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II. Bachelor's thesis details

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Control design of a quadcopter with suspended load

Bachelor's thesis title in Czech:

Návrh řízení kvadrokoptéry se zavěšeným břemenem

Guidelines:

1. Perform state of the art on control of a quadcopter alone and in configuration with a suspended load
2. For a given simulation model of a quadcopter with suspended load, propose and test various control schemes, including PID cascade and state feedback controllers.
3. Based on simulation results, propose a control scheme for implementation on a physical quadcopter
4. Summarise the results

Bibliography / sources:

- [1] Ogata, Katsuhiko, and Yanjuan Yang. Modern control engineering. Vol. 4. India: Prentice hall, 2002.
- [2] Potter, J. J., Adams, C. J., and Singhose, W., 2015. "A planar experimental remote-controlled helicopter with a suspended load". IEEE/ASME transactions on mechatronics, 20(5), pp. 2496-2503.
- [3] Klausen, K., Fossen, T. I., and Johansen, T. A., 2017. "Nonlinear control with swing damping of a multirotor uav with suspended load". Journal of Intelligent & Robotic Systems, 88(2-4), pp. 379-394.

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III. Assignment receipt

The student acknowledges that the bachelor's thesis is an individual work. The student must produce his thesis without the assistance of others, with the exception of provided consultations. Within the bachelor's thesis, the author must state the names of consultants and include a list of references.

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Statement

I declare that I have worked out this thesis independently assuming that the results of the thesis can also be used at the discretion of the supervisor of the thesis as its co-author. I also agree with the potential publication of the results of the thesis or of its substantial part, provided I will be listed as the co-author.

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Abstract

Purpose of this bachelor work is to design different controllers through different methods such as PID cascade and state feedback for a quadcopter with suspended load. Construction of the model is done through software MATLAB and its add-on Simulink. Further on it will observe the results obtained through the designed models. The quadcopter model used in this thesis was provided by Ing. Kuře Matěj.

Abstrakt

Účelem této bakalářské práce je navrhovat pomocí různých metod, jako například kaskády PID a stavové zpětné vazby, různé ovladače pro kvadrokoptéru se zavěšeným břemenem. Výroba modelu byla provedena prostřednictvím software MATLAB a jeho doplňku Simulink. Dále si povšimne výsledků získaných pomocí navržených modelů. Model kvadrokoptéry použitý v této práci dodal Ing. Kuře Matěj.

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1. Introduction

With prevailing popularity of quadcopters with its simple structure, quadcopters obtained a lot of attentions from researchers, companies, governments and public. It is facing various regulation across the borders, even though quadcopters have become the one of the most common unmanned vehicle around. For an example, after quadcopters been commercialized, it has become the first option to shoot a aerial footages which previously required manned aerial vehicles such as fixed wing airplanes or helicopters. In such manner, numerous studies on and about the quadcopters are still popular.

For the theoretical part of this bachelor work, first it will discuss about the unmanned aerial vehicles, definition, types and use of UAVs. Then this work will discuss about the quadcopters as a subordinate branch of UAV, and also quadcopter's type, use, mechanical components quadcopters require and will discuss about those specific components and various sensors. Further on it will debate on the demanded conditions and hardships to develop quadcopters with suspended load and its necessity. Than will review on several controller design technologies and tactics, in following three categories: Linear, nonlinear and Learning-based controllers.

From those controller design tactics, PID cascade and state feedback control method will be applied to design the controller for the given model of a quadcopter with a suspended load in 2 dimensional space with help of software MATLAB and its add-on Simulink. The way how the controllers are constructed and tuned will be stated, and the response and behavior of PID cascade and state feedback controller to different conditions like following the simple trajectory and the response to the signal with noise, will be debated and compared.

2. UAV

A term unmanned aerial vehicle can cover from pilotless remote-controlled airplanes to fully autonomous rotorcrafts. According to International Civil Aviation Organization's (ICAO) circular, for both manned and unmanned vehicles can perform autonomous operations such as keeping the aircraft on course, balancing fuel use, various communications with ground facilities, acknowledging conflict traffic, plotting and executing optimum descent profiles and take-off and landing. ICAO predicts that all kinds of aircraft, like balloons, gliders, airplanes and rotorcrafts can be automatized in the future. UAVs are facing many regulations and prohibitions in the many countries. For safe integration of UAVs into the non-segregated airspace, ICAO demands UAVs to have an ability to act and response as manned aircraft do, mainly the interaction ability with air traffic control[1].

Like many other technologies do, the needs of UAVs have been raised during the war times. Compare with the time and capital need to be spent to train a pilot then develop and using a replaceable UAVs were more economic. Nowadays most of the armed forces are developing and employing UAVs as their asset. At initial stage, UAVs were used like cruise missiles but UAVs' purpose moved more into surveillance, later. One of the most well known military UAV is USA's RQ-4 Global Hawk. Global Hawk is a surveillance and reconnaissance UAV, which can fly high as 60000 feet, for 34 hours, a time which is difficult for an single pilot to handle [2].



Fig. 1. RQ-4 Global Hawk[2]

Other than cost effectiveness and better duration of operations, another advantage of UAVs is that they can be used in more hazardous environment where humans cannot enter or have difficulties to perform its work properly. For example, in 2011, three Fukushima Daiichi nuclear powerplant's reactors got flooded by the tsunami. At this incident, many unmanned vehicles were introduced. For example, Honeywell's T-Hawk. It is a small unmanned aerial system for a nuclear disaster which can keep the track of radiation levels and provides visuals of the disaster sites for an analysis[3].

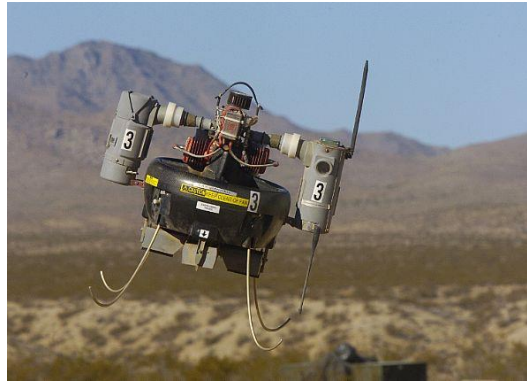


Fig. 2. Honeywell RQ-16 T-Hawk[4]

3. Quadcopter

The limitations what quadcopters are facing nowadays are related to lessening of its weight compare to its thrust while maintaining its stability and increasing its flight time which is associated with development of the battery technology. Even with the abundant numbers of researches in few decades, quadcopters are still required to have higher level of controllability to be used among civilians[1]. Regarding controlling quadcopter, a lot of different methods are introduced and applied on quadcopters to see what kind of controller is most convenient, stable and simple.

Main difference between quadcopter with convectional aerial vehicles is that quadcopters can operate in restrained space, unlike fixed wings which requires runways to fly, or helicopters requiring some extra spaces for the tail rotor. Also, quadcopters are relatively cheap, easy to design and construct. These reasons make quadcopters attractive candidate for the multipurpose or multiuse aerial vehicles[5].

As stated in chapter 1, autonomous quadcopters can be categorized as one type of UAV, so quadcopters can be applied in many different field, such as surveillance, reconnaissance and research data collecting like UAVs do. From quadcopter's economical and constructive advantages, quadcopters are also being used casually in many other ways such as aerial photography, art and sports. Unlike the fixed wing UAVs, quadcopters are able to takeoff and land without runways, there is numerous tests to use quadcopters for deliveries. For example, Amazon and DHL have a project to use quadcopters in their delivery systems[6].

The basic construction of quadcopter is 4 actuators (2 clockwise and 2 anticlockwise), frame which has 4 equivalent length arms, various sensors and a power source. Material for quadcopter frame including 4 arms, can be any material if it's light and hard enough to hold the other components and bear various stresses. For private designed quadcopters, lot of plastic frames have been used since it is easy to design and produce the desired shape with help of 3D printers. Aluminum is also widely used because of its lightness and relatively low price. Like, Dà-Jiāng Innovations' (**DJI**) Inspire 2, for the most developed commercial quadcopters use carbon fibers or composite fibers are used to make their frame and arms[7].



Fig. 3. DJI Inspire 2 Teardown[8]

3.1. Actuator

Most of the available commercial quadcopters are using electric motors as their actuators. The most widely used types of motor is brushed DC motor and brushless DC motors. Brushed DC motors can be easily found on low priced quadcopters or budget private made ones. For expensive quadcopters like DJI's, use brushless DC motors (**BLDC**). Brushed DC motors requires more torque to achieve certain speed than BLDC motors, while BLDC can achieve higher speed and longer lifespan with better energy consumption. But these differences can be overcome with software feedback[9].

3.2. Battery

Battery is the most common power source used to drive rotors. Battery capacity is proportional to the flight time, therefore in order to have longer flight endurance it is necessary to have larger battery with better efficiency. But the quadcopter's overall weight is one of the critical factor in designing quadcopter, it is also important to lessen the weight of battery to be as light as possible. And also by using the better software it is possible to achieve higher battery life.

There are studies on quadcopters using different power sources, such as quadcopters using combustion engines. These gas powered quadcopters are usually developed to have longer flight endurance. But these gas powered quadcopters are tend to be heavier than electric quadcopters and more difficult to control due to the vibration generated from the engine[10]. Many combustion engine powered quadcopters are using variable pitch rotors but this way quadcopter loses one of its advantage: mechanical simplicity.

3.3.Propeller

Propellers are fans attached to the rotor, converting rotational motion into thrust. It is classified with its diameter and pitch. Quadcopter's four propellers should be identical to each other, and usually motors will provide suitable specifications of propeller to have optimum power consumption[11].

3.4.Sensors

For controlling a quadcopter, sensor is the essential in order to provide feedback to form a closed loop. In quadcopter, numerous sensors are used to measure current state of it such as its current position, angle, acceleration, altitude and direction. These sensors are usually integrated into one component called inertial measurement unit. As a result of developments in micro system technologies, we can find some of the sensors like gyroscope or accelerometers in miniature sizes with an affordable price. The closest examples nowadays are the smartphones, which are normally equipped with gyroscope, accelerometer and global positioning system.

3.4.1. Gyroscope

Gyroscope is the crucial sensor to keep the quadcopter stable. It measures the angular rate of quadcopter, which can be controlled with the counteraction of the rotor. For the quadcopters with suspended load, it is also required to apply sensors to the load to measure its position and behavior. The first considerable micromachined gyroscope by Draper Laboratory appeared in 1988 and after various types of gyroscopes were emerged. Most of micromachined vibration gyroscopes are belongs to the categories of tuning fork gyroscopes (**TFG**). TFGs have relatively high precision[12].

3.4.2. Accelerometer

Mechanical principle and design of micromachined accelerometers are not far from the ones of gyroscopes. There are many different types of accelerometers like TFGs do. Micromachined accelerometers consist of silicon circuit, tuning fork tine and calibration mass. One end of the fork will be subjected to external forces and will generate different frequency while the other end generates natural frequency. The difference between two frequencies will be the output of this specific double-ended tuning fork used accelerometer[13].

3.4.3. Positioning

Other than accelerometer and gyroscope, quadcopter can be equipped with other sensors to specify its current position. Global Positioning System (**GPS**) is one of the most prevail method which defines quadcopter's position according to satellites. Since GPS is using satellites its application is limited when quadcopters are being used somewhere it cannot communicate with the satellite, such as indoors or underground. To unravel this problem other methods were introduced such as ultrasonic, infrared and visual positioning systems[14].

3.4.4. Barometer

Barometer is a device to measure the atmospheric pressure. With the fact that pressure decreases as the above sea level increase, it can also be used as an altimeter. Barometer can be used to improve the altimeter accuracy levels when GPS cannot[15].

3.4.5. Magnetometer

While compass can indicate the direction to the north and south poles of the Earth, modern magnetometer can tell the direction and strength of the field. Micromachined magnetometers can be divided into, measuring field strength exceeding the Earth's field, measuring disturbances in the Earth's field and measuring changes in generated or induced field[16].

3.4.6. Example – Pixhawk

On the market, diverse control units, boards or panels are available. One of the example of them is Pixhawk, originated from PIXHAWK student project. Since it is a control unit, it consists different sensors to provide measurements to control the quadcopter. Pixhawk unit can have sensors like, gyroscope, accelerometer, magnetometer and barometer[17].



Fig. 4. Pixhawk[17]

3.4.7. Quadcopters with Suspended Load

With its advantages, quadcopter is been used in various fields such as military, research and aerial cinematography. To assign different works to quadrotor, cameras, grippers were attached according to its need. For the logistical usage, it is more common to use grippers than sling loads. For example, recently announce Boeing's prototype cargo drone[18] carries its load attached right below the frame. Because of its instability, it is considered more difficult to design external suspended load carrying applications[19].

4. Control

Quadcopters are underactuated and inherently unstable. One of the example showing the necessity of control is, when quadcopters moves horizontally, its roll or pitch angle will be tilted and to prevent quadcopter flipping upside down, thrusts must be controlled. 3 main categories of quadcopter flight controllers are: Linear control system, nonlinear control system and learning-based control system[20]. There exist various strategies to make the control of quadcopters, it can be one simple method applied to a quadcopter or can be multiple method combined and used.

For quadcopters with the suspended load, when the arm length of the suspended load is long enough, then the requirements for modeling is low. But if the load is attached close to the quadcopter and if the load is larger and heavier than the quadcopter, the importance of modelling is emphasized as the coupling effect is also strengthened[21].

4.1. Linear Controller

The basic and traditional way to design a controller is using linear controls. The first fully autopiloted helicopter was CH-53A which was able to waypoint navigate and follow the terrain, in 60-70s. It was using classical linear controller with the method of sequential loop closure[20][22]. There are different ways to design linear controllers, such as Proportional-Integral (**PI**), Proportional-Derivative (**PD**) and Proportional-Integral-Derivative (**PID**) controllers, Linear quadratic (**LQ**), H-infinite, or gain scheduling method.

4.1.1. Proportional Integral Derivative

A PID controller is combination of three different control terms. **P**, proportional control is the simplest controller from the conventional linear controllers which response immediately when the error occurs. If the gain of the controller is high than its offset will get smaller, response will get quicker, but it may end up with underdamped motion. One of the problems of P controller is that, it cannot acknowledge the gradual disturbances or changes. The calculated output, $u(t)$ of P controller can be defined as:

$$u(t) = u_b + K_p e(t)$$

where u_b is bias constant, K_p is the proportional gain and $e(t)$ is the error.

I, integral control is being used to decrease the offset but also it has destabilizing effect on control loop. Stand-alone I controllers are only applied on few applications. The output of I controller is:

$$u(t) = u_b + K_i \int_0^t e(\tau) d\tau$$

where K_i is integral constant, and τ is integration variable.

D, derivative control cannot be used alone. It provides damping effect and inhibit the overshoot. It can provide better response than the proportional controller but if the gain is too high it can worsen the response.

$$u(t) = K_d \frac{de}{dt}$$

If P and I controllers are combined it will make PI controller, which has advantages of P controller with lesser offset, and for PD controller it is allowed to have higher proportional gain with the stability of D controller. And if all three control methods are used, it makes PID controller. Output of D controller used with error signal and combined PID is as following:

$$u(t) = u_b + K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d e'(t)$$

PID controller is one of the most basic technology, widely applied to control UAVs[20].

4.1.2. Linear Quadratic

The linear quadratic controller is arising from optimal control theory field, which tries to optimize system's function emerged from initial to final state. BERGERMAN, Marcel, et al.'s study shows that with the LQ regulator it gave the better alignment of the poles which were unstable[24].

4.1.3. Gain Scheduling

Gain scheduling is a method used to control a nonlinear model, but since it consists of a set of linear models[20], gain scheduling is dealt in linear controllers. Designing a gain scheduled controller is done in following order: computation of linear parameter-varying model, linear design method applied on linear parameter-varying model, implementing family of linear controllers which have scheduled controller gains according to current scheduling variable and performance assessment[25].

4.1.4. State Feedback Control

State feedback controllers observe its state variables from the measured outcome and from this feedback it controls the state. The easiest way of state feedback control is done by pole placement which controls the system by setting a gain so that the poles of the system to be in the desired position[26].

4.2. Nonlinear Controller

Having a perfect linearization in a reality is absurd, there are limitations in describing the nature of the quadcopters purely with the linear approaches. To solve these constraints involved with nonlinearity, nonlinear controllers are introduced. For examples of nonlinear control methods, there exists feedback linearization, adaptive control and backstepping methods[20].

4.2.1. Feedback Linearization

Feedback linearization transforms the system's nonlinear state variables into linear system which gives linear dynamics. For these linearized systems, it is possible to apply techniques used for linear system, and then these controlled linearized systems will be inverse transformed back into its original state.

4.2.2. Adaptive Control

Some control tasks may have uncertainties or unpredictable parameters, and if those parameters are not resolved it may cause inaccuracy or instability. Adaptive controller estimates the uncertain parameters of controller from measured system signals, and use the estimations for control input calculations. Adaptive control can also be applied in some process controls with hundreds of control loops to reduce the numbers of manually tuned parameters and increase engineering efficiency and practicality[27].

4.2.3. Backstepping Method

Backstepping is a recursive method for the control of underactuated linear and nonlinear systems[20]. Backstepping integrates a full system design problem into sequence of design problems of lower order subsystems. Under less restrictive conditions, backstepping can solve stabilization, tracking, and robust control problems, more sufficiently than other methods[28].

4.3. Learning-based Controller

Learning-based type of controllers do not need quadcopter's dynamic model but requires trials and flight data to train the system. Popular used learning-based systems are controllers using fuzzy logic and neural network. For fuzzy logic controllers they translate the human pilot's information and knowledges into rules that fuzzy control system can use. Controllers with neural networks generally use the network to identify some unknowns and then combined with general control theories[20].

5. Model

5.1. Notations and Dynamics

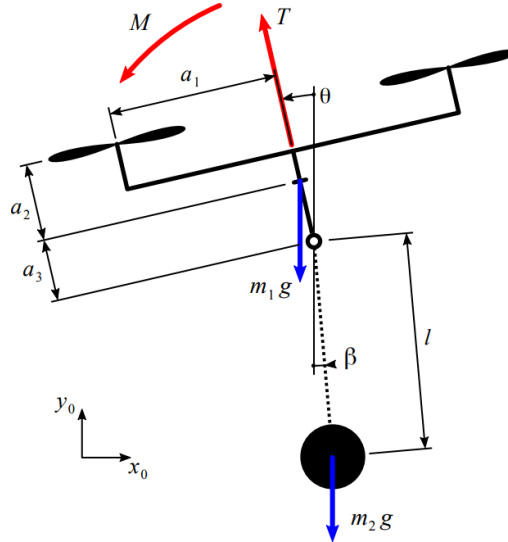


Fig. 5. Figure of the model, Ing. Kuře Matěj and Ing. Bušek Jaroslav

For this paper, two models of quadcopter with suspended load with 2 degrees of freedom as shown in fig.5. were provided by Ing. Kuře Matěj. One model is derived without the external forces to give a better view on the model and another one is with the air drag so that the quadcopter will behave more realistically.

For the model following notations and its parameters were given and chosen:

Notation	Explanation	Parameter
m_1	Mass of the quadcopter	10 [kg]
m_2	Maas of the suspended load	5 [kg]
l	Length of the shaft	0.4 [m]
a_1	Length of quadcopter arms	0.25 [m]
a_2	Length from center of mass to propellers	0.05 [m]
a_3	Length from center of mass to the joint	0.07 [m]
g	Gravitational acceleration	10 [m.s ⁻²]
c	Damping coefficient	0.001 [-]
k	Friction coefficient	0.001 [-]
I_1	Moment of Inertia of quadcopter	0.0196 [kg.m ²]
I_2	Moment of Inertia of suspended load	0.1 [kg.m ²]

According to the provided model, dynamics of the quadcopter with suspended load in 2D is described in matrix form:

$$M(t)\ddot{x}(t) + C\dot{x}(t) + Kx(t) + Q(t) = L(t)u(t) + F(t)f$$

For this model, state vector is given as, $x(t) = [x(t) \ y(t) \ \theta(t) \ \beta(t)]^T$. Where $x(t)$ is horizontal and $y(t)$ is vertical coordinate of the center of mass of the quadcopter in the global system. $\theta(t)$ describes the pitch of the quadcopter and $\beta(t)$ describes the angle between vertical axis of the quadcopter and the suspended load shaft. The input vector, $u(t) = [T(t) \ M(t)]^T$ is consist of combined thrust of the actuators, $T(t)$ and the total torque generated $M(t)$. There is given 2 types of dynamic matrices, one without air drag to have better insight, and one with air drag. $M(t)$ matrix shows how are the forces and torques were distributed with the acceleration of the quadcopter, C , K are non-time variant damping and friction coefficients matrices of the joint between quadcopter and the suspended load shaft, respectively. So C , K matrices only have the elements associated with the angular movements. $L(t)$ matrix distributes input thrust and torque, and $F(t)$ matrix is for the external forces which has been neglected for this paper.

5.2.Linearization and State Space Model

For the linearization of the dynamic model, equilibrium state, when the quadcopter is in a hovering condition need to be considered. During the hovering stage, all the linear and angular displacements and velocities are set to be equal to zero, while thrust of all actuators are identically distributed against the gravitational force so that the quadcopter can stay in air but not moving in vertical direction neither in positively or negatively.

$$x_0, y_0, \theta_0, \beta_0, \dot{x}_0, \dot{y}_0, \dot{\theta}_0, \dot{\beta}_0 = 0$$

$$F_{1,0} = F_{2,0} = \frac{g(m_1 + m_2)}{2}$$

In dynamics of the model, what gives it the nonlinearity are sine and cosine functions. In the equilibrium state, θ and β values lie in vicinity of zero. Around zero for example, $\cos(\theta)$ behaves like 1 and $\sin(\theta)$ like θ .

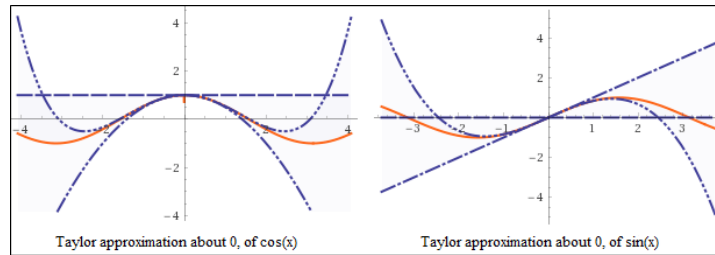


Fig. 6. Taylor approximations of cosine and sine[29] Image generated from the ref.

There are 2 provided linearized dynamic models, one with the air drag and one without. Following is the linearized quadcopter model with air drag.

$$\bar{M} = \begin{bmatrix} m_1 + m_2 & 0 & a_3 m_2 & l m_2 \\ 0 & m_1 + m_2 & 0 & 0 \\ a_3 m_2 & 0 & m_2 a_3^2 + I_1 & a_3 l m_2 \\ l m_2 & 0 & a_3 l m_2 & m_2 l^2 + I_2 \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} 2c & 0 & a_3 c & cl \\ 0 & 2c & 0 & 0 \\ a_3 m_2 & 0 & c a_3^2 + 2c & a_3 cl - c \\ cl & 0 & a_3 cl - c & cl^2 + 2c \end{bmatrix}$$

$$\bar{K} = \begin{bmatrix} 0 & 0 & (m_1 + m_2)g & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & k + a_3 g m_2 & -k \\ 0 & 0 & -k & k + g l m_2 \end{bmatrix}$$

$$\bar{L} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Formation of state space model from the linearized dynamic model is also provided as following

$$\begin{aligned} \begin{bmatrix} I & 0 \\ 0 & \bar{M} \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \end{bmatrix} &= \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ L \end{bmatrix} u(t) \\ \begin{bmatrix} I & 0 \\ 0 & \bar{M} \end{bmatrix}^{-1} &= \begin{bmatrix} I^{-1} & 0 \\ 0 & \bar{M}^{-1} \end{bmatrix} \rightarrow \begin{bmatrix} I^{-1} & 0 \\ 0 & \bar{M}^{-1} \end{bmatrix} \begin{bmatrix} 0 & I \\ -\bar{K} & -\bar{C} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -\bar{M}^{-1} \bar{K} & -\bar{M}^{-1} \bar{C} \end{bmatrix} = \bar{A} \\ \bar{B} &= \begin{bmatrix} 4 \times 2, \text{zero matrix} \\ \bar{M}^{-1} \bar{L} \end{bmatrix} \end{aligned}$$

New state vector of the model and state space representation are introduced as follow

$$\bar{x}(t) = [x(t) \ y(t) \ \theta(t) \ \beta(t) \ \dot{x}(t) \ \dot{y}(t) \ \dot{\theta}(t) \ \dot{\beta}(t)]^T$$

$$\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}u(t)$$

6. Simulation

For the simulations of the quadcopter with suspended load model, software MATLAB version R2017a and its add-on product Simulink are used. two types of linear controllers are designed and simulated. PID cascade controller and state feedback controller. To have better comparisons, identical conditions are applied to both controllers. To see which controller endures better and maintains the stability against the noise with same intensity in both vertical and horizontal direction.

Specific conditions for the simulation are, quadcopter with suspended load is assigned to move to 5m horizontally and 5m vertically in positive direction, and second condition would be just like the first condition but with some noises are applied in x and y direction.

6.1. PID Cascade Controller

For the PID cascade controller, 2 PID controllers and 1 PD controller are placed. 1 PID is used to control thrust of the quadcopter and other PID and PD are placed in series to control the torque of the quadcopter. Subsystem is formed based on state space model, which takes the controlled input vector $u(t)$ through controllers and state vector $x(t)$ is obtained through integrators within the subsystem.

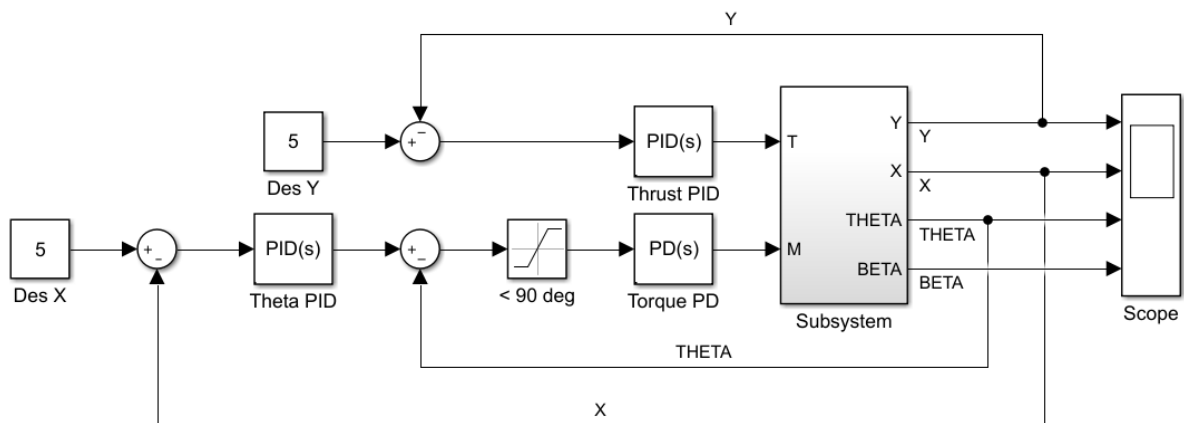


Fig. 7. Scheme of the PID cascade

Since the PID controllers are functioning in a way to eliminate the differences between desired value and controlled variable, it is connected together by subtracting controlled variable from the desired position signal. Tuning of the controllers is done from the inner loop to outer loop,

with trial and error. For the inner loop PD controller one saturator is placed since in ideal case the quadcopter's pitch should not exceed more than it should. For this controller saturator with $(-0.6;0.6)$ limit range is applied, which is in degrees $(-34^{\circ};34^{\circ})$.

Tuned parameters of each controller are listed in tuned order.

Controller	Type	Value
Torque PD	Proportional	0.46
	Derivative	0.53
	Filter coefficient	66
Thrust PID	Proportional	0.05
	Integral	0.01
	Derivative	40
	Filter coefficient	100
Theta PID	Proportional	-0.05
	Integral	-0.00002
	Derivative	-0.08
	Filter coefficient	100

For the tuning of the thrust PID, it is not effected by other variables so it shows similar responses like regular PID does, but for the cascade torque and theta controllers, they are effected by one another, since they are placed in series it wouldn't give the expected response like thrust PID does..

6.2.State Feedback Controller

For this bachelor work, pole placement method is used to find the state gain of the state feedback controller. With help of MATLAB it is possible to calculate the state gain of the model by using the place command, which requires A and B matrices and the desired positions of the pole.

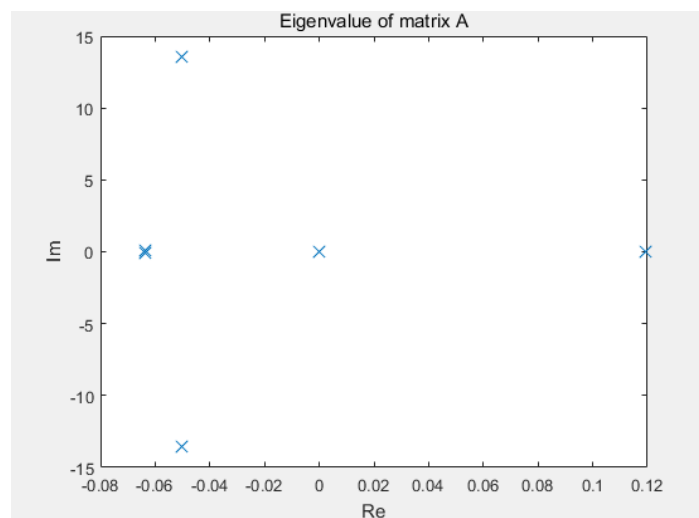


Fig. 8. Eigenvalue of matrix A

Eigenvalues of matrix A is $[0; -0.05 + 13.6j; -0.05 - 13.6j; 0.12; -0.06 + 0.1j; -0.06 - 0.1j; 0; -0.0001]$. Its consist of 2 zeroes, 2 real numbers and 4 complex numbers. Except for zeros and 0.12, all of A matrix's eigenvalue lies in negative area.

Response of the model asked to move to designated coordinate $x, y = [5, 5]$, with the state gain calculated with eigenvalues of matrix A is shown in Fig.9.

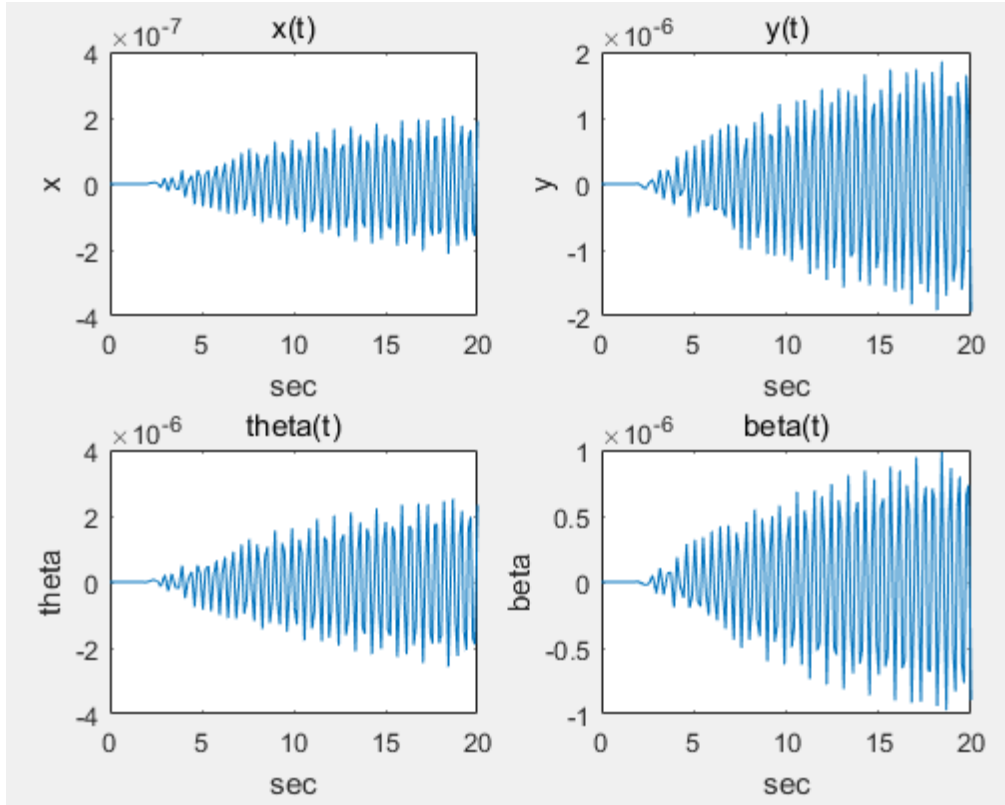


Fig. 9. Response to eigenvalue of matrix A

Its oscillation amplitude is very minor, it vibrates in micrometer range but like it is shown, its amplitude is increasing as time does. And it is very clear that the quadcopter is not reaching the desired coordinates in x, y axis.

To choose the appropriate coordinates of the poles, at first put all the poles as close as possible and like the PID cascade, few different positions were tried. And the following is the coordinate of the desired pole position to get the sufficient state gain

$$P = [-2 + 4j; -2 - 4j; -1.9 + 3.8j; -1.9 - 3.8j; -2.1 + 3.8j; -2.1 - 3.8j; -2.2; -1.8]$$

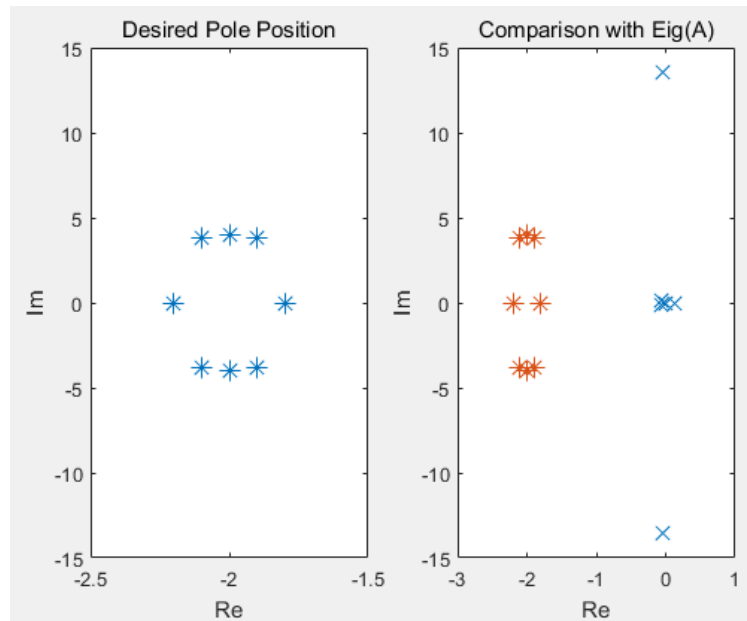


Fig. 10. Desired pole position and its comparison with eigenvalue of matrix A

As shown in Fig.10, all the poles are within the negative area and placed close together. Model controlled with recalculated gain according to desired poles is shown in Fig.11.

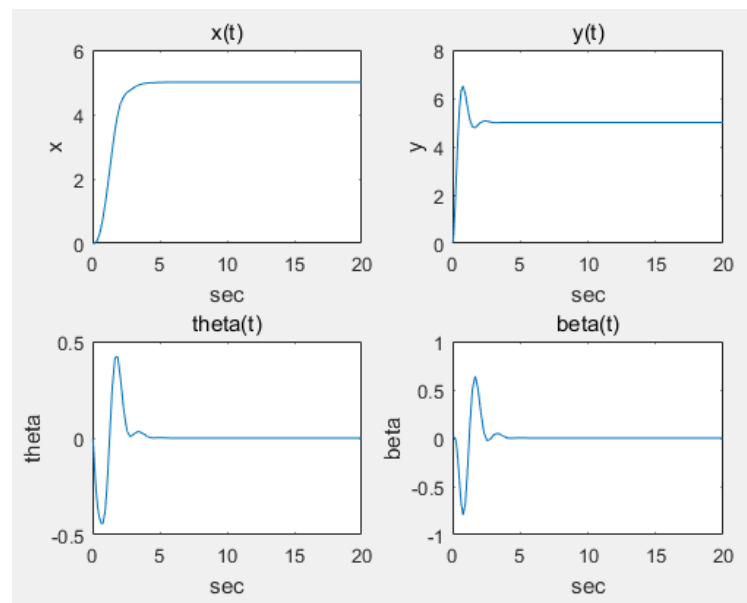


Fig. 11. With state gain calculated with desired pole coordinate

7. Results

For the first comparison of PID cascade and state feedback controller, quadcopter is assign to move to $x,y = [5,5]$ coordinate.

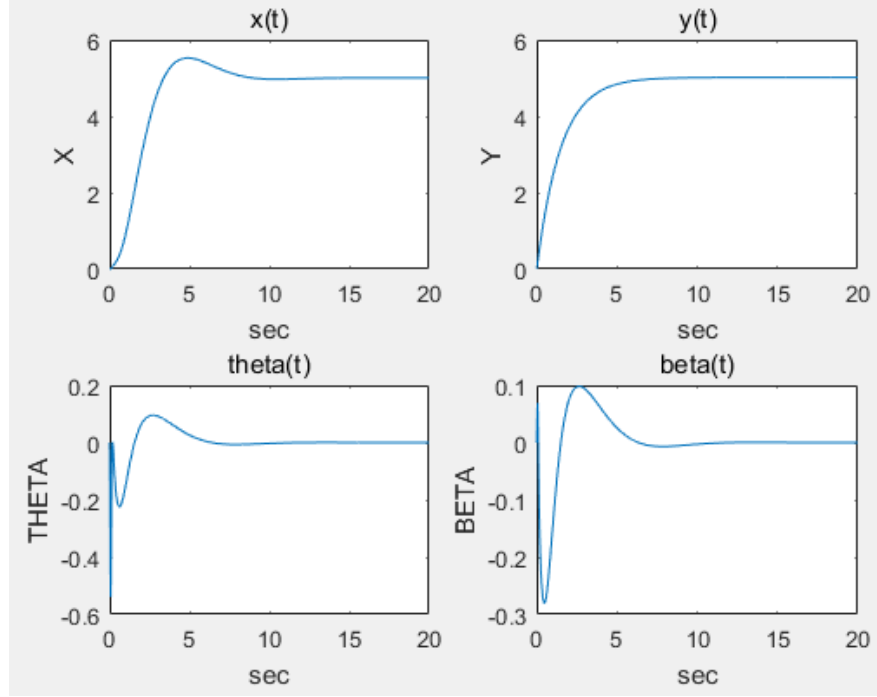


Fig. 12. PID cascade's response to first condition

Fig.11 and 12 were asked the exact same work, but different controllers, for PID cascade controller, in x-axis it took 10 seconds to reach its desired position and y-axis took 6 seconds. For state feedback controller it took 5 seconds for x-axis and 3 seconds in y-axis. Getting to the designated coordinate, PID cascade controller has half the speed of state feedback does. For the angular displacement, pitch of the PID cascade's quadcopter changes from about -30° to $+5^\circ$, but it keeps the β from -17° to $+5^\circ$. But for state feedback control, range of β is even more higher than the range of θ .

For the second condition, noise with 0.001 power and 0.5 sample time is applied.

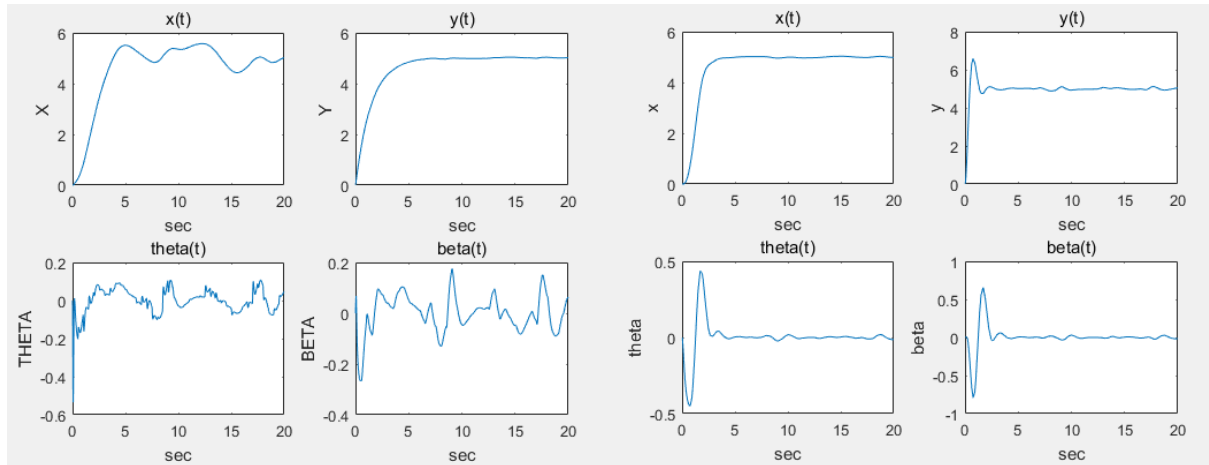


Fig. 13. Response with noise, PID cascade(Left), State feedback(Right)

In Fig.13. shows the response of each controller to the exact same condition as Fig.11 and 12, but noise applied to x and y axis. For PID cascade, it seems response in y axis is the only stabled variable and for state feedback controller every variable look alike to the one without the noise.

8. Conclusion

Designing controllers for the 2 dimensional quadcopter with suspended load, the method used was trial and error, which seems very intuitive therefore had a skeptical expectation on the results of the model. For an example, if the proportional gain of the PID cascade controller was increased, then it might have had a similar control time like state feedback control. As a payback there is a possibility of an overshoot. The point is that if the tuning was done more carefully than it is possible to have similar response tendencies for both controllers. Further research is required for more precise methods of tuning PID controllers and selection of appropriate pole positions for state feedback controllers.

Even though, time to reach the equilibrium point were different for two controllers, the behaviors of the angle between quadcopter's vertical axis and the suspended load shaft were different. PID cascade controller had narrower range of β angle while state feedback controller's β oscillated almost as much as the quadcopter pitch did. With a consideration about the usage and purpose of quadcopters with suspended load, having a better stability of β angle can be an advantage of PID cascade controller.

When noise was applied, PID cascade and state feedback controller showed remarkable differences. While state feedback controller had similar tendency of response as it did for the condition without the noise, PID cascade controller seem much more unstable except for the $y(t)$. It is possible to estimate the reason why $y(t)$ remained so still while the other variables were fluctuating from the structure of it. Because thrust is controlled with the changes of y . From the result, it is reasonable to say that state feedback controller is more suitable for the environment exposed with external force.

For future it is possible to further investigate in other methods and tactics to design the controller for example non-linear ways which were not introduced in this bachelor thesis, or try to combine different tactics together to see if it controls the behave better or incase it does have better response than debate whether it is worthies with the complexity.

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