Czech Technical University in Prague
Faculty of Electrical Engineering
Department of Control Engineering

Master's Thesis

Cyber-physical One-pipe Hydronic Heating Testbed

Author: Bc. David Zelenka
Supervisor: Ing. Jiří Dostál

Prague, January 2019

ZADÁNÍ DIPLOMOVÉ PRÁCE

I. OSOBNÍ A STUDIJNÍ ÚDAJE

Příjmení: **Zelenka** Jméno: **David** Osobní číslo: **406519**

Fakulta/ústav: **Fakulta elektrotechnická**

Zadávající katedra/ústav: **Katedra řídicí techniky**

Studijní program: **Kybernetika a robotika**

Studijní obor: **Kybernetika a robotika**

II. ÚDAJE K DIPLOMOVÉ PRÁCI

Název diplomové práce:

Kyberneticko-fyzický testbed pro jednopotrubní otopné soustavy

Název diplomové práce anglicky:

Cyber-physical one-pipe hydronic heating testbed

Pokyny pro vypracování:

1. Nastudujte problematiku základní termodynamiky, hydrodynamiky a prediktivních řídicích algoritmů. Seznamte se s předloženým simulátorem budov.
2. Navrhněte testbed umožňující přípravu vnějších podmínek pro jednu fyzicky realizovanou odbočku jednopotrubní sítě.
3. Navrhněte prediktivní regulátor pro rychlé sledování referencí ze simulovaného otopného systému.
4. Ověřte schopnosti navrženého regulátoru ve spojení se simulátorem budovy. Pokud bude dostupný fyzicky realizovaný testbed, ověřte jeho schopnosti, jinak použijte simulaci testbedu i jednopotrubní odbočky.

Seznam doporučené literatury:

[1] Borrelli F., Bemporad A., and Morari M. – Predictive control for linear and hybrid systems – Cambridge University Press 2011.
[2] Diehl, M., et al. – Real-time optimization and nonlinear model predictive control of processes governed by differential-algebraic equations. – Journal of Process Control 2002.
[3] Předota J. – Model Predictive Control for Buildings with One-pipe Hydronic Heating – Distributed Predictive Control – CTU 2018.

Jméno a pracoviště vedoucí(ho) diplomové práce:

Ing. Jiří Dostál, katedra řídicí techniky FEL

Jméno a pracoviště druhé(ho) vedoucí(ho) nebo konzultanta(ky) diplomové práce:

Datum zadání diplomové práce: **27.09.2018** Termín odevzdání diplomové práce: **08.01.2019**

Platnost zadání diplomové práce: **19.02.2020**

Ing. Jiří Dostál podpis vedoucí(ho) práce	prof. Ing. Michael Šebek, DrSc. podpis vedoucí(ho) ústavu/katedry	prof. Ing. Pavel Ripka, CSc. podpis děkana(ky)

III. PŘEVZETÍ ZADÁNÍ

Diplomant bere na vědomí, že je povinen vypracovat diplomovou práci samostatně, bez cizí pomoci, s výjimkou poskytnutých konzultací.
Seznam použité literatury, jiných pramenů a jmen konzultantů je třeba uvést v diplomové práci.

.

Datum převzetí zadání Podpis studenta

© ČVUT v Praze, Design: ČVUT v Praze, VIC

Declaration/Prohlášení

I hereby formally declare that the paper submitted is entirely my own and all resources (literature, projects, software etc.) used are cited appropriately with the corresponding bibliographical references provided.

Prohlašuji, že jsem předloženou práci vypracoval samostatně a že jsem uvedl veškeré použité informační zdroje v souladu s Metodickým pokynem o dodržování etických principů při přípravě vysokoškolských závěrečných prací.

In Prague, date / V Praze dne: .... Signature / Podpis: ....

Acknowledgements

First of all, my deepest thanks is addressed to my supervisor Ing. Jiří Dostál for all of his advices, necessary suggestions and his endless patience.

Furthermore, I am very grateful for a big encouragement from my family as well as for their tolerance. I would like to thank to my friends.

Last but not least, I appreciate support from Ondra and Tomáš who have always made a good mood in the office.

Abstract

The aim of this thesis is to propose a testbed for one-pipe hydronic heating device. The testbed will allow us to measure the pump performance, interactions between devices in a hydronic networks and acquire parameters of the heating medium. The testbed consists of a cybernetic and a physical part, which are connected via a data link.

The Cybernetic part comprises a computer simulation of a one-pipe hydronic network in a real building, whereas the physical part constitutes of the tested device, and an external conditions preparation unit. The physical part prepares the external conditions based on the simulation results, the external conditions are the temperature and flow of the heating medium. The testbed design combines two paralelly connected centrifugal pumps, a flow-through heater and a tank for cold medium storage. The usage of a 3-way valve enables water flow routing, which helps to minimise the stored medium temperature. Because of the valve, we have to use a testbed swithed model. Moreover, parallel pump connection results in algebraic constraint of the model in the hydraulic and thermal domain.

We design two controllers for the testbed. The first is a standard feedback loop, which operates in different state conditioned modes. Further, a model predictive controller is presented along with the explanation of the algebraic constraints and discontinuity application. The testbed model and the controllers are implemented in Matlab and Simulink. The testbed model functionality is presented both with the standard controller as well as with the predictive controller on different artificial situations. At last we demonstrate the controller performance on a precise building simulation of a one-pipe hydronic network.

Key words

Hydronic heating system, one-pipe heating, MPC, nonlinear DAE, centrifugal pumps, CasADi

Abstrakt

Cílem práce je navržení testovací stolice pro jednopotrubní otopné zařízení, jehož funkci potřebujeme otestovat. Stolice nám umožní měřit výkon testovaného zařízení, zkoumat jeho vliv na další zařízení v otopné soustavě a získávat parametry otopného média. Stolice se skládá z kybernetické a fyzické části, které jsou navzájem popojeny datovým přenosem.

Kybernetická část se skládá z počítačové simulace jednopotrubní otopné soustavy v reálné budově, zatímco fyzická část je složena z testovaného zařízení a jednotky na přípravu vnějších podmínek. Fyzická část má za úkol na základě informací ze simulace připravit vnější podmínky pro testované zařízení, tedy teplotu otopného média a jeho průtok. Konstrukce stolice obsahuje paralelně zapojená odstředivá čerpadla, průtokový ohřívač a nádrž na ukládání studeného média. Díky použití trojcestného ventilu a možnosti přesměrování toků média, můžeme minimalizovat teplotu média v nádrži. Tento ventil způsobuje nutnost použití přepínaného modelu stolice. Paralelní zapojení čerpadel má za následek vznik algebraických omezení v hydraulické i tepelné doméně.

Pro řízení stolice jsou navrženy dva regulátory. Prvním regulátorem je standardní zpětnovazební smyčka, která pracuje v několika módech řízení na základě měřených stavů. Poté se věnujeme návrhu prediktivního regulátoru včetně realizace algebraických rovnic a přepínaného modelu. Model zařízení je včetně řízení implementován v Matlabu a v Simulinku. Funkce modelu testovací stolice je prezentována na různých skutečných situacích, které mohou nastat. Demonstrován je základní i prediktivní regulátor. Nakonec je předvedeno řízení stolice na výstupních datech ze simulací jednopotrubní otopné soustavy reálné budovy.

Klíčová slova

Hydronické otopné soustavy, jednopotrubní vytápění, MPC, nelineární DAE, odstředivé čerpadlo, CasADi

Contents

List of Figures

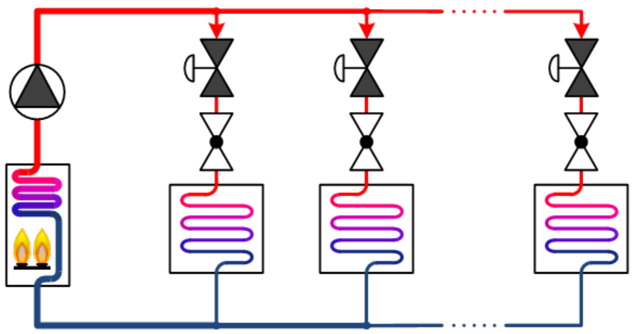
1. Introduction

1.1. State of the Art in Hydronic Heating Systems

Buildings have been heated since a long time ago, and nowadays, it is unimaginable for people to be living in unheated homes. The demand of having a warm place to stay belongs nowadays in modern world among the satisfaction of basic comfort needs of humans. The fundamental principle of heating has not changed for many decades. The principle is to heat up a heat transfer medium which is, in most cases, water, and distribute it to desirable places, e.g. rooms. Water is used for its good accessibility, and desirable properties especially its heat capacity. Heating systems have been continuously developing, but the main division of their topology stays almost the same. There exist two-pipe and one-pipe networks, both of which can either be active or passive.

Two-pipe Hydronic Network Most common type of heating water distribution today is a two-pipe system. Generally, hot water from boiler is fed to all parallel connected heating radiators, thus the input temperature is the same in each radiator. Cold water is then returned back to the boiler. Heating power of each radiator is controlled either passively or actively. The passive variant uses hydraulic valves on each radiator to change hydraulic resistance to influence the water flow. Pressure is generated by a pump in the circuit. This type of network is well established in the industry and is practitioned by projectants and plumbers nowadays. On the other hand drawbacks such as need for hydrostatic balancing and high pressure losses are undesirable. Among positives is the same temperature for all heat exchangers in the network, and a fact that it is a widely used mature technology.

The active two-pipe variant utilises multiple pumps to regulate water flow through each radiator, instead of valves. This solution provides pumping energy savings, increased power controllability and does not require hydrostatic balancing. Nevertheless, compared to the pasive type, it is at the moment still more expensive to install and it needs check valves in order to eliminate reverse flow.



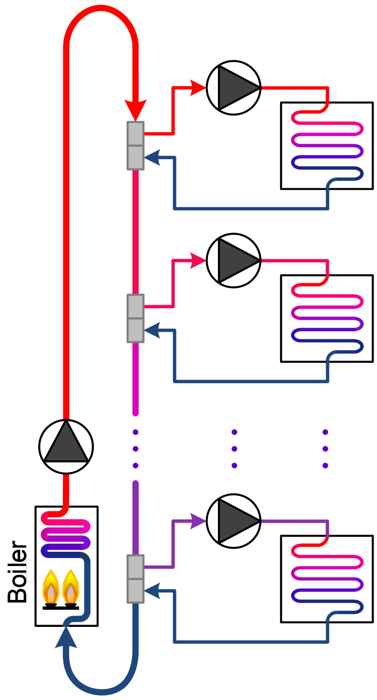
Obrázek 1.1.: Passive two-pipe heating network. [12]

One-pipe Hydronic Network Concerning hot water preparation, it is the same as in the two-pipe network. The main difference is in the distribution to the heated zones. Schematic of an active one-pipe network is depicted in Fig. 1.2. Hot water is prepared in a boiler and circulated in a primary circuit. Heated zones are connected in series in the primary circuit. Each zone has an influence on following zones by removing heat from the primary circuit. Hence the bigger the heat demand in a zone is, the colder water is available for the subsequent zones. Various alternative arrangements of passive one-pipe networks have also been developed, but they are not used anymore due to the design complexity, and a difficult thermal and hydrostatic balancing [16].

The active one-pipe variant outperforms the passive variant in numerous aspects, so the active will be described here. Heated zones are connected to a primary circuit with a double T fitting, one inlet to the secondary circuit and one outlet from the secondary back to the primary pipe. Secondary flow through a heat exchanger is induced by a local secondary pump. The pump sucks in hot primary water, which then transfers some of its heat to a heated zone, and the now cold water is returned back to the primary circuit. By controlling the secondary water flow, the desired heat transfer into the zone can be reached. Even though only heating is described in this work, the same principles apply to cooling as well.

It is desirable that the ports of the double T are close one to the other to ensure neglectable pressure difference in between them. As a consequence, the secondary circuit is hydraulically separated from the primary circuit and power is drawn from the primary pipe only by means of a secondary pump operation.

The advantages of one-pipe design are an easy instalation, less material usage, and predicted lower maitenance costs. On the other hand, active one-pipe systems require are undergoing development at the moment. Benefits of the technology are not well know yet also neither well verified.



Obrázek 1.2.: Active one-pipe heating system scheme. [12]

1.2. One-pipe Heat Exchanger Management Unit (HEMU)

This project is a part of a larger project focused on the development of heat exchanger management unit (HEMU). Multiple studies and works have already been done. An electronic pump control device was developped [14]. Techniques of water flow estimation and control [18] as well as heat exchangers' heat flow control has been uncovered [21]. On top of that, our aim is to be able to control effectively and precisely not only separate heated zones but also climatic environment of whole buildings by using intelligent predictive algorithms [11, 17, 13] based on identified building models [10]. Several additional publications have also been written as the project continues.

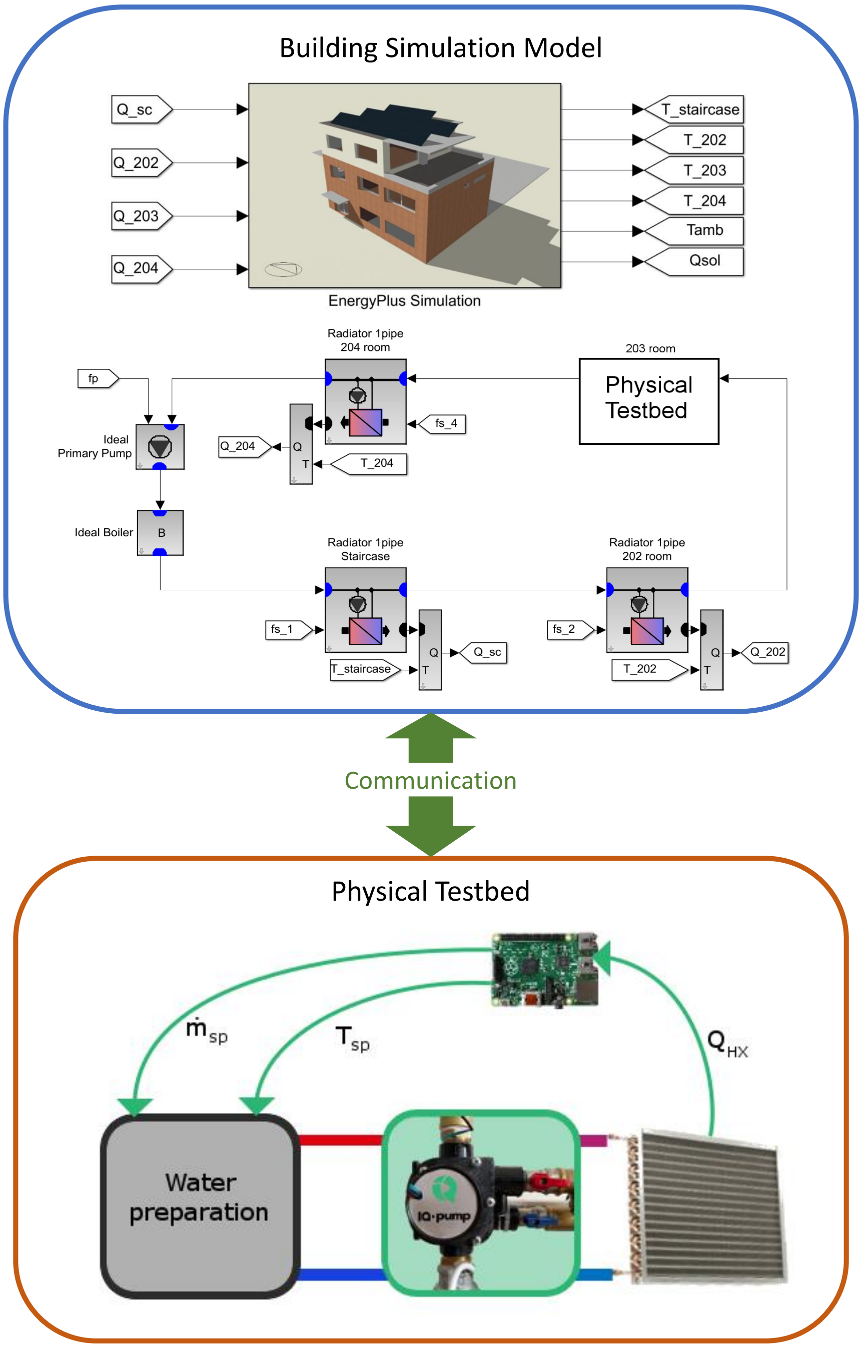
HEMU is comprised of an off-the-shelf wet rotor centrifugal pump mounted on a specifically designed housing that integrates double T piece with temperature sensor mountings. It is controlled by a circuit board that includes micro controller, motor driver, and other periferies like communication modules. The idea behind it is that all pumps in one building would communicate with each other which gives the opportunity to implement an efficient building heating system. For the sake of this work, the secondary flow of HEMU is controlled by a PI controller according to the temperature difference between the desired and zone temperatures. HEMU device was patented in year 2018 [2].

1.3. Cyber-physical Testbed

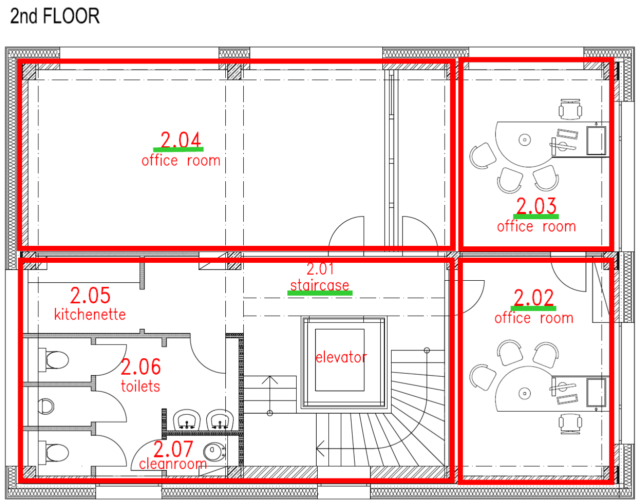
The main aim of this work is to design a testing device for HEMU to validate its performance in laboratory conditions. The testbed will allow us to measure the pump performance, interactions between primary and secondary water parameters and other valuable data such as water recirculation etc. The testbed (in Figure 1.3) can be split into two related parts: cybernetic and physical. The physical part is formed of a HEMU and a water preparation unit, whereas the cybernetic part constitutes an EnergyPlus building simulation as well as a simulation of the rest of the one-pipe hydronic network.

The cybernetic part consists of many zones, e.g. rooms or offices, in which we need to control climate parameters (temperature, humidity, air quality etc.). Floor plan of the simulated building is in Fig. 1.4. The goal is to control zone temperature. Each zone is defined by its heat capacity, heat interactions between zones, and the outside environment. More precise models also taking into account for a wall capacity can also be used. Heat is transfered into a zone via a model a of heat exchanger (HX). Heating system is modeled as a one-pipe network. Heat flows into individual zones are modulated through controlling secondary flow in HEMUs. In order to test HEMU in real world conditions heating of one simulated zone is replaced by a physical device. Simulated parameters (external conditions) of the zone are communicated to the physical part and it is the testbed job to prepare and track these external conditions accurately.

The physical part is represented by the tested HEMU including a heat exchanger and water preparation unit. The task of the water preparation unit is to transform simulated conditions into a physical domain to provide realistic conditions for the HEMU. The variables to track are in particular the primary water flow and primary water temperature. Primary water is then fed into the HEMU. This way we replicate real application of HEMU while not having to physically realize the whole one-pipe network of a building. In an ideal case, there should be no difference between the testbed primary water properties and a real application of the HEMU in terms of external conditions. Testbed measurements, most importantly the current heat flow and the return water temperature etc. will be sent back to the simulation. We plan to connect the physical HEMU to the simulation over BACnet protocol to provide the most realistic conditions. BACnet is a communication standard used in Building Automation and Control.



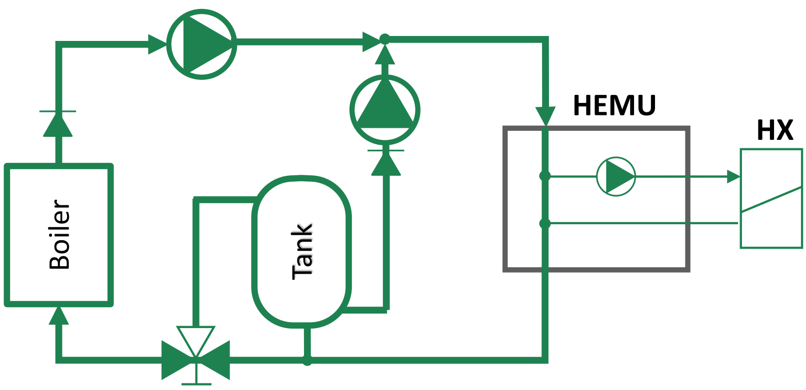
Obrázek 1.3.: Cyber-physical testbed scheme.



Obrázek 1.4.: Floor plan of the modeled building. 4 heated zones illustrated by red frames.

1.3.1. Testbed Concept

Firstly, we had to come up with the testbed layout to ensure effective and precise control, fast response and reachability of the desired external conditions. The former idea of a testbed [21] was to heat up the return water exiting HEMU to a desired temperature and then feed it back into HEMU using a pump to reach the required primary water flow. This design was not able to track fast reference changes due to the boiler dynamics and a limited heat source.



Obrázek 1.5.: Scheme of the physical part of the testbed. Water flow direction is symbolized by arrows.

To improve the design, we added a parallel pipe branch with a water tank, where cold water would be stored (in Figure 1.5). Having cold and hot water supplies allow to mix water arbitrarily in order to achieve desired temperatures and water flows. The most significant advantage is in fast reference changes. We are able to preheat the boiler prior to a reference temperature increase, and at the same time follow the reference of a lower temperature by mixing in cold water from the water tank. This also works in the opposite case if there is a reference temperature drop. In this case the boiler power is reduced and the cold water is mixed to reach the dropped reference temperature. This effect is caused by the dynamics of the boiler which is not able to drop its temperature imediately because of heat accumulation in the heating elements and in the water mass stored in the boiler.

We are using two electrical centrifugal pumps. The centrifugal pumps in general are used as a source of pressure and their main advantage is simplicity. They consist of only a minimum moving parts, thus only low maintenance is required. Among the disadvantages, there is a low suction power. As a result, the pumps need to be primed or put under water. In our case of using a closed hydraulic loop, this is not a big problem. We will focus on theory of centrifugal pumps later in this work.

Having two parallel branches with centrifugal pumps could lead into reverse water flow. This would happen if one of the pumps produced considerably lower water pressure. The water flow direction and speed depend on the differences between pump pressures and on pressure drops in the primary loop. We added a check valve to both parallel branches in order to prevent the testbed from the reverse flow.

To utilise the residual water heat most effectively, we added a 3-way valve into the network. This type of valve allows us to choose whether the boiler will be sourced directly from the HEMU or from the water tank. This way, the temperature in the tank can be kept low in order to maximize the operational range of the inlet water temperature.

1.4. Control Design Theory

To ensure the best performance, in other words the smallest error from the simulation reference, a controller has to be designed. Firstly, a condition based closed loop controller was created. Its results were satisfactory for a steady state operation, but it failed to follow fast changes of reference. This is a perfect example where Model predictive controller (MPC) is suiteble. Since a majority of buildings is controlled by a fixed schedule we can assume future references or even have predictions from the building MPC controller. That is why the testbed MPC can prepare for sudden changes ahead of time, and track the reference tighter.

1.4.1. Standard Feedback Controller

A proportional controller is used as a sufficient base case for the testbed control. Its principle is fairly simple and the design is not different from the linear system feedback [8]. It continuously calculates a difference between the desired reference r and the measured states y. We denote this difference an error vector e, where $e(t) = r(t) - y(t)$. The control input is defined as

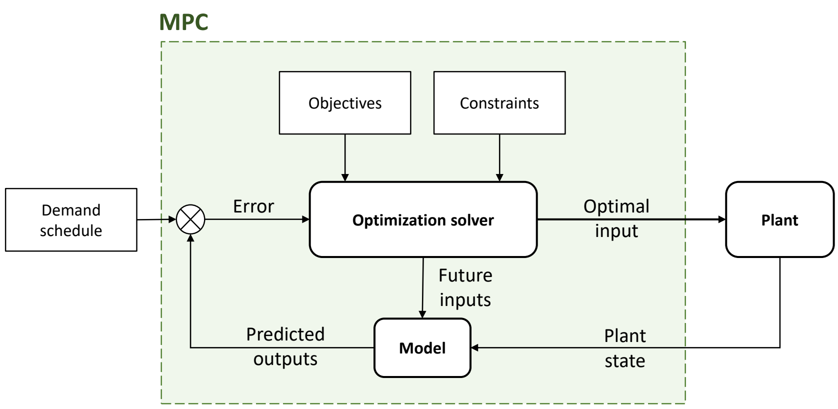
$$u(t) \quad = \quad \boldsymbol{K_p} e(t). \tag{1.1}$$

The compensating effect is tuned by selecting a gain K_P.

The main disadvantage is in not having the future prediction. Therefore so some states, like boiler temperature, might not not heat up quickly enough to fulfill the reference. Another drawback is that there is always some residual controll error which is needed to generate the output. It can be eliminated by using integral component for the controller if the error was too significant.

1.4.2. Model Predictive Controller

In the previous section we reffered to the standard feedback control. This section will focus on a more advanced technique of a Model predictive controller in a Fig. 1.6. As the name suggest, MPC uses a system model to predict the future states. This kind of constrained optimal control [9] enables to design an optimal controller to reduce the error between reference and real signals. It is important to mention that MPC is optimal in terms of chosen control criteria on a finite prediction horizon.



Obrázek 1.6.: MPC controller scheme.

Prediction horizon is a finite time period beginning at present time, in which the objective function J is minimized. The prediction horizon is being shifted continuously forwards, thus we can call MPC a receding horizon controller. The prediction horizon is devided into N sample periods of a time T_s. By objective function we define control goals that we need to be met. It is necessary to provide future estimated goals to the controller. Both inputs and the system states can be limited by constraints that represent physical or any other limits of the variables, e.g. maximum heating power, minimum and maximum pump flow etc. MPC uses measured states in current time t_0 and finds inputs u into the dynamic model to minimize objective function all along the prediction horizon. Then the input $u(t_0)$ is sent to the real system, the other future actions are stored for initialization in the next time step. This cycle is executed periodically, therefore it can be described as a closed loop control. System dynamics is embedded into a function $f\left(x, z, u, t\right)$ and the output is defined by $h\left(x, z, u, t\right)$ where x are dynamic states, z are algebraic states and t is time.

A typical objective function for reference tracking and input effort minimisation is

$$J = \sum_{k=0}^{N} \|y_{ref}(l(k)) - h(x, z, u, l(k))\|_{\boldsymbol{Q}}^2 + \|u(l(k))\|_{\boldsymbol{R}}^2 + \|u(l(k)) - u(l(k-1))\|_{\boldsymbol{P}}^2 \quad (1.2)$$

J is minimized with respect to $x(l(1))$..., $x(l(k))$ and $u(l(0))$,..., $u(l(k-1))$ subject to

$$x\left(l(k+1)\right) = f\left(x, z, u, l(k)\right) \quad (1.3)$$

$$x(t_0) = \text{current states} \quad (1.4)$$

where $l(k) = t_0 + kTs$ and which is restricted by constraints

$$x_{LB}(l(k)) \leq x(l(k)) \leq x_{UB}(l(k)) \quad (1.5)$$

$$u_{LB}(l(k)) \leq u(l(k)) \leq u_{UB}(l(k)). \quad (1.6)$$

The objective function consists of weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ which penalise deviation of particular states as well as control inputs respectively. The third term limits fast input changes, and $\boldsymbol{P}$ is the weighting matrix of this term.

The norm symbolises

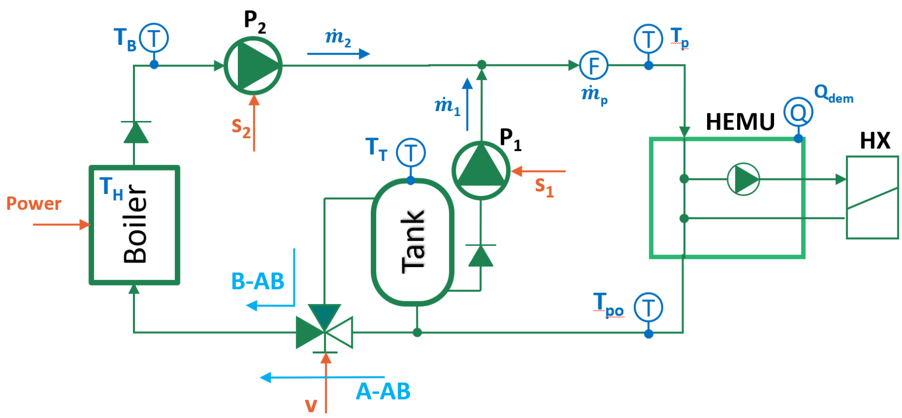
$$\|u - v\|_{\boldsymbol{M}}^2 = (u - v)^T \boldsymbol{M} (u - v)$$

for arbitrary in size matching vectors v, w and matrix $\boldsymbol{M}$.

2. Testbed Model

Nowadays, it is absolutely essential to be able to design products as fast as possible. Years ago, people used to apply so called trial and error method. When they constructed a machine which did not work properly they had to redesign it. It used to take a very long time to manufacture new parts for such machine. In order to avoid the lenghty redesign process, nowadays, we use dynamic models for development. It is much cheaper to compute behaviour of a machine using their models and to make improvements. Moreover, use of dynamic models is essential for optimal control design and necessary for MPC implementation. This work is primarily about MPC so in this chapter we will extract the testbed mathematical model.

This chapter is devided in five sections: two main sections are describing the hydraulic and the thermal flow in our testbed. These two subsystems are closely dependent in terms of primary water parameters. Following sections go more into detail of the specific subsystems. The manipulated and measured variables of the testbed are schematicaly depicted in Figure 2.1.



Obrázek 2.1.: Testbed parameters in a scheme. Orange - manipulated variables, Dark blue - measured variables.

2.1. Hydrostatic Domain

In this section, we will introduce the governing hydrostatic equations. Water dynamics is neglected since we assume that it is faster than the thermal dynamics. These steady state equations put into relation hydraulic pressure in water pipes with water flows. This approach is equivalent to electrical circuit equations and is in detail described in [15, 11]. The relation between a water pressure Δp_x [Pa] and a mass flow $\dot{m}_x$[kg/h] is

$$\Delta p_x \quad = \quad \dot{m}_x^2 R_x \tag{2.1}$$

where R_x $\left[\mathrm{Pa\,h^2kg^{-2}}\right]$ stands for total hydraulic resistance of the water line. Hydraulic resistance is an energy dissipation in water that causes water pressure to drop. It occurs due to friction between fluid and pipe walls as well as due to fast changes of the flow direction. The testbed pressure balance is described by

$$\Delta p_{pL} \quad = \quad \Delta p_1 - \Delta p_{1L} \tag{2.2}$$

$$\Delta p_{pL} \quad = \quad \Delta p_2 - \Delta p_{2L} \tag{2.3}$$

and each equation describes one parallel branche. Pressure generated by pumps is represented by Δp_i [Pa], $i = 1, 2$. Remaining terms represent pressure losses, Δp_{pL} [Pa] stands for a pressure loss in common primary pipe and Δp_{iL} [Pa] symbolizes pressure losses in parallel branches. Installing (2.1) into (2.2) and (2.3) leads to

$$R\dot{m}_p^2 \quad = \quad \Delta p_1 - R_1\dot{m}_1^2 \tag{2.4}$$

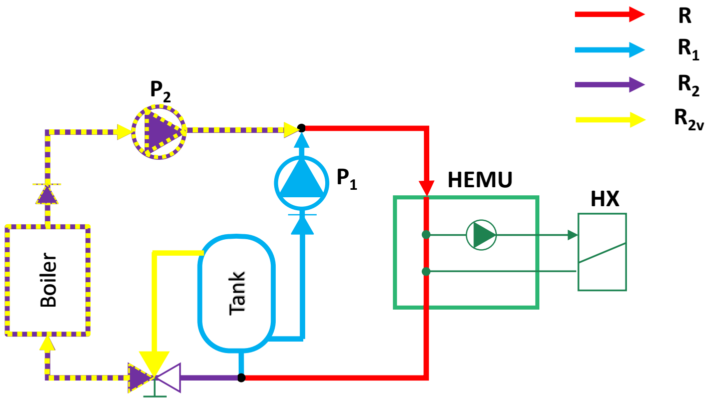
$$R\dot{m}_p^2 \quad = \quad \Delta p_2 - R_2\dot{m}_2^2. \tag{2.5}$$

R, R_1, R_2 and alternatively R_{2v} $\left[\mathrm{Pa\,h^2kg^{-2}}\right]$ stand for total hydraulic resistances of pipes in each testbed segment. Mass water flows are labeled $\dot{m}_i$ [kg/h], $i = 1, 2$ for parallel branches and primary mass flow $\dot{m}_\mathrm{p}$[kg/h] is determined by

$$\dot{m}_p \quad = \quad \dot{m}_1 + \dot{m}_2. \tag{2.6}$$

$\dot{m}_\mathrm{p}$ is the flow that enters HEMU.

The location of segments is shown illustratively in Figure 2.2. Hydraulic resistances also include the resistance of T fittings, boiler, check valves, switch valve, and inlets and outlets in the water tank.



Obrázek 2.2.: Total hydraulic resistance in segments.

2.1.1. Centrifugal Pump

To extend the equations above, we need a model of the used centrifugal pumps. This type of pump is a source of hydraulic pressure, but our main concern is the mass flow $\dot{m}_i$. Performance of this type of pumps is usually described by pump head h_i which characterises the height in which the pump can deliver a liquid. This is why we need to introduce pump head-flow characteristics described usually by a 2$^{\text{nd}}$ order polynomial

$$h_i(\dot{m}_i) = a_i\dot{m}_i^2 + b_i\dot{m}_i + c_i. \tag{2.7}$$

This equation is valid for the nominal pump speed. For our purposes we need to control the pressure in a wide operational range. This is realised by applying affine laws presented in [11] which describes the usage of the speed scaling ratio

$$s_i = \frac{n_i}{n_{i,nominal}},\ s_i \in \langle 0; 1\rangle\,. \tag{2.8}$$

The scale factor tells the ratio of pump shaft speed n_i compared to nominal velocity $n_{i,nominal}$ and by installing it into equation (2.7) we get

$$h_i(\dot{m}_i, s) = a_i\dot{m}_i^2 + b_i\dot{m}_i s_i + c_i s_i^2. \tag{2.9}$$

Terms a_i and b_i describe pump losses due to friction, leakage or recirculation. The last term c_i describes the maximum head available at zero flow. The pressure is calculated with a formula

$$\Delta p = \varrho g h \tag{2.10}$$

which we can substitute into (2.9) to obtain

$$\frac{\Delta p_i}{\varrho g} = a_i \dot{m}_i^2 + b_i \dot{m}_i s_i + c_i s_i^2. \tag{2.11}$$

ϱ[kg m^{-3}] and g[m s^{-2}] being water density and gravitational acceleration respectively. Rearranging (2.4), (2.6) and (2.11) to obtain

$$R\dot{m}_p^2 = a_1 \varrho g \dot{m}_1^2 + b_1 \varrho g \dot{m}_1 s_1 + c_1 \varrho g s_1^2 - R_1 \dot{m}_1^2 \tag{2.12}$$

$$R\dot{m}_p^2 = a_2 \varrho g \dot{m}_2^2 + b_2 \varrho g \dot{m}_2 s_2 + c_2 \varrho g s_2^2 - R_2 \dot{m}_2^2 \tag{2.13}$$

$$\dot{m}_p = \dot{m}_1 + \dot{m}_2. \tag{2.14}$$

These equations help us calculate the steady state water flows of both parallel branches based on the input scale factors s_i.

2.1.2. Solving Hydrostatic Equations

In this section, we would like to show how the affine laws work for parallely connected pumps. Mixing is realised by flow contribution of each pump. For simulations and control, we need to be able to determine both desired control signal and resultant output flow.

Calculation of Mass Flows Given pump speeds, s_i we will in our model typically need to compute the resulting system mass flows $\dot{m}_i$ and $\dot{m}_p$. These are obtained by solving the set of algebraic equations (2.12) - (2.14).

Calculation of Scale factors Analogically we compute the speed scale factors s_i. Equations (2.12) - (2.14) are rearranged into

$$0 = G_{i2} s_i^2 + G_{i1} s_i + G_{i0}, \tag{2.15}$$

from where, the

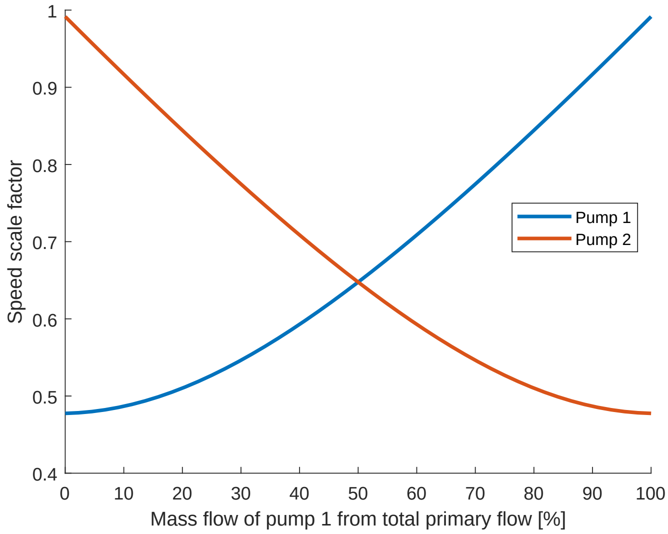
$$\begin{aligned} G_{i0} &= a_i \rho g \dot{m}_i^2 - R_i \dot{m}_i^2 - R\dot{m}_p^2 \\ G_{i1} &= b_i \rho g \dot{m}_i \\ G_{i2} &= c_i \rho g. \end{aligned}$$

are computed as

$$s_{i1,2} = \frac{-G_{i1} \pm \sqrt{G_{i1}^2 - 4G_{i0}G_{i2}}}{2G_{i2}}.$$

This expression can generate multiple solutions. We will select only real non negative numbers.

Figure 2.3 shows an example of solution for a constant primary flow. Constants used for this example are discussed in section 2.6. It can be seen that the solution exists for a resulting flow with different flow contributions of each of the pumps. This proves that we can arbitrarily mix water from both branches. Notice the scale factor for zero flow is not zero. This is the scale factor needed for preventing reverse flow by generating pressure. We have to bear this in mind for controller design, as the reverse flow is prevented by using check valves so that the concerned pump can be swithed off.



Obrázek 2.3.: Flow dependence of two parallel centrifugal pumps. Sum of flows is constant.

2.2. Thermal Domain

This section explains physical rules and dependence in the thermal domain of our testbed based on results in [11]. It contains water mixing equations and both sources of water, i.e. boiler and tank. HEMU is introduced separately in section 2.3. Heat exchange behaviour is much slower, thus differential equations are used to describe dynamic models.

2.2.1. Water Tank Model

Water tank is a reservoir for water storage which is thermally isolated from the surrounding environment. The isolation material used is usually certain kind of foam, e.g. polyurethane foam. However, for our purposes, the isolation layer will not be of a high importance, since the temperature gradient between stored cold water and outside environment will be lower than if we stored hot water.

For simplicity of the model, we assume the tank water to be ideally mixed. Nevertheless, in reality hot water will tend to stay up, and cold water will stay at the bottom of the tank. This effect would have only a minor influence on the control algorithms, but the correct location of inlet and outlet ports will help to keep the water temperature low. Hence the water entry port shoud be located in the bottom and the exit port on the top.

The inlet water flow $\dot{m}_T$ [kg/s] equals the outlet flow, the mass of stored water m_T [kg] is a constant. The coefficient α_T describes the heat exchange rate between tank water T_T [K] and outside temperature T_{out} [K]. The heat exchange coefficient is derived in [17]. If the tank was ideally isolated, α_T would be zero. c_p $\left[\mathrm{J\,kg^{-1}K^{-1}}\right]$is a specific heat capacity of water. Notice that different mass flow units are used in this section than in section 2.1.

Tank dynamics is described by equation

$$m_T c_p \frac{d}{dt} T_T = \dot{m}_T c_p (T_{Tin} - T_T) + \alpha_T (T_{out} - T_T), \tag{2.16}$$

which demonstrates the heat influence on the tank water temperature. The left term is a total heat change inside the tank. The right side of the equation express the dependence of T_T on entry water parameters and on the outside temperature. You can check that for $T_{Tin} > T_T$ the T_T rises as well as for $T_{out} > T_T$. Also, the bigger the temperature difference is, the faster the T_T changes. Similarly, the higher the T_{out}, the faster the tank water temperature rises.

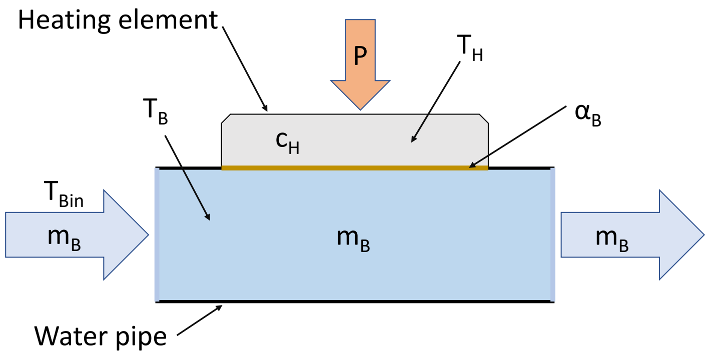
2.2.2. Boiler Model

We decided to model the boiler with a second order system. In particular, a water heater will be used (for reference see Fig. 2.4). Such a heater consists of an electric heating element that heats the water, and of a water pipe. It is able to heat water instantly as it flows through. Due to this principle, the response to input power is noticably fast in comparison to water storage boilers.

The equation

$$c_p m_B \frac{d}{dt} T_B = \alpha_B (T_H - T_B) + \dot{m}_B c_p (T_{Bin} - T_B) \tag{2.17}$$

describes heat flows in the heating element. The temperature of the heater T_H [K] is influenced by the input power P [W], and by the temperature of boiler water T_B [K], i.e. water in the pipe. C_H $\left[\mathrm{J\,K^{-1}}\right]$ is a heat capacity of the heating element which also includes heat capacity of the pipe. Heat exchange coefficient α_B defines heat transfer rate between water and heating element.



Obrázek 2.4.: Water heater scheme.

The equation

$$C_H \frac{d}{dt} T_H = P + \alpha_B (T_B - T_H) \tag{2.18}$$

represents heat exchange of water and heating element. On the left, side we have a change of energy of water mass m_B [kg] stored in the pipe. It behaves similarly to a small water storage. The right side of the equation represents influence of the heater and inlet water. Inlet water is characterised by its mass flow $\dot{m}_B$ [kg/s] and temperature T_{Bin} [K].

2.2.3. Mixing Equation

Water from the tank and the boiler is mixed which results in the primary water temperature T_p. Mixing is described by an energy balance equation

$$\dot{m}_p c_p T_p = \dot{m}_1 c_p T_T + \dot{m}_2 c_p T_B, \tag{2.19}$$

which can be simplified to

$$\dot{m}_p T_p = \dot{m}_1 T_T + \dot{m}_2 T_B. \tag{2.20}$$

The primary temperature is a product of T_T, T_B and $\dot{m}_i$:

$$T_p = \frac{\dot{m}_1 T_T + \dot{m}_2 T_B}{\dot{m}_1 + \dot{m}_2}. \tag{2.21}$$

2.3. HEMU Model

For our model, we also need to define the HEMUs' influence on the returning water which is then again heated up or stored in the tank.

The secondary water flow $\dot{m}_s$ [kg/s] has no pressure influence on the primary flow, so we will only take into consideration the hydraulic resistance of the double T fitting. The contribution is included in the primary circuit resistance R.

More important is the thermal effect since the returning water temperature is reduced when flowing through the heat exchanger (HX) which is located in the heated zone. The thermal influence of the HX will now be described.

2.3.1. Heat Exchanger

Secondary water is cooled down when there is a secondary flow, assuming $T_p > T_{HXin}$, and heated if $T_p < T_{HXin}$. T_{HXin} is the air temperature entering the HX. The temperature drop depends on the heat demand Q_{dem} [W], which is an input into the system. We assume HEMU is able to regulate the secondary flow ideally such that Qdem is removed from the secondary fluid exactly. Relation of primary water temperature and the returning water temperature (primary out) T_{po} [K] can be described by the heat balance equation

$$\dot{m}_p T_{po} \quad = \quad \dot{m}_p T_p - \frac{Q_{dem}}{c_p} \tag{2.22}$$

which we can simplify to

$$T_{po} \quad = \quad T_p - \frac{Q_{dem}}{\dot{m}_p c_p}. \tag{2.23}$$

On the real testbed Q_{dem} of the HX is controlled by changing the temperature gradient between the secondary water and the inflow of air into the HX. Heat flow can be controlled either by changing $\dot{m}_s$ or air flow.

2.3.2. Primary Water Flow Limitation

Having an electrical boiler that could handle all different simulated references during the long term testing would be too expensive and inefficient. Therefore, we will have to restrict one of the simulated variables. Since primary temperature T_p is more important to be tracked precisely, the primary flow will be allowed to be also altered. This means that by lowering the $\dot{m}_p$ the return water temperature is also lower and can be stored in the water tank.

The minimal primary water flow $\dot{m}_{pmin}$ [kg/s] has to be selected higher with respect to the maximal secondary flow $\dot{m}_{smax}$ [kg/s] to prevent recirculation of secondary water in the double T fitting. A coefficient K_{sr} puts the flows into relation

$$K_{sr}\dot{m}_{pmin} \quad = \quad \dot{m}_{smax} \tag{2.24}$$

and it will have to be chosen so that recirculation does not occur. The rule of thumb is $K_{sr} > 1$.

If we use the $\dot{m}_{pmin}$ different than the reference value of the $\dot{m}_p$ then the actual testbed return water temperature will differ from the expected T_{po}. As a result a calculated parameter T_{poSIM} [K] has to be sent back to the building simulation. It is calculated as

$$T_{poSIM} \quad = \quad T_p - \frac{Q_{demTB}}{\dot{m}_{sp} c_p}, \tag{2.25}$$

where Q_{demTB} [W] is an actual heat consumption.

2.4. DAE System Representation

For the control description of our testbed, a state space model is needed. For various physical processes a set of ordinary linear equations is sufficient. However, a more complex behaviour has to be represented by nonlinear differential equations. Most physical systems can be explained by those equations, but there is a special group of systems that have to be described by differential algebraic equations (DAE). Our testbed is modeled as a nonlinear DAE system.

Nonlinear DAE systems are generally formulated by

$$\dot{x} \quad = \quad f(x, z, u, t) \tag{2.26}$$

$$0 \quad = \quad g(x, z, u, t) \tag{2.27}$$

$$y \quad = \quad h(x, z, u, t), \tag{2.28}$$

where x is a vector of differential state variables, z is a vector of algebraic state variables, u is an input vector, and y is a vector of outputs. f, g and h are arbitrary nonlinear functions. Nonlinear differential algebraic systems require special approach for their simulation as well as the controller synthesis.

2.4.1. System Variables

Firstly the vectors of state variables is defined. The differential state variables

$$x = \left[T_{\mathrm{T}},\, T_{\mathrm{H}},\, T_{\mathrm{B}} \right]^T \tag{2.29}$$

are clear, since we have already derived differential equations of subsystems. The algebraic states are

$$z = \left[\dot{m}_1,\, \dot{m}_2 \right]^T. \tag{2.30}$$

The inputs

$$u = [Q_{\text{dem}},\, P,\, s_1,\, s_2,\, T_{\text{out}}]^T \tag{2.31}$$

are determined by the components used in the testbed. We are able to control both pumps, heater power and the heat consumption of secondary circuit. A valve control will also be added, but we will not include it in the DAE, because of its discontinous nature.

As the output variables description, we chose the ones that will be measured on the real testbed

$$y = [\dot{m}_{\text{p}},\, T_{\text{p}},\, T_{\text{po}},\, T_{\text{T}}, T_{\text{B}}]^T\,. \tag{2.32}$$

2.4.2. System Equations in a Compact Form

By merging subsystem equations from sections 2.1, 2.2 and 2.3, we obtain the differential algebraic model of the testbed.

$$\frac{d}{dt}T_T = \dot{m}_T \frac{c_p}{m_T c_p}(T_{Tin} - T_T) + \frac{\alpha_T}{m_T c_p}(T_{out} - T_T) \tag{2.33}$$

$$\frac{d}{dt}T_H = \frac{P}{C_H} + \frac{\alpha_B}{C_H}(T_B - T_H) \tag{2.34}$$

$$\frac{d}{dt}T_B = \frac{\alpha_B}{c_p m_B}(T_H - T_B) + \dot{m}_B \frac{1}{m_B}(T_{Bin} - T_B) \tag{2.35}$$

$$0 = a_1 \varrho g \dot{m}_1^2 + b_1 \varrho g \dot{m}_1 s_1 + c_1 \varrho g s_1^2 - R_1 \dot{m}_1^2 - R \dot{m}_p^2 \tag{2.36}$$

$$0 = a_2 \varrho g \dot{m}_2^2 + b_2 \varrho g \dot{m}_2 s_2 + c_2 \varrho g s_2^2 - R_2 \dot{m}_2^2 - R \dot{m}_p^2 \tag{2.37}$$

$$0 = \dot{m}_p - \dot{m}_1 - \dot{m}_2 \tag{2.38}$$

$$0 = T_p \dot{m}_p - \dot{m}_1 T_T - \dot{m}_2 T_B \tag{2.39}$$

$$0 = T_{po} \dot{m}_p \gamma - T_p \dot{m}_p \gamma + \frac{Q_{dem}}{c_p} \tag{2.40}$$

To match different mass flow units in the hydraulic and in the thermal part, a constant $\gamma = \frac{1}{3600}$ is introduced.

The equations (2.38) - (2.40) can be installed into equations (2.33) - (2.37). The equations are presented in this specific way in order to ensure clarity of the system representation.

This set of equations is not complete without the valve influenced variables which change flow or the temperature of inflow to both the tank and the boiler. We present the valve effects in the next section.

2.4.3. Valve Function

The 3-way valve is used to reroute the return water from HEMU between the tank and boiler inlets. The purpose is to control the water tank inflow to get the lowest possible stored water temperature T_T. Therefore, we need to identify variables that change by switching the valve, and introduce the necessary principles. We assume a fast valve to be used. The two settings are described in the following text and are schematically depicted in Figure 1.5. The constant γ is also added.

A-AB Valve Position

This setting will be the one used most of the time. The boiler is fed directly by the HEMU output. The tank flow is equal to $\dot{m}_1$. This situation is defined by the following assignment:

$$\begin{aligned} T_{Tin} &= T_{po} \\ \dot{m}_T &= \dot{m}_1 \gamma \\ T_{Bin} &= T_{po} \\ \dot{m}_B &= \dot{m}_2 \gamma. \end{aligned}$$

In the model this valve position is identified by a value $V = 0$.

B-AB Valve Position

This setting changes the inlet water temperature of the boiler to T_T, and accordingly the tank flow increases to $\dot{m}_p$, because $\dot{m}_2$ no longer bypasses the tank. As a result, T_T is in this case influenced more, because of the higher $\dot{m}_T$ flow. This case is defined by:

$$\begin{aligned} T_{Tin} &= T_{po} \\ \dot{m}_T &= \dot{m}_p \gamma \\ T_{Bin} &= T_T \\ \dot{m}_B &= \dot{m}_2 \gamma. \end{aligned}$$

The hydraulic resistance of the boiler branch changes to

$$R_2 = R_{2v},$$

due to the different pipe lengths and fittings. This valve position is identified by a value $V = 1$.

2.4.4. Hydraulic Inertance Model

We had to modify the system equations (2.36) - (2.37) in order to make the system easier to simulate. Implementation of pure algebraic hydraulic equations with switched parameters was not succesful. We added an inertance term describing fluid dynamics which was presented in [18]. This effect generates a pressure difference Δp_I [Pa] of the fluid in the network by changing the mass flow. It is defined by

$$\Delta p_I = I\frac{\mathrm{d}}{\mathrm{dt}}\dot{m}. \tag{2.41}$$

I stands for inertance

$$I = \frac{L}{A} \tag{2.42}$$

where A is the pipe cross section and L is the pipe length. Adding (2.41) into (2.36) and (2.37) results into new differential equations set

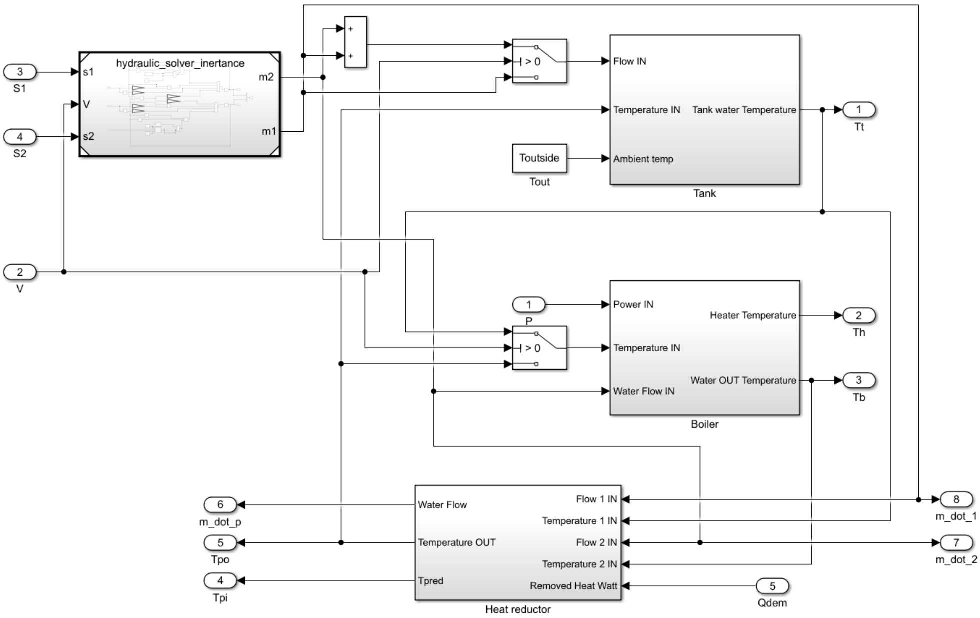
$$\frac{d}{dt}\dot{m}_1 = \frac{a_1\varrho g}{I_1}\dot{m}_1^2 + \frac{b_1\varrho g}{I_1}\dot{m}_1 s_1 + \frac{c_1\varrho g}{I_1}s_1^2 - \frac{R_1}{I_1}\dot{m}_1^2 - \frac{R}{I_1}\dot{m}_p^2 \tag{2.43}$$

$$\frac{d}{dt}\dot{m}_2 = \frac{a_2\varrho g}{I_2}\dot{m}_2^2 + \frac{b_2\varrho g}{I_2}\dot{m}_2 s_2 + \frac{c_2\varrho g}{I_2}s_2^2 - \frac{R_2}{I_2}\dot{m}_2^2 - \frac{R}{I_2}\dot{m}_p^2. \tag{2.44}$$

These equations are used for the testbed model only. For MPC implemetation, algebraic equations are preserved. Inertance values are set to get fast dynamics, as in reality inertance will expectably have only a minor effect.

2.5. Model Implementation

Matlab Simulink is used for the implementation of the model. The model is split into several subsystems as shown in Figure 2.5. Tank and boiler equations are connected as subsystems, so is the secondary circuit model. The hydraulic part is implemented in a dedicated referenced model. Based on the inputs, the resultant hydraulic flows are distributed among the tank and boiler. Influence of the 3-way valve is executed via switches routing the temperature and the water flow signals according to the valve control signal.



Obrázek 2.5.: Testbed model Simulink scheme.

2.6. Real Testbed Components and Parameters

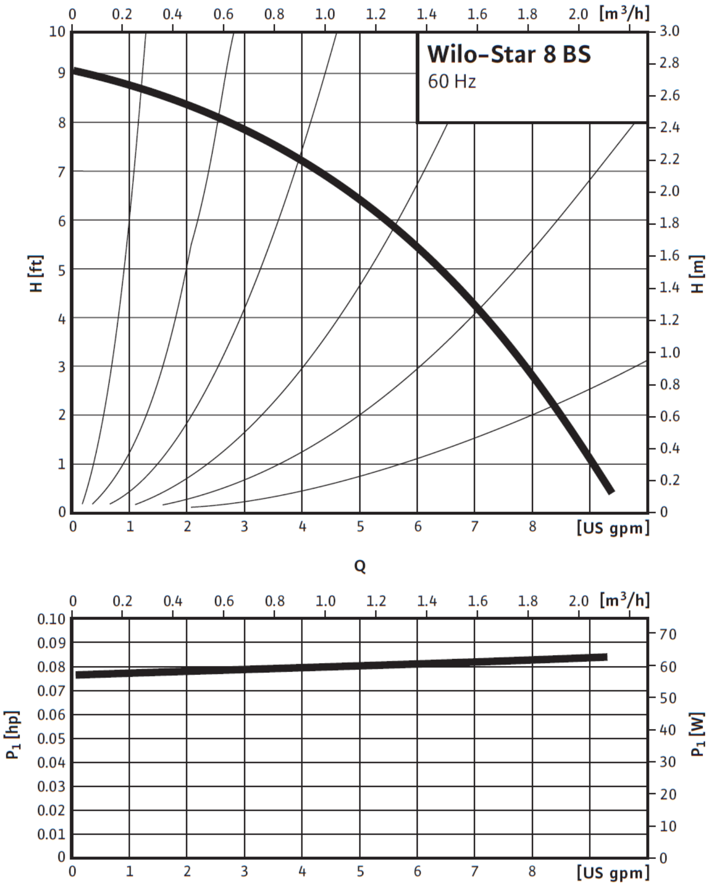
In this section we will introduce particular components to be used in the real testbed. This will provide more accurate results of simulations, under which their performance and suitability will be verified.

2.6.1. Pumps

For our testbed, water pumps have to be chosen. For practical reasons, both pumps are of the same type. A device manufactured by a worldwide leader in the pump industry, Wilo, comes into consideration, particularly Wilo Star 8 BS [6]. It is a maintenance free centrifugal pump with bronze body used for domestic hot water applications. It has a maximum delivery head of 2.7 m, a maximum volume flow rate of 2.7 m^3/h and a maximum working pressure of 140 psi. Head - flow diagram of this pump is presented in Figure 2.6. The working temperature range is sufficient, from -10 °C to 110 °C. The polynomial parameters are $a_i = -5.3875 \cdot 10^{-7}$, $b_i = -5.7957 \cdot 10^{-5}$, $c_i = 2.7368$, and were fitted to the datasheet diagram.

2.6.2. Water Tank

We decided to use a water tank of the capacity of 100 liters, i.e. $m_T = 100$ kg. The heat transfer between the water in the tank and ambient environment was estimated from parameters



Obrázek 2.6.: Wilo Star 8 BS head - flow diagram.[6]

available from various manufacturers. The tank used has a heat radiation of 60 W when $T_T = 60°\text{C}$ and $T_{\text{out}} = 22°\text{C}$. This means that it cools to $T_T = 55°\text{C}$ in approximately 10 hours. We identified a coefficient $\alpha_T = 3$ which corresponds to the experiment.

2.6.3. Boiler

First, we specified the maximum heating power to a reasonable value of $P_{max} = 5000$ W. For example, a flow-through heater Drazice PTO 1733 meets this main requirement. It is usually used for domestic hot water applications. Than, we defined parameters which from its material properties. The water mass in boiler pipes was estimated to $m_{WB} = 2$ kg. Then, a heater heat capacity was estimated to a capacity of 1 kg of steel which is $C_B = 450$ J/K, since such heater weights in total around 1.3 kg of a dry weight. The heat transfer parameter $\alpha_B = 10^4$ was identified in the same way as in case of the water tank.

Obrázek 2.7.: Wilo Star series centrifugal pump.[6]

Obrázek 2.8.: Flow-through water heater Drazice PTO 1733 rated to 5 kW of power.[4]

2.6.4. Valve

Our testbed design includes a 3-way valve with a very fast transition time. The faster valve, the better heat economy, because we can react faster to T_{po} changes, and store cold water in the tank. Ideally, a solenoid valve would be used, but because of its high price and poor availabillity, a slower valve will be used in the real application. For example, a 1" fitting Regulus VZP 325-230-1P [3] presented in Figure 2.9. It is controlled by the input voltage. Without power it is normally opened B-AB. The transition time is 6 and 10 seconds to reach B-AB and A-AB respectively. Hydraulic resistance of such valve is $R_{valve} = 4.73 \cdot 10^{-3} \mathrm{Pa\,h^2kg^{-2}}$.

Obrázek 2.9.: 3-way valve Regulus VZP 325-230-1P.[3]

2.6.5. Check Valve

This component is very simple, but helps us to make the testbed more efficient. For simulation purposes, we chose a fitting FIV.08030 (Figure 2.10) which is available in a 1” variant with a hydraulic resistance of $R_{check} = 4.94 \cdot 10^{-3} \mathrm{Pa\,h^2kg^{-2}}$ [1]. It is made of brass, and it should withstand PN20 (Pressure nominal) and a maximum temperature of 80°C which is eqiuvalent to the domestic hot water network. This value is suficient for our purposes.

Obrázek 2.10.: Check valve FIV.08030.[1]

2.6.6. Pipes

We will be using a flexible PEX (cross-linked polyethylene) pipes which are currently replacing traditional metal pipes. The two parameters concerning pipes for testbed model are specific hydraulic resistance of the pipes and their length. A specific resistence for a 1” PEX pipe is for the expected testbed water flows of approximately $1.78 \cdot 10^{-4} \mathrm{Pa\,h^2kg^{-2}m^{-1}}$ [5].

For the simulation purposes, we will set the parameters followingly: $R = 2 \cdot 10^{-4}\mathrm{Pa\,h^2kg^{-2}}$, $R_1 = 5,5 \cdot 10^{-3}\mathrm{Pa\,h^2kg^{-2}}$, $R_2 = 10 \cdot 10^{-3}\mathrm{Pa\,h^2kg^{-2}}$ and $R_{2v} = 10,5 \cdot 10^{-3}\mathrm{Pa\,h^2kg^{-2}}$. These values are the sums of resistance of the components in every segment.

3. Controllers Implementation

The main aim of the control is a precise tracking of reference values for T_p and $\dot{m}_p$. Reference parameters for the primary water are T_{ref} for temperature and $\dot{m}_{ref}$ [kg/h] for mass flow. In an ideal case, we would have an unlimited source of hot and cold water, so that $T_B > T_{ref}$ and $T_{ref} > T_T$ would always apply. Under these conditions the water parameters would be reached by changing $\dot{m}_{1,2}$. However, our closed water circuit solution requires a different attitude. We assume both pumps to be able to reach the maximum $\dot{m}_{ref}$ separately. First, a simple closed loop controller is presented, followed by an MPC design.

3.1. Standard Controller

The design of the first controller was an essential step prior to designing a more complicated MPC. It also helped to validate the testbed model. Due to the fact that the ideal conditions $T_B > T_{sp}$ and $T_{sp} > T_T$ cannot be fulfilled at all times, we had to come up with a condition-based controller. It uses different modes of control to achieve the best possible results based on actual conditions. By splitting the controller into modes, we have the ability to keep T_T as low as possible. Cold water will be then very useful in case we need T_p to drop. This is reached by using only minimal $\dot{m}_T$ when $T_{po} > T_T$. The controller modes will now be described. They are devided according to temperatures T_B, T_T and T_{ref}.

3.1.1. Mode 1

In this case, $T_B > T_{ref}$ and $T_{ref} > T_T$. Required flows $\dot{m}_{1sp}$ [kg/h] and $\dot{m}_{2sp}$[kg/h] are computed by solving the seto of equations (2.38), (2.39). The analytical solution is

$$\begin{aligned}\dot{m}_{1sp} &= \dot{m}_{ref}\frac{T_p - T_T}{T_B - T_T} &(3.1)\\ \dot{m}_{1sp} &= \dot{m}_{ref}\frac{T_p - T_B}{T_T - T_B}. &(3.2)\end{aligned}$$

The calculated flows should guarantee that the reference parameters are met. The pump inputs $s_{1,2}$ are then obtained from the solution of equations (2.36), (2.37).

Boiler power is controlled in a closed loop where T_B and T_{ref} are compared. The control law is

$$P = K_p(T_{ref} - T_B). \tag{3.3}$$

The power is limited by a maximum power P_{max}. This technique brings the boiler water temperature close to the temperature reference, and so $\dot{m}_T$ is minimised.

3.1.2. Mode 2

The second mode is used in case $T_B < T_{ref}$ and $T_{ref} > T_T$. This condition requires the highest possible increase in temperature, thus $\dot{m}_1$ is set to zero, and the boiler flow is set to $\dot{m}_{2sp} = \dot{m}_{ref}$. The s_2 scale factor is calculated from (2.37). The boiler power is set same as in the previous mode by applying the control law (3.3).

3.1.3. Mode 3

The last mode handles the situation when $T_{ref} < T_T$. The reference temperature cannot be reached immediately, and there is need for cold water preparation. Cold water is prepared by circulating primary water through the water tank until T_{ref} is achieved. The temperature of the circulated water is reduced in the heat exchanger. The water flows are set to $\dot{m}_1 = \dot{m}_{ref}$,$\dot{m}_2 = 0$ and the scale factor s_1 is calculated.

3.1.4. Valve Logic

The valve setting depends on T_{po} and T_T. The valve is set to A-AB position if $T_{po} > T_T$ and the return water is fed to the boiler. Flow into the tank is present only if the pump 1 is operating. This ensures that we do not heat up tank water too much if T_{po} is high.

The second valve position B-AB is set in case where $T_{po} < T_T$. In this position the T_T is being reduced thanks to lower inflow water temperature compared to A-AB.

3.2. MPC

In this part, we will describe the application of MPC to our testbed problem. The main aim is to significanty reduce the tracking error. This is done by predicting future states of the testbed, while having a future plan of references T_p, $\dot{m}_p$ and Q_{dem}.

3.2.1. Problem Construction

First of all, the control problem needs to be assembled. System equations, objective function, and constraints are prepared for an NLP solver. In this way, we give the solver information about the system and what are the required control goals.

Objectives

Setting the objective function correctly is essential for MPC. Different objectives can have a similar impact on the result. The final objective function is a sum of all partial objectives

$$J = \sum_i \lambda_i J_i$$

Each partial objective is weighted with a coefficient λ_i.

Reference Tracking Desired parameters are reached by minimising the error between desired and measured values.

$$J_1 = \|y(k) - y_{ref}(k)\|^2$$

Tank temperature minimisation Minimising tank water temperature helps us preserve cold water for a future use. It is done by the means of minimising the pump 1 flow. This helps to reduce demand for cold water.

$$J_2 = \dot{m}_1(k)^2$$

Minimal Heating Usage We use this objective for the case if we would like to limit the power used for heating.

$$J_3 = P(k)^2$$

Input Signal Smoothness This objective ensures that the difference between steps is minimised.

$$J_4 = \|u(k) - u(k-1)\|^2$$

Constraints

We use two types of constraints. They need to be introduced during problem construction. However, these constraints are updated only during MPC run. Without having to construct the problem every iteration, it is computationally more efficient. Constraints are devided into equality constraints and variable constraints. The first type of constraints are equality contraints and they include plant design and its dynamics, whereas the second type enables us to limit variables such as maximum heating power.

System Dynamics The most important constraint is the testbed model description. It ensures that the state at the end of each interval needs to be equal to the beginning of the next one. The differential equations are constrained by an integrator, whereas the algebraic equations are captured by ordinary equation constrains. The testbed differential equations are nonlinear, hence we need a suitable integrator - Runge-Kutta 4 for example. The system dynamics constraint is described by

$$x(k+1) = F\left(x(k),\, u(k)\right).$$

Runge-Kutta 4 (RK4) [19] is an explicit method for differential equation integration. Having initial conditions, the future values are computed. Function

$$\frac{\mathrm{d}}{\mathrm{dt}} x(t) \quad = \quad f(x(t),\, u)$$

yields approximations of differential the equations where h is an integration time step.

$$\begin{aligned}
k_1 &= f\left(x(k),\, \bar{u}(k)\right) \\
k_2 &= f\left(x(k) + \frac{h}{2}k_1,\, \bar{u}(k)\right) \\
k_3 &= f\left(x(k) + \frac{h}{2}k_2,\, \bar{u}(k)\right) \\
k_4 &= f\left(x(k) + hk_3,\, \bar{u}(k)\right)
\end{aligned}$$

These partial steps are summed up to obtain a value of the next integration step

$$x(x+1) = x(k) + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) = F\left(x(k),\, \bar{u}(k)\right).$$

DAE Equation Constraints In order to capture the algebraic equations, we implemented DAE as equality constraints. They include thermal and hydraulic equations (2.38) and (2.39).

Initial State At the beginning of each optimisation, system states are set to the measured or estimated values

$$x(0) = x_{estimated},$$

which act as the initial conditions for model differential equations.

States' Box Constraints Both x and z states are limited by these constraints:

$$\begin{aligned} 5 \leq &\quad T_H &\leq 500 \\ 5 \leq &\quad T_B &\leq 80 \\ 5 \leq &\quad T_T &\leq 80 \\ 0 \leq &\quad \dot{m}_1(k) &\leq \dot{m}_{1\,max}(k) \\ 0 \leq &\quad \dot{m}_2(k) &\leq \dot{m}_{2\,max}(k) \end{aligned}$$

Boiler Power Input to the boiler is limited by the maximum available power P_{max}.

$$0 < P < P_{max}$$

Primary Return Temperature The first value is the measured temperature.

$$T_{po}(0) = T_{poMeasured}$$

For the purpose of faster optimisation, the T_{po} values are estimated based on the reference values

$$T_{poref}(k) = T_{ref}(k) - \frac{Q_{dem}(k)}{\dot{m}_{ref}(k) \cdot c_w}, \tag{3.4}$$

$$T_{po}(k) = T_{poref}(k).$$

Due to the occurance of $\dot{m}_{ref}$ in the denominator, we set

$$T_{ref}(k) = T_{po}(k)$$

if $\dot{m}_{ref} = 0$. In experiments, we discovered that difference of T_{po} from real value does not have an extensive influence on the model prediction.

Valve Influenced Variables The variables $\dot{m}_{Tin}$ and T_{Bin} are influenced by the valve position. Therefore, constraints need to be adapted according to the estimated valve position. The estimated valve position is determined by comparing $T_{poref}(k)$ and $T_T(0)$. We assume that T_T does not change during the prediction horizon.

At time k when valve is A-AB for $T_{poref}(k) > T_T(0)$:

$$\begin{aligned} \dot{m}_{Tin}(k) &= \dot{m}_1(k) \\ T_{Bin}(k) &= T_{po}(k). \end{aligned}$$

At time k samples when valve is B-AB for $T_{poref}(k) < T_T(0)$:

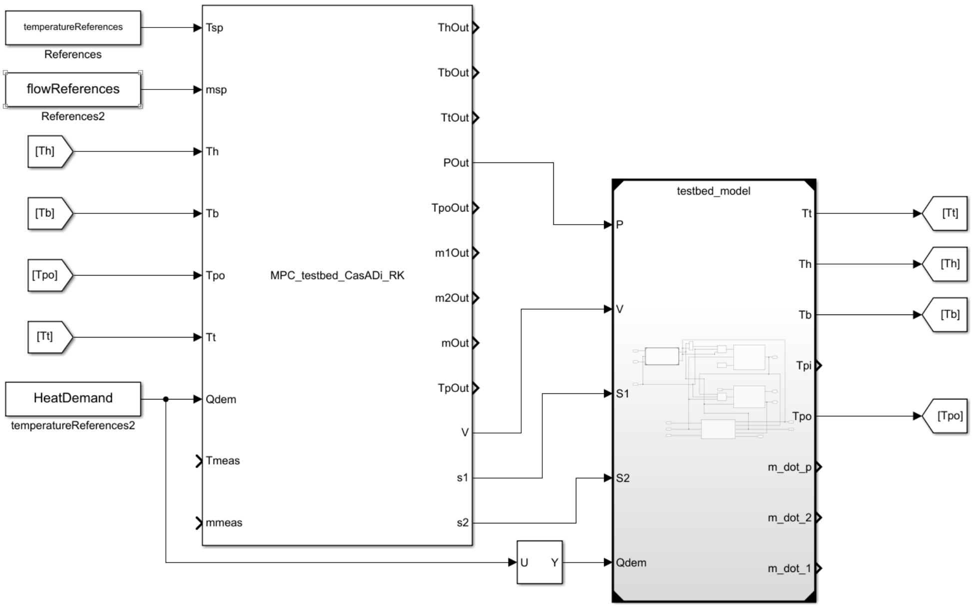
$$\begin{aligned} \dot{m}_{Tin}(k) &= \dot{m}_p(k) \\ T_{Bin}(k) &= T_T(k). \end{aligned}$$

3.3. Matlab Implementation

The basic feedback controller was implemented into the Matlab function block which provides enough options for such a controller. The Matlab function is written in a Matlab code, and is called once in a simulation step.

For implementation of our MPC a CasADi [7] framework was chosen. It is an open-source tool for nonlinear optimisation and algorithmic differentiation. It is perfectly suitable for solving optimal control problems including MPC and is widely used through all different fields of application, both academic and industrial. CasADi includes support for Matlab, and is relatively easy to use. This facts makes learning using the tool much faster. Our particular MPC is implemented as a Matlab System object which can be easily used in a Simulink scheme. Optimal results are computed by an IPOPT [20] nonlinear solver. Furthermore, the Matlab system sample time shall be set to the selected Ts and initialise all system variables.

In the Matlab system object, the MPC is firstly initialised by setting various parameters and assembling the control problem. Then, the MPC runs with the simulation and at each step the constraints are updated and the solver is called. The block update speed has to be explicitly set to the MPC time sample interval to prevent calling the solver at each integration step. The Simulink MPC scheme is presented in Figure 3.1.



Obrázek 3.1.: Testbed Simulink scheme with the Matlab system MPC.

4. Simulation Experiments

This part deals with the application of the theoretical results in this work. Simulation results are demonstrated firstly on representative scenarios, and then on simulated data of a one-pipe network.

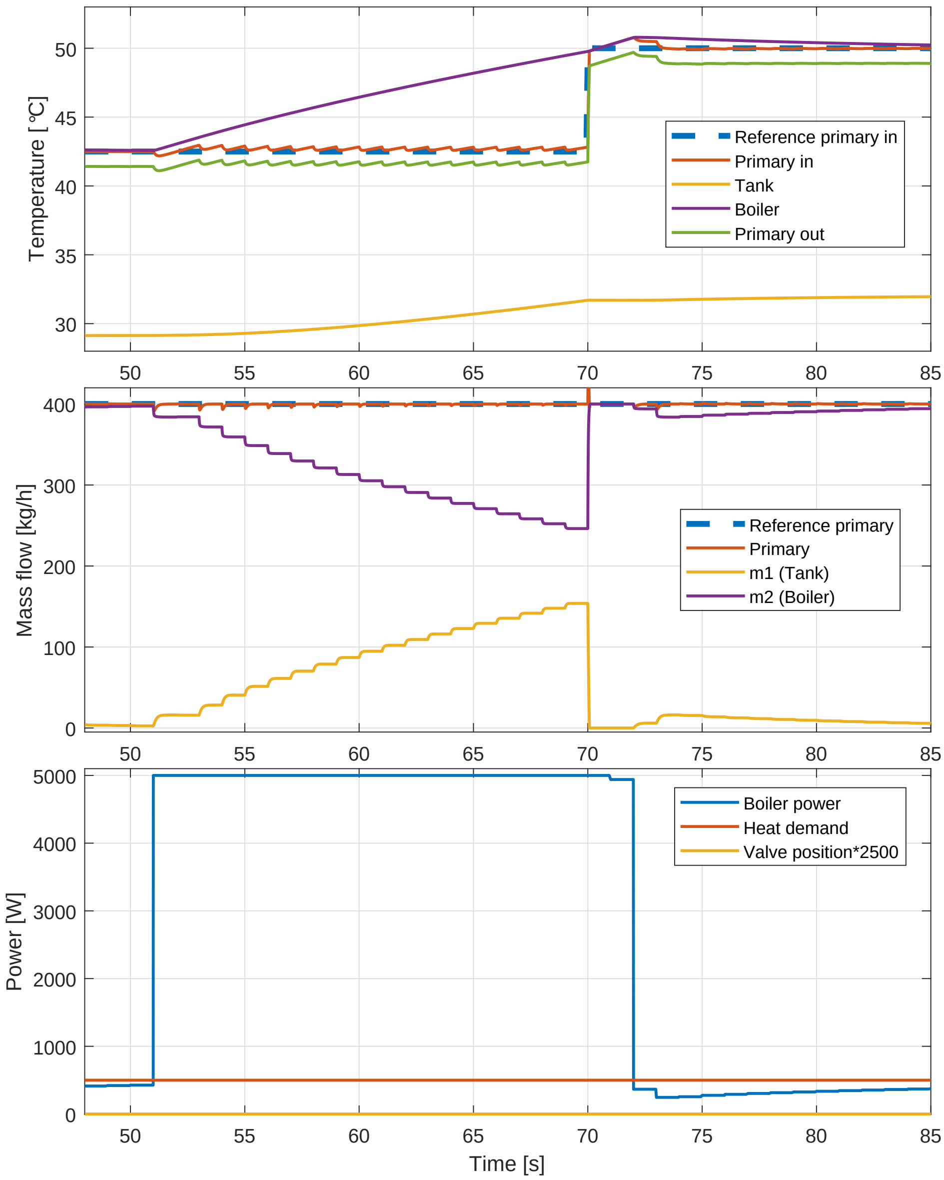
4.1. Exemplary Scenarios

In this section, control results are demonstrated. We include normal operation situations, such as reference changes as well as some special situations including e.g. the valve function. Both controllers are presented.

We present several scenarios which might occur on the real testbed. MPC sample time is set to $T_\mathrm{s} = 1\mathrm{s}$, the prediction horizon to $N = 20$. The controller then captures 20 seconds of reference signal in advance. For the sake of more evident changes of T_T in the experiments, the boiler capacity was reduced to $m_T = 2\,\mathrm{kg}$.

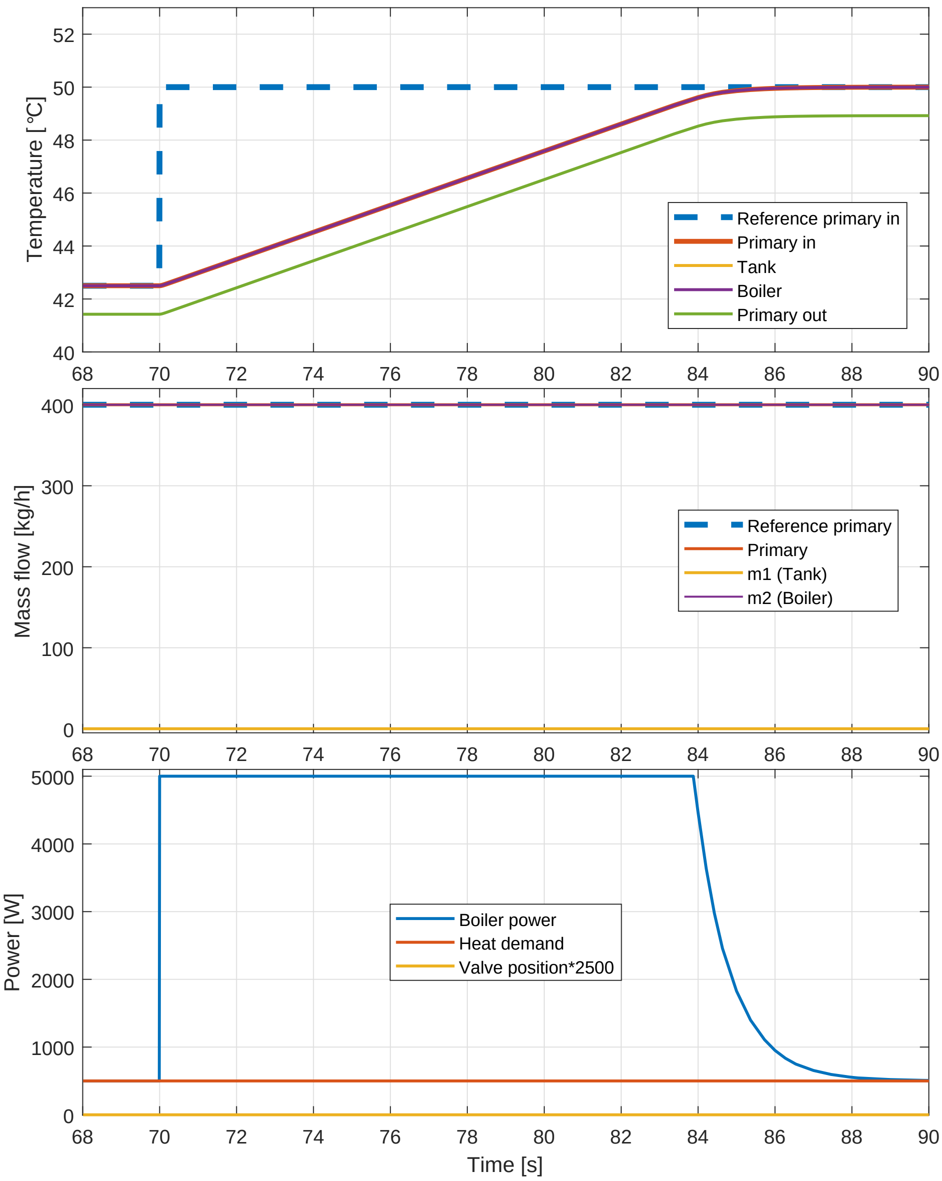
4.1.1. Primary Temperature Reference Increase

Along with the temperature drop, this is the most common circumstance the testbed will occur in. Also, the difference between the MPC in Figure 4.1 and the basic controller in Figure 4.2 is the most significant for this type of the reference change. The primary flow will be constant at 400 kg/h. The MPC starts to heat the boiler at the time 51 s, using maximum power, and at the same time it starts to mix in cold water to sustain the primary reference temperature. At the time 70 s, the reference temperature changes from 43°C to 50°C, and cold water delivery is stopped. The reference signal is tracked perfectely on time. A small temperature overshoot occurs, but then, T_B is reduced to track the reference in order to minimise the boiler flow. Higher temperature reference changes would require either less primary flow, more powerful boiler or longer prediction horizon. Notice that the tank water temperature slightly rises as $\dot{m}_1$ increases.



Obrázek 4.1.: Simulated result of MPC for primary temperature increase.

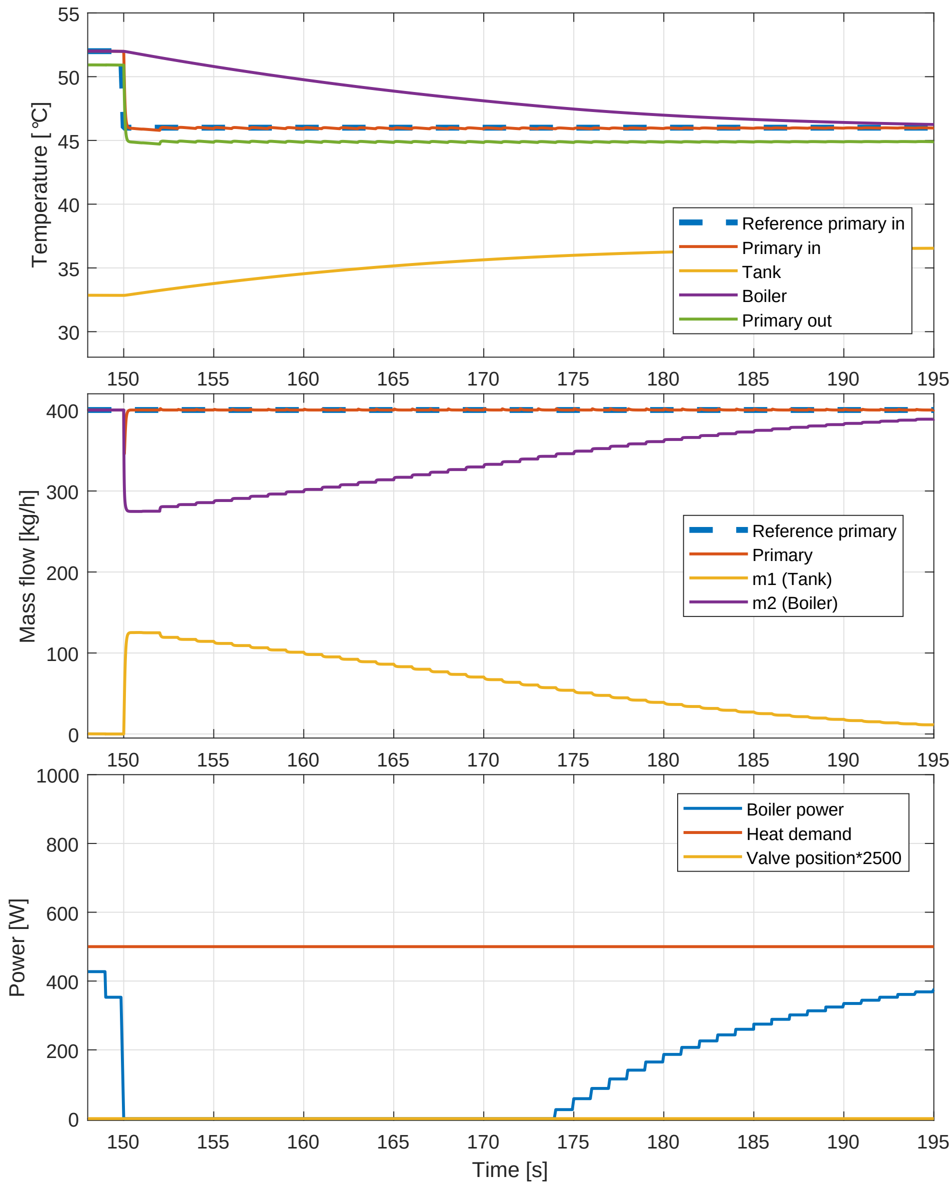
In the case of a standard controller, the same reference signals are used (Figure 4.2). The boiler starts to heat the water at the time of the reference change, and it takes 15 s until the new reference is reached. This situation clearly demonstrates the advantage of MPC operation. The controller operates in MODE 2 which uses boiler pump only, while there is no need for any cold water mixing.



Obrázek 4.2.: Simulated result of standard controller for primary temperature increase.

4.1.2. Primary Temperature Reference Drop

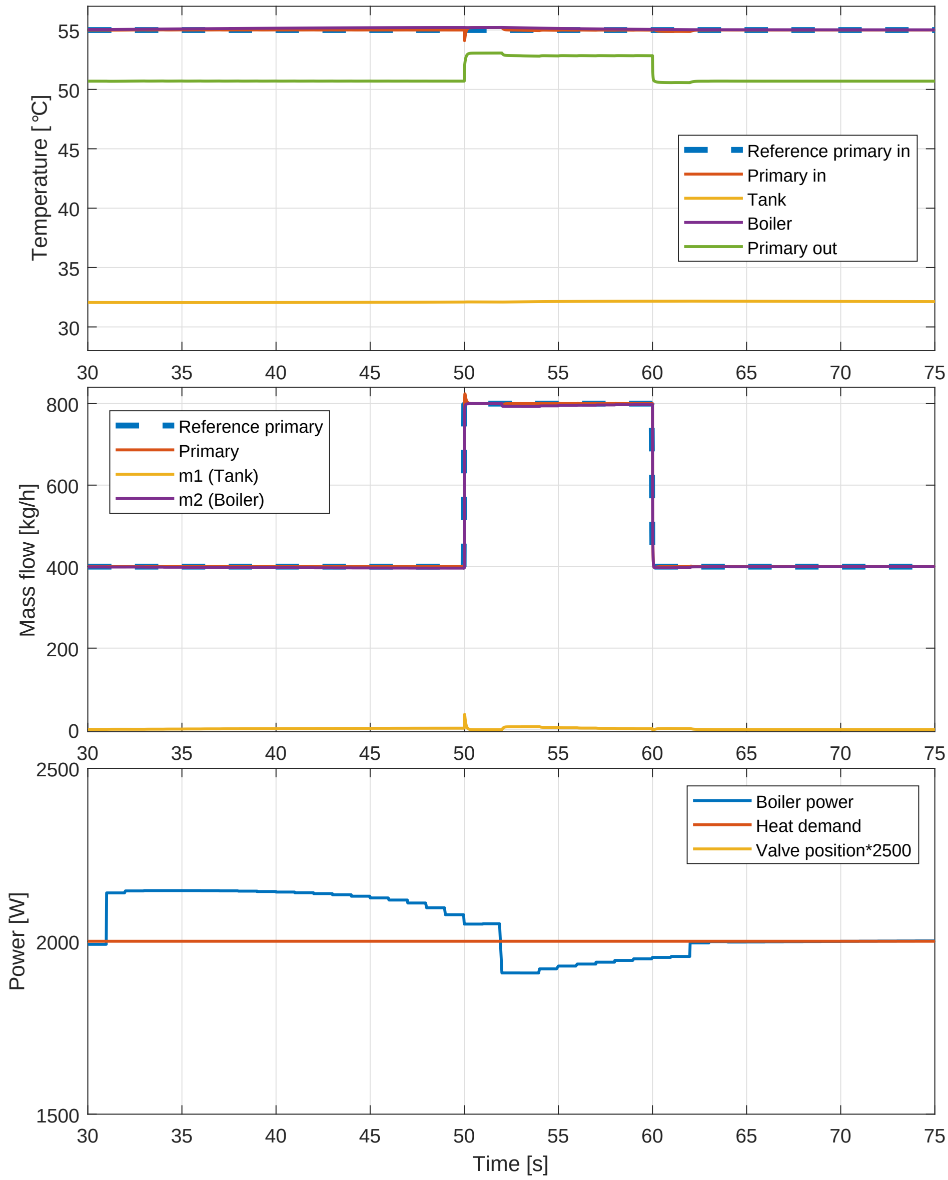
A drop of the reference primary temperature works in the same way for the MPC in Figure 4.3 and for the standard controller. At the time of the drop, the boiler power is reduced, while the cold water mixes in. The actual primary temperature is cooled, and reference temperature is reached immediately. As the boiler cools down, $\dot{m}_1$ is reduced and the boiler power is resumed.



Obrázek 4.3.: Simulated result of MPC for primary temperature reference drop.

4.1.3. Primary Reference Flow Changes

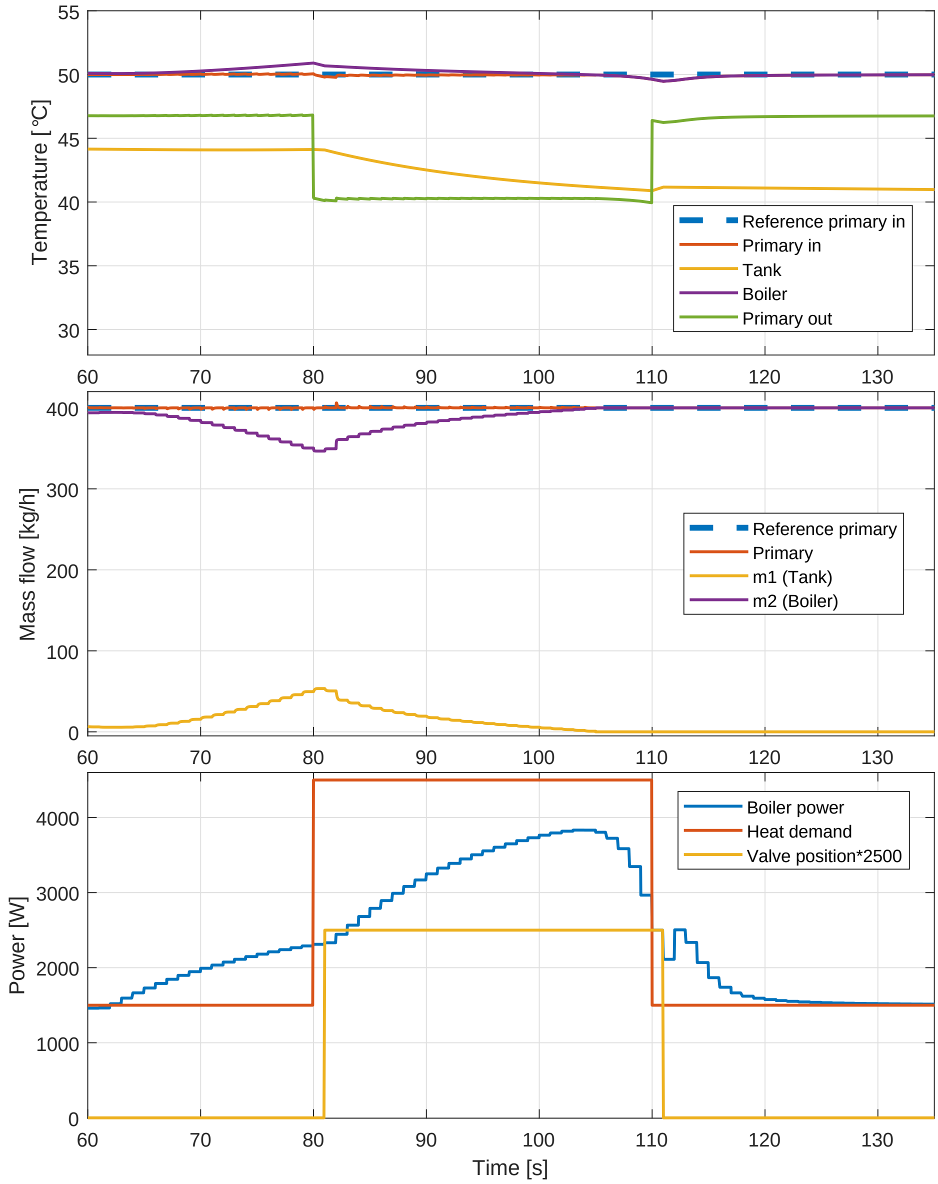
Both controllers respond to a primary flow change equally. Therefore, only MPC results are demonstrated in Figure 4.4.Water flow is an algebraic variable, thus it is changed immediately. The pumps act without any delay. As an effect of the increased primary flow, the return water temperature T_{po} becomes apparentely higher.



Obrázek 4.4.: Simulated result of MPC for primary reference flow increase and drop.

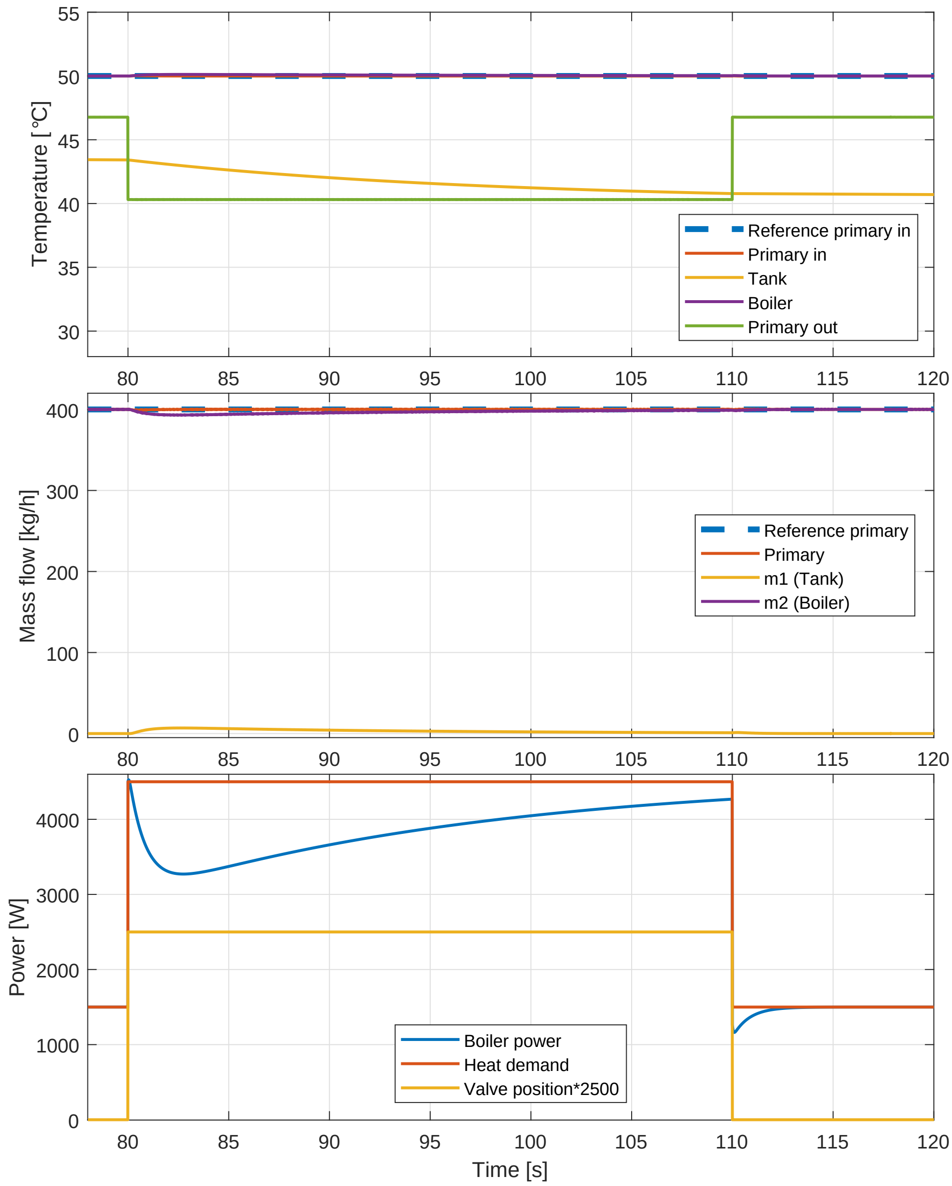
4.1.4. Heat Demand Change and Valve Function

In this section, we demonstrate changes of a HX heat demand together with the 3-way valve function. In the time 80 s, the heat demand is rised. The MPC (in Figure 4.5) preheats the water slowly in advance, while the standard controller (in Figure 4.6) increases power rapidly, once it detects T_{po} to drop.



Obrázek 4.5.: Simulated result of MPC for changing heat demand of heated zones including demonstration of 3-way valve function.

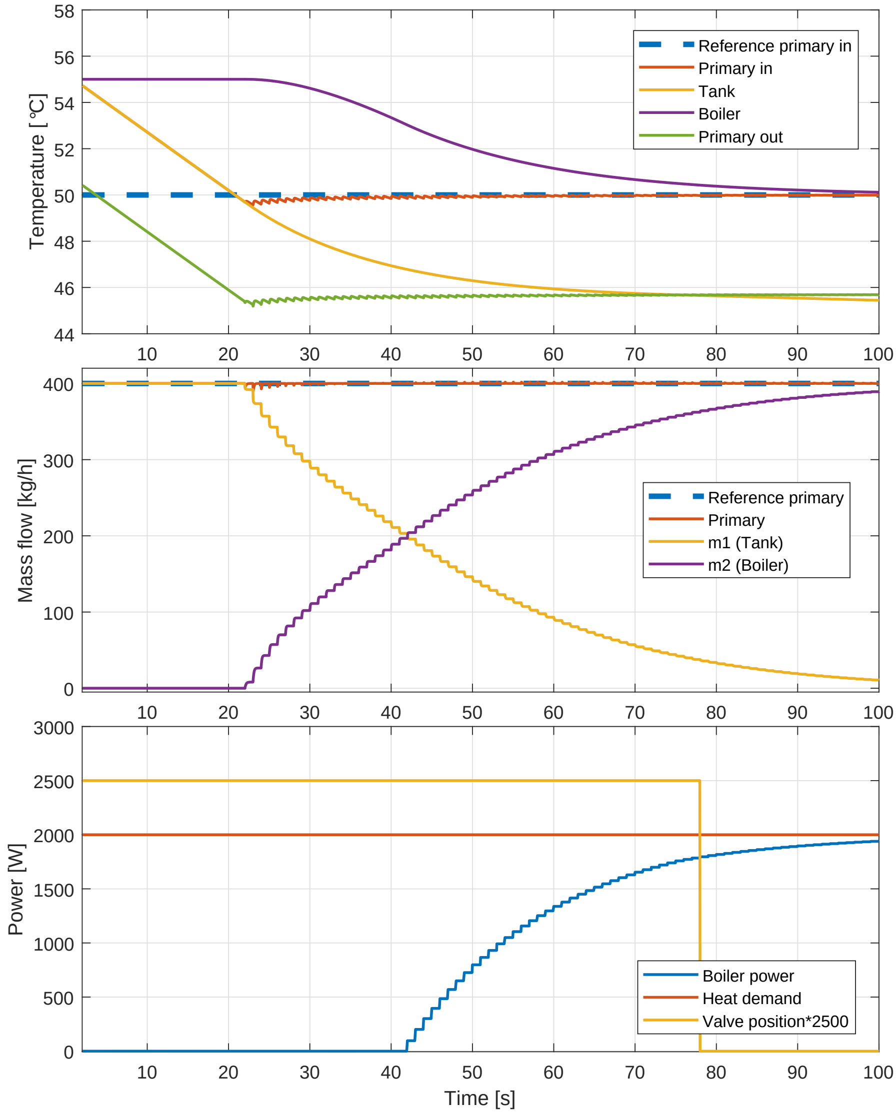
In both cases, the return water temperature falls below T_{T}, so the valve swithes from A-AB to B-AB. We can clearly see, the tank temperature is cooled by the cold return water. It is also worth mentioning that the boiler uses less power when heating the warmer tank water in the case B-AB. Once the heat demand rises again, the valve switches back to the default position A-AB.



Obrázek 4.6.: Simulated result of standard controller for changing heat demand of heated zones including demonstration of 3-way valve function.

4.1.5. Tank Water Temperature Minimisation Scenario

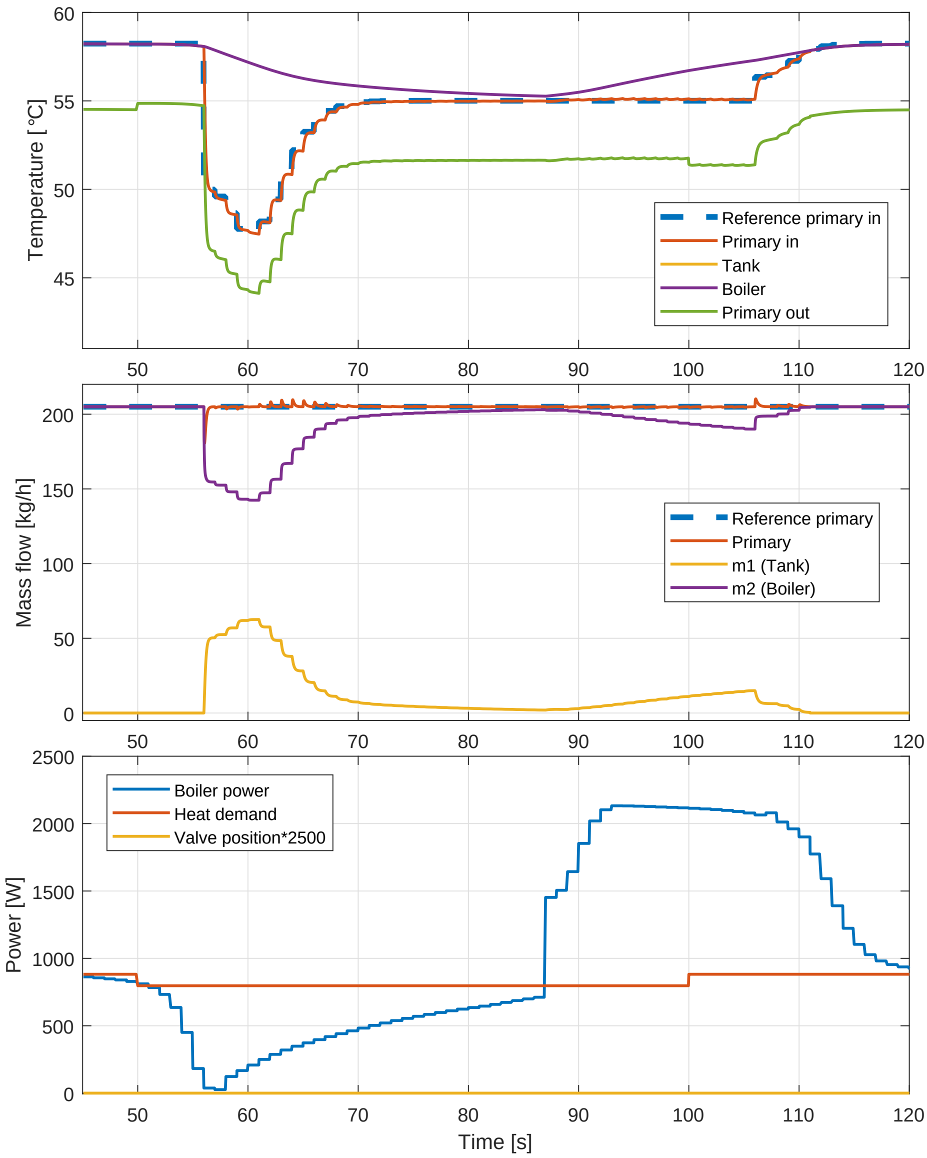
An experiment of a testbed cooling is shown in Figure 4.7. In case reference temperature is below both T_{B} and T_{T}, both controllers will aim to cool down the tank water by inducing $\dot{m}_1$ flow. As soon as $T_{\mathrm{T}} < T_{ref}$, the $\dot{m}_1$ is reduced and complemented with $\dot{m}_2$, a normal operation is entered. The valve is set as needed based on T_{po} and T_{T} relation. The Figure depicts significant cooling of the water tank by the outside environment of the temperature $T_{out} = 22$°C, since we set the m_T low for clarity of the signals, and the α_T remains the same.



Obrázek 4.7.: Simulated result of MPC when water temperature in the testbed network is higher than reference temperature.

4.2. Building Simulation Reference

In the end, we show the testbed simulation results of the MPC controller on building simulation model in Figure 4.8. The data were generated from a simulated one-pipe network on a precise EnergyPlus building simulation model. The reference signals had to be sampled to the MPC sample time $T_s = 1\text{s}$. The building simulator was already shown in Figures **??** and **??** as well as the floor plan in Fig. 1.4.



Obrázek 4.8.: Simulated MPC performance on a real building simulation model. Tank water temperature is not shown and it is around 25 °C.

In the depicted situation room 203 is replaced by the testbed. All heated zones are working constantly except for the first zone in the network which is the staircase. The first zone is initially disabled. In time 50 s the first zone HEMU initiates its secondary flow thus injects cold water from the HX into the primary pipe. This results in a primary water temperature drop in the following rooms. In time 56 s the temperature drop arrives to the testbed zone 3. The testbed tracks such a reference very precisely.

5. Conclusion

This thesis deals with the development of a cyber-physical testbed for one-pipe hydronic heating systems. In the beginning, a short preview of existing hydronic networks is made and them main differences are discussed. That is followed by presenting the testbed concept that consists of a cybernetic part (computer simulation of a building) and a physical part (real hydronic network containing the tested device). The cybernetic part sends simulated parameters (external conditions) to the physical part which makes the testing process more realistic. Then, control approaches of the physical part are presented for both the standard feedback controller and the MPC.

The physical part was designed with respect to the precision of tracking of reference parameters. The hydronic network used for preparation of these external conditions consists of two sources of water, water heater along with water tank for cold water storage. The presence of cold water storage together with application of MPC gives us the opportunity to track fast changes of references. A 3-way valve is added to improve the tank temperature management. Mathematical model of the testbed is described in detail and devided into thermal domain and hydraulic domain. The final model is a set of switched differential algebraic equations. This type of system require a special approach both in modeling and control compared to ODE systems.

The work discusses components to be used for the testbed realisation. The model was implemented as a Matlab Simulink model and applies parameters of the chosen components. We decided to use CasADi framework for the MPC implementation. This tool is compatible with Matlab and provides all necessary functionalities.

In the end, controllers' performances are demonstrated firstly on exemplary scenarios and then on building simulated references. Results show great advantages of the predictive control. Knowledge of the future reference states allows the MPC to track fast reference temperature increases which is not possible when using the standard controller. By the submission date of this work the physical testbed was not available.

Future work Our controller design assumes that the hydraulic model is perfectely identified which can be difficult to do. Otherwise a deviation of pump flows can occur. We suggest to design an estimator of $\dot{m}_{1,2}$ flows that will allow to implement local feedback control of pumps. It is not an easy task but such an improvement would make the testbed perform better. As an alternative a flow sensor to each pump could be added.

Bibliography

[1] https://www.ivarcs.cz/katalog/vytapeni-ivartrio/zpetny-ventil-eura-lehky-p139461/, Accessed on 09.11.2018.

[2] Patent nr. 31889: A device for heat exchanger control in a single-pipe heating network. https://isdv.upv.cz/webapp/webapp.pts.det?xprim=10429433&lan=cs&s_majs=&s_puvo=Ji%C5%99%C3%AD%20Dost%C3%A1l&s_naze=&s_anot=1, Accessed on 03.01.2019.

[3] Regulus. https://www.regulus.cz/cz/ventil-zonovy-tricestny-vzp-325-230-1p, Accessed on 09.11.2018.

[4] Dražice. http://www.dzd.cz/ohrivace-a-zasobniky-teple-vody/elektricke/zavesne/pto, Accessed on 09.11.2018.

[5] Friction loss tables. https://www.plumbingsupply.com/ed-frictionlosses.html, Accessed on 09.11.2018.

[6] Wilo. http://www.wilo-canada.com/fileadmin/ca/Downloads/Technical_Data_Sheets/6-WILO_Star_Product_Data_Sheets.pdf, Accessed on 09.11.2018.

[7] Joel A E Andersson, Joris Gillis, Greg Horn, James B Rawlings, and Moritz Diehl. CasADi – A software framework for nonlinear optimization and optimal control. *Mathematical Programming Computation*, In Press, 2018.

[8] Panos J. Antsaklis and Anthony N. Michel. *A linear systems primer*. Birkhaeuser, 2007.

[9] Francesco Borrelli. Constrained optimal control of linear and hybrid systems. *Springer*, 2003.

[10] Tomáš Bäumelt. Distributed building identification. Master's thesis, CTU, 2016.

[11] Jiří Dostál. Decentralized control of hydronic building systems. Dessertatioin thesis study, CTU, 2015.

[12] Jiří Dostál. Studie proveditelnosti OPPPR - Koncept Praha, 2018.

[13] Jan Hauser. Distributed Predictive Control of Buildings. Master's thesis, CTU, 2017.

[14] Jiří Kubeš. Návrh elektroniky čerpadla se sférickym motorem. Master's thesis, CTU, 2016.

[15] Bruce Roy Munson, Theodore Hisao Okiishi, Wade W. Huebsch, and Alric P. Rothmayer. *Fundamentals of fluid mechanics*. John Wiley et Sons, Inc., 2013.

[16] O. Zlevor, J. Dostál. Demand-oriented hydronic heating system and the active one-pipe system design tool. *in CLIMA 2019, Bucharest*, Submitted, 2019.

[17] Jan Předota. Model predictive control for buildings with one-pipe hydronic heating. Master's thesis, CTU, 2018.

[18] Jiří Valtr. Mass Flow Estimation and Control in Pump Driven Hydronic Systems. Master's thesis, CTU, 2017.

[19] W. H. Press, B. P. Flannery, S. A Teukolsky, and W. T. Vetterling. Runge-kutta method. *Numerical Recipes in FORTRAN: The Art of Scientific Computing, 2nd ed. Cambridge, England: Cambridge University Press, pp. 704-716*, 1992.

[20] A. Wächter and L. T. Biegler. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Mathematical Programming 106(1), pp. 25-57*, 2006.

[21] Ondřej Zlevor. Heat flow control of water-to-air heat exchanger. Master's thesis, CTU, 2017.

A. Resistance Unit Transformation

Since component suppliers and manufacturers usually publish parameters of their products as a hydraulic conductance $K\left[\text{bar} \to \text{m}^3/\text{h}\right]$, we need to perform a transformation. Hydraulic conductance is defined by a nonlinear mapping

$$q\left[\text{m}^3/\text{h}\right] = K\sqrt{\Delta p\,[\text{bar}]}$$

Hydraulic resistance, on the other hand, is in our framework defined by

$$\Delta p\,[\text{Pa}] = R\left(\dot{m}\,[\text{kg/h}]\right)^2.$$

Transformation between K and R is calculated from equating the flow in the same units (or pressure in the same units), and results in

$$R = \frac{1}{10K^2} \tag{A.1}$$

$$K = \frac{1}{\sqrt{10R}}. \tag{A.2}$$

B. Contents of the CD attached

The CD attached contains a pdf version of the thesis.