applicable conclusions are important too. Besides the general reminder of those mostly forgotten methods, the following tasks and topics in consecutive sections are solved in this thesis.

The first task is to briefly introduce the compressible gas dynamics problematics, its basics and general principles as well as possible solutions to subsonic, supersonic and transonic problems and flow fields.

As the foundation of the theoretical analytical methods for high speed design methods may be aging, the known and well described Guderley's cusp and its lifting variations [3] problem can serve as a perfect test case for comparison of the hodograph-based methods, classical gas dynamics principles and modern computational techniques.

After understanding the principles and managing all the methods for transonic flow analysis and design, the next task is to preview the idea of combination of the flow field numerical computation and theoretical methods. In other words, to outline the revitalization of the rheograph transformation method [5] and elliptic continuation using modern computational tools on a mathematically valid theoretical example for internal supercritical nozzle throat flow.

To demonstrate the ability, functionality and possible benefits of such method on a relevant practical case, it is applied on a real geometry and real flow field of the blade cascade SE 1050 [7]. This cascade is well know for it's specific flow pattern and the goal here is to show how the results obtained from numerical simulations can be used as the initial condition for the rheograph transonic flow analysis ans propose possible shape modification.

1.3 Thesis Goals

In general, the main point to prove is that the hodograph based methods with a fresh touch of computational techniques can provide a working, powerful and even today relevant analysis and design tool for both academic and practical cases. The thesis goals to verify this hypothesis are therefore formulated as:

- To validate described solutions to the compressible and transonic flows and their functionality and accuracy on a case of sonic cusped airfoils.
- To reduce the need for manual analytical solution and complex mathematics and extend the rheograph transformation method using computational techniques for solution of linearized flow equations and method of characteristics.
- To describe the application for internal aerodynamics and apply the solution on a novel academic example of a supercritical nozzle.
- To test the abilities of such approach on a practical case of the blade cascade SE 1050. To analyze the specific flow field pattern using the rheograph transformation and propose possible design solutions.
- To discuss the functionality of developed method for wide usage in relevant academic and practical applications.

2 The Theory and Solutions of the Compressible Fluid Flows

The effect of compressibility brings the high speed aerodynamics to the new level of understanding due to reaching the natural physical limits. The barrier of sound distinguishes the flow in two differently behaving and differently described regimes. The only way of analytical solutions to the transonic flow fields leads to transformations into linearized nonphysical planes. Modern computational era arrives then with new possibilities and numerical methods and computational fluid dynamics to solve directly the general equations.

Before solving the concrete applications, it is appropriate to introduce the known theory and describe general state of knowledge. Therefore, the overview of compressible flow basic principles [8], [9] and introduction to later used flow solutions is described in this chapter.

2.1 Compressible Fluid Flow and Gas Dynamics

Compressible fluid flow implies variation of density in the flow field resulted principally from pressure changes between two points in the flow. The rate of change in density with respect to pressure is then closely connected with the velocity propagation of small pressure disturbances, or in other words the speed of sound. The barrier of sound $M = 1$ distinguishes the flow regimes into subsonic regime for $M < 1$ and supersonic regime for $M > 1$. For velocities close to the speed of sound, both types of fluid can occur together and form a transonic flow field. If subsonic flow is accelerated enough, it can reach sonic speed and continue supersonic to form expansion or shock waves. Transonic regime is extremely sensitive to any changes mainly in areas closest to the sonic conditions, where only minor changes in geometry mean dramatic changes in the flow field. And while the basics of subsonic and supersonic theory can be described by linear theory, much more difficult situation as a transonic regime always leads to nonlinear description with various theoretical difficulties.

2.2 Near Sonic Flow and Hodograph Based Methods

The proper full mathematical solution of transonic flow phenomena resulting in the co-existence of elliptic and hyperbolic basic differential equations have to bring together classical hydraulic methods with the wave propagation solutions. To understand the basics of such theory, the flow can be restricted, so that the velocity magnitude is close to speed of sound and analytically exact solutions can be obtained [4].

Velocity potential ϕ and stream function ψ are defined with $q = |\vec{c}\,|$ and ϑ the flow angle.

$$\phi_x = \frac{\rho_0}{\rho}\psi_y = u = qcos\vartheta \tag{2.1}$$

$$\phi_y = \frac{\rho_0}{\rho}\psi_x = u = qsin\vartheta \tag{2.2}$$

To avoid the nonlinearity of the system, the transformation of the solution to the hodograph plane can be applied replacing the physical coordinates with the flow angle ϑ and Prandtl-Meyer turning angle ν.

$$\nu = \int_{a^*}^{q} \sqrt{|M^2 - 1|}\frac{dq}{q} \tag{2.3}$$

These new variables lead to define a hodograph plane wherein the basic system becomes linear Beltrami system:

$$\phi_\nu = K\left(\nu\right)\psi_\vartheta \left(\nu \geq 0, M \geq 1\right) \tag{2.4}$$

$$\phi_\nu = -K(\nu)\,\psi_\vartheta\,(\nu \le 0, M \le 1) \tag{2.5}$$

with

$$K = K(M(\nu)) = \frac{\rho_0}{\rho}\sqrt{|M^2 - 1|} \tag{2.6}$$

ϑ and ν are also functions of a computational working plane obtained from the basic ν, ϑ hodograph by conformal (subsonic) or characteristic (supersonic) mapping. For subsonic including sonic conditions conformal mapping defines working plane ζ. E is the mapping function.

$$\zeta_0 = \nu + i\vartheta \tag{2.7}$$

$$\zeta = s + it = E(\zeta_0) \tag{2.8}$$

The basic system in ζ becomes

$$\phi_s = -K(\nu(s,t))\,\psi_t \tag{2.9}$$

$$\phi_t = K(\nu(s,t))\,\psi_s \tag{2.10}$$

$\nu(s,t)$ is then the real and $\vartheta(s,t)$ imaginary part of $E^{-1}(\zeta)$. These equations, Eq. (2.9) and Eq. (2.10) form the linear Beltrami system and elimination of ψ and ϕ leads to linear Poisson equations:

$$\phi_{ss} + \phi_{tt} = \frac{K_s}{K}\phi_s + \frac{K_t}{K}\phi_t \tag{2.11}$$

$$\psi_{tt} + \psi_{tt} = \frac{K_s}{K}\psi_s + \frac{K_t}{K}\psi_t \tag{2.12}$$

New characteristic variables occur for supersonic region with a suitable mapping function H.

$$\xi = H(\vartheta + \nu) \tag{2.13}$$

$$\mu = H(\vartheta - \nu) \tag{2.14}$$

The system is then valid in ξ, μ plane

$$\phi_\xi = K(\nu(\xi,\mu))\,\psi_\xi \tag{2.15}$$

$$\phi_\mu = -K(\nu(\xi,\mu))\,\psi_\mu \tag{2.16}$$

or

$$\left(\frac{d\psi}{d\phi}\right)_{\xi,\mu=const.} = \pm K^{-1}. \tag{2.17}$$

That is the basic relation to integrate the flow equations for method of characteristics for supersonic flow.

By eliminating ϕ and ψ a system for physical plane coordinates can be obtained. The transonic similarity laws containing a similarity parameter σ for reduction of variables for place and state x, y, q, ϑ are:

$$s = \pm 2 \cdot 3^{-1} \cdot \sigma^{-1} \left(\kappa - 1\right)^{1/2} a|1 - \frac{q}{a^*}|^{3/2} \tag{2.18}$$

$$t = \sigma^{-1}\vartheta \tag{2.19}$$

$$x = \phi/a^* \tag{2.20}$$

$$y = \sigma^{-1/3} \cdot 3^{1/3} \left[2^{-1} \left(\kappa - 1\right)\right]^{\frac{1}{\kappa-1}+\frac{1}{3}} \cdot \psi/a^* \tag{2.21}$$

s is positive for supersonic, negative for subsonic and equal to zero for sonic conditions. The basic system (2.4) and (2.5) then yields to corresponding Beltrami system for reduced physical plane parameters s and t.

$$X_s = \pm|s^{1/3}|Y_t \tag{2.22}$$

$$X_t = \pm|s^{1/3}|Y_s \tag{2.23}$$

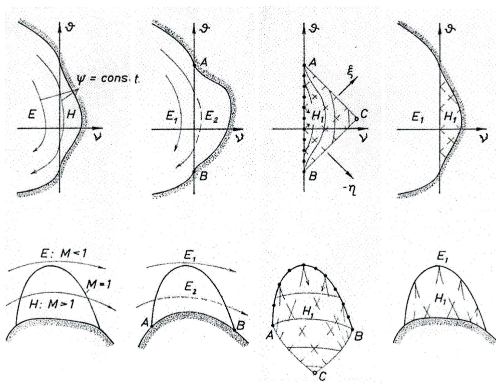

Figure 2.1: Conformal and characteristic mapping [4]

The idea of whole process is symbolized in the Figure 2.1. The specific structure of supercritical flow containing singularities and mathematically problematic patterns requires specific treatment. Sonic lines and isotachs saddle points regularly appearing in the transonic flow fields represent very weakly singular behavior and due to this, the flow surface may fold into multivalued hodograph. This effect is why the special rheograph plane needs to be used in order to obtain single-valued characteristics grid. The idea also led to the analogy between the plane electric conductor and the coefficient K [4].

2.3 Numerical Methods for Compressible Fluid Flows

Modern computational abilities allow to solve directly the partial differential equations numerically using computational fluid dynamics methods [10]. These methods in general transform differentials to differentiations using finite computational grids.

To demonstrate the functionality of a numerical flow simulation, an example of oblique shock [11] is used here to compare different schemes available for compressible fluid flow, from simple Lax-Friedrichs and MacCormac [12] to advanced AUSM scheme [13]. AUSM is advanced high order scheme used in ANSYS Fluent CFD code as well as many others suitable for compressible flows and will be used in the later flow simulation cases below. Due to an exact analytical solution, this example is perfect for testing the accuracy of the methods as well.

Results in form of pressure contours for the AUSM scheme are shown in the Figure 2.2. The wave angles and pressure values in formed regions correspond to the analytically calculated values.

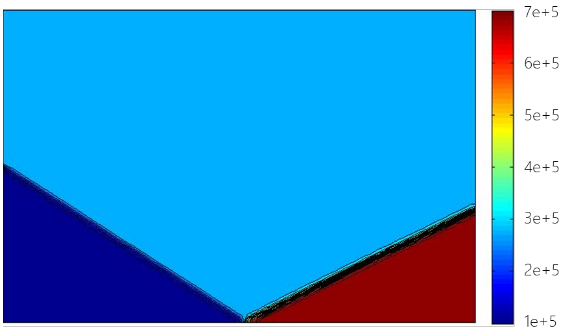

Figure 2.2: Contours of pressure [Pa] - AUSM

The pressure distribution on the wall for each code is shown in the Figure 2.3. The solid line represents the analytical solution.

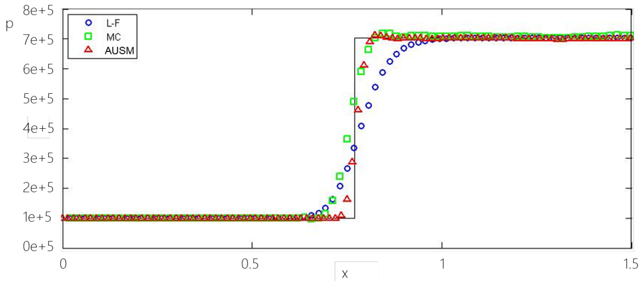

Figure 2.3: Wall pressure distribution [Pa]

The previous results lead to a conclusion that the Lax-Friedrich scheme shows its natural characteristic of a strong dissipation, even with very low artificial viscosity coefficients. On the other hand the MacCormac Lax-Wendroff scheme acts much more accurately in this way but thanks to strong oscillations near the shocks the influence of the artificial viscosity term had to be raised what causes that the pressure change along the wave is still not as steep as the AUSM scheme results show. That proves the AUSM as a reasonable choice for upcoming CFD simulations.

3 Lifting Guderley's Cusp as a Validation Case

This section is dedicated to remembering of some analytically defined methods and airfoil geometries as a test cases and to comparison of analytical flow description with numerical results [14], making use of a commercially available computational fluid dynamics code and with classical gas dynamics analysis.

The main point of the use of Guderley's case is to arrive at a high degree of understanding typical transonic challenges and remind on this example that accelerated optimization strategies may be carried out with a reduced set of input parameters, before costly experiments may focus directly on optimum design cases.

3.1 Exact Solution

Finding analytical solutions to above hodograph relations described in Chapter 2.2 allows to derive the formulae defining the shape, flow conditions and pressure coefficient for cusped airfoils in a uniform sonic flow $M_\infty = 1$ [3], [15]. The solution of such airfoil is described in the Figure 3.1 and the schematic view on a cusp with all defined parameters is in the Figure 3.2.

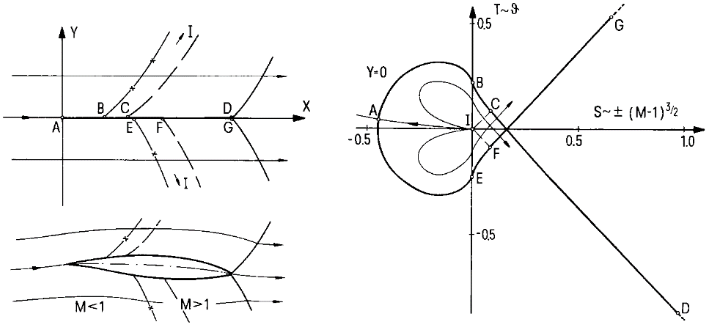

Figure 3.1: Cusp solution in the real and the rheograph plane [3]

Such solution results in the cusp shape described with two parameters - thickness and camber parameter. Thickness to chord ratio is τ and camber to chord ratio ω. The solution is exact for $\tau \to 0$ and practically valid for slender airfoils with $\tau \leq 0.5$. The case with camber to chord ratio $\omega = 0$ is the symmetrical one also known as "Guderley's cusp". Limit of validity for cambered airfoils is given by ratio $\omega/\tau \leq 0.5$.

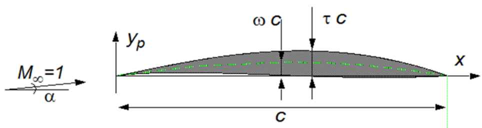

Figure 3.2: The cusp solution and parameters [15]

The theory [3] gives the camber/thickness parameter $P\left(\frac{\omega}{\tau}\right)$, the angle of attack α, geometry vertex data and finally the pressure coefficient c_p. Knowing these, a complete analytical solution of this problem of sharp cusped airfoil in a sonic free stream is known.

3.2 Numerical Solution

For flow simulations, the inviscid – Euler model with ideal gas implemented in a commercial CFD software ANSYS Fluent [16] was used. The numerical flux scheme was the AUSM scheme with second order upwind. The only outer boundary condition was set as the non-reflective pressure far field boundary condition. Contours of Mach number to visualize the simulated flow field are shown in the Figure 3.3. The flow is obviously smooth along the whole length as the theory suggests.

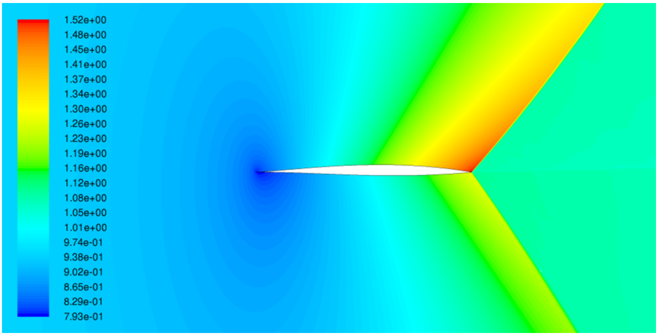

Figure 3.3: Contours of Mach number: $\tau = 0.05$ and parameter $\omega/\tau = 0.02$

Figure 3.4 shows the comparison with the analytical data via the pressure coefficient c_p distribution, dashed line represents the theory and solid line is a results from the numerical simulation.

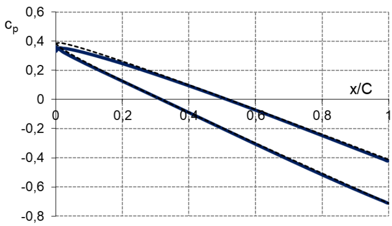

Figure 3.4: Pressure coefficient distribution

From correct flow behavior already obvious from contour figure, it is expectable that the pressure coefficient distribution along the airfoil surface will also correspond at least tendentiously with the theoretical lines and data in the Figure 3.4 confirm that. In areas where the pressure coefficient values lay between 0.3 and -0.8 are numerical results almost identical with dashed theoretical. As the values rise from these bounds, noticeable deviations appear. This may be given by the fact that the flow velocities start to differ from a near sonic flow.

3.3 Clasical Gas Dynamics Analysis

The situation of this case in terms of shock waves formation was analyzed using classical gas dynamics relation and the shock polars for this configuration were calculated [17].

Shock polars for the airfoil with the solid vertical line representing the trailing edge angle are depicted in the Figure 3.5. Both polars intersect twice and considering that the stable solution is the natural one, the result is the lower point of intersection. The flow turning angle in absolute value on the upper side of the profile is approximately $10.8°$ and on the lower side $4.3°$. To compare these numbers with the CFD results, the values from the nearest cell of the shock are approximately $10.9°$ for the upper side and $4.0°$ for the lower side. That is very satisfying result considering finite character of the computational mesh on one side and ideal gas dynamics theory on the other.

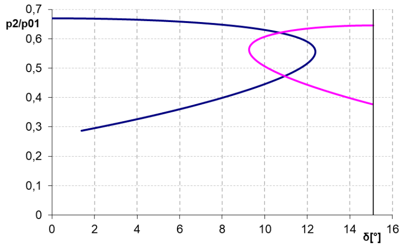

Figure 3.5: Shock polars for Case I

3.4 Off-Design Conditions

Using the sonic free stream condition, it has been already proved that it is possible to correctly simulate the analytical solutions, but the whole problem was limited by an ideal state of exact flow velocity. To get closer to the real world, even when this case is meant to be an academic example, it is always a benefit to think about some off-design conditions [18]. Especially here, when talking about very special and sensitive flow speed value, it can be very easily imagined that the velocity will oscillate around value of Mach number $M = 1$ and the problem can turn into a supersonic or on the other hand subsonic free stream case.

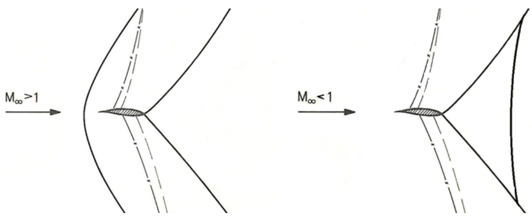

Figure 3.6: Supersonic (left) and subsonic (right) flow field [3]

These two off-design cases are symbolized in the Figure 3.6. On the left is the airfoil in mildly supersonic free stream what causes a creation of detached bow wave in front of the cusp. Detailed shape, strength and location of detached shock depending on the free stream Mach number is more explained in [3] and [15]. On the right side is the opposite case in slightly subsonic free stream.

Graphical result of the numerical simulation for this setup is shown then in the Figure 3.7 with displayed Mach number isolines.

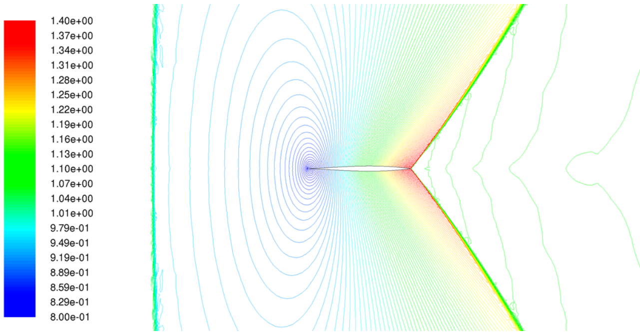

Figure 3.7: Mach number isolines $M_\infty = 1.05$

The solution for opposite problem, the subsonic flow field, is displayed again using Mach number isolines in the Figure 3.8.

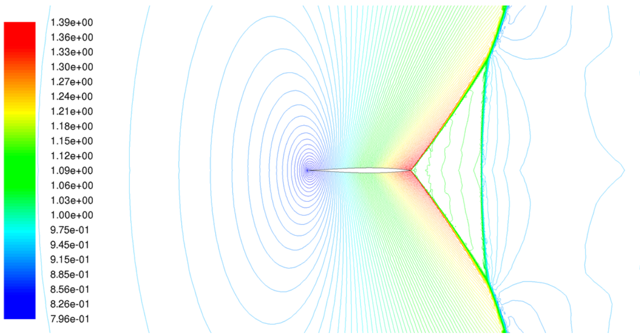

Figure 3.8: Mach number isolines $M_\infty = 0.95$

These two off-design setups extend the problematic of developed near sonic flow theory past sharp profile and describe the sensitivity of the transonic problems as a whole. It is clear how important is precise analysis when solving problems around sonic values and how a very small change in flow properties can change the whole behavior. And it has been confirmed here, that the numerical simulations followed the right trend as expected form theoretical analysis and proved as fast and reliable tool for design or verification. Vice-versa, analytical tools also prove their relevancy, as the predicted behavior is correct with high precision compared to all the simulations results.

4 Supercritical Symmetric Nozzle

Classical transonic hodograph-based design methods can be employed and revitalized as fast modern numerical tools and can be used to serve as tools to substitute analytical models for solution of differential Laplace/Poisson equations and the method of characteristics. The concept of elliptic continuation is applied to solve transonic boundary value problems avoiding the inherently nonlinear nature of the basic equations and obtaining transonic flow examples using the method of characteristics in an inverse mode.

This contribution makes use of particular solutions to the gasdynamic equations focusing on the transonic regime [19], as rheograph representation of the well known Laval throat accelerated flow allows for some extensions resulting in a new type of nozzle flow.

4.1 Rheograph Formulation of Laval Nozzle Flow

A general rheograph solution to the potential flow was already described in previous chapter. For flows with only small deviations from sonic velocity only a simplified perturbation potential equation may be used instead of the full potential equation [6]:

$$m\,(1-l)^l\,|\phi_x|^l\phi_{xx} - \phi_{yy} - \frac{k\phi_y}{y} = 0 \tag{4.1}$$

with the three switch parameters k, l, m which can then convert the equation accordingly. Later hodograph transformation [6] converts Equation (4.1) into a set of coupled Beltrami equations for velocity variables U, V and physical coordinates X, Y, valid in a parametric “rheograph” plane (s, t). The use of this technique can be shown on the example of transonic area near the nozzle throat. For two-dimensional planar flow in the Laval nozzle flow for subsonic domain $(k = 0, l = 1, m = -1)$ and elimination of V or U and X or Y yields

$$U_{ss} + U_{tt} = 0 \tag{4.2}$$

$$V_{ss} + V_{tt} = 0 \tag{4.3}$$

$$X_{ss} + X_{tt} - \frac{1}{3}\left[\frac{U_s}{U}X_s + \frac{U_t}{U}X_t\right] = 0 \tag{4.4}$$

$$Y_{ss} + Y_{tt} + \frac{1}{3}\left[\frac{U_s}{U}Y_s + \frac{U_t}{U}Y_t\right] = 0 \tag{4.5}$$

Laplace (Poisson) equations (Eqs. (4.2) - (4.5)) are now formulated for subsonic region and are ready to be solved numerically using finite difference methods for further purposes. It is good to mention that this particular set of equations can be very helpful for code and results validation because they also give a simple analytical solution in form of

$$U = 2st \tag{4.6}$$

$$V = t^2 - s^2 \tag{4.7}$$

$$X = \left(t^{4/3} + s^{4/3}\right) \tag{4.8}$$

$$Y = -2^{2/3}\left(t^{2/3} - s^{2/3}\right) \tag{4.9}$$

This solution is valid for subsonic domain, that means for $s < 0$, according to rheograph transformation, the $s = 0$ represents the sonic line.

4.2 Elliptic Continuation and Method of Characteristics

The solution of the whole problem now consists of two parts (see the Figure 4.1). At first, solving the elliptical domain in order to obtain the flow field up to the sonic line including the sonic line data, by an elliptic continuation of the boundary value problem beyond the sonic line location $(s = 0)$. Next is the extension into the hyperbolic supersonic domain, using the previously obtained sonic line data as initial conditions.

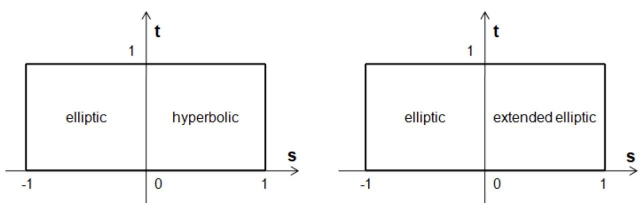

Figure 4.1: Rheograph s, t plane: Mapping of real transonic and partly fictitious flow

In direct CFD simulation of transonic flow in physical space, the location of the sonic line, dividing the subsonic from the supersonic part of the flow, is usually unknown, while a hodograph formulation a priori prescribes the location of the sonic line.

In an effort to understand the physical meaning of elliptic continuation as an auxiliary step to solve transonic flow problems, the solution within the continuation domain can be interpreted as a „Fictitious Gas“ subsonic flow, later to be corrected by a „Real“ supersonic flow calculation. Once the parameters along the sonic line are established, it can easily continue with the method of characteristics for the supersonic flow solution.

The equations system for the planar nozzle geometry for fictitious part with application of Eq. (4.6) transforms the equations for X and Y means that they can now be solved separately:

$$U_{ss} + U_{tt} = 0 \tag{4.10}$$

$$V_{ss} + V_{tt} = 0 \tag{4.11}$$

$$X_{ss} + X_{tt} - \frac{1}{3}\left[\frac{1}{s}X_s + \frac{1}{t}X_t\right] = 0 \tag{4.12}$$

$$Y_{ss} + Y_{tt} + \frac{1}{3}\left[\frac{1}{s}Y_s + \frac{1}{t}Y_t\right] = 0 \tag{4.13}$$

This system of perturbed variables can be now solved on a simple rectangular grid describing the whole domain (elliptic + extended elliptic) in the rheograph s, t plane using finite difference methods with one of iteration schemes. Boundary conditions can be prescribed as Dirichlet conditions, for the special case of nozzle flow taken from the analytical solution. For this case, the Successive over-relaxation iteration method has been chosen.

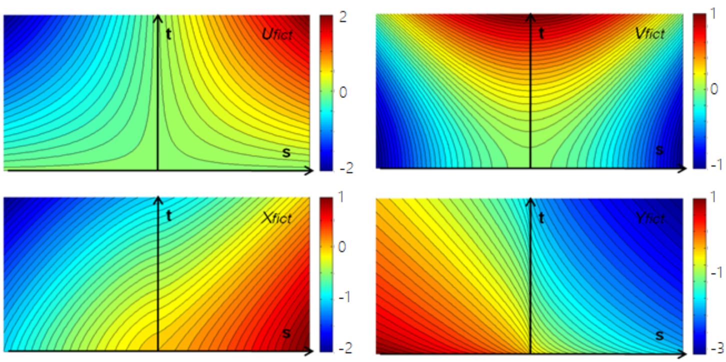

Figure 4.2: Fictitious gas results

Now, noticing that the left half of the figures ($s < 0$) represents the valid subsonic solution, the data along t-axis are the data of the sonic line. To continue into supersonic domain ($s > 0$), the method of characteristic can be used but staying in described rheograph plane. With already known solution of U and V from (Eqs. (4.6) - (4.9)), the X and Y can be solved as a initial value problem in the characteristic triangle

$$\frac{dY}{dX}_{\eta=const} = |U^{-1/3}| \tag{4.14}$$

$$\frac{dY}{dX}_{\xi=const} = -|U^{-1/3}| \tag{4.15}$$

The sketch of the investigated domain is shown in the Figure 4.3.

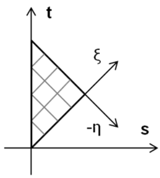

Figure 4.3: Characteristic domain in the s, t plane

The resulting domain visualized in the physical X, Y plane in the Figure 4.4 shows the local supersonic domain from the sonic line to the neutral characteristics.

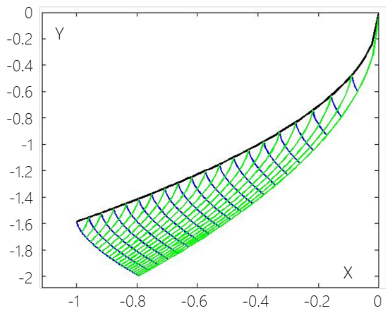

Figure 4.4: Characteristic domain in the X, Y coordinate system

4.3 Shock-free Supercritical Nozzle Shape Integration

The space downstream the limiting characteristic is now free to continue without changing the upstream solution, but the ξ characteristic itself is not sufficient for continuing the characteristic pattern and some of the flow characteristics need to be prescribed. For a simple accelerating (basic Laval) nozzle, usually the parameters along the flow X axis are being used.

For this case of symmetrical accelerating-decelerating nozzle, the symmetrical character of the rheograph solution can be used to calculate the mirrored quadratic region BCFC'. The sketch of the transformation of the middle region is shown in the figure (Fig. 4.5).

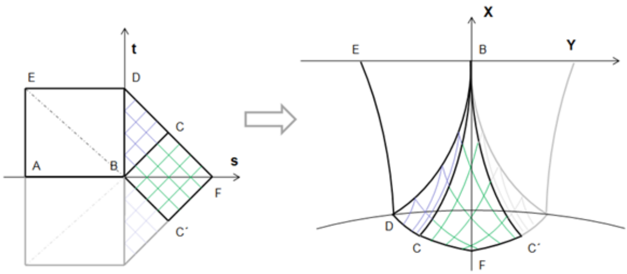

Figure 4.5: Rheograph to physical plane transformation of composing a flow using symmetry properties of the solution

Now, the attention is paid to one flow-bounding streamline starting in the upstream accelerated Laval nozzle model and ending in its mirror image representing a decelerated outlet, both parts connected by fitting in the gap pattern. With the well-known Laval contours to be parabolas, the connection within the gap is still missing.

To obtain streamlines from the computed characteristic region, the values need to be evaluated along a constant stream function. For the chosen near sonic formulation, lines $Y = const$ represent such stream function. Interpolation of $V(X)$ along $Yc = const$ (see Fig. 4.6) defines the contour angle and allow for streamline shape integration. The scaling parameters A, B in Eqs. (4.16) - (4.19) represent transonic similarity parameters and allow for obtaining scaled flow fields $u, v(x, y)$ from the parametric solution $U, V, X, Y(\xi, \eta)$ in the rheograph.

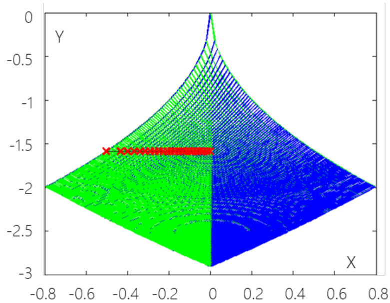

Figure 4.6: Shape integration

For the purpose of creating a new flow example, the upstream accelerated Laval flow and a mirror solution as downstream decelerated exit flow were used. The new symmetrical gap solution fits then exactly between the neutral characteristics, bridging the Laval portions of the flow and forming a complete accelerated and decelerated flow model, with chosen contour arc fitting together without curvature irregularities at the contact characteristics.

Real geometric coordinates (x, y) and flow parameters (u, v) are obtained by the following equations, representing the transonic similarity laws:

$$x = AX \tag{4.16}$$

$$y = AB^{(-l/3(1+2k))}\left(\kappa+1\right)^{-l/2}Y \tag{4.17}$$

$$\frac{u}{u_{ref}} - 1 = B^{(1+k-(1-k)l/3)}\left(1^{l}U\right)^{1-l/3} \tag{4.18}$$

$$y^{k}\frac{v}{v_{ref}} - 1 = A^{k}B^{(1+k)}\left(\kappa+1\right)^{(l(1-k)/2)}\left(1-l/3\right)V \tag{4.19}$$

The final shape of a symmetric shock free accelerating-decelerating scaled nozzle is shown in the Figure 4.7.

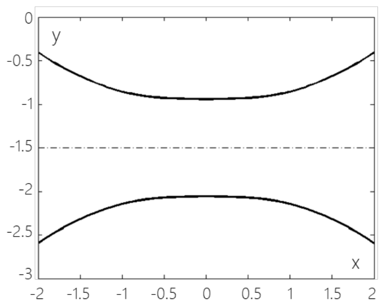

Figure 4.7: Symmetric accelerating-decelerating nozzle

Velocity distribution along a nozzle wall shows two characteristic slope discontinuities at the contact locations, which stem from the weakly singular behavior in the nozzle throat, Fig. 4.8.

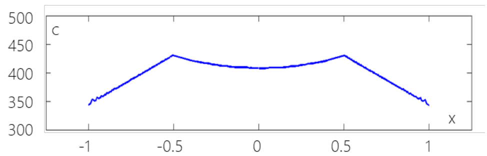

Figure 4.8: Velocity distribution along nozzle contour [m/s]

4.4 Validity and Resume

The rheograph transformation method used together with modern computational and evaluation techniques may even nowadays still be a working tool for creative aerodynamic design. The much simpler numerical solution of differential equations as showed above now makes the approach simpler, more user friendly and applicable even for engineers with lesser background in advanced mathematics. The outcome is a special novel nozzle solution of up to sonic conditions accelerating nozzle that subsequently decelerates the flow to subsonic regime without formation of the shock.

On the top of that, this particular case can now possibly serve as an academic validation or a test case for specialized compressible flow numerical codes.

5 SE 1050 Blade Cascade Analysis

As a great example to demonstrate the abilities of the above described methods, a well-known turbine blade cascade SE 1050 from ERCOFTAC database as an application challenge AC 6-12 [7] can be used.

The specific shaping of this blade cascade leads to the formation of recompression area in the supersonic section caused by rapid change in curvature followed by the discontinuity and straightening of the surface in the expansion section. The goal here, in general, is to show the possible way of situation analysis and possible solution to change the current flow field.

5.1 Case Description

The chosen profile SE 1050 was designed for the last stage of a steam turbine and it is a section of 1085 mm long rotor blade at the distance 320 mm from the root. It was designed primarily to operate in the transonic regime as the initial design is designed for exit isoentropic Mach number 1.208.

Interferogram image of the flow field for this transonic regime is visualized in the Figure 5.1. A visible area of compressed flow appears in the expansion section and forms very noticeable hump in isolines. Various CFD simulations and analysis were also carried out using different models and approaches, showing good correspondence to measured and visualized data.

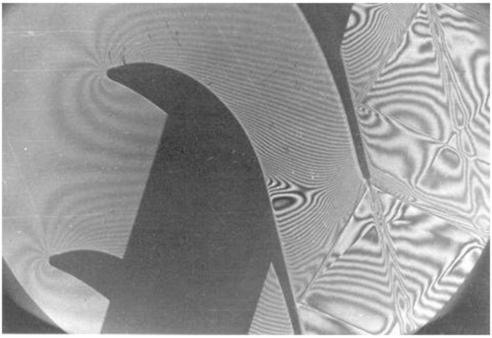

Figure 5.1: SE1050 blade cascade interferogram [7]

The original purpose of the ERCOFTAC case is to provide obtained unique data from experimental measurements [20] to help to validate CFD codes used for numerical simulations. For purposes of this thesis, the main concentration aims on the small disturbed section in the expansion region caused by unsensitive blade shaping and intense changes in the blade surface curvature.

In the next sections, the rheograph transformation method will be used to analyze the situation and to outline possible solutions in order to get rid of or minimize this issue. The actual flow field will be computed via CFD to get all the necessary data to see if there is any possibility of minor geometry modification that will improve the situation without drastically changing the geometrical and flow parameters.

5.2 Numerical Simulation

For the simulation itself, the well tested ANSYS Fluent code was used. Boundary conditions for pressure inlet and pressure outlet were calculated to correspond with the operating conditions from the experimental test.

Graphical output of the simulation is shown in the Figure 5.2 in form of Mach number isolines. The flow accelerates smoothly up to the sonic line, but right after here, the geometry causes the flow to slow down to be later again accelerated until the interaction of the surface and shock leaving the trailing edge of the neighboring blade. The wobble in flow velocity forms the kink obvious from the contour lines and shows the rough flow behavior resulting in the compression that occurred during the flow expansion.

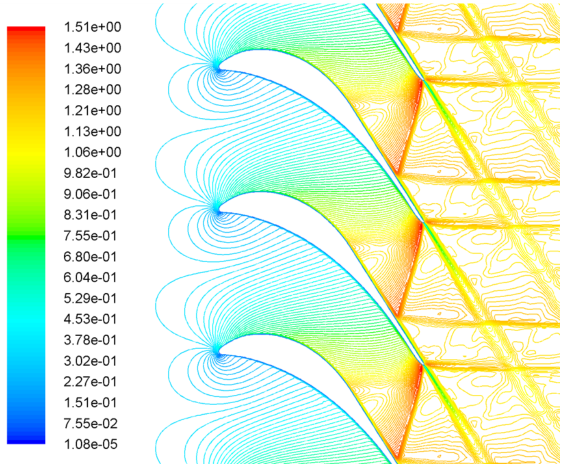

Figure 5.2: Mach number isolines

The same can be observed also in the pressure field. The insensitive shape design causes very rapid flow expansion which is not followed by further concave surface shape and leads to the formation of the compression region.

5.3 Flow Analysis and Design Modification

For detailed flow and shape analysis, a similar approach as in the previous chapter solving Laval nozzle flows can be applied, only this time the simulated flow field will be used as the initial condition for rheograph study. Flow data from extracted sonic line can be transformed and provide a perfect initial condition for building the characteristic region.

The possible continuation is described in [5] as a non-symmetrical nozzle exit design by prescribing velocity distribution along the nozzle axis. This approach once more demonstrates the difference between classical hodograph and characteristic or rheography plane, where the structure may be controlled to show a single-valued characteristic grid instead of multivalued “folded” hodograph. The sketch of the situation is depicted in the Figure 5.3.

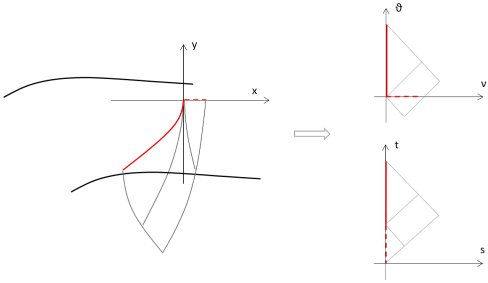

Figure 5.3: Physical plane to rheograph transformation

After transformation of this characteristic pattern back in to physical plane, the mapped region can be easily combined with obtained flow filed results. In the Figure 5.4, both, the results from CFD simulation and interferogram image, are put together in one visualization with added rheograph solution highlights. Red solid line is the extracted sonic line and dashed blue line is the calculated neutral characteristics. The yellow point then shows the change in curvature of the blade surface shape. Until this point, the surface is curved more progressively, but suddenly, the curvature goes down to zero to the trailing edge. This discontinuity leads to an infinite pressure coefficient gradient and later supersonic re-compression.

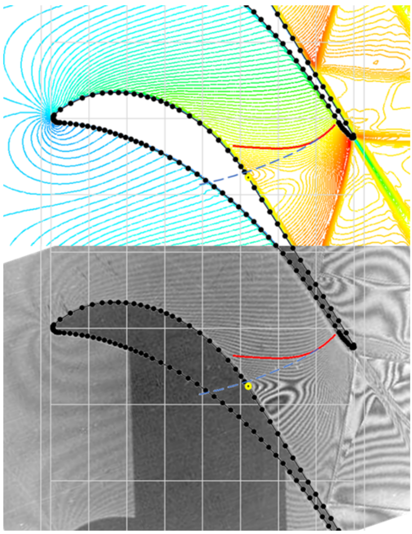

Figure 5.4: Results with sonic line and neutral characteristic position

Considering these facts, shape modification of this particular case is very challenging task. If the problematic area was further away, the method from Chapter 4 could be used to easily find appropriate streamline shape downstream from the neutral characteristic, without disturbing the sonic line and thus the upstream conditions. There is a theoretical way how to reshape the supersonic section of the blade to obtain expanding nozzle-like solution, but the real thickness of the blade and a trailing edge fixed position makes only a little room for such intensive intervention.

For relevant conditions, the shape must be changed in more sensitive manner manually and the update of the subsonic and near sonic regions is inevitable. One way to improve current state is just a minor change in area of the sonic line and surface curvature discontinuity, that in very sensitive phenomenon like transonic flow has major effect on the flow field. In general, the goal is to make the shape curvature slightly more moderate and extend it further downstream.

Mach number isolines in the Figure 5.5, in comparison with the original flow field, show some improvement, but the situation is not fully resolved. This is due to the still persistent presence of a straight surface in the expansion area.

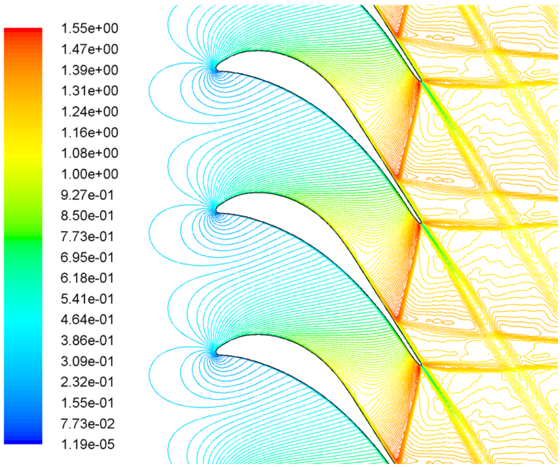

Figure 5.5: Isolines of Mach number

Another possibility is the whole blade shape modification or optimization. Using a parametric tool [21], for e.g. a PARSEC application [22], a shape very similar to the SE1050 can be quickly generated with paying extra attention to the previous discontinuity location.

Flow field around the optimized blade (Fig. 5.6) is obviously even further in terms of re-compression elimination, but the of use the complex parametric description changes the overall shape of the blade and thus the flow parameters in other sections as well.

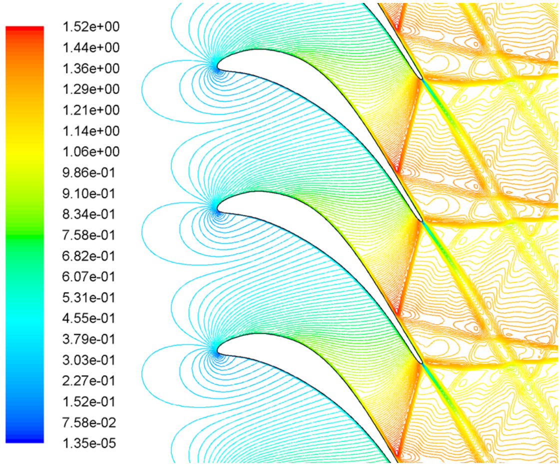

Figure 5.6: Isolines of Mach number

Figure 5.7 shows static pressure distribution on the blade surface with dashed grey line representing the curvature discontinuation point. Local modification of the blade and more sensitive shaping in terms of curvature leads to dispersal of the expansion-compression region. The fully optimized blade eliminated the rapid expansion, what disposed the strong re-compression to occur, but the shape does not avoid minor oscillation in the problematic area.

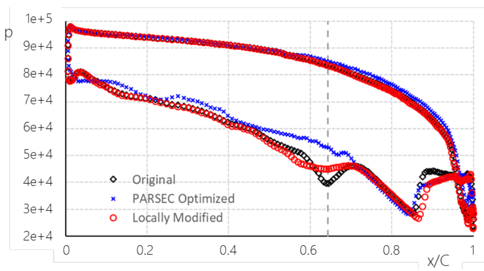

Figure 5.7: Surface static pressure distribution [Pa] - comparison

In terms of efficiency, or losses, both new shapes show some improvement over original geometry, the best values are presented by the locally modified blade shape. Individual values for static to static loss coefficient are shown in the Table 5.1.

Table 5.1: Loss coefficient

original	locally modified	optimized
0.033	0.029	0.032

5.4 Results and Evaluation

The SE1050 blade cascade case shows a practical potential of revitalized hodograph based transformation methods together with numerical solutions. The flow field calculated from numerical simulation can serve as a great input for rheograph analysis and as the initial condition for design modification. Using such approach on the case of SE1050 blade cascade confirms a specific behavior of the flow and computation of a characteristic field unveiled that the insensitive shape change and curvature discontinuity lies, specifically due to the transonic flow sensitivity, at the inappropriate location very close to the sonic line almost right at the position of the neutral characteristics. This fact means that there is no simple solution to resolve the noncontinuous expansion.

The upstream undisturbing design based on after-sonic line characteristics build up is very limited. From the practical point of view, this issue is impossible to eliminate by modification of just supersonic section of the blade. Therefore, in order to arrive with relevant solution without changing the basic cascade characteristics, the geometry modification has to encroach upstream to the subsonic region. The shape can be modified locally to secure continuous curvature change or reshaped as a whole using parametrization or optimization tool. These solutions may more or less smooth the flow field and decrease the losses, but hardly improve aerodynamic forces.

6 Summary

High speed aerodynamics and the nature of the compressible fluid flows brings specific problems and phenomena that requires special solutions, but only some of them can be directly described by simplified relations and laws of the gas dynamics. The need for complete analytical solution of the transonic flow fields requires deeper understanding and methods standing on the basis of potential flow transformed in linearized hodograph based planes were developed. Modern computational era then arrived with new solutions which can simulate the flows by numerical methods and direct discretization of basic differential equations. This also allowed very rapid rate of various cases simulations and data, but also led to the lack of some of the general knowledge of the compressible flow characteristics.

The all three approaches, the classical gas dynamic relations, rheograph transformation method and numerical modeling, were described and validated on the case of Guderley's cusp. Obtained results confirmed well the accordance between numerical and exact analytical data by means of described near sonic flow theory. And finally, detailed analysis based on classical gas dynamics proved a good accordance of shock waves parameters at the trailing edge for regular interaction of supersonic flows. Off-design condition setups added a bit more practical view on the problematic and proved that even here, it is still possible to predict and simulate the flow behavior correctly.

To show that the rheograph analysis and design methods used with modern computational techniques may even nowadays still be a working tool for creative aerodynamic design, the special case of symmetric supercritical nozzle was introduced and solved. The numerical modification of the elliptic continuation method combined with a method of characteristics have been shown to calculate the hitherto unknown mathematical inviscid flow example of a symmetric accelerating-decelerating nozzle throat. This computational extension removed one of the significant drawbacks of the analytical methods, the mathematically challenging manual solution of initial underlying transformed flow equations. The nozzle geometry has been designed as an accelerated flow Laval nozzle reaching supersonic conditions, but continuing using a novel solution and subsequently decelerates to the subsonic regime without creation of a shock. This example shows that not only wing design for aeronautics, but also internal high speed aerodynamics is a field where analytical flow models can still be useful for design and provide a deeper understanding of the underlying flow phenomena.

As a proof that rheograph solutions may be beneficial to practical problems, the simulated flow through the SE1050 blade cascade known as an ERCOFTAC test example with specific flow pattern with the re-compression area was analyzed. The flow field data, as a result of numerical simulation, can be used as an initial condition for the method of characteristics to obtain a neutral characteristics position necessary for situation analysis and following solution proposal. The rheograph analysis shows that the curvature discontinuity and subsequent shape change lies right at the position of the extremely sensitive near sonic region and thus, there is no simple upstream flow independent design solution to this issue. Nevertheless, a hint of possible ways how to improve the situation were proposed. Local curvature based shape modification and full geometry parametric optimization were suggested with positive results in terms of re-compression area elimination, in slightly improved losses but neutral to more or less negative effect on the overall aerodynamic forces.

6.1 Conclusions

To conclude the work, the main goals defined for this thesis needs to be evaluated. Various solutions of transonic flows described in all previous chapters may follow different path to the result, but in the end nicely correspond with each other. This was proved on the Guderley's cusp example, where predicted flow field and parameters correspond with numerical simulation data. The supercritical symmetric nozzle case shows that the renovation of hodograph based methods, even now just on academic example, is possible. Combination of rheograph transformation and modern computational techniques can lead to interesting design tool which is also easier to understand and use than the original analytical approach. This method is also fully applicable not only on airfoil geometries, but for internal flows as well. The application on a practical case of SE 1050 blade cascade proves the functionality on relevant real life problems. The rheograph analysis shows that the only effective design must intervene the subsonic domain upstream the sonic conditions. Previous cases are a good example of usage range and a proof, that this approach can provide real benefits for different practical and academical problems. It may not be fully redeeming, sometimes the issue just does not have a possible simple solution, but it is also important to explain and understand the origin of a unwanted behavior. All the conclusions previously mentioned may not be easy to acquire in today's fast times, but if so, the results could extend the possibilities and deliver new sights for various topics.

Throughout the whole thesis, unique characteristics of transonic flow are solved and discussed, but all the non trivial individual examples or cases and their solutions repeatedly show the importance of deeper understandings of the problematics. Without the knowledge, many obtained results, no matter if from simulation or experiment, can be interpreted incorrectly because of at first sight hidden origin of the problem, especially at the regime of high speed aerodynamics. Any scientist or engineer working in this field of study should keep this in mind.

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Author's Publications

Publications with direct connection to the thesis

Publications in Journals with Impact Factor:

[JS1] Stodůlka, J., Sobieczky, H., Šafařík, P.: *Analytical and Numerical Modifications of Transonic Nozzle Flows,* Journal of Thermal Science, Vol 27, Issue 4, pp. 382-388, 2018

Peer Reviewed Publications (in WOS or SCOPUS):

[JS2] Stodůlka, J., Šafařík, P.: *Analysis of Transonic Flow Past Cusped Airfoil Trailing Edge,* Acta Polytechnica, Prague, pp. 193-198, 2015

[JS3] Stodůlka J., Sobieczky, H.: *On Transonic Flow Models for Optimized Design and Experiment,* Experimental Fluid Mechanics 2013, EPJ Web of Conferences 67, 02111, 2014

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Others:

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Publications without direct connection to the thesis

Publications in Journals with Impact Factor:

[JS6] Klir, D., Shishlov A.V., Kokshenev, V.A. ... Stodůlka, J., et al.: *Ion Acceleration Mechanism in Mega-ampere Gas-puff Z-pinches,* New Journal Of Physics 2018, Vol. 20, no. 053064

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[JS8] Stodůlka, J., Vitkovičová, R.: *Estimation of the Drift Eliminator Efficiency Using Numerical and Experimental Methods,* Experimental Fluid Mechanics 2015, EPJ Web of Conferences, Vol. 114, 02111, 2016

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