CZECH TECHNICAL UNIVERSITY IN PRAGUE
Faculty of Civil Engineering
Thákurova 7, 166 29 Prague 6, Czech Republic

MASTER'S THESIS PROPOSAL

study programme: Civil Engineering

study branch: Advanced Masters in Structural Analysis of Monuments and Historical Constructions

academic year: 2017/2018

Student's name and surname: Pablo Bañasco Sánchez

Department: Department of Mechanics

Thesis supervisor: Petr Fajman

Thesis title: Calibration of finite elements model of St. Jakub in Cirkvice by modal testing

Thesis title in English see above

Framework content: Structural dynamics

The objective of the work is to develop a calibrated FE model that would allow in the future to evaluate the possibility of reinstalling the original bells of the tower.

The parameters obtained through this calibration have been compared with those extracted from the MQI method.

Furthermore, it is also an aim of this dissertation to compare the dynamic properties of the bell-tower of St. Jakub in Cirkvice with the homonymous one in Kutná Hora and with those of other bell-towers.

Finally, it has been studied the influence of the bell swinging on the dynamic behavior of the bell-tower.

Assignment date: 9/04/2018 Submission date: 02/07/2018

If the student fails to submit the Master's thesis on time, they are obliged to justify this fact in advance in writing, if this request (submitted through the Student Registrar) is granted by the Dean, the Dean will assign the student a substitute date for holding the final graduation examination (2 attempts for FGE remain). If this fact is not appropriately excused or if the request is not granted by the Dean, the Dean will assign the student a date for retaking the final graduation examination, FGE can be retaken only once. (Study and Examination Code, Art 22, Par 3, 4.)

The student takes notice of the obligation of working out the Master's thesis on their own, without any outside help, except for consultation. The list of references, other sources and names of consultants must be included in the Master's thesis.

.... Master's thesis supervisor

.... Head of department

Date of Master's thesis proposal take over: July 2018

.... Student

This form must be completed in 3 copies – 1x department, 1x student, 1x Student Registrar (sent by department)

No later than by the end of the 2nd week of instruction in the semester, the department shall send one copy of BT Proposal to the Student Registrar and enter data into the faculty information system KOS. (Dean's Instruction for Implementation of Study Programmes and FGE at FCE CTU Art. 5, Par. 7)

DECLARATION

Name:	Pablo Bañasco Sánchez
Email:	plbs1988@gmail.com

Title of the Msc Dissertation:	Calibration of finite element model if St. Jakub church in Cirkvice by modal testing
Supervisor(s):	Petr Fajman
Year:	2017/2018

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

I hereby declare that the MSc Consortium responsible for the Advanced Masters in Structural Analysis of Monuments and Historical Constructions is allowed to store and make available electronically the present MSc Dissertation.

University:	Czech Technical University in Prague
Date:	02/07/2018
Signature:	___________________________

This page is left blank on purpose.

ACKNOWLEDGEMENTS

I do not hesitate to acknowledge my supervisor, Petr Fajman, for being always available to attend my multiple questions, and Petr Cicvárek, from the church management team for helping me to visit the case study. I would like also to acknowledge Petr Kabele, always ready to help all SAHC students. Especially, I acknowledge SAHC Consortium for the scholarship that allowed me attend and complete the program. And, finally, to my colleagues, Bledian, Gary, Jacopo and Nigar, for helping me during the development of this dissertation.

This page is left blank on purpose.

ABSTRACT

The assessment of material mechanical properties is one of the classical problems in the structural analysis of existing buildings and historical constructions. Between all the possible non-destructive or minor-destructive tests, the modal analysis, and then, the calibration of a finite elements model, is the only one that experimentally measures parameters related to the global structural behavior.

The objective of the work is to develop a calibrated FE model that would allow in the future to evaluate the possibility of reinstalling the original bells of the tower. Furthermore, to compare the dynamic properties of the bell-tower of St. Jakub in Cirkvice with the homonymous one in Kutná Hora.

The Church of St. Jakub in Cirkvice has masonry bell-tower located in the opposite side of the apse, aligned with the longitudinal axis of the nave. The developed FE model has been calibrated through the provided results of modal testing. Later, the parameters obtained through this calibration have been compared with those extracted from the MQI method. Those results have been also compared with other dynamic studies of bell-towers, among them, the bell-tower of St. Jakub in Kutná Hora. Finally, it has been studied the influence of the bell swinging on the dynamic behavior of the bell-tower.

This page is left blank on purpose.

ABSTRAKT

Kalibrace konečněprvkového modelu kostela Sv. Jakuba v Cirkvici pomocí měření vlastních frekvencí

Vyhodnocení materiálových vlastností je jeden z klasických problémů statické analýzy stávajících historických konstrukcí. Z nedestruktivních zkoušek je experimentální měření vlastních frekvencí nejlepší možnost, jak získat parametry pro výpočet celé konstrukce.

Cílem práce je vytvořit model věže kostela Svatého Jakuba v Církvici pro metodu konečných prvků (FEM). Pokusit se zjistit materiálové vlastnosti a okrajové podmínky ze změřených vlastních frekvencí a vyhodnotit, jestli je možné znovu zavěsit zvon do věže. Výsledky frekvencí jsou porovnány s frekvencemi dalších kostelních věží včetně věže v Kutné Hoře.

Jednolodní kostel Svatého Jakuba má zděnou věž, která je umístěna na konci lodě naproti oltáři. Vytvořený FE model byl kalibrován pomocí změřených vlastních frekvencí. Parametry získané z dynamických měření byly porovnány s hodnotami získanými MQI metodami. Poté byly porovnány i výsledné frekvence věže s hodnotami z jiných věží včetně věže sv. Jakuba v Kutné Hoře. Nakonec je vyhodnocen vliv zvonění na dynamické chování věže, což je klíčový faktor pro možnou reinstalaci zvonu.

This page is left blank on purpose.

RESUMEN

Calibración del modelo de elementos finitos de la Iglesia de San Jakub en Cirkvice a través del test modal

La definición de las propiedades mecánicas de los materiales es uno de los problemas clásicos del análisis estructural de edificios existentes y construcciones históricas. Entre todos los posibles test no destructivos o ligeramente destructivos, el análisis modal y, por tanto, la calibración de modelos de elementos finitos es el único test que toma en cuenta parámetros relacionados con el comportamiento global de la estructura.

El objetivo de este trabajo es desarrollar un modelo de elementos finitos que en el futuro permita evaluar la posibilidad de reinstalar las campanas originales del campanario. Además, compara las propiedades dinámicas del campanario de la Iglesia de San Jakub en Cirkvice con su homónimo de Kutná Hora.

La Iglesia de San Jakub en Cirkvice posee un campanario localizado en el lado opuesto del ápside, ambos alineados con el eje longitudinal de la nave. El modelo de elementos finitos desarrollado ha sido calibrado gracias a los resultados de un análisis modal. A continuación, los parámetros obtenidos de la calibración han sido comparados con los resultados de la aplicación del método del índice de calidad mural (MQI) y con otros estudios relacionados con el comportamiento dinámico de otros campanarios, entre otros, el la Iglesia de San Jakub en Kutná Hora. Finalmente, se ha estudiado la posible influencia de la oscilación de las campanas en la dinámica estructural del campanario.

This page is left blank on purpose.

TABLE OF CONTENTS

LIST OF FIGURES

1. INTRODUCTION

The church is in the village of Jakub (today Jakub - Cirkvice), 90km far from Prague (fig. 1.0.1). It is a Romanesque building located on a low hill called the Hill of St. Jakub and dedicated to St. James the Greater. The importance and location of this church could be explained in two hypotheses. On one hand, in the 12th century Svatojakubské pilgrimages culminated in Santiago de Compostela (Spain) to the tomb of St. James. The pilgrims came from various places in Europe, visiting churches for praying, many of which served as landmarks. St. Jacob, visible from far, over the Haberdask Road linking Western Moravia with Polabi. On the other hand, the church building could be a request for redemption of Slavibor of Svabenice, perhaps the husband of the church's benefactor – Mary. Nevertheless, the age of construction of the church and the characters involved on it are not clearly defined.

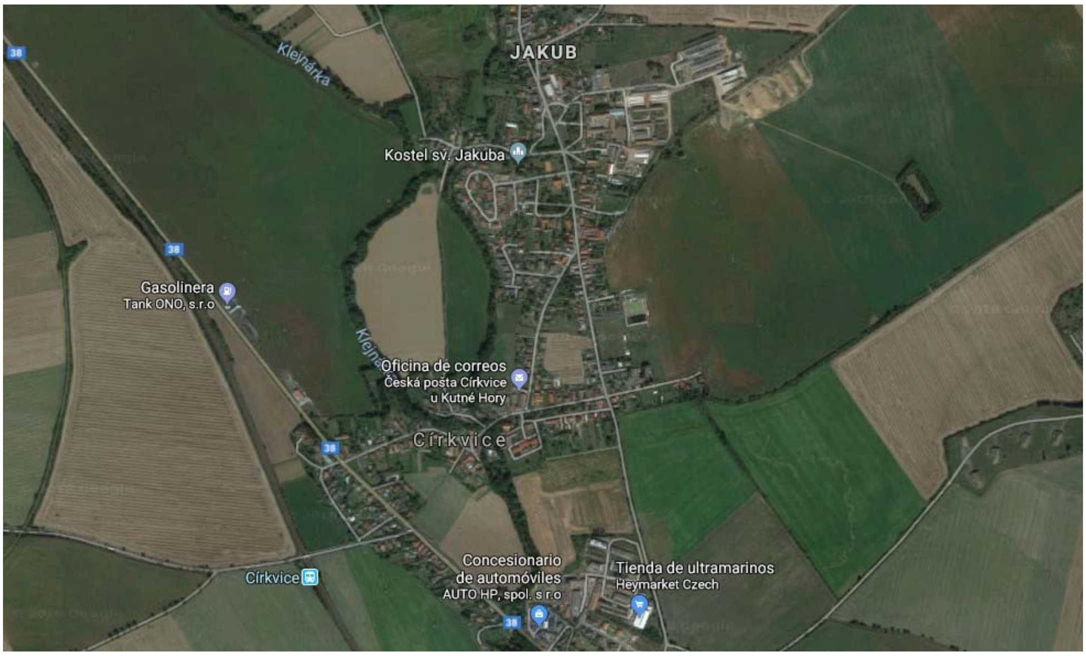


Figure 1.0.1. Jakub-Cirkvice, location. Google Maps.

The principal interest of this church is its main façade, which is unusually decorated in comparison with other Romanesque buildings in the country (fig. 1.0.2). The non-modular organization of the main façade, most probably, is due to its decorative character. The set of sculptures found in this elevation is the largest preserved set of Romanesque sculptures in the country. Because of its historical value, the church was declared a National Cultural Monument in April 2008. However, the few information found about this heritage source is everything written in Czech language, complicating the analysis for a non-Czech speaker. Basically, the historical information is based on the Visit Guide published in 2015 and written by Anna Vanková where is possible also to find some theories of the origin based on archive research and hypothesis of the significance of the sculptures of the main façade (Vanková, 2015).

Figure 1.0.2. St Jakub in Cirkvice, main façade. Image from the author.

In this dissertation, the main objective is to develop a calibrated FE model that would allow in the future to evaluate the possibility of reinstalling the original bells of the tower, currently stored inside the bell-tower. Considering the possibility of re-installing them, it has been studied the influence of the bell swinging on the dynamic behavior of the bell-tower. In past year, it has been developed similar studies for the case of St. Jakub in Kutná Hora, making possible in this dissertation to compare the dynamic properties of the bell-towers of St. Jakub in Cirkvice and Kutná Hora. It is also applied the MQI method to characterize the masonry quality, however, it will be shown that the stiffness of the studied bell-tower provides results in the calibration that are not considered in the scales of the MQI method, making more necessary if possible to evaluate the results in comparison with other known towers.

1.1 BUILDING DEFINITION AND MAIN HISTORICAL ALTERATIONS

The church itself was built in grey sandstone and covered by yellow and red plaster. These original colors are still visible on one of the openings of the main façade, nowadays closed (Vanková, 2015) (fig. 1.1.1). The church is composed by a rectangular nave 8,00m by 6,00m with a semicircular apse at east and a six stories high bell-tower at west (fig. 1.1.2). The access to the nave is through its main façade south oriented. The backside façade is almost symmetrical to the main one but in this case is not decorated with sculptures. The apse is crowned by a dome and externally by a cone composed by a timber structure and rooftiles. On its behalf, the nave, almost 8,00m high, is covered by a groin vault with the main barrel east-west oriented and two smaller perpendicular barrels with the key point slightly raised. Externally is a gable roof supported by a timber truss structure and covered by plane rooftiles.

Figure 1.1.1. Remains of the original colored plaster.

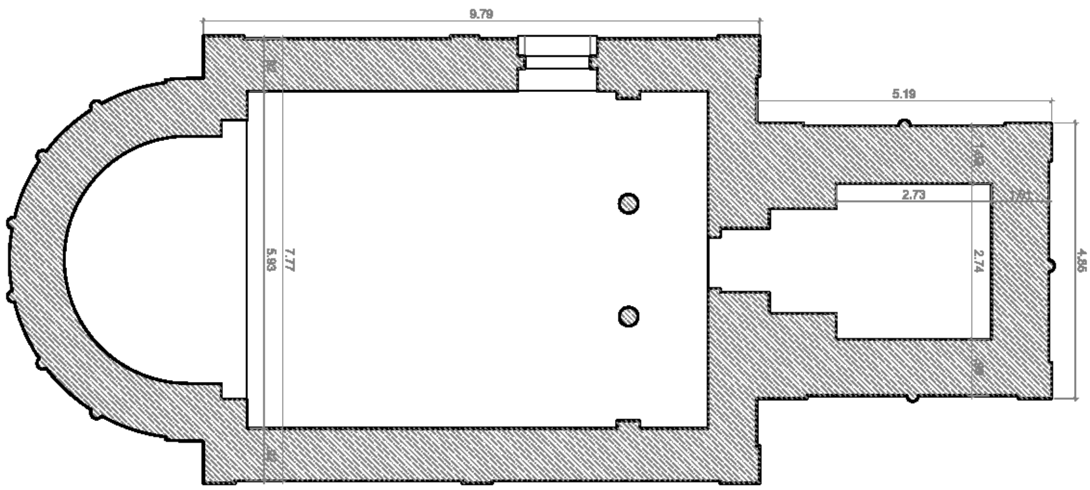


Figure 1.1.2. Ground floor, CAD drawing. Pablo Bañasco, 2018.

Inside, at east, in the semicircular apse is found the altar. In 1874 the existing altar was replaced for a new one. The wooden part of today's altar was made by carver Pavel Cerny, according to the design of Josef Kranner. Along with the altar, frescoes are found in the apse. Their author is Petr Maixner. In the center in the upper part of this apse is painted on a golden background a blessing Christ with Virgin Mary on the right, and Saint John the Baptist on the left. Below, four paintings from the life of St. James (Vanková, 2015) (fig. 1.1.3).

Figure 1.1.3. Altar.

At west, in the second level of the tower, the choir is supported by three arches, with two central columns with Romanesque capitals and bases (fig. 1.1.4a). The shanks of the columns are different, one is smooth and the other is decorated with intertwined ropes (fig. 1.1.4b). There was originally existing a spiral staircase in one of the extremes of this set of arches, however, it was removed after the construction of an outer staircase that accessed the choir externally through the tower (Vanková, 2015). Currently, there is a timber staircase in the interior of the nave.

Figure 1.1.4a & 1.1.4b. 1.1.4a, Choir elevation. 1.1.4b, Columns detail.

The strongest modifications are found in the tower, where the two top levels were modified in the 19th century introducing bigger openings and changing the covering solution. Based on some historical drawings, the original roof followed a more singular onion shape commonly found in the region (fig. 1.1.5a), currently is simply a pyramid shaped timber supported roof. During this period, a major reconstruction under the leadership of Josef Mocker took place in the spirit of Restauro Stilistico, an attempt to restore the historic object to the purity of its original style, taking out even some baroque additions and introducing the current cornices (Vanková, 2015) (fig. 1.1.5b).

Figure 1.1.5a & 1.1.5b. 1.1.5a, historical drawing. 7b, the church tower nowadays.

The tower body, with a squared plan, is composed by six stories; The first level, accessible from the lower part of the choir, is covered by a groin vault and has no vertical accesses. The second level is the choir composed by a barrel vault that ends on a staircase that gives access to the next level. The side walls of this staircase have a slimmer cross-section than the rest of the building, 400mm instead of about 1000mm. The third level is a 3,65m high space with a disorganized timber structure that gives access to the extrados of the nave ceiling and its timber truss structure. The next level is a transition floor to finally reach the last space, a two stories high element with four sides openings composed by three arches and two columns each. In this top part should be found the two bells; a bigger-one dated on 1504 and cast by Ondřej Ptáček, the most significant bell manufacturer from Kutná Hora, and a smaller-one, cast in 1883 and rebuilt in 1917. However, in 1968 the two surviving bells were located on the vault of the second floor of the tower where they are waiting for the construction of a new hanging structure (Vanková, 2015) (fig. 1.1.6).

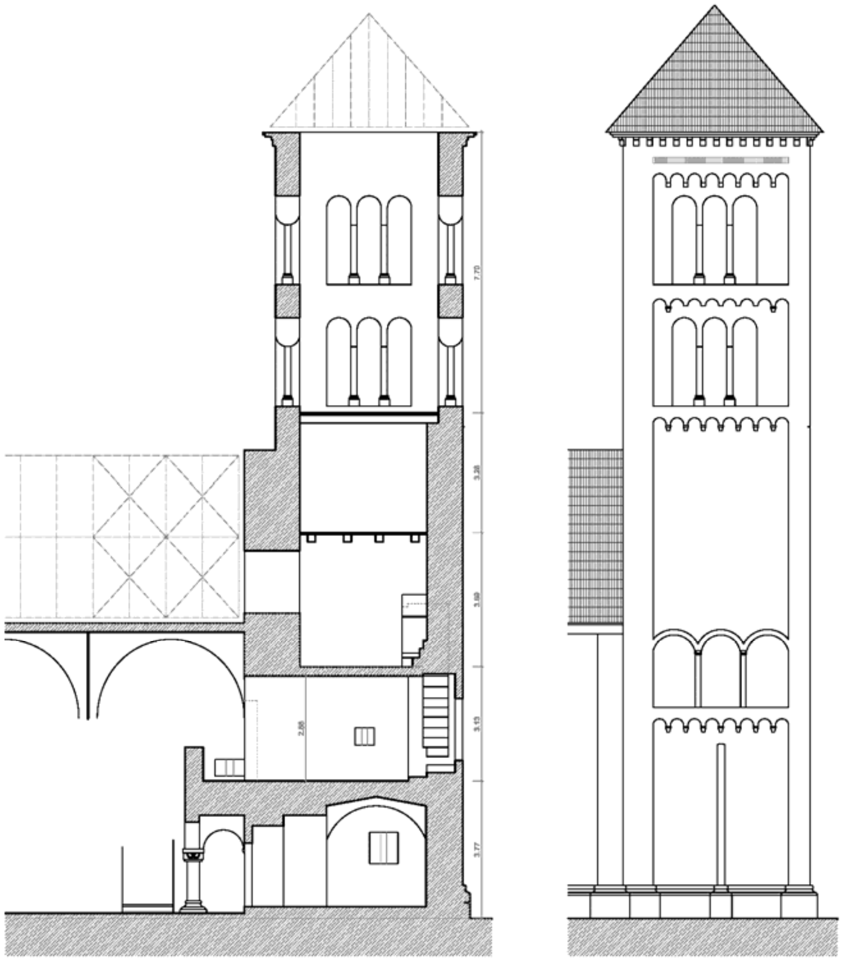


Figure 1.1.6. Longitudinal section & north elevation, CAD drawing. Pablo Bañasco, 2018.

2. LITERATURE REVIEW

The following text aims to point out the main theoretical concepts applied in this dissertation, clarifying them to let the next chapters have a fluent explication of the steps carried out. This dissertation mainly concerns about the definition of the mechanical properties on a masonry bell-tower through the modal testing. Therefore, first, it is necessary to present the concept of Natural Frequencies & Modal Shapes to understand the relation between the mechanical and dynamic properties of the bell-tower. Subsequently, it is pointed out the concepts behind the Structural Modal Testing, a non-destructive method to obtain the natural frequencies of a structure, among other data. Furthermore, since the case study is a masonry building, it is explained the Masonry Quality Index (MQI) developed by Italian researchers, a method to qualitatively characterize mechanical properties of a masonry element. Finally, it is analyzed the forces produced by the bell swinging and its possible influence in the structure due to mechanical resonance.

2.1 NATURAL FREQUENCIES AND MODAL SHAPES

The free vibration of a structure is the mechanical oscillation that occurs at its natural frequency, without any driving force. If the oscillation is not natural is considered a forced vibration.

In the mechanical field, occurs structural resonance when the forced vibration matches the natural frequency of a system. In this situation, relatively small driving forces could produce large amplitude oscillations. Therefore, it is important to study the natural frequencies of the structure and those produced by other secondary elements to avoid mechanical and acoustic resonance that could cause damages in the structure.

Another important effect inherent to any real vibrating system is the damping, the tendency of those systems to reduce the amplitude of oscillation by dissipating energy in each cycle of vibration. Damping in structures is usually represented by its equivalent viscous damping so that the energy dissipated by damping is equivalent to the energy that the system dissipates. This assumption allows a linear differential equation of motion (Eq. 2.1.1).

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = 0 \quad \text{Eq. 2.1.1}$$

Where m is the mass, c is the damping coefficient and k is the stiffness of the system. The displacement is represented by the time dependent equation $u(t)$, then its first derivative is the velocity and the second one is the acceleration. Therefore, it is needed a complementary solution to solve the system (Eq. 2.1.2). Where A and B are constants depending on the initial conditions and ω_n is the natural circular frequency, directly dependent on the stiffness k and inversely dependent on the mass m (Eq. 2.1.3). Since the natural circular frequency is directly dependent on the stiffness k it can be stated that is also directly dependent on the Young's modulus. Stablishing, thus, the two more important mechanical parameters for the analysis of the natural frequencies of a structure. However, as it will be exposed subsequently, the parameters obtained from the structural modal testing are the natural linear frequencies (Eq. 2.1.4).

$$u(t) = A \cos \omega_n t + B \sin \omega_n t \quad \text{Eq. 2.1.2}$$

$$\omega_n = \sqrt{\frac{k}{m}} \quad \text{Eq. 2.1.3}$$

$$f_n = \frac{\omega_n}{2\pi} \quad \text{Eq. 2.1.4}$$

This description, so far expressed in the SDOF-field, must be concluded jumping to the N-DOF-field where it will be an equation of motion for each eigenvalue, meaning as many equations as degrees of freedom (Eq. 2.1.5 & 2.1.6), where ϕ_n is the deformed shape of each mode and $q_n(t)$ provides the modal amplitude. Each system (or structure) has its individual ways of vibrating, called natural modes of vibration, corresponding to harmonic motions and characterized by a modal frequency; Mode 1 (1st bending in X), Mode 2 (1st bending in Y), Mode 3 (Torsion along Z), Mode 4 (2nd bending in X), Mode 5 (2nd bending in Y), etc.

$$u(t) = \phi_n q_n(t) \quad \text{Eq. 2.1.5}$$

$$q_n(t) = A_n \cos \omega_n t + B_n \sin \omega_n t \quad \text{Eq. 2.1.6}$$

2.2 STRUCTURAL MODAL TESTING

The structural modal testing is a non-destructive technique used to achieve and advanced knowledge of a structure. The modal analysis is the only non-destructive way to experimentally measure parameters related with the global structural behavior. It is the field of measuring and analyzing the dynamic response of structures when excited by an input. However, as far as masonry constructions are concerned, the so-called output-only identification technique must be applied. Even if it is called output-only, the real input is the ambient vibration which is a random input.

The aim of the modal testing is to get the Frequency Response Function $H(\omega)$. Defined in frequency domain, corresponds to the ratio between the response of the system and the excitation force. In other words, $H(\omega)$ could be considered the amplification factor of the response of the system when subjected to an external excitation.

The utility of this function es based on the assumption that, near a resonant frequency or a peak of the function, the response is dominated by the resonant mode shape and, consequently, there are no contribution of other non-resonant modes. Once the frequency response function is known it must be applied at least two methods to subtract the natural frequencies and its corresponding modal shape. In frequency domain; Peak Picking, Frequency Domain Decomposition, Enhanced Frequency Domain Decomposition or Polimax. And in time domain; Random Decrement, Recursive Techniques, Maximum Likelihood Methods or Stochastic Subspace Identification Methods.

Modal analysis, in the field of historical structures, may be considered for:

- Evaluation of dynamic characteristics of buildings
- Validation of behavioral models, typically finite elements models, in their elastic range, for successive structural verifications
- Troubleshooting of structures experiencing problems in response
- Monitoring. Control systems for structures
- Checking repair efficiency

2.3 MASONRY QUALITY INDEX

The mechanical behavior of masonry elements depends on many factors, among others:

- Compressive strength (mortar & units)
- Shear strength (mortar & units)
- Units shape
- Volumetric ratio between components
- Wall texture

The definition of the mechanical properties of masonry elements is one of the most important concerns in civil engineering in the field of historical constructions. The quality of masonry has a fundamental role in determining the bearing capacity of a construction. Especially when it is the case of a multi-leave wall, the monolithic behavior acquires a particular importance.

In-situ test always involves an economical investment and sometimes they are not possible to be carried out. Then, it is necessary to assess masonry's properties through a qualitative analysis. The masonry quality index aims to develop a simple and systematic approach to analyze masonry structures. Based on the consideration of the behavior of an "ideal" masonry wall and the mechanical properties of the constituent materials (stones, bricks, mortars, etc.) (Borri et al., 2015).

This method for the analysis and classification of historic masonry is based on the analysis of seven parameters; Mechanical properties and conservation state (SM), Units dimensions (SD), Units shape (SS), Wall leaf connections (WC), Horizontality of bed joints (HJ), Stagger properties of vertical joints (VJ) and Mortar properties (MM). One general parameter, three regarding the units and leaf connection and three regarding the mortar. Based on the criterions exposed in the tables 1 to 7, each parameter must be classified according to three categories; Fulfilled (F), Partially fulfilled (PF) and Not fulfilled (NF). Table 8 assign a numerical value for each category and parameter. These values are implemented in the MQI equation (eq. 2.3.1).

$$MQI = SM * (SD + SS + WC + HJ + VJ + MM) \quad \text{Eq. 2.3.1}$$

With the obtained value from the equation in Table 9 the masonry quality will be categorized in Good quality behavior (A), Average quality behavior (B) or Inadequate quality behavior (C) for three different situations; Vertical actions (V), Out-of-plane actions (O) and In-plane actions

(I). Finally, the graphs and equations presented in fig. 10 of the method, provide upper and lower bounds for the elastic modulus, shear strength and compressive strength.

2.4 MECHANICS OF THE BELL SWINGING

The swinging of bells inside a tower must be considered as an oscillating system with harmonic frequency. According to Ivorra et al., 2006, there are three types of bell swinging systems; Central European, English and Spanish. The Central European system oscillates between a maximum angle while the English one completes full circles alternating the sense of rotation. Both systems are unbalanced, meaning that the center of gravity does not match with the center on rotation. Meanwhile, the Spanish system is better balanced, and the bells can be even anchored to the windows frames, rotating continuously in the same direction. In the case of St. Jakub in Cirkvice, in case of installation of the bells, they would be installed according to the Central European system with a maximum angle of oscillation between 70° and 80°.

It must be studied the bell swinging influence in the dynamic behavior of the bell-tower due to the mechanical resonance phenomenon. This phenomenon, in the case of bell-towers, is the tendency of the system to vary its response with a greater amplitude when the frequency of the bell swinging is similar to the natural frequency of the bell-tower. Considering that the bell swinging system has a harmonic frequency, it must be compared not only the fundamental frequency but also its *n* multiples.

Currently, there are available two approaches. The first one is defined in the German standard DIN 4178: 2005-04 (Bell-towers: calculation and execution). The second one is defined by Ivorra et al., 2006. Both approaches require a great knowledge of the geometrical characteristics of the bell and its swinging system. However, in Lunga & Solar, 2010 (Church-towers and Bell-tower: Campanology, design, disturbance, reconstruction and rehabilitation), there are proposed empirical relations based on the Czech traditions to obtain the parameters needed in the mentioned German standard to calculate the oscillating frequency and the vertical and horizontal forces.

According to the standard DIN 4178, the horizontal and vertical forces resulting from the bell swinging are given by equation 2.4.1 & 2.4.2.

$$H = \frac{m*g}{1+k^2} * \left(2 * \frac{\cos\varphi_0}{\cos\varphi} - 3\right) * \cos\varphi \sin\varphi \quad \text{Eq. 2.4.1}$$

$$V = \frac{m*g}{1+k^2} * (k^2 + 3\cos^2\varphi - 2\cos\varphi \sin\varphi_0) \quad \text{Eq. 2.4.2}$$

Where $k = \frac{i_s}{r}$; the mass radius of inertia or radius of gyration divided by the eccentricity or unbalance of the system. Therefore, it can be also calculated the fundamental period of the harmonic oscillation of the bell applying equation 2.4.3.

$$T_0 = \sqrt{\frac{4\pi^2 * r * (1+k^2)}{g}} \quad \text{Eq. 2.4.3}$$

3. GEOMETRICAL SURVEY

The geometrical survey is a basic requirement of any intervention in the historical and cultural heritage. Its importance has been clearly pointed out by the Italian chart “Carta del Rilievo Architettonico”. A geometrical survey is one of the primitive steps for the knowledge of cultural heritage. It consists on the graphic representation of the built reality, which is performed by a set of activities to comprehend the actual architectural configuration including its dimensional, metrical, historic, structural and constructive characteristics. A good geometrical survey should provide the precise and truthful morphologic and dimensional configuration of the object in study, in its actual physical state. The technical, technological and material acknowledgement of the object helps to understand both, the constructive phases and the actual conditions of alterations and degradation. The ability to edit and update the geometrical survey allows to show past phases and possible future interventions (Jimenez Martín & Pinto Puerto, 2003).

In the case study of St. Jakub in Cirkvice, a traditional geometrical survey methodology has been adopted, meaning that the drawing methods are not based in the photogrammetric restitution. The methodology consisted of the following steps:

A. Preliminary analysis of the basic geometry of the structure to allow taking measurement data since the first visit.
B. Measurements taken with a high precision laser meter and development of the photographic report.
C. Detailed geometrical analysis by free hand drawing of the singular architectural elements, specially the columns and other Romanesque architectural detailing existing in the church.
D. Drawing in CAD. The set of five A3 in scale 1:100 includes:
    - Sheet 1: Plan drawings 1. (Ground floor – Choir level – Upper choir or vaults level – Trusses level)
    - Sheet 2: Plan drawings 2. (Bells level – Bell-tower roof-structure – Bell-tower roof)
    - Sheet 3: Sections & elevations 1. (Transversal bell-tower section – Transversal nave section)
    - Sheet 4: Sections & elevations 2. (Longitudinal section – North elevation)
    - Sheet 5: Sections & elevations 3. (West elevations – South elevation)

The full set of drawings is included in annex in scale 1:100, anyway, in this section are included miniatures of the set of drawings to let the reader have a graphical idea of the information available.

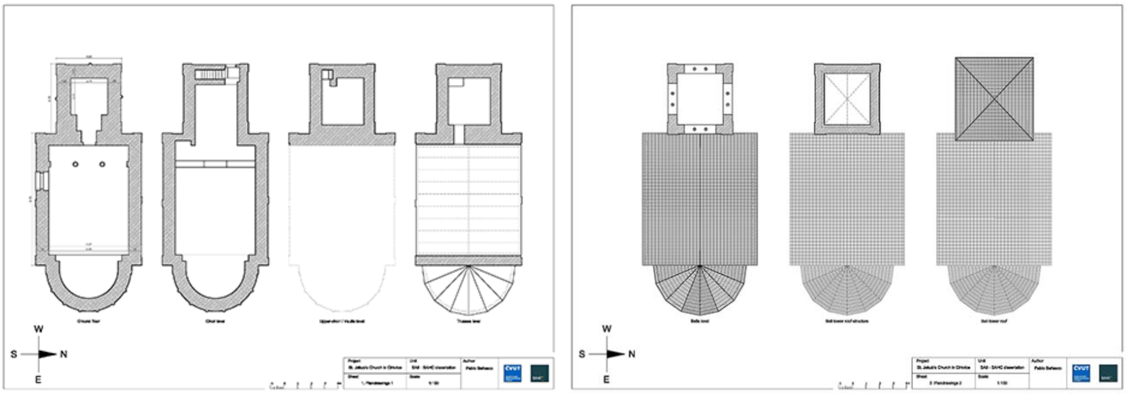


Figure 3.1. Plan drawings. Pablo Bañasco, 2018.

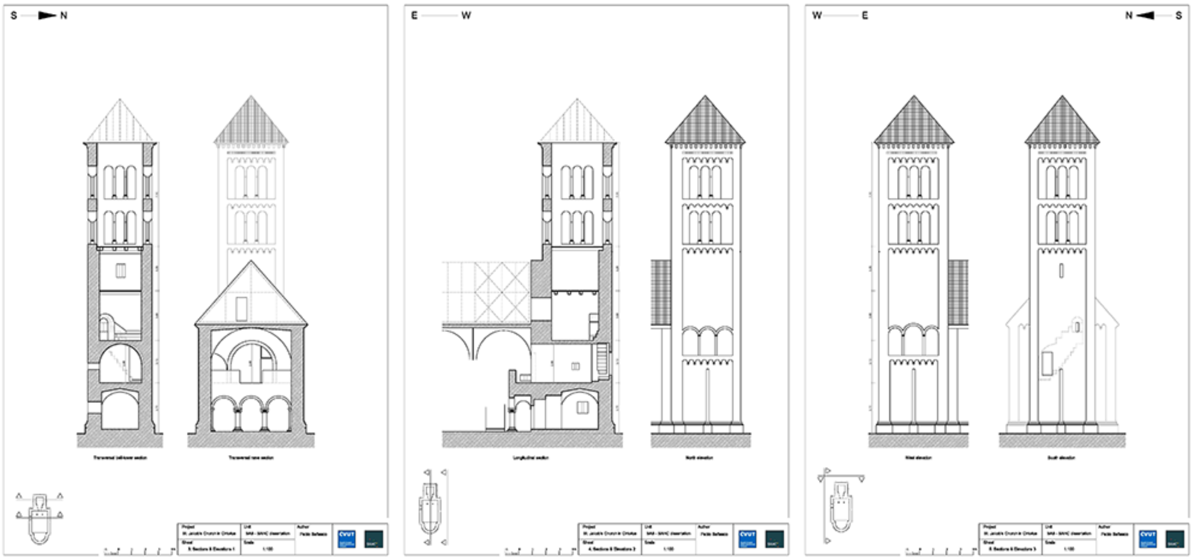


Figure 3.1. Sections & Elevations. Pablo Bañasco, 2018.

4. STRUCTURAL MODELING

The aim of the numerical model developed in this dissertation is to be calibrated through the modal testing to finally obtain a better knowledge about the material properties affecting the general behavior of the bell-tower. This non-destructive method was developed before the dissertation was started so in this analysis it will be only considered in detail the results but not the process to obtain them.

4.1 GEOMETRY SIMPLIFICATION & FE MODELLING

As mentioned, the aim of the numerical model is to study the dynamic behavior of the tower. In this direction, each step and simplification of the reality have been done considering its possible influence in the oscillations of the bell-tower. First, it was modelled the tower itself, implementing those singularities that affect the general behavior in the way shown by the relation between the three first modes of vibration. Initially, it is considered a rigid foundation to simplify the problem in the early stages. Later, it is added nave with a simplification of its timber roof-structure that allows to consider the self-weight of this element. Finally, there are considered some scenarios about the foundation behavior.

A. Bell-Tower

The first level is composed by one-meter thick walls in three sides, only the east wall (nave side) is different with 1,40m thickness. This space is covered by a groin vault. The walls have been modelled with plane and vertical shell elements aligned in the geometrical center of the real wall. Meanwhile, the groin vault is simplified as a plane shell element aligned with the key point of the existing groin vault. There are two openings in this level; one is the entrance from the nave and the other one is the window opening facing south (fig. 4.1.2.1).

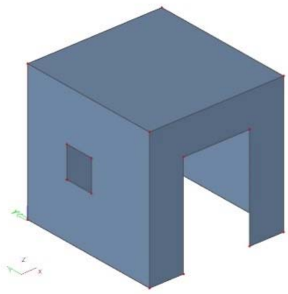


Figure 4.1.2.1. Bell-tower level 1.

Subsequently, in the second level, the thickness of the wall changes from the lower wall. The north and south walls are 0,85m thick, the east one must be considered as 0,67m and at west there is a staircase aligned with the façade. This discontinuity of the geometrical center of the walls makes necessary to connect both levels with a rigid connection (line rigid arm). This level is covered by a barrel vault modelled with a curved shell element aligned with the intrados of the existing vault. This vault makes necessary to model the lateral walls (north and south) in two parts, one above the other, to create nodes in the starting height of the vault and to introduce rigid connections between the wall and the barrel vault.

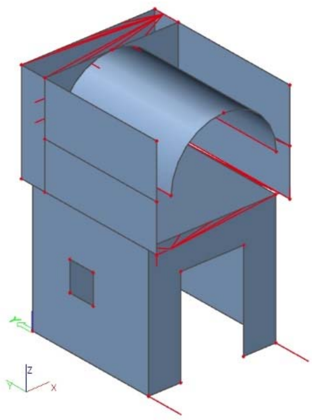


Figure 4.1.2.2. Bell-tower level 2.

According to the SCIA Engineer manual, a rigid link is a displacement and/or rotation dependence between a master node and slaves nodes. The dependence is determined by the position of both nodes and on the selected type of rigid link. The software gives the possibility of inputting hinges in one or both nodes, master and slaves. However, in all the situations in which this connection is necessary in the model of St. Jakub church, it is decided to input a rigid-rigid connection, so either the displacement and the rotations will be identical in both types of nodes.

In this second level occurs one of the singularities of the structure in terms of simplification. It is just the staircase that brings to the next level, aligned with the façade in the west side of this space. This element is sided in west and east by two 0,40m thick walls and it has a free internal space of 0,70m. Everything seems to indicate that the lower part, below the steps, is the continuation of the lower one-meter thick wall. In the upper part is not easy to define the construction technique applied to create the transition between the staircase side walls and the upper wall. Apparently, there are stone pieces settled like lintels that transfer the load from the upper wall (1,00m) to the staircase elements (0,40+0,40m) (fig. 4.1.2.3). At the beginning was considered the possibility of model this area as a continuous wall, without considering the staircase, however, it was concluded that this singularity has influence in the second frequency (bending in Y). In the FE model this reality is simplified implementing two walls of 0,40m continuous in the second level, without considering that the lower wall is continuous up to the steps and that the staircase lands in the next level.

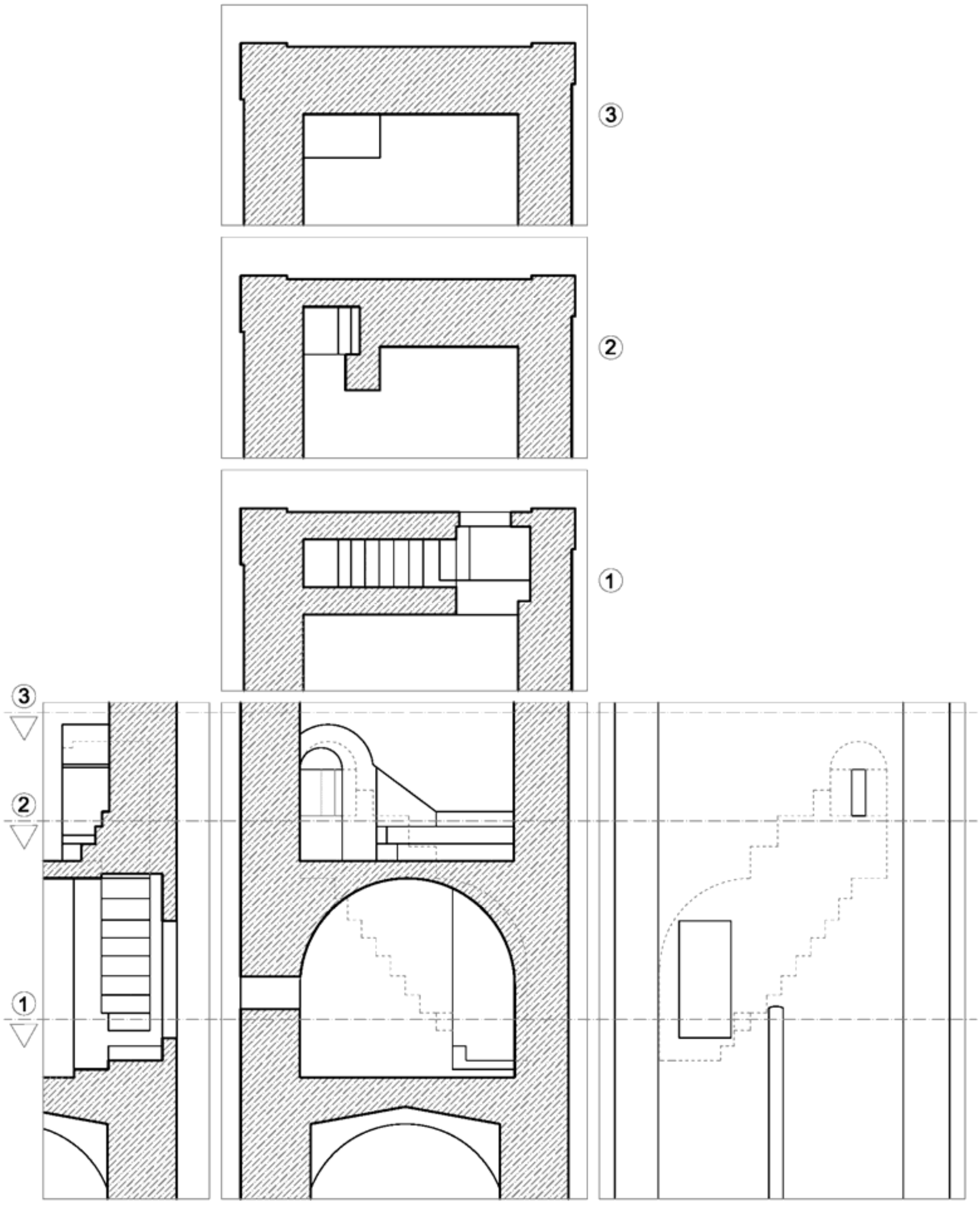


Figure 4.1.2.3. CAD detailing of the staircase in the second level of the bell-tower. Pablo Bañasco, 2018.

In the next two levels, 3rd and 4th, the walls do not change the thickness and there is only one opening in the fourth level facing west. Afterward, in the top part of the bell-tower, all the walls are created with the same thickness, 0,67m, been again necessary to implement the rigid connection between elements. The windows are modelled as rectangular openings with two columns inside, instead of designing the three arches supported by two columns. Finally, a simplification of the roof has been modelled with beam elements (fig. 4.1.2.4).

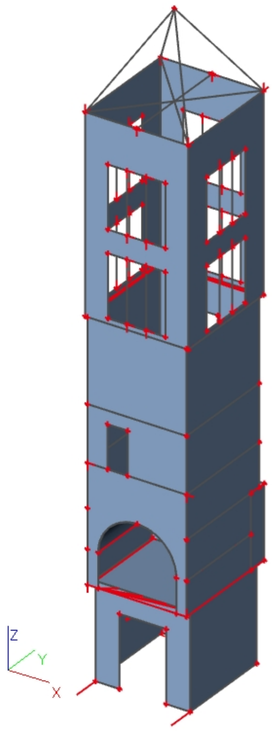


Figure 4.1.2.4 Full bell-tower.

B. Nave

By nave is meant the nave body plus the apse. However, in the purpose of this dissertation, to study the dynamic behavior of the bell-tower, the nave acts as a rigid body influencing the natural oscillations of the studied body along the interface existing between the wall belonging to the nave and the one to the bell-tower. In that regard, the nave has been simplified as much as possible, so the openings and apse are not modelled, however, it is considered that the groin vault helps the box behavior of the nave, so it has been modelled aligned with the intrados of the real element with an accurate shape thanks to the simple tools provided by the selected software, SCIA Engineer. The roof's timber structure has been modelled to provide the self-weight and an appearance closer to the real shape of the real building.

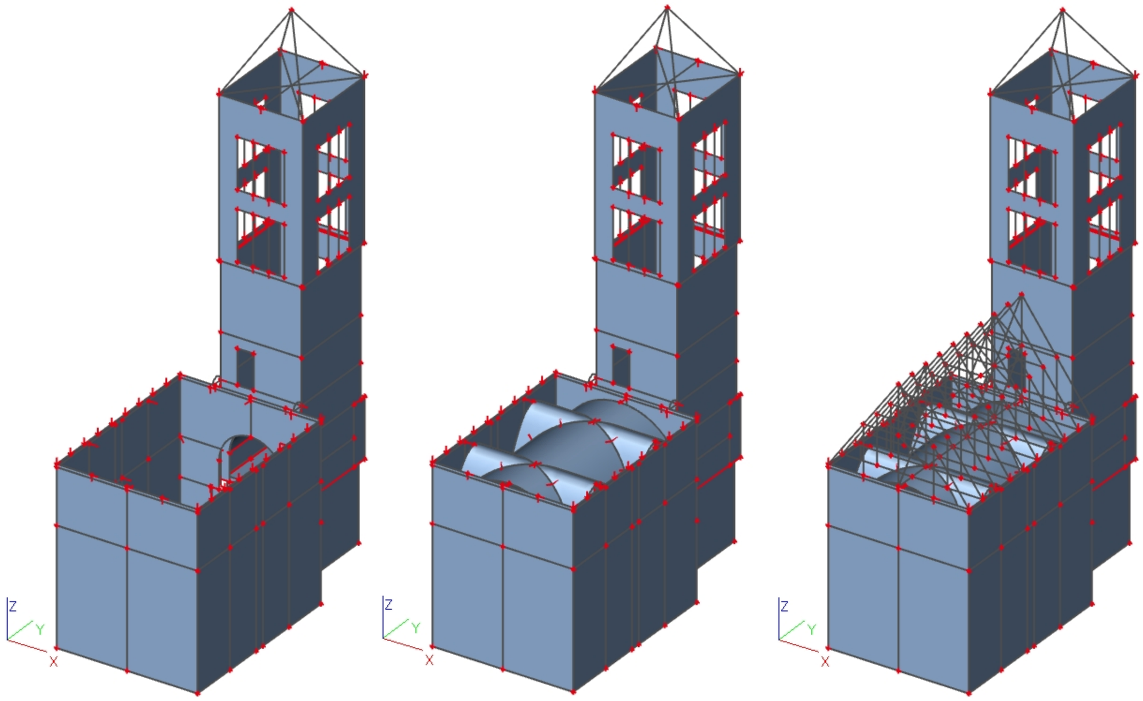


Figure 4.1.2.5. Nave. Left, nave walls. Center, nave vault. Right, Nave timber trusses.

C. Foundation

The main purpose of this dissertation is not to study in detail foundation or soils properties or its influence in the dynamic behavior of the bell-tower. Because of that, no further investigation on foundation characteristics or soil properties were carried out. However, both are factors that influence the oscillations of the bell-tower, so it is necessary to develop some hypothesis or considerations. The foundation has been modelled as a one-meter height continuous element with a width 10% thicker in each side than the existing supported wall. This means a foundation 10cm thicker in each side, total 1,20m and 1,00m height all along the structural elements. On its behalf, the supports are modelled in the manner that it is possible to modify later the stiffness of the soil in the three directions (x, y & z).

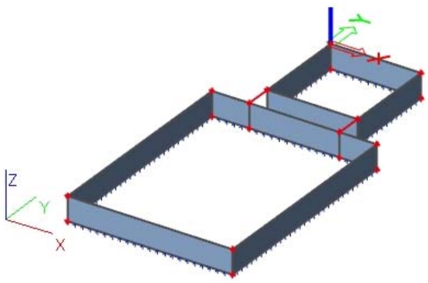


Figure 4.1.2.6. Foundation.

D. Connection between Bell-tower & Nave-body

The interface existing between the nave and the bell-tower has been demonstrated to have a non-linear influence in the bell-tower natural frequencies. However, the non-linear analysis of wall interface is currently a wide field of research making impossible to be studied in this dissertation which is based on the dynamic behavior through a linear structural analysis. In any case, to be able to calibrate the harmonic natural oscillation of the bell-tower it is necessary to model the connection between both bodies. In this study, it is proposed to model this interface with four hypothetical beams acting only with axial forces and whose Young's modulus can be modified independently from other structural materials to calibrate the known natural frequencies.

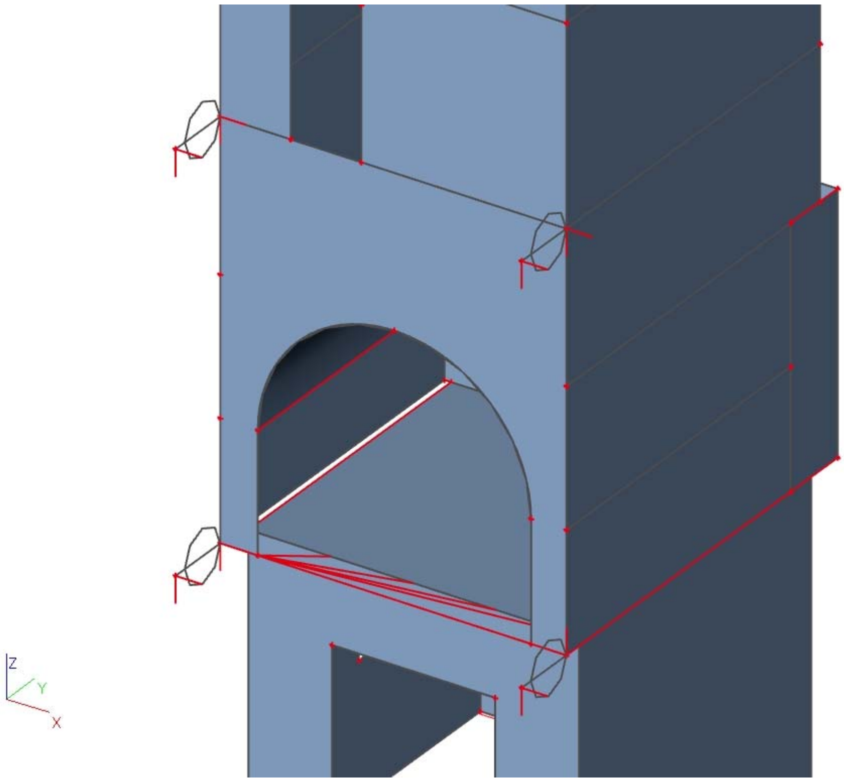


Figure 4.1.2.7. Non-linear interface simplification.

According to the SCIA Engineer manual, from the finite element analysis point of view, the 1D member can act like a standard 1D member or like member acting only with axial forces. The difference is that the standard 1D member can transfer all the internal forces, while the latter variant only acts under tension or compression.

After these simplifications, it is checked that the first three modes provided by the model match the expected modes of vibration; Mode 1 (1st bending in X – fig. 4.1.2.8), Mode 2 (1st bending in Y – fig. 4.1.2.9), Mode 3 (Torsion along Z – fig. 4.1.2.10).

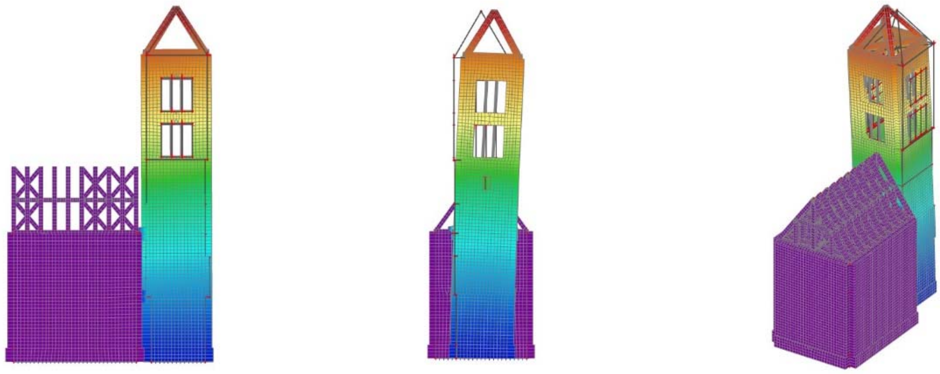


Figure 4.1.2.8. Mode 1, 1st bending in X.

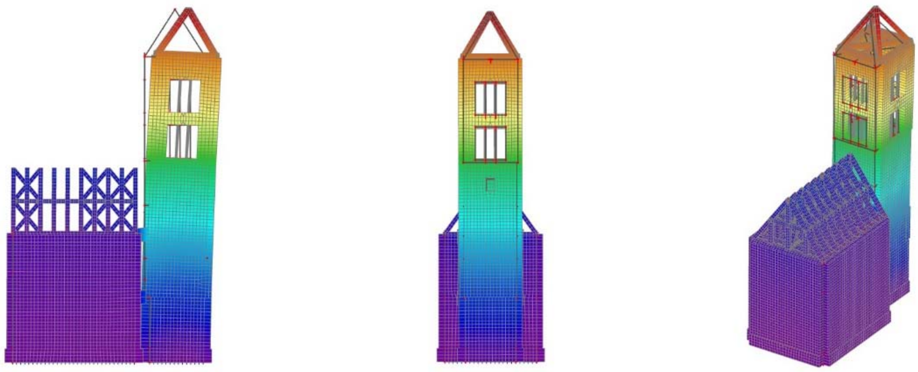


Figure 4.1.2.9. Mode 2, 1st bending in Y.

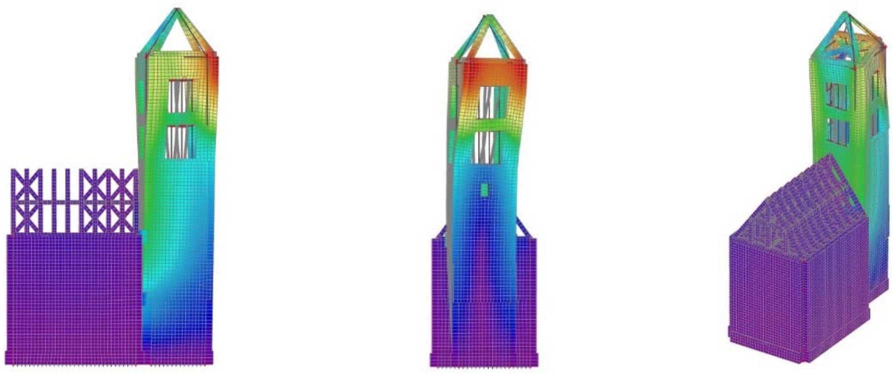


Figure 4.1.2.10. Mode 3, Torsion along Z.

4.2 MATERIAL MECHANICAL PROPERTIES

The church, consisting in a nave with circular apse and a bell-tower with rectangular base, is mainly built with masonry composed by sandstone and natural hydraulic mortar. Internally, is covered by plaster, technique historically applied also externally as it is known due to some remains and based on the documentation analyzed in section 1.1. (Building definition and main historical alterations). The other structural material is timber, used in the supporting structures of the roofs of the nave, apse and tower.

As seen in section 2.1. (Natural frequencies and modal shapes), the most relevant parameters affecting the dynamic behavior of the bell-tower are the mass *m* and the stiffness *k*. This last parameter is, likewise, dependent on the geometry of the element and directly proportional to the Young's modulus *E*.

4.2.1 MASONRY & MQI ANALYSIS

As mentioned above, in-situ test always involves an economical investment and the masonry quality index proposes a simple and systematic approach to analyze masonry structures based on the consideration of the behavior of an "ideal" masonry wall (Borri et al., 2015).

As a qualitative analysis, the estimation of masonry quality requires an in-depth knowledge of historical construction methods. However, sometimes is not possible to classify the parameters with assurance enough. Because of that, in this analysis two of the seven parameters will be considered in two possible scenarios, providing a range of MQI values instead of a single value.

The method is organized in tables that provide comments about the construction techniques need to match with the "ideal" masonry wall. Table 1 (fig. 4.2.1.1) provides Criteria for analysis of stone/brick mechanical properties and conservation state (SM). Apparently, any wall made of "Sandstone or tuff elements" must be classified as "Partially Fulfilled", however, the interventions executed in the 19th century and those more recent occurred in the sixties of the 20th century, provides a good state of conservation with "Un-damaged elements or degraded/damaged elements <10%", that would bring the classification of the parameter SM to the "Fulfilled" level. Based on these two considerations, both levels will be considered in the following.

Table 1 Criteria for analysis of stone/brick mechanical properties and conservation state (SM)

NF	Degraded/damaged elements (>50 % of total number of elements)
	Hollow bricks (solid < 30 %)
	Mud bricks
	Unfired bricks
PF	Presence of degraded/damaged elements (≥10 %, ≤50 %)
	Hollow bricks (55 ≥ solid ≥ 30 %)
	Sandstone or tuff elements
F	Un-damaged elements of degraded/damaged elements <10 %
	Solid fired bricks
	Hollow bricks (55 % < solid)
	Concrete blocks
	Hardstone

Figure 4.2.1.1. Table 1. Criteria for analysis of stone/brick mechanical properties and conservation state (SM). Borri et al., 2015: p. 2649.

Table 2 (fig. 4.2.1.2) addresses Criteria for analysis of stone/brick dimensions (SD). In case of “Co-presence of elements of different dimensions”, the SD parameter should be settled as “Partially Fulfilled”, however, if more than 50% of the units have their larger dimension (stretcher) longer that 40cm, then, the parameter should be considered as “Fulfilled”. In the case of St. Jakub in Cirkvice, there are co-existing units of different dimensions, nevertheless, more than 50% of them are longer that 40cm. Based on these two considerations, both levels will be considered in the following.

Table 2 Criteria for analysis of stone/brick dimensions (SD)

NF	Presence of more than 50 % of elements with large dimension < 20 cm
	Brick bond pattern made of only head joints
PF	Presence of more than 50 % of elements with large dimension 20–40 cm
	Co-presence of elements of different dimensions
F	Presence of more than 50 % of elements with large dimension > 40 cm

Figure 4.2.1.2. Table 2 Criteria for the analysis of stone/brick dimensions (SD). Borri et al., 2015: p. 2650.

Table 3 (fig. 4.2.1.3) stablishes Criteria for the analysis of stone/brick shape (SS). For “Barely cut stones or perfectly cut stones on both masonry leaves”, the parameter SS is classified as “Fulfilled”. In this case of study, are found both situations, barely cut and perfectly cut. Most probably because some of the areas or all of them where constructed to be plastered. Or even because of the co-presence of older and newer units, some from the first ages of constructions and other belonging to the interventions occurred in the last two centuries (fig. 4.2.1.4).

Table 3 Criteria for analysis of stone/brick shape (SS)

NF	Rubble, rounded or pebble stonework (predominant) on both masonry leaves
PF	Co-presence of rubble, rounded or pebble stonework and barely/perfectly cut stone and bricks on both masonry leaves
	One masonry leaf made of perfectly cut stones or bricks
	Masonry made of irregular (rubble, rounded, pebble) stones, but with presence of pinning stones
F	Barely cut stones or perfectly cut stones on both masonry leaves (predominant)
	Brickwork

Figure 4.2.1.3. Table 3 Criteria for the analysis of stone/brick shape (SS). Borri et al., 2015: p. 2652.

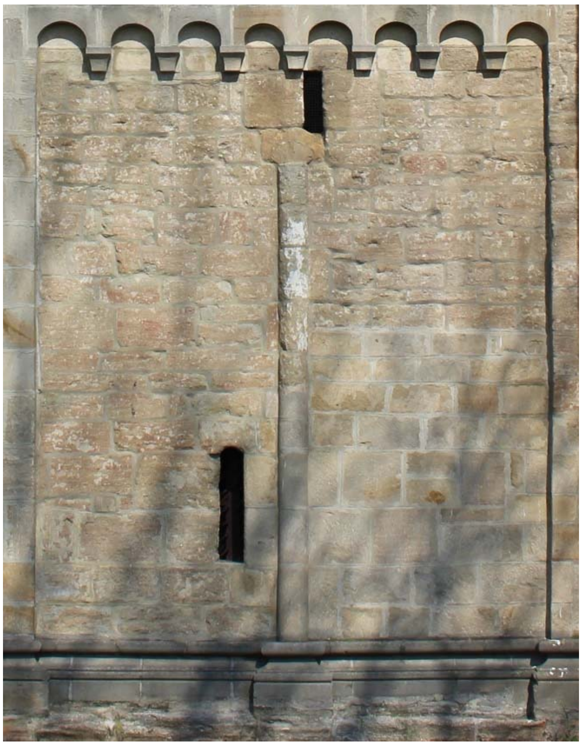


Figure 4.2.1.4. Presence of two different stone finishes.

Table 4 (fig. 4.2.1.5) gives Criteria for analysis of wall leaf connections (WC). This parameter mostly influences the out-of-plane behavior. There are two approaches, quantitative when the wall section is visible and qualitative when it is not. In the case of St. Jakub in Cirkvice, the wall cross-section is not visible, and no complementary non-destructive test has been developed so the qualitative approach must be followed. For double-leaf stone walls the outcome "Partially fulfilled" is assumed when the wall thickness is larger than the stone stretcher and when are present a limited number of headers (2-5 headers each m^2). In this case of study, there are some units that most provably are acting as wall leaf connections, however, they are not found following a systematic organization, and their presence is not enough to consider this parameter as "Fulfilled". In any case, in this order, further in-situ test should be developed to get an in-depth knowledge of the wall leaf connections.

Table 4 Criteria for analysis of wall leaf connections (WC)

	Visible section (quantitative analysis)	Non-visible section (qualitative analysis)
NF	$M_l < 1.25$	Small stones compare to wall thickness
	Small stones (for any M_l value)	No headers
PF	$1.25 < M_l < 1.55$	For double-leaf wall
		Presence of some headers
		Wall thickness larger than stone large dimension
F	$M_l > 1.55$	Wall thickness similar to stone large dimension
		Systematic presence of headers

Figure 4.2.1.5. Table 4 Criteria for the analysis of wall leaf connections (WC). Borri et al., 2015: p. 2652.

Table 5 (fig. 4.2.1.6) settles Criteria for the analysis of horizontality of bed joints (HJ). This parameter may highly affect the lateral and compression strength of a masonry wall panel. In the case of the studied church, even if the rows are not always done with the same unit size, they follow a continuous and horizontal line. This line is eventually interrupted by those units that are supposed to be the wall leaf connection's headers. However, in this analysis this parameter will be considered "Fulfilled" since apparently the criterion of horizontality in the rows is respected in the construction techniques found in the walls of this church.

Table 5 Criteria for analysis of horizontality of bed joints (HJ)

NF	Bed joints not continuous
PF	Intermediate situation between NF and F
	For double-leaf wall
	Only one leaf with continuous bed joints
F	Bed joints continuous
	Stone masonry wall with bricks courses (distance between courses $< 60\,\text{cm}$)

Figure 4.2.1.6. Table 5 Criteria for the analysis of horizontality of bed joints (HJ). Borri et al., 2015: p. 2653.

Table 6 (fig. 4.2.1.7) offers Criteria for the analysis of stagger properties of vertical joints (VJ). As shown in figure 4.2.1.8, the vertical joints in the considered wall does not follow a strict criterion of vertical alignment, however, two consecutive rows rarely present coinciding vertical joints so the VJ parameter is considered as “Partially fulfilled” in this analysis.

Table 6 Criteria for analysis of stagger properties of vertical joints (VJ)

	Quantitative analysis	Qualitative analysis
NF	Single-leaf wall ($M_l < 1.4$)	Aligned vertical joints
	Double-leaf wall ($M_l < 1.4$ for one masonry leaf, $M_l < 1.6$ for the second one)	Aligned vertical joints for at least 2 large stones
	Wall made of very small stones	Solid brick wall made of only headers
PF	Single leaf wall $1.4 < M_l < 1.6$	Partially staggered vertical joints
	Double-leaf wall	(vertical joint between 2 brick is not placed in the middle of adjacent upper and lower brick)
	(a) Both leaves $1.4 < M_l < 1.6$	
	(b) For at least one leaf $M_l > 1.6$	
	(c) First leaf $M_l > 1.6$	
	(d) Second leaf $1.4 < M_l < 1.6$	
F	Single leaf wall $M_l > 1.6$	Properly staggered vertical joints
	Double-leaf wall (both leaves $M_l > 1.6$)	(vertical joint between 2 stones is placed in the middle of adjacent upper and lower stone)

Figure 4.2.1.7. Table 6 Criteria for the analysis of stagger properties of vertical joints (VJ). Borri et al., 2015: p. 2654.

Figure 4.2.1.8. Vertical joints analysis.

Finally, table 7 (fig. 4.2.1.9) proposes Criteria for analysis of mortar properties (MM). The quality of the bonding between mortar and stones/bricks influences the load transferring along the wall and subsequently the general behavior of the building. In this case of study, the mortar seems to be in good quality and generally is not degraded, furthermore, the bed joints have a homogeneous thickness. In some areas, seems that the so-called repointing technique was applied, however, to conclude this, further information should be collected in-situ. In any case, the load transferring along the units through the mortar joints seems to be guaranteed so in this analysis the parameter MM will be considered as “Fulfilled”.

Table 7 Criteria for analysis of mortar properties (MM)

NF	Very weak mortar, dusty mortar with no cohesion No mortar (rubble or pebble stonework)
	Large bed joints made of weak mortar (thickness comparable to stone/brick thickness) Porous stones/bricks with weak bonding to mortar
PF	Medium quality mortar, with bed joints not largely notched
	Masonry made of irregular (rubble) stones and weak mortar, but with presence of pinning stones
F	Good quality and non-degraded mortar, regular bed joint thickness or large bed joint thickness made of very good quality mortar Masonry made of large perfectly cut stones with no mortar or very thin bed joint thickness

Figure 4.2.1.9. Table 7 Criteria for the analysis of mortar properties (MM). Borri et al., 2015: p. 2654.

Table 8 assigns numerical values to each parameter, also depending on the loading case considered; Vertical loading (V), Horizontal in-plane loading (I) and Horizontal out-of-plane loading (O) (figure 4.2.1.10). This numerical correlation of the parameters allows to apply the equation $MQI = SM * (SD + SS + WC + HJ + VJ + MM)$ that provide an MQI total value (figure 4.2.1.11). In this analysis, since one founds some uncertainty in two parameters (SM & SD), in figure 4.2.1.11 are presented the four possibilities considering two options for each of those parameters.

MQI (numerical values)		Vertical loading (V)	Horizontal in-plane loading (I)	Horizontal out-of-plane loading (O)
SM	Partially fulfilled	0.7	0.7	0.7
	Fulfilled	1	1	1
SD	Partially fulfilled	0.5	0.5	0.5
	Fulfilled	1	1	1
SS	Fulfilled	3	2	2
WC	Partially fulfilled	1	1	1.5
HJ	Fulfilled	2	1	2
VJ	Partially fulfilled	0.5	1	0.5
MM	Fulfilled	2	2	1

Figure 4.2.1.10. Numerical correlation for the MQI parameters given by table 8 in Borri et al., 2015: p. 2655. Note that parameters SM & SD are considered in two possible scenarios so subsequently four values of MQI will be given for each load case (V, I & O).

MQI (total value)		Vertical loading (V)	Horizontal in-plane loading (I)	Horizontal out-of-plane loading (O)
SM Partially fulfilled	SD Partially fulfilled	6.30	5.25	5.25
	SD Fulfilled	6.65	5.60	5.60
SM Fulfilled	SD Partially fulfilled	9.00	7.50	7.50
	SD Fulfilled	9.50	8.00	8.00

Figure 4.2.1.11. MQI values for the four possible scenarios given by the equation $MQI = SM * (SD + SS + WC + HJ + VJ + MM)$, Borri et al., 2015: p. 2651.

Applying the equations given in figure 10 (Borri et al., 2015: p. 2658) is possible to calculate upper and lower bounds for the compressive strength, shear strength and Young's modulus. Because of the purpose of this dissertation is the analysis of the dynamic behavior of the bell-tower and in this field the most influencing mechanical property of this three is the Young's modulus, here it is presented only this last parameter (figure 4.2.1.12). These values of the elastic modulus are the aim of the application of the MQI method in this study. They are presented in a graph distinguishing between Young's modulus corresponding to a Vertical loading case and those for the Horizontal loading case. It possible to graphically analyze the influence of the MQI parameters SM & SD that provide a range of Young's Modulus from 1,36GPa to almost 4GPa, a great difference (fig. 4.2.1.13). This graphical representation will allow in the next sections to compare the results from the MQI method with the calibration of the model based on the experimental modal testing.

Young's modulus (MPa)			Vertical loading (V)		Horizontal in-plane loading (I)		Horizontal out-of-plane loading (O)	
SM Partially fulfilled	SD Partially fulfilled	Upper bound	A	2299	C	1937	F	1937
		Lower bound	B	1639	D	1366	G	1366
	SD Fulfilled	Upper bound	H	2434	J	2051	L	2051
		Lower bound	I	1742	K	1451	M	1451
SM Fulfilled	SD Partially fulfilled	Upper bound	O	3574	Q	2797	S	2797
		Lower bound	P	2620	R	2019	T	2019
	SD Fulfilled	Upper bound	U	3878	W	3035	Y	3035
		Lower bound	V	2858	X	2202	Z	2202

Figure 4.2.1.12. Young's Modulus calculation based on the equations given by Figure 10 Borri et al., 2015: p. 2658.

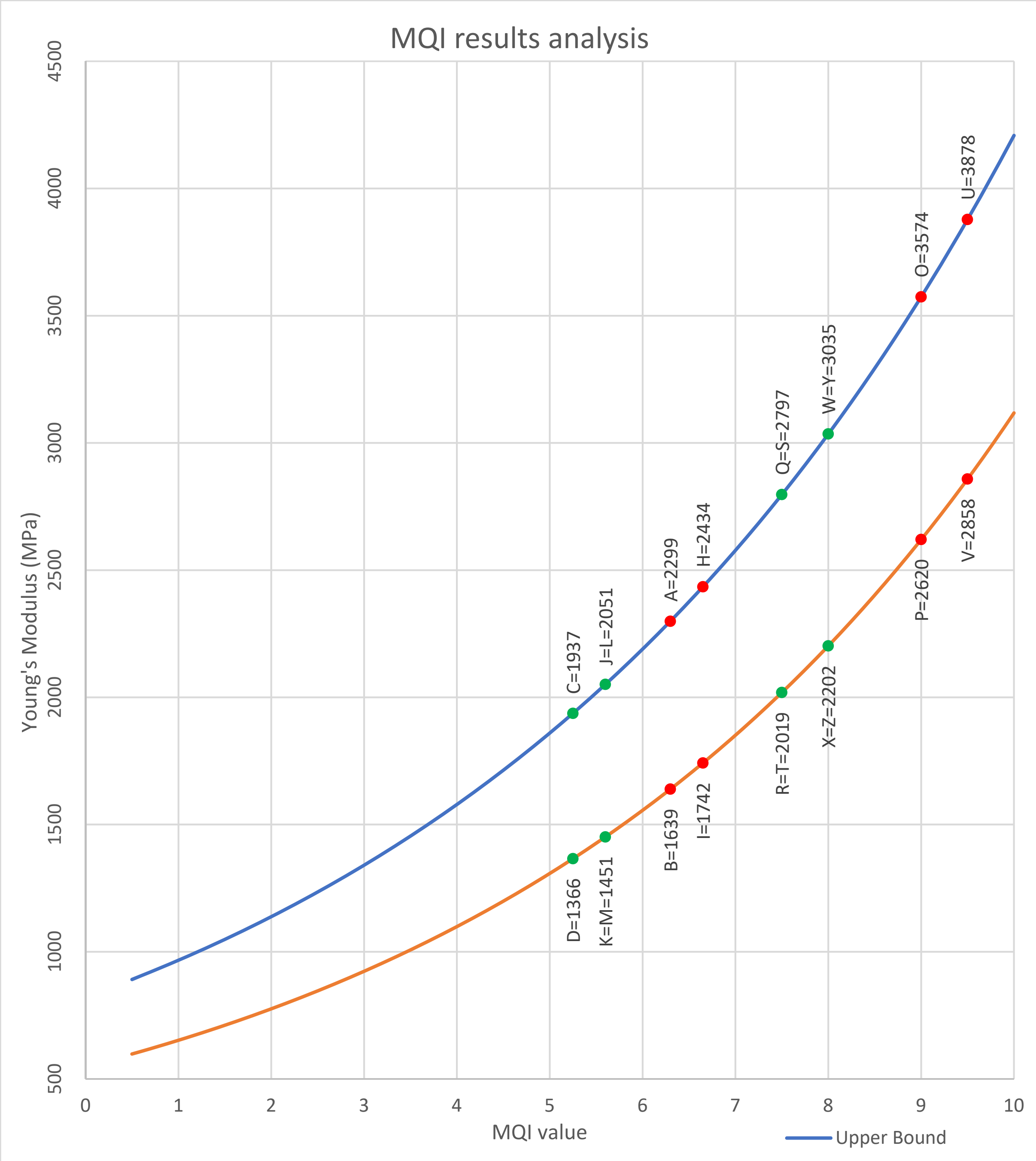


Figure 4.2.1.13. Young's modulus vs. MQI. The graph represents in blue the Upper Bound ($E=821,24*e^{0,1634x}$) and in orange the Lower Bound ($E=548,31*e^{0,1738x}$), where "x" is the MQI total value. Meanwhile, in red are represented the results for the Vertical loading case (V) and in green the results for the Horizontal in-plane & out-of-plane loading case (I & O).

4.2.2 TIMBER

The modelled timber structures are found in the roof support of the nave and the bell tower. These elements are not influencing the first three modes of vibration further than with its self-weight. Then, in the case of timber mechanical properties, the determinant parameter is the density and not the Young's modulus. In the FE model it is settle as "Material type: Timber", density = 530 kg/m^3 and E = 9,00 GPa.

4.3 MODAL TESTING & FE MODEL CALIBRATION

Since the MQI method was published, the procedure widely applied is to obtain the masonry quality index method to know upper and lower bounds for the mechanical properties related to the existing walls. This procedure was successfully applied in the previous case study of St. Jakub in Kutná Hora. However, the geometry and dynamic properties of the bell-tower of St. Jakub in Cirkvice do not allow to follow that procedure. This is because the necessary Young's modulus to match the experimentally obtained natural frequencies is higher in most of the cases than any value considered in the equations provided by the MQI.

Due to this fact, in this section, the mechanical properties needed to calibrate the model are considered independently from MQI results. The study and comparison of the results it is provided in the next chapter (5. Results analysis).

4.3.1 EXPERIMENTAL MODAL TESTING RESULTS

The natural linear frequencies extracted from the in-situ experimental modal testing are those corresponding to the first three modal shapes; 1st mode: Bending of the tower in the axis perpendicular to the nave longitudinal axis. 2nd mode: Bending of the tower in the axis parallel to the nave longitudinal axis. 3rd mode: Twisting of the tower.

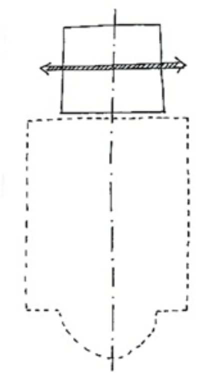
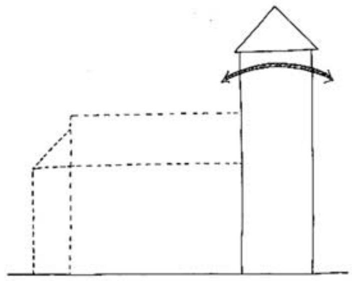
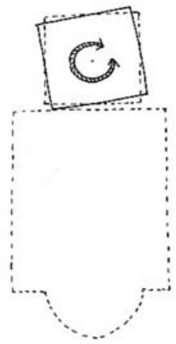
MODE 1	MODE 2	MODE 3
Bending of the tower in the axis perpendicular to the nave longitudinal axis (X)	Bending of the tower in the axis parallel to the nave longitudinal axis (Y)	Torsion along Z
2,38 Hz	**2,88 Hz**	**6,15 Hz**
		

Figure 4.3.1.1. Experimental modal testing results provided by Doc. Ing. Petr Fajman, CSc.

4.3.2 FE MODEL CALIBRATION

Given the model with the geometry simplifications and assumptions exposed in section 4.1 (Geometry simplification and FE modelling), it is necessary to know how each parameter or assumption influences the calibration of the natural frequencies in the FE model. Recalling some concept already exposed above, the circular natural frequency and, consequently, the linear natural frequency, is inversely dependent on the mass m and directly dependent on the stiffness k, $\omega_n = \sqrt{\frac{k}{m}}$, $f_n = \frac{\omega_n}{2\pi}$.

Thus, the mass of the sandstone wall is one of the most determinant parameters and the values chosen in this dissertation are based on laboratory testing developed on a CTU PhD dissertation, it presents a table where are given density values for different quarries near Cirkvice and Kutná Hora (Kott, 2012). In the model calibration this parameter will be fixed considering two situations; a lower scenario: density = 2100 kg/m^3 (solutions 1 to 6), and a higher scenario: density = 2350 kg/m^3 (solutions 1' to 6'). Later, this range of solutions will be compared with the MQI method and with dynamic properties of other bell-towers to evaluate the assumptions.

In the case of the stiffness, will only be modified the Young's modulus to calibrate the frequencies, since the geometry will be similar in all scenarios. This parameter modifies all the frequencies; however, it will be used to match, in first term, the Mode 1 of vibration because it is not deeply influenced by the presence of the nave or the support properties. The Mode 2 of vibration, in the case of St. Jakub in Cirkvice, is more dependent on the interface between the nave and the tower, besides the Young's modulus. The calibration of the mechanical properties of the modelled springs basically controls the relation between Mode 1 & 2.

The third mode of vibration is typically the most difficult one to be calibrated. In this analysis one has developed a hypothesis in which is possible to get a closer calibration for Mode 3. Finally, after several trials, the only way to control the torsion along the axis Z has been found in the stiffness of the supports in X & Y direction. This would be the case of horizontal sliding. However, if flexibility in the support is given to the whole building in the three directions is not possible to calibrate the third mode. Then, the developed hypothesis consists in allow the sliding only in the bell-tower, providing full rigid foundation in the nave. The results from this situation must be treated as such hypothesis and evaluated with further in situ tests. However, it is observed that any consideration of the soil behavior and its influence in the third mode has strong impact in the values of the Young's modulus.

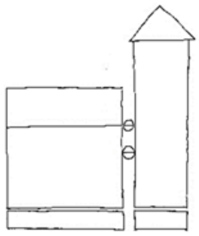
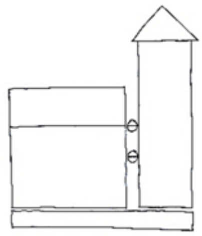
When analyzed the results of the calibration, the mechanical properties of the springs representing the interface nave-tower and the stiffness of the supports in X & Y directions must be understood as both; an input and a result of calibrating the model. However, they represent two complex fields of structural analysis and it is not the aim of this dissertation to study these concepts into detail. Probably, it would be interesting to develop further non-linear analysis combined with soil studies and other NDT's in future SAHC's dissertations.

The construction technique applied in the foundation, besides the depth of the element, could have also an influence on the dynamic behavior of the tower. On one hand, the foundation could be considered as a thicker continuation of the walls, as it is modelled. On the other hand, when the foundation was built, it could have been done as a continuous element. In this case, the deformations of the foundation belonging to the tower and the nave must be same in the point of their connection. According to this, one has modelled a rigid connection between the two sides of the foundation to study the influence of this consideration in the results of the calibration.

Based on the characteristics of the developed FE model, there is more than one final solution for the key parameters determinants of the dynamic behavior. Below, are presented those solutions organized according to the possible assumptions on the soil behavior. 1 & 4: full rigid foundation. 2 & 5: flexible in Z axis and rigid in X & Y. 3 & 6: rigid in Z and flexible in X & Y with full rigid foundation of the nave. Furthermore, in solutions 1 to 3 it is considered that there is an independent behavior in the foundations Nave-Tower, and, in solutions 4 to 6 the two sides of the foundations are connected through a rigid arm. Meanwhile, in solutions 1 to 6 is considered density equal to 2100 kg/m^3 and in solutions 1' to 6' the density is 2350 kg/m^3. For each solution is presented the FEM frequencies of the first three modes as a prove of calibration and accuracy of the result. The Young's modulus of the springs representing the interface shows how determinant is the interface in the global dynamic behavior and, more precisely, in the second mode of vibration. Finally, the Young's modulus of the masonry elements is the parameter that must be understood as a result of the calibration process (fig. 4.3.2.1 & 4.3.2.2).

In solutions 1, 2, 4 & 5, in both considerations of the density value, as it is considered the foundation to be rigid in X & Y, is not possible to calibrate the Mode 3. This make this group of solutions less faithful, however, is possible to extract that, in solutions 1 & 4, the third frequency is closer to the desired value (around 7,00 Hz from the desired 6,15 Hz) meaning that the behavior of the soil could be close to the rigid case. Meanwhile, considering the deformation of the soil only in the vertical direction, solutions 2 & 5, brings the third frequency to a value too far from the desired one to consider these results as reliable.

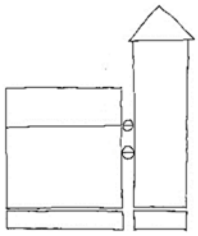
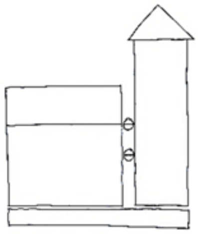
In solutions 3 & 6, it is possible to approximate the frequency of the Mode 3. This make this group of solutions more reliable, in any case, must be always reminded that this hypothesis should be further studied with in situ studies. It is noted that the consideration for the relation of the foundation relation nave-tower, with less influence in the previous cases, now has a determinant impact in the calibration, demonstrating the importance of further studies about the soil and foundation characteristics.

Independent foundations Nave-Tower			Connected foundations Nave-Tower		
					
1 Full rigid foundation			4 Full rigid foundation		
Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
2,37 Hz	2,87 Hz	6,98 Hz	2,38 Hz	2,88 Hz	7,00 Hz
Masonry	E = 3,55 GPa		Masonry	E = 3,55 GPa	
Interface (springs)	E = 200 MPa		Interface (springs)	E = 200 MPa	
2 Flexible foundation A Kz = 1400 MN/m² K x & y = rigid			5 Flexible foundation A Kz = 1400 MN/m² K x & y = rigid		
Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
2,38 Hz	2,88 Hz	8,01 Hz	2,38 Hz	2,89 Hz	7,76 Hz
Masonry	E = 4,95 GPa		Masonry	E = 4,60 GPa	
Interface (springs)	E = 85 MPa		Interface (springs)	E = 85 MPa	
3 Flexible foundation B Kz = Rigid K x & y = 40 MN/m²			6 Flexible foundation B Kz = 1400 MN/m² K x & y = 45 MN/m²		
Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
2,38 Hz	2,91 Hz	6,18 Hz	2,38 Hz	2,85 Hz	6,88 Hz
Masonry	E = 4,10 GPa		Masonry	E = 3,70 GPa	
Interface (springs)	E = 90 MPa		Interface (springs)	E = 85 MPa	

Results from modal testing		
Mode 1	Mode 2	Mode 3
2,38 Hz	2,88 Hz	6,15 Hz

Figure 4.3.2.1. Density = 2100 kg/m^3. Possible solutions of FEM calibration through the modal testing in St. Jakub, Cirkvice.

The provided range on density, from 2100 to 2350 kg/m^3, affects the calibration of the Young's modulus in almost half GPa (3,55 to 3,97 and 4,10 to 4,45) being less necessary develop further in situ test to evaluate the density than the soil properties and foundation behavior.

Independent foundations Nave-Tower			Connected foundations Nave-Tower		
					
1' Full rigid foundation			4' Full rigid foundation		
Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
2,38 Hz	2,89 Hz	6,96 Hz	2,38 Hz	2,89 Hz	6,99 Hz
Masonry	E = 3,97 GPa		Masonry	E = 3,97 GPa	
Interface (springs)	E = 270 MPa		Interface (springs)	E = 200 MPa	
2' Flexible foundation A Kz = 1400 MN/m² K x & y = rigid			5' Flexible foundation A Kz = 1400 MN/m² K x & y = rigid		
Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
2,38 Hz	2,90 Hz	8,13 Hz	2,38 Hz	2,89 Hz	7,83 Hz
Masonry	E = 5,70 GPa		Masonry	E = 5,25 GPa	
Interface (springs)	E = 100 MPa		Interface (springs)	E = 90 MPa	
3' Flexible foundation B Kz = Rigid K x & y = 50 MN/m²			6' Flexible foundation B Kz = Rigid K x & y = 30 MN/m²		
Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
2,37 Hz	2,88 Hz	6,24 Hz	2,37 Hz	2,87 Hz	6,87 Hz
Masonry	E = 4,45 GPa		Masonry	E = 4,1 GPa	
Interface (springs)	E = 100 MPa		Interface (springs)	E = 150 MPa	

Results from modal testing		
Mode 1	Mode 2	Mode 3
2,38 Hz	2,88 Hz	6,15 Hz

Figure 4.3.2.2. Density = 2350 kg/m^3. Possible solutions of FEM calibration through the modal testing in St. Jakub, Cirkvice.

5. RESULTS ANALYSIS

As seen in the preceding chapter, in this dissertation two methodologies have been developed to obtain the mechanical properties of masonry elements, MQI method and FEM calibration. However, facing difficulties to introduce directly the obtained values of MQI method in the FE model, now both results must be compared among them and with other studied bell-towers. Finally, it is studied too the forces generated by the bell swinging in case it is decided to restore and reinstall them in the bell-tower.

5.1 COMPARISON BETWEEN MQI & MODAL TESTING

The aim of compare the results from MQI method with those of the FE model calibration through the modal testing is to analyze the reliability of the obtained values for Young's modulus and to evaluate the MQI analysis performed in section 4.2.1 Masonry & MQI analysis.

The initial purpose of the MQI, mostly due to the high seismicity of Italy, is to provide a fast and easy method to obtain mechanical properties of masonry walls without the economical investment and time consuming needed in the non-destructive and minor-destructive tests. Also, it is an approach that must give an answer to any kind of masonry buildings, from good state of conservation to damaged structures, from very refined construction technologies to those traditional and local.

An Italian research group published their results as a paper named Masonry walls with irregular texture of L'Aquila (Italy) seismic area: validation of a method for the evaluation of masonry quality (Rovero et al. 2016). It shows the comparison between the MQI method and the Flat-jack MDT that provides a reliable value for compressive strength and Young's modulus. In this paper are selected 5 types of masonry walls to implement on them the MQI method. Then, in three of those walls was applied the Flat-jack MDT. The values for Young's modulus resulted to be between the upper and lower bound expressed by the MQI method and they compared those values in terms of percentage with the average within this to values (figures 5.1.1 & 5.1.2)

MQI values	Vertical loading (V)	Horizontal in-plane loading (I)	Horizontal out-of-plane loading (O)
Fj C1 (Mu1 type)	4	4	3.5
Fj S9 (Mu1 type)	4	4	3.5
Fj S10 (Mu4 type)	2	2	1.5
Fj S21 (Mu5 type)	4.5	5.5	4

Figure 5.1.1. MQI values for the three masonry types compared with flat-jack test (Rovero et al., 2016).

Young's modulus (MPa)	MQI		Flat-jack	% to average
Fj C1 (Mu1 type)	Upper bound	1578.79	1437	-6.83%
	Lower bound	1098.87		
Fj S9 (Mu1 type)	Upper bound	1578.79	1236	8.32%
	Lower bound	1098.87		
Fj S10 (Mu4 type)	Upper bound	1138.67	990	-3.29%
	Lower bound	776.22		
Fj S21 (Mu5 type)	Upper bound	1713.20	1518	-4.09%
	Lower bound	1198.64		

Figure 5.1.2. Young's modulus comparison. (Rovero et al., 2016).

$$\% = \frac{Average\ theoretical\ value\ for\ MQI - Flat\ jack\ experimental\ result}{Flat\ jack\ experimental\ result}$$

In other order, in Borri et al., 2015, their method is compared with the values given by the Italian standard, where sometimes are higher or lower than the equations provided, meaning that is reliable to obtain results out of the expressed range. In this dissertation, the results obtained from the FEM calibration are compared with the theoretical expected values from the MQI to evaluate the structural modelling assumptions.

To correlate the results from MQI with the FEM calibration, it is applied the same approach considered in Rovero et al., 2016, as above mentioned, for the comparison among the theoretical and experimental results. In the mentioned validation of the MQI, the Italian research group obtain the results of the flat-jack between the proposed upper and lower bound, so they compare that result with the average between both limits. However, here most of the values are above the upper bound so, probably, would be more appropriate to correlate the results only with the upper bound. In any case, in figure 5.1.3 & 5.1.4, both relations are expressed.

Young's modulus (MPa)			Vertical loading (V)		FE Calibration result		% to average	% to Upper bound
SM Fulfilled	SD Fulfilled	Upper bound	U	3878	**E1**	3550	-5.12%	9.24%
		Lower bound	V	2858				
SM Fulfilled	SD Fulfilled	Upper bound	U	3878	**E4**	3550	-5.12%	9.24%
		Lower bound	V	2858				
SM Fulfilled	SD Fulfilled	Upper bound	U	3878	**E2**	4950	-31.96%	-21.65%
		Lower bound	V	2858				
SM Fulfilled	SD Fulfilled	Upper bound	U	3878	**E5**	4600	-26.78%	-15.69%
		Lower bound	V	2858				
SM Fulfilled	SD Fulfilled	Upper bound	U	3878	**E3**	4100	-17.85%	-5.41%
		Lower bound	V	2858				
SM Fulfilled	SD Fulfilled	Upper bound	U	3878	**E6**	3700	-8.97%	4.82%
		Lower bound	V	2858				
SM Fulfilled	SD Fulfilled	Upper bound	U	3878	**E1'**	3970	-15.16%	-2.31%
		Lower bound	V	2858				
SM Fulfilled	SD Fulfilled	Upper bound	U	3878	**E4'**	3970	-15.16%	-2.31%
		Lower bound	V	2858				
SM Fulfilled	SD Fulfilled	Upper bound	U	3878	**E2'**	5700	-40.91%	-31.96%
		Lower bound	V	2858				
SM Fulfilled	SD Fulfilled	Upper bound	U	3878	**E5'**	5250	-35.84%	-26.13%
		Lower bound	V	2858				
SM Fulfilled	SD Fulfilled	Upper bound	U	3878	**E3'**	4450	-24.31%	-12.85%
		Lower bound	V	2858				
SM Fulfilled	SD Fulfilled	Upper bound	U	3878	**E6'**	4100	-17.85%	-5.41%
		Lower bound	V	2858				

Figure 5.1.3. Correlation for Young's modulus obtained from MQI and from FEM calibration. E1 to E6; density = 2100 kg/m^3. E1' to E6'; density = 2350 kg/m^3.

$$\%\, from\ average = \frac{Average\ theoretical\ value\ for\ MQI\ -\ FEM\ calibration\ result}{FEM\ calibration\ result}$$

$$\%\, from\ Upper\ bound = \frac{Upper\ bound\ theoretical\ value\ for\ MQI\ -\ FEM\ calibration\ result}{FEM\ calibration\ result}$$

In the MQI analysis developed in previous sections some uncertainties where found. In the case of stone/brick mechanical properties and conservation state (SM), based on the given criteria was possible to consider the parameter as Partially Fulfilled or Fulfilled. Also, in the case of stone/brick dimensions (SD), was possible to understand the parameter as Partially Fulfilled or Fulfilled based on the co-presence of different dimensions or in the percentage of large units. In this table and graph, it is decided to compare the results in terms of percentages with those extracted from the MQI (V) = 9,5 (see figure 4.2.1.11), however, is not possible to conclude that the SM & SD parameters should be considered in both cases as Fulfilled without further studies on the walls of St. Jakub in Cirkvice.

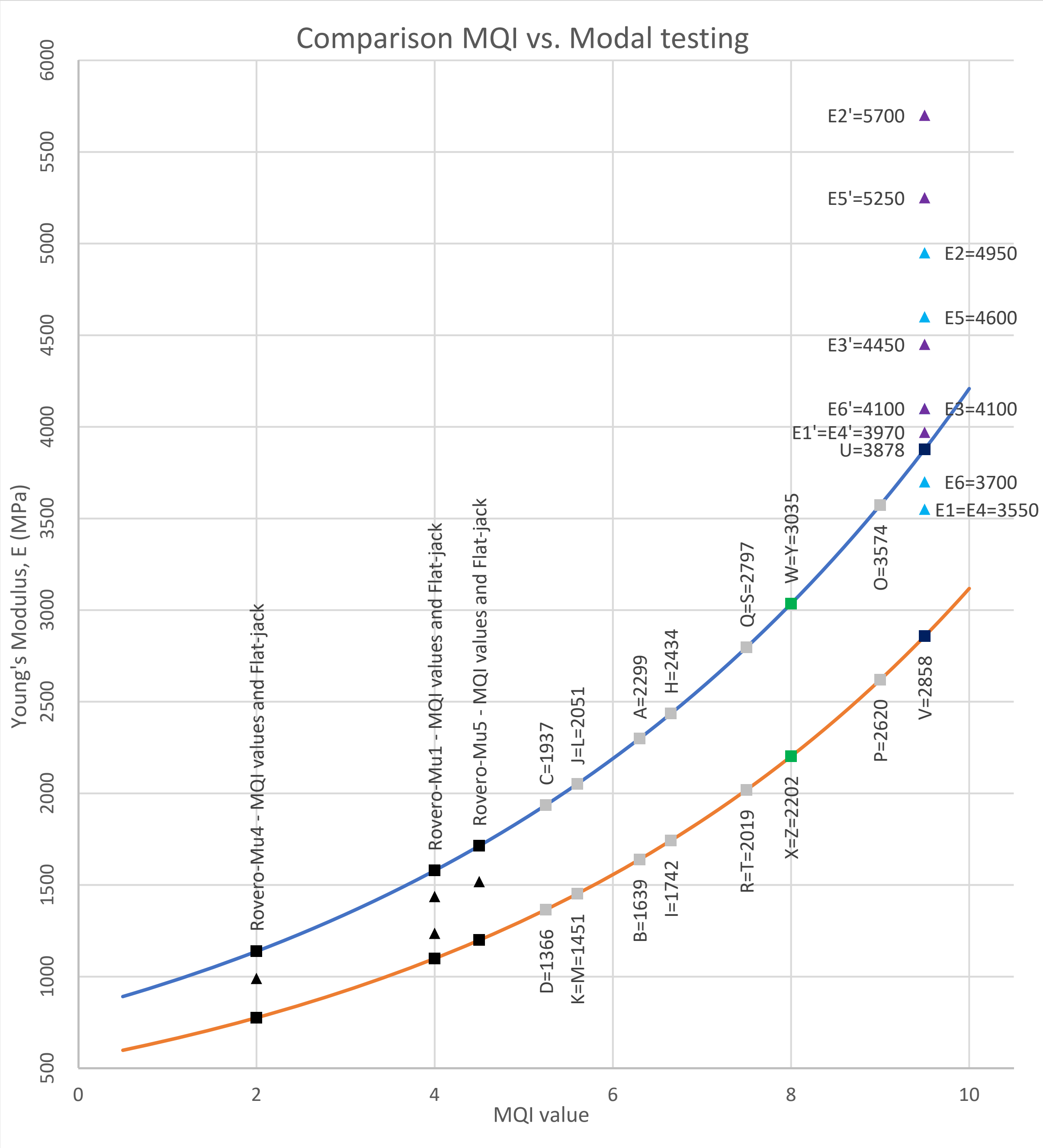


Figure 5.1.4. Graphic representation of the results from MQI and experimental analysis. Squares: MQI values. Triangles: Experimental & FEM calibration values. Black: Rovero et al., 2016. Grey: Lower values for MQI considered in section 4.2.1. Dark Blue (squares): MQI(V) = 9,5. Green (squares): MQI(I&O) = 8. Light blue (triangles): FEM calibration, density = 2100 kg/m^3. Purple (triangles): FEM calibration, density = 2350 kg/m^3.

For density equal to 2100 kg/m^3, solutions 1 & 4, are in the range of percentages accepted by Rovero et al., 2016, even related with the average between upper and lower bounds. However, the third mode of vibration is not calibrated in these solutions. Meanwhile, with density equal to 2350 kg/m^3, the case of full rigid foundation, solutions 1' & 4', could only be related with the upper bound to be considered as reliable as per the expressed methodology of comparison. Those solutions in which is considered only flexible foundation in the vertical direction, in both scenarios of wall density, have been demonstrated to be too far from reliable values when compared with the MQI method.

In the case of the developed hypothesis about the sliding of the tower, considering the masonry wall density equal to 2100 kg/m^3, the resulted Young's modulus is still in the ranges accepted by Rovero et al., 2016. However, with denser masonry walls, the comparison indicates that could be more appropriate to consider lower densities.

5.2 COMPARISON WITH OTHER BELL-TOWERS

Similar studies on bell-tower dynamics have been developed and published many times before. As a part of further considerations on this field, Ivorra et al., 2011 provides a compilation of bell-tower properties, some of them extracted from non-linear analysis. There are some tables procuring height, compressive and tensile strength, Young's modulus, natural frequencies of the first three modes of vibration and damping ratio, this last parameter given by a modal testing with forced vibration. However, not all the parameters are available for all the bell-towers. Furthermore, it is included in the next table the results of this dissertation and that belonging to St. Jakub in Kutná Hora (fig. 5.2.1).

Source of information	Tower height (m)	Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)	Young's modulus (MPa)
St. Jakub in Cirkvice	21.50	2.38	2.88	6.15	3550-3970 or 4100-4450
Julio et al., 2008	33.00	2.13	2.47	6.56	4800
Abruzzese & Vari, 2004	33.10	2.00	2.17	6.70	4000
Ivorra & Ballares, 2007	35.50	2.30	2.40	5.50	-
St. Jakub in Kutná Hora	57.00	0.97	1.24	2.90	2100 or 4500
Gentile & Saisi, 2007	74.00	0.59	0.71	2.46	1750
Binda et al., 2000	112.00	0.44	0.45	1.66	-

Figure 5.2.1. Compiled bell-towers properties (Ivorra et al., 2011).

The closest natural frequencies to those of St. Jakub in Cirkvice are given by the tower of the university of Coimbra (Julio et al., 2008) and Capocci's tower in Rome (Abruzzese & Vari, 2004). It is possible to extract from this table that with higher frequencies, necessarily, the Young's modulus is higher. On its behalf, St. Jakub in Kutná Hora, a higher tower with lower frequencies, has a lower Young's modulus when considered rigid foundations but, if there are considered springs in its foundation, the results are very close. These comparisons make more reliable the elastic modulus provided in solutions 3 & 3' (E = 4,10 & 4,45 GPa). Therefore, after comparing the FEM calibration results with MQI and with similar bell-towers, these solutions seem to be more appropriate than those of rigid foundations, however, one must emphasize that this hypothesis should be evaluated with further in situ studies.

5.3 BELL SWINGING INFLUENCE

As mention in section 2.4 (Mechanics of the Bell Swinging), it is important to study the influence of this harmonic oscillating system in the dynamic response of bell-towers due to the phenomenon on mechanical resonance. Due to the shape of the bells, it is difficult to calculate accurately some parameters that are needed in the equations given in the standard DIN 4178 (eq. 2.4.1-2.4.3). Therefore, it is necessary to base these parameters in some empirical relations. In the present analysis these relations are given by Lunga & Solar, 2010 as follows:

a. Mass of the bell: $m_b = d^3 * k$ .Where d is the diameter of the bell and k is a constant that may vary from 560 to 600, however in Czech Republic is found acceptable to apply 598
b. Center of gravity of the bell, r_b: May vary from *0,58*$*d$ to *0,64*$*d$. Where d is the diameter of the bell
c. Mass radius of inertia, i_s: May vary from *0,80*r* to *0,90*r*. Where are is the eccentricity of the system.

As mention in section 1.1 (Building definition and main historical alterations), nowadays the bells are located on the vault of the second floor of the tower so there is now existing swinging system (fig. 5.3.1a). However, the bells of St. Jakub in Kutná Hora where installed recently (fig. 5.3.1b). Based on the design of the yoke installed in that homonymous church, in this analysis is supposed a similar yoke and swinging system (fig. 5.3.2). In any case, if in the future is decided to restore the bells and relocate them in the top body of the bell-tower, all the empirical relations above mentioned should be re-evaluated.


Figure 5.3.1a & 5.3.1b. 5.3.1a, Bells of St. Jakub in Cirkvice in their current location. 5.3.1b, St. Jakub in Kutná Hora during the installation if its bell.

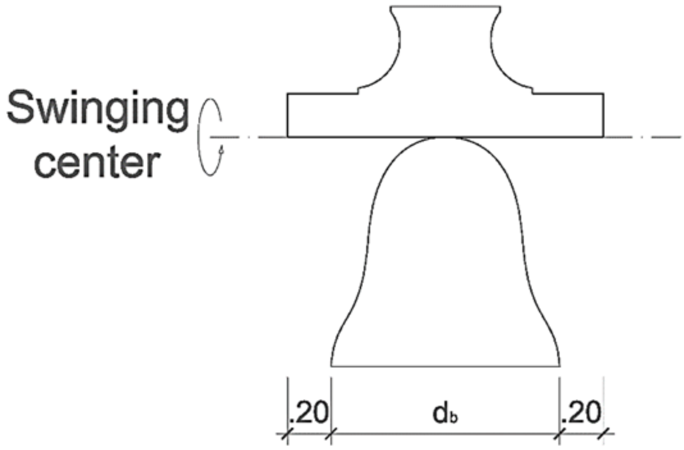


Figure 5.3.2. Schematic representation of the bell swinging system and yoke based on the recently installed system in St. Jakub in Kutná Hora.

Based on the above mentioned empirical relations and on the schematic hypothesis of the swinging system that could be installed in St. Jakub in Cirkvice, the next tables show the values of each parameters needed in the equations 2.4.1, 2.4.2 & 2.4.3 and the result of those equations.

$$H = \frac{m * g}{1 + k^2} * \left(2 * \frac{\cos\varphi_0}{\cos\varphi} - 3\right) * \cos\varphi \sin\varphi \qquad \text{Eq. 2.4.1}$$

$$V = \frac{m * g}{1 + k^2} * (k^2 + 3\cos^2\varphi - 2\cos\varphi\cos\varphi_0) \qquad \text{Eq. 2.4.2}$$

$$T_0 = \sqrt{\frac{4\pi^2 * r * (1 + k^2)}{g}} \qquad \text{Eq. 2.4.3, where } k = \frac{i_s}{r}$$

Yoke 1	Area (m^2)		0.49
	Width (m)		0.20
	Volume (m^3)		0.10
	Density (kg/m^3)		560.00
	Weight (kg)	m_y	55.15
	Distance from center of gravity to center of rotation	r_y	-0.22

Yoke 2	Area (m^2)		0.30
	Width (m)		0.15
	Volume (m^3)		0.05
	Density (kg/m^3)		560.00
	Weight (kg)	m_y	25.30
	Distance from center of gravity to center of rotation	r_y	-0.16

Bell 1	Diameter (m)	d	1.05
	Weight (kg)	m_b	692.26
	Empirical relations for mb	$m_b = d^3 * k$	
	k may vary from 560 to 600 but for Czech Republic is empirically found acceptable 598		
	Distance from center of gravity to center of rotation	r_b	0.63
	Empirical relations for r_b	$r_b=0.58*d$	0.61
		$r_b=0.64*d$	0.67

Bell 2	Diameter (m)	d	0.815
	Weight (kg)	m_b	323.72
	Empirical relations for mb	$m_b = d^3 * k$	
	k may vary from 560 to 600 but for Czech Republic is empirically found acceptable 598		
	Distance from center of gravity to center of rotation	r_b	0.50
	Empirical relations for r_b	$r_b=0.58*d$	0.47
		$r_b=0.64*d$	0.52

Swinging system 1	Total mass	$m=m_b+m_y$	747.41
	Eccentricity or Unbalance	r	0.57
	mass radius of inertia	i_s	0.47
	Empirical relations for i_s	$i_s=0.8*r$	0.45
		$i_s=0.9*r$	0.51
	Kappa	*k*	0.83
		$(m*g)/(1+k^2)$	4.35

Swinging system 2	Total mass	$m=mb+m_y$	349.02
	Eccentricity or Unbalance	r	0.45
	mass radius of inertia	i_s	0.38
	Empirical relations	$i_s=0.8*r$	0.36
		$i_s=0.9*r$	0.41
	Kappa	*k*	0.84
		$(m*g)/(1+k^2)$	2.01

Figure 5.3.3. Highlighted in blue are those parameters that are actually measured in situ or that are taken between the bounds given in the empirical relations. $g = 9{,}81\, {}^{m}/_{s^2}$

Swinging system 1 (80°)

φ(°)	φ(rad)	V (kN)	H (kN)
0	0.00	14.52	0.00
5	0.09	14.43	-1.00
10	0.17	14.15	-1.97
15	0.26	13.70	-2.87
20	0.35	13.09	-3.68
25	0.44	12.33	-4.36
30	0.52	11.46	-4.89
35	0.61	10.50	-5.26
40	0.70	9.48	-5.45
45	0.79	8.44	-5.46
50	0.87	7.40	-5.27
55	0.96	6.41	-4.89
60	1.05	5.49	-4.34
65	1.13	4.67	-3.63
70	1.22	3.99	-2.77
75	1.31	3.47	-1.80
80	1.40	3.11	-0.74

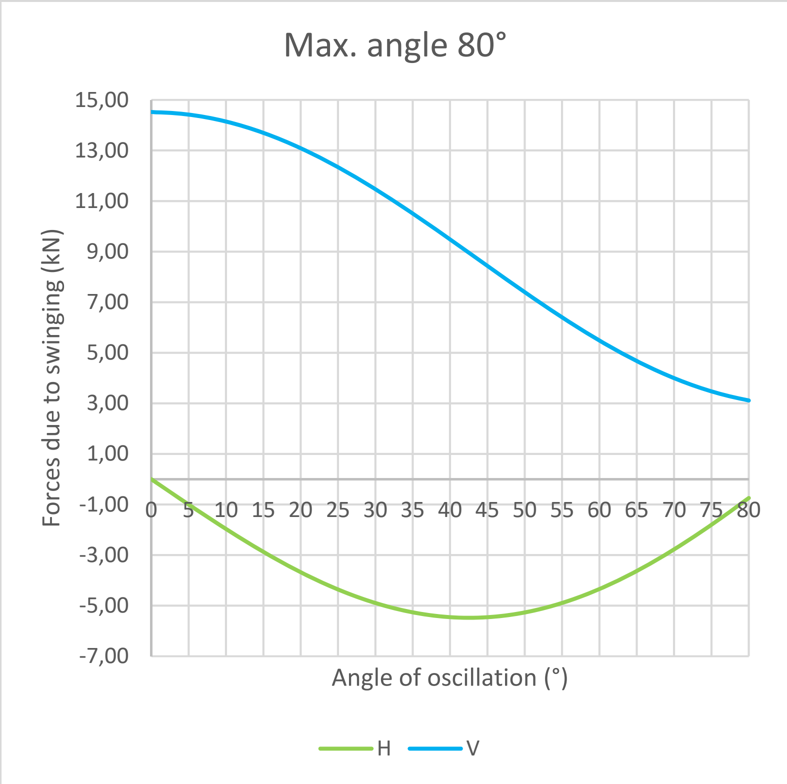


Figure 5.3.4. Forces due to swinging (H & V). Bell 1 (d=1,05). Maximum swinging angle 80°.

Swinging system 1 (70°)

φ(°)	φ(rad)	V (kN)	H (kN)
0	0.00	13.06	0.00
5	0.09	12.97	-0.87
10	0.17	12.71	-1.71
15	0.26	12.28	-2.49
20	0.35	11.71	-3.18
25	0.44	11.00	-3.74
30	0.52	10.19	-4.16
35	0.61	9.30	-4.42
40	0.70	8.36	-4.51
45	0.79	7.40	-4.42
50	0.87	6.46	-4.15
55	0.96	5.57	-3.69
60	1.05	4.76	-3.07
65	1.13	4.06	-2.30
70	1.22	3.49	-1.40

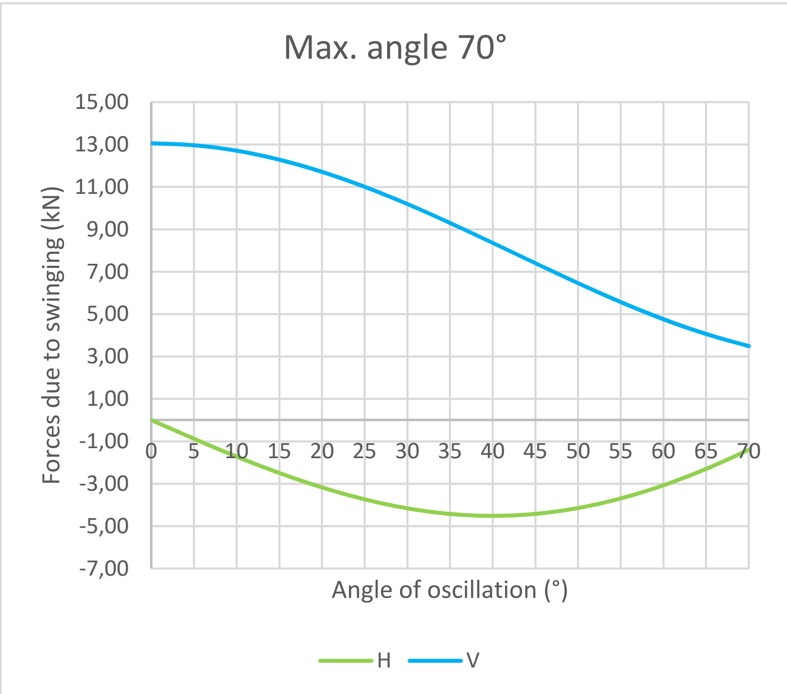


Figure 5.3.5. Forces due to swinging (H & V). Bell 1 (d=1,05). Maximum swinging angle 70°.

Swinging system 2 (80°)

φ(°)	φ(rad)	V (kN)	H (kN)
0	0.00	6.74	0.00
5	0.09	6.70	-0.46
10	0.17	6.57	-0.91
15	0.26	6.36	-1.32
20	0.35	6.08	-1.70
25	0.44	5.73	-2.01
30	0.52	5.33	-2.26
35	0.61	4.89	-2.43
40	0.70	4.42	-2.52
45	0.79	3.93	-2.52
50	0.87	3.46	-2.43
55	0.96	3.00	-2.26
60	1.05	2.57	-2.00
65	1.13	2.20	-1.67
70	1.22	1.88	-1.28
75	1.31	1.64	-0.83
80	1.40	1.48	-0.34

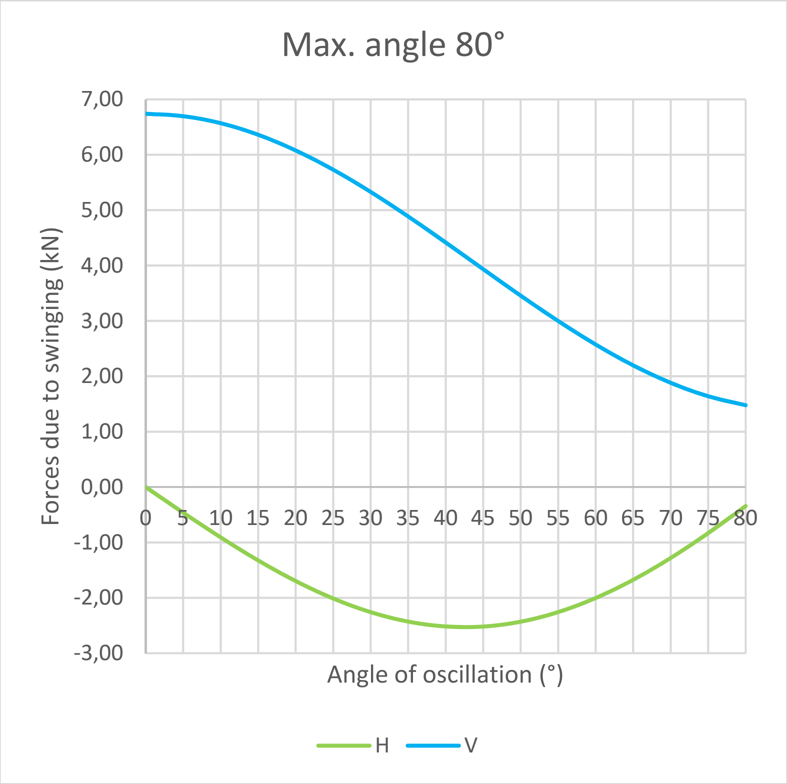


Figure 5.3.6. Forces due to swinging (H & V). Bell 2(d=0,815). Maximum swinging angle 80°

Swinging system 2 (70°)

φ(°)	φ(rad)	V (kN)	H (kN)
0	0.00	6.06	0.00
5	0.09	6.02	-0.40
10	0.17	5.90	-0.79
15	0.26	5.71	-1.15
20	0.35	5.44	-1.47
25	0.44	5.12	-1.73
30	0.52	4.74	-1.92
35	0.61	4.33	-2.04
40	0.70	3.90	-2.08
45	0.79	3.46	-2.04
50	0.87	3.02	-1.91
55	0.96	2.61	-1.70
60	1.05	2.24	-1.42
65	1.13	1.91	-1.06
70	1.22	1.65	-0.64

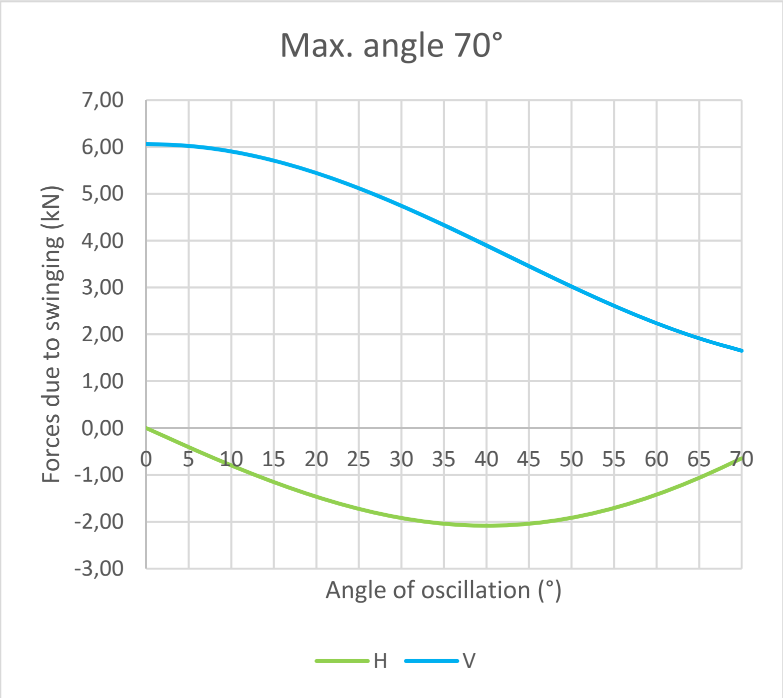


Figure 5.3.7. Forces due to swinging (H & V). Bell 2(d=0,815). Maximum swinging angle 70°

From tables 5.2.4 to 5.2.7, could be extracted that, for the cases studied, the maximum vertical force occurs between 40° and 45° and has a value that varies between 60% to 70% of the weight of the swinging system, depending on the maximum angle of oscillation considered. Meanwhile, the maximum horizontal force occurs always at 0° and it is almost twice of the swinging system weight (fig 5.2.8).

	Swinging system 1 (d=1.05)					
	System weight (kN)					7.33
	80°			70°		
	φ(°)	kN	F/ system weight	φ(°)	kN	F/ system weight
Max. Vertical force	0	14.52	198.03%	0	13.06	178.12%
Max. Horizontal force	40-45	5.5	75.01%	40	4.51	61.51%

	Swinging system 2 (d=0.815)					
	System weight (kN)					3.42
	80°			70°		
	φ(°)	kN	F/ system weight	φ(°)	kN	F/ system weight
Max. Vertical force	0	6.74	196.85%	0	6.06	176.99%
Max. Horizontal force	40-45	2.6	75.94%	40	2.08	60.75%

Figure 5.3.8. Maximum forces generated by bell swinging and comparison with the weight of the systems.

This analysis of the influence of the bell swinging is even more important, if possible, to be studied in terms of frequencies. The mechanical resonance phenomenon could occur if the fundamental frequency of the harmonic oscillating system or any of its *n* multiples matches the natural frequency of the bell tower in the direction of the swinging. Considering the possible swinging direction parallel to the Mode 1 of vibration of the bell-tower, figure 5.2.9 shows the comparison of the frequencies for the swinging system 1 & 2.

Swinging system 1

T_o (s)	1.96	Mode 1
f_o (Hz)	0.510	2.38
f_o*n (n=2)	1.135	
f_o*n (n=3)	1.703	
f_o*n (n=4)	2.270	
f_o*n (n=5)	2.838	
f_o*n (n=6)	3.405	

Swinging system 2

T_o (s)	1.76	Mode 1
f_o (Hz)	0.568	2.38
f_o*n (n=2)	1.135	
f_o*n (n=3)	1.703	
f_o*n (n=4)	2.270	
f_o*n (n=5)	2.838	
f_o*n (n=6)	3.405	

Figure 5.3.9. Fundamental frequency of the oscillating system and its first five *n* multiples.

T_o is dependent on the eccentricity of the system and on the mass radius of inertia, both parameters highly dependent on the empirical relations given in the literature. Even if the bells are totally different, their frequency of oscillation is very similar, sign of the great importance of the analysis of the bell swinging influence in the bell-towers. This fact should be considered when developing a project for the re-installation of the bells. However, in the conditions expressed in this analysis, the frequency of the bell swinging is only near the mode 1 natural frequency for n = 4, meaning that it is a very low periodicity to influence significantly in the global structural behavior. Anyhow, in case of deciding to re-install the bells in the bell-tower, as mention above, all the parameters expressed in figure 5.2.3, should be re-evaluated. Additionally, the current timber structure should be further studied to know how the possible deformations due to swinging would affect to the masonry structure and even if its current state of conservation would allow the installation of a new bell swinging system.

6. CONCLUSIONS

After a general overview of the case study, description of the building and definition of its main historical alterations, it has been reviewed those key theoretical concepts needed to frame the following steps of the work done.

It is considered that the geometrical survey is a basic requirement of any intervention in the historical and cultural heritage. It consists on the graphic representation of the built reality, which is performed by a set of activities to comprehend the actual architectural configuration including its dimensional, metrical, historic, structural and constructive characteristics. According to this consideration, the set of drawings performed has been demonstrated to be totally necessary to develop the following steps.

The Church of St. Jakub in Cirkvice is a masonry building with its bell-tower aligned with the longitudinal axis of the nave. Therefor, one has decided to apply the masonry quality index (MQI) method to obtain values for the mechanical parameters of the masonry elements.

With the aim of developing a calibrated FE model that would allow in the future to evaluate the possibility of reinstalling the original bells of the tower, a FE model with shell elements has been performed. Different possible scenarios have been considered to try answer some uncertainties like soil properties or the foundation behavior.

Then, the FE model has been calibrated to match the natural frequencies resulted from the experimental modal testing. However, the obtained values from the MQI method for the Young's modulus have been demonstrated to be too low in most of the considered scenarios in the case St. Jakub in Cirkvice. Therefor, the results from the calibration have been compared with the MQI, following the methodology of correlation applied by Rovero et al., 2016, and with other published dynamic studies of bell-towers (Ivorra et al., 2011).

The considered scenario of slightly sliding of the bell-tower along the horizontal axis seems to provide the more appropriate results when compared with other studied bell-towers. However, the developed hypothesis considers that the nave is not sliding in respect to the bell-tower. Consideration that would need further studies of soil & foundation behavior.

During the application of the MQI method, some uncertainties regarding the quality of the units and the considerations regarding its dimensions. Still after obtaining results from the FEM calibration, in the case studied, is not possible to conclude about these uncertainties.

Finally, the natural frequencies of the bell-tower have been compared with those extracted from the bell swinging analysis. It is possible to conclude that the oscillating frequency of the bell swinging is near to the first mode of vibration in its $n = 4$ multiple. Then, the probability that the mechanical resonance phenomenon takes place is low. However, there are many parameters based on empirical relations that could be re-evaluated if it is decided to reinstall the bells.

In general terms, the state of conservation is good except for some moisture issues in the lower part of some walls. However, it would be important to further study the internal timber structure regarding its structural behavior and state of conservation to guaranty its stability in case of reinstalling the bell swinging system.

In those cases that is needed to assess mechanical properties of structural materials, the non-destructive and minor-destructive techniques are selected based on the observed problem, the cost limits, the applicability of the techniques, the time available, etc. Usually in most of the cases is not one technique but a set of them. However, in this dissertation the calibration of the FE model it is only based on the results of the experimental modal testing. This point makes necessary to develop further considerations and comparisons, among others, about the soil properties, the interface nave-tower. In fact, the Church of St. Jakub in Cirkvice would probably be an interesting case study for researchers analyzing the interaction between two parallel masonry walls.

7. REFERENCES

A. Borri, M. Corradi, G. Castori, A. de Maria (2015). A method for the analysis and classification of historic masonry. *Bull Earthquake Eng* nº13, p. 2647-2665.

S. Ivorra, MªJ. Palomo, G. Verdú & A. Zasso (2006). Dynamic forces produced by swinging bells. *Meccanica* nº41*,* 47-62.

S. Ivorra, F. Pallarés, J. Adam (2011). Masonry bell towers: Dynamic considerations. *Structures and buildings,* volume 164, p. 3-12.

A. Jiménez Martín & F. Pinto Puerto (2003). Levantamiento y análisis de edificios. Tradición y futuro. (Geometry survey and building's analysis. Tradition and future).

R. Lunga, J. Solař. (2010). Kostelní věže a zvonice: kampanologie, navrhování, poruchy, rekonstrukce a sanace. (Church-towers and Bell-tower: Campanology, design, disturbance, reconstruction and rehabilitation).

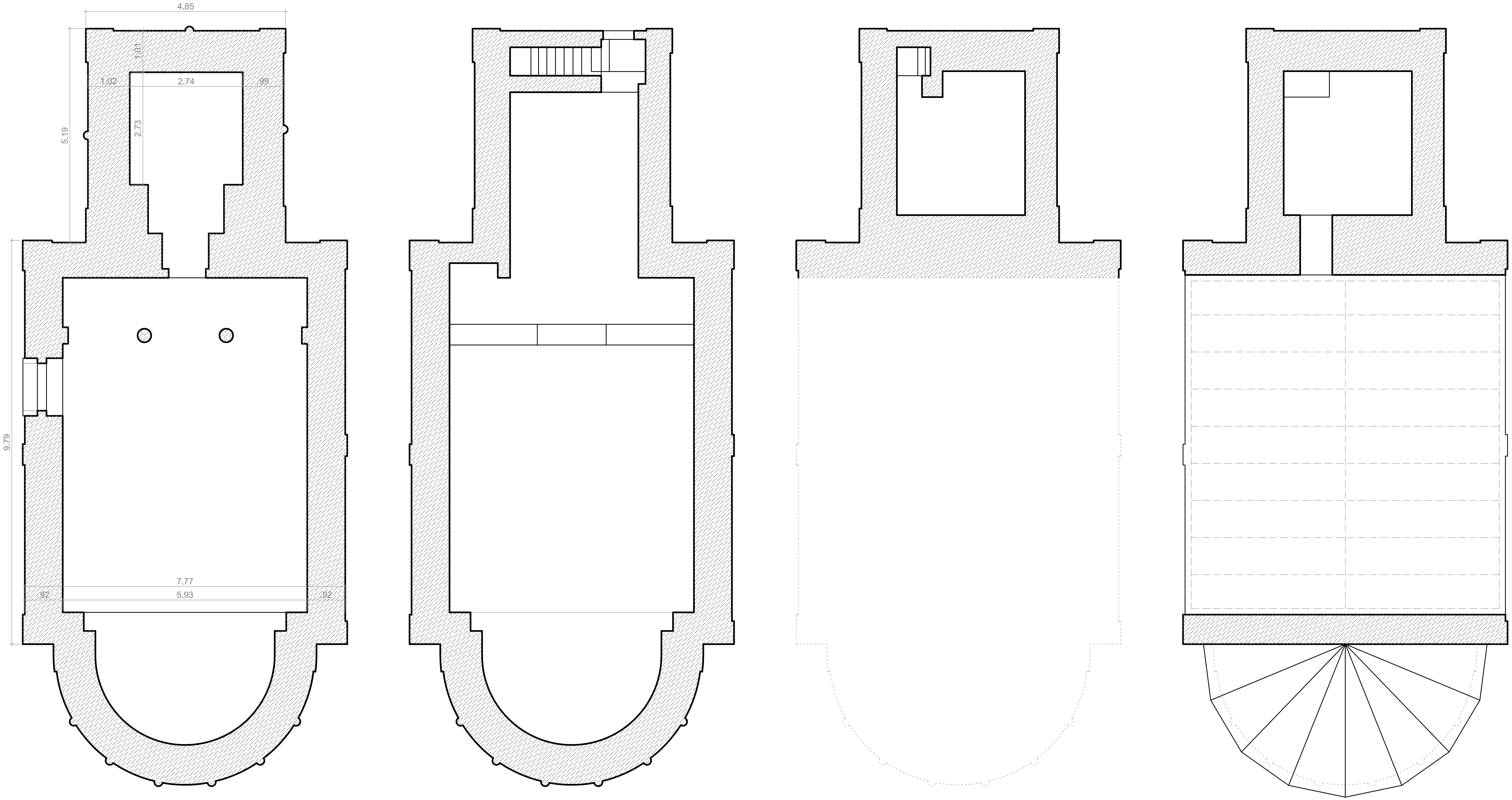
L. Rovero, V. Alecci, J. Mechelli, U. Tonietti & M. de Stefano (2016). Masonry walls with irregular texture of L'Aquila (Italy) seismic area: validation of a method for the evaluation of masonry quality. *Materials and structures* nº49, p. 2297-2314.

A. Vanková (2015). Kostel Sv. Jakuba Většího (The Church St. James the Greater).

DIN 4178: 2005-04. Bell-towers: calculation & execution (German standard).

8. ANNEX

8.1 ST. JAKUB IN CIRKVICE SHOPDRAWINGS (1:100)

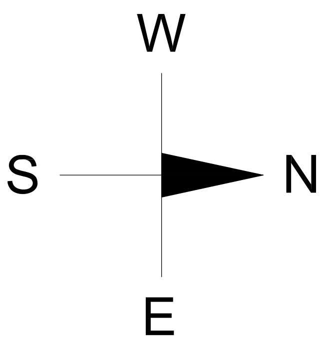


Ground floor

Choir level

Upper-choir / Vaults level

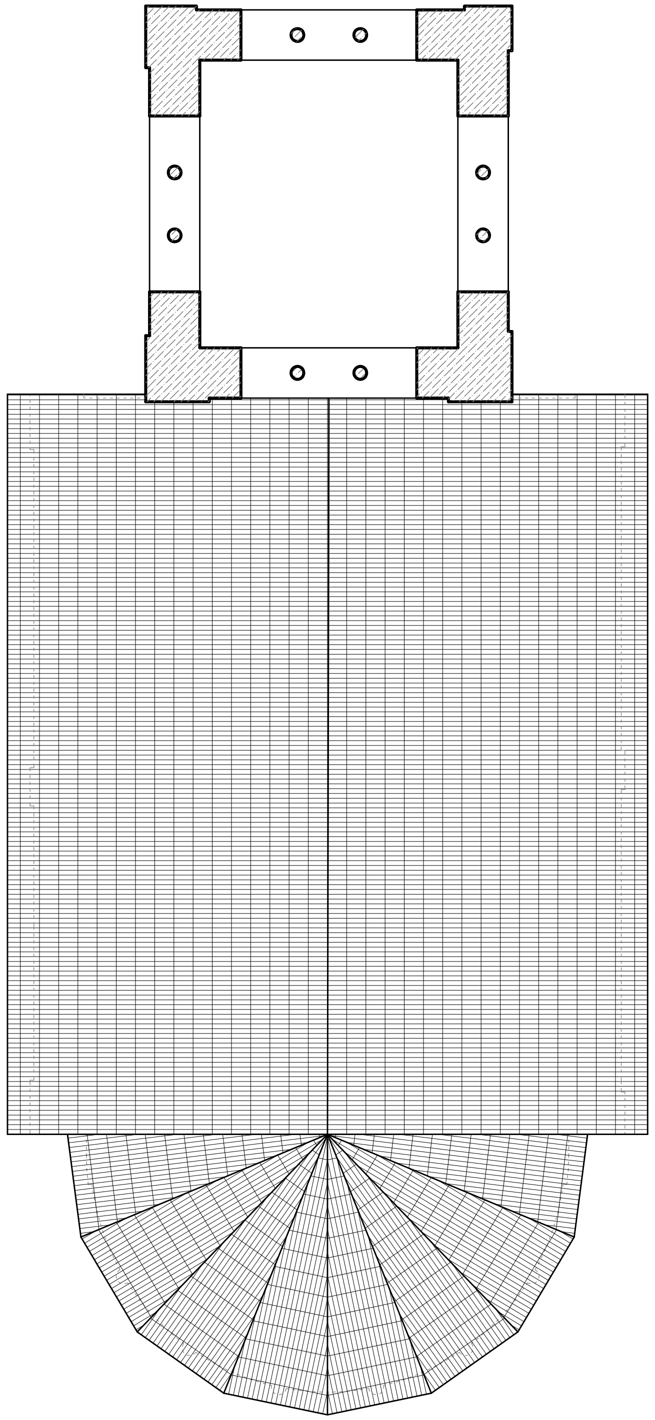
Trusses level



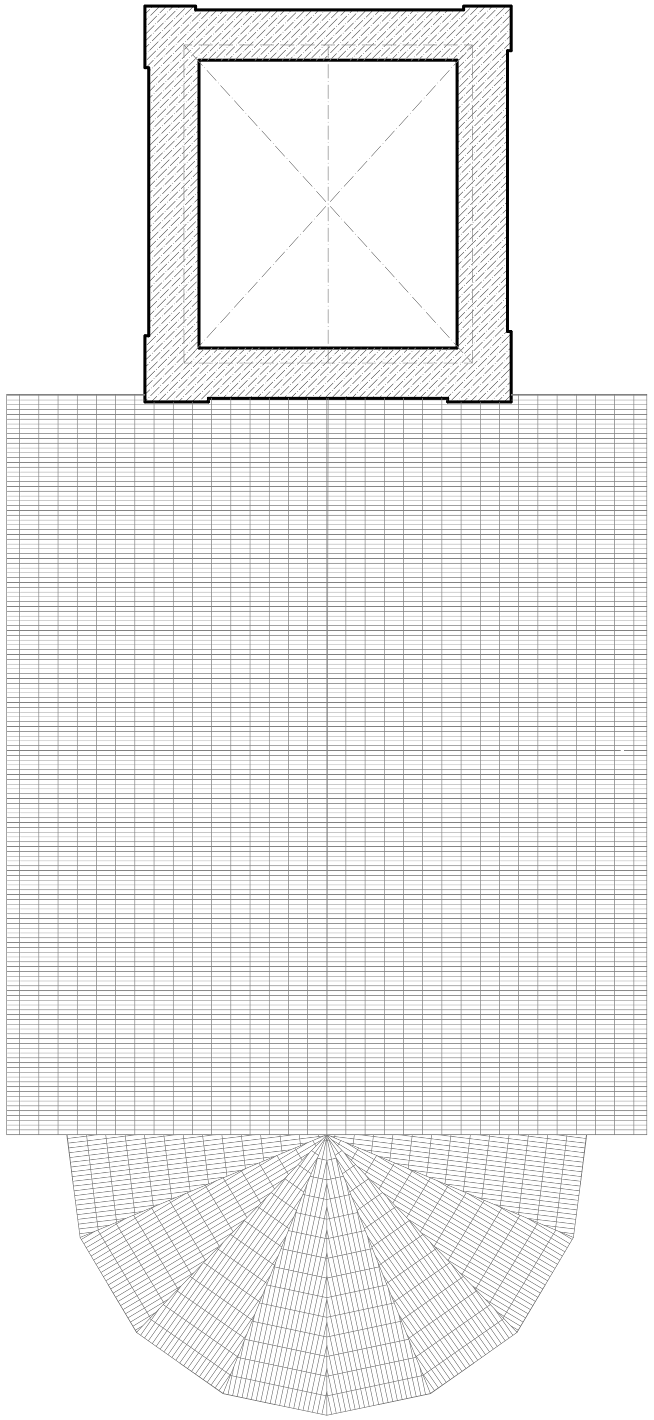
Project	Unit	Author
St. Jakub's Church in Cirkvice	SA8 - SAHC dissertation	Pablo Bañasco
Sheet	Scale	
1. Plandrawings 1	1:100	

ČVUT ČESKÉ VYSOKÉ UČENÍ TECHNICKÉ V PRAZE

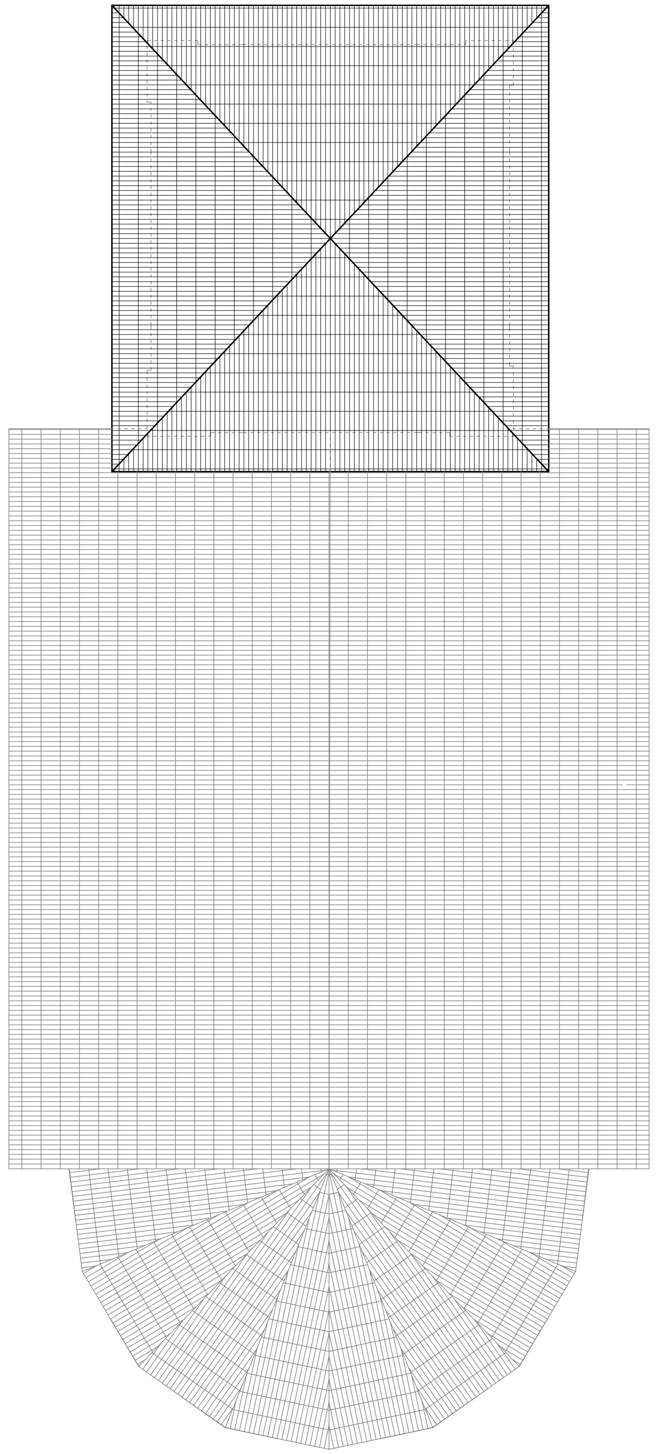
SAHC



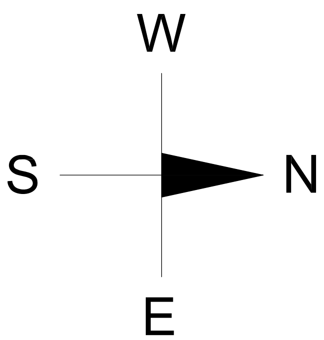
Bells level



Bell-tower roof-structure



Bell-tower roof



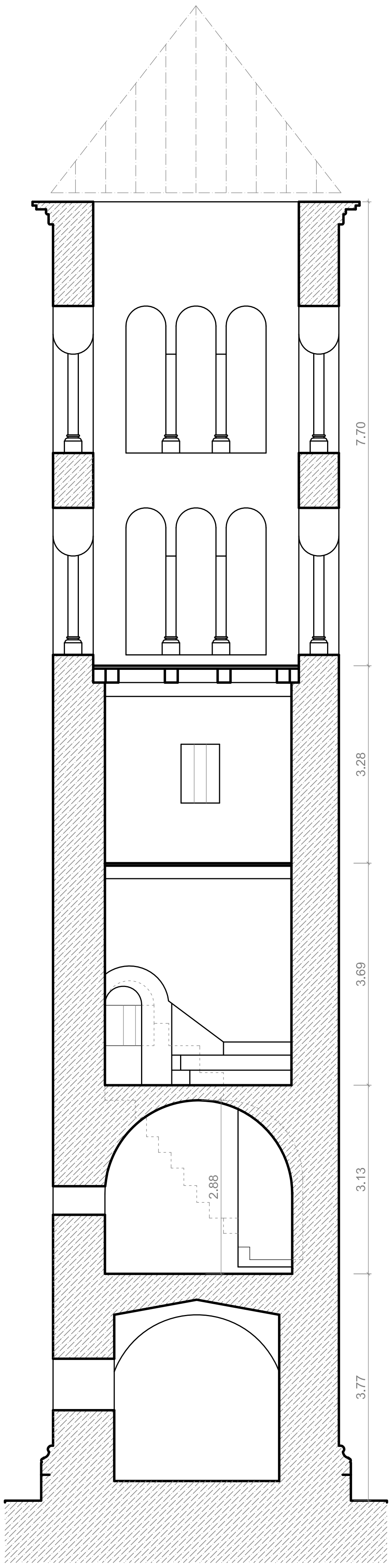
-1 0 1 2 3 4m

Project	Unit	Author
St. Jakub's Church in Cirkvice	SA8 - SAHC dissertation	Pablo Bañasco
Sheet	Scale	
2. Plandrawings 2	1:100	

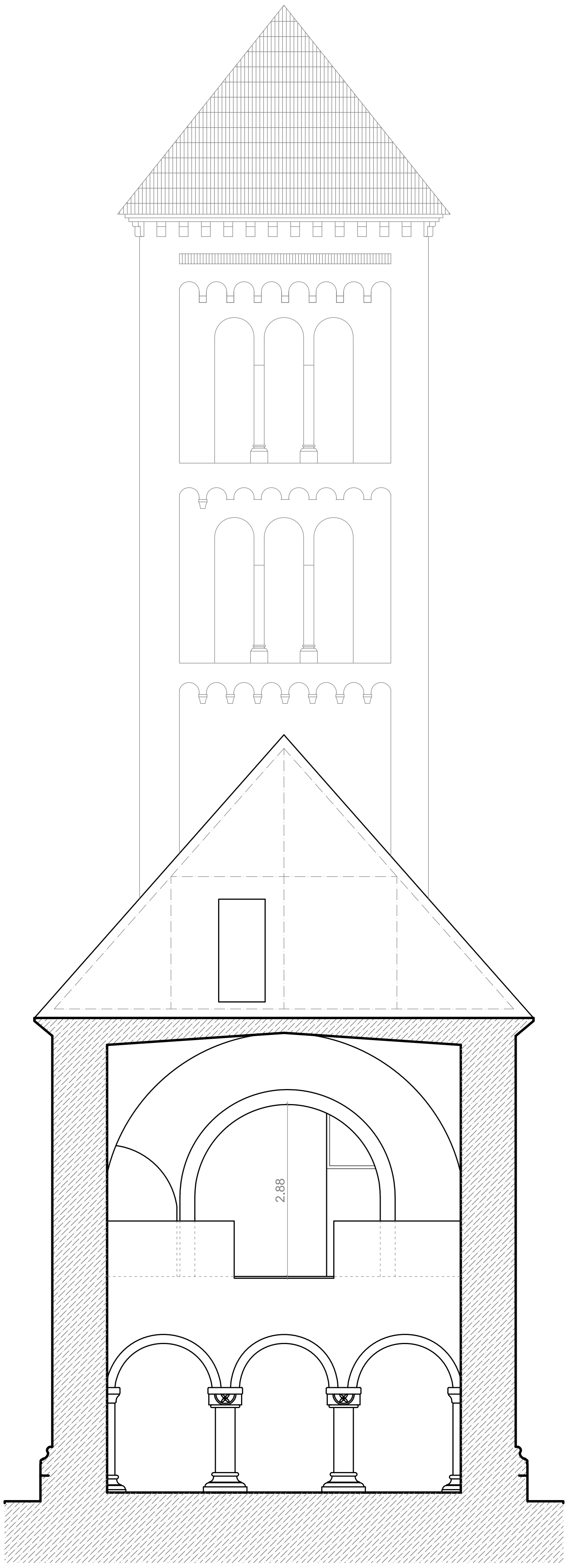
ČVUT ČESKÉ VYSOKÉ UČENÍ TECHNICKÉ V PRAZE

SAHC

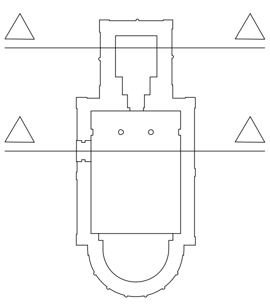
S → N



Transversal bell-tower section



Transversal nave section



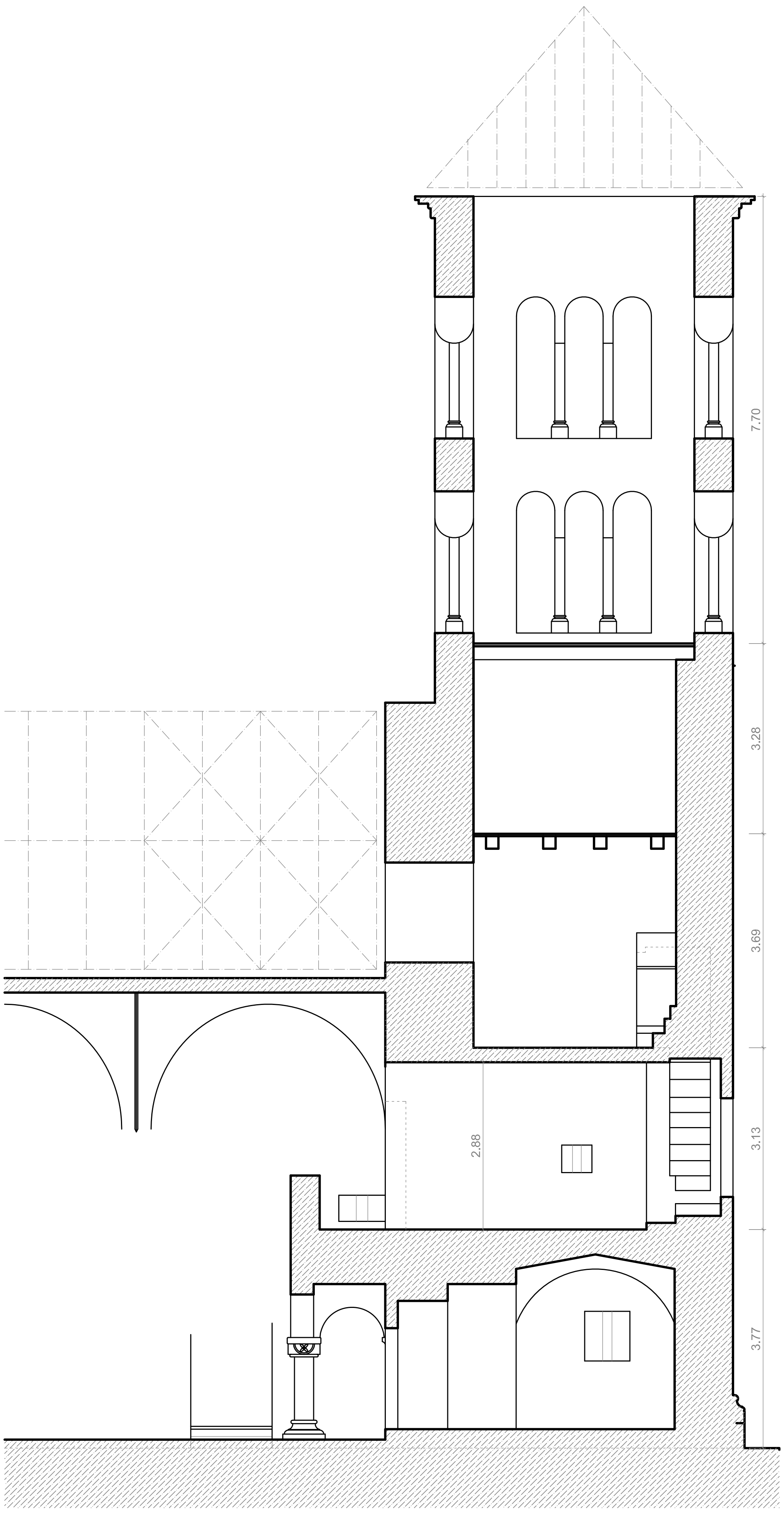
-1 0 1 2 3 4m

Project	Unit	Author
St. Jakub's Church in Cirkvice	SA8 - SAHC dissertation	Pablo Bañasco
Sheet	Scale	
3. Sections & Elevations 1	1:100	

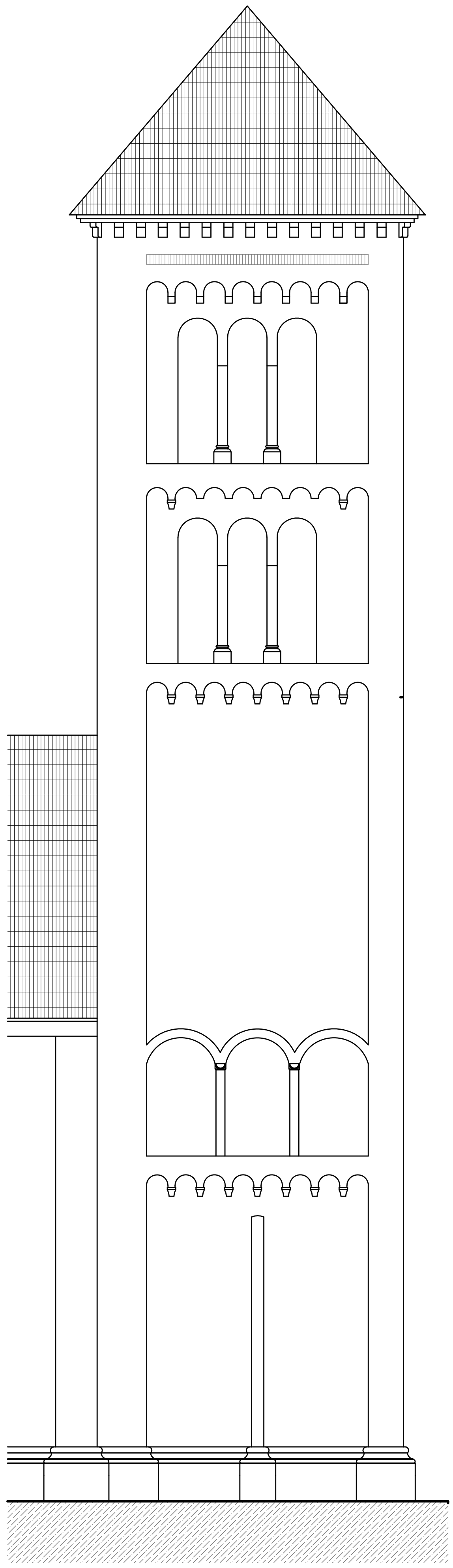
ČVUT ČESKÉ VYSOKÉ UČENÍ TECHNICKÉ V PRAZE

SAHC

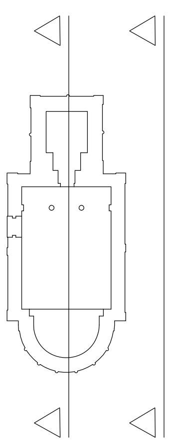
E —— W



Longitudinal section



North elevation



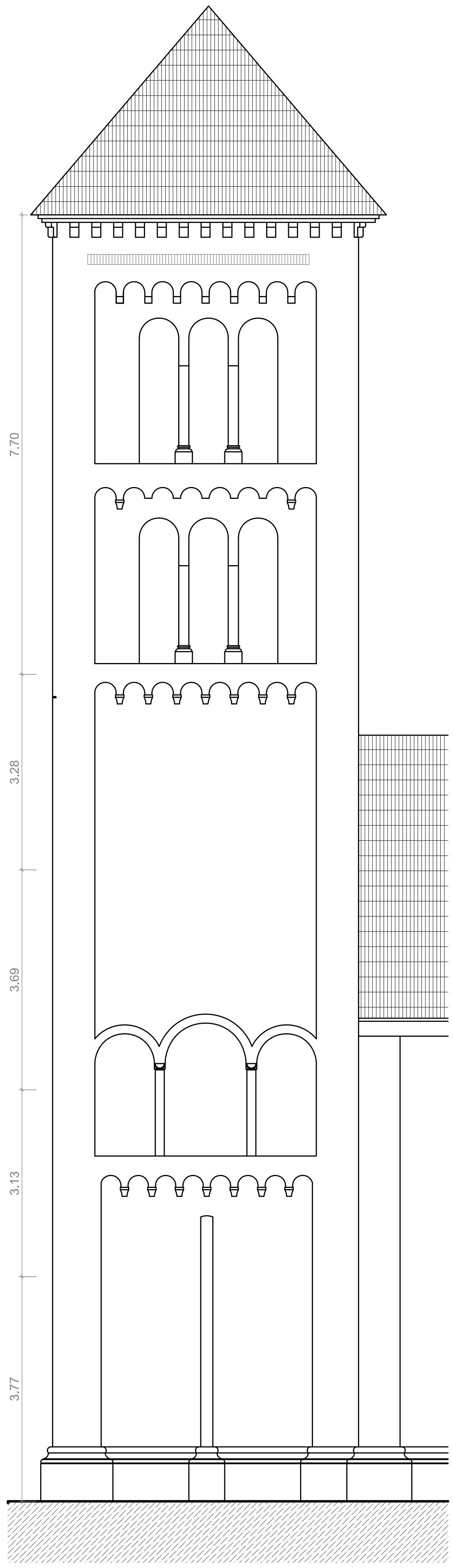
Project	Unit	Author
St. Jakub's Church in Cirkvice	SA8 - SAHC dissertation	Pablo Bañasco
Sheet	Scale	
4. Sections & Elevations 2	1:100	

ČVUT ČESKÉ VYSOKÉ UČENÍ TECHNICKÉ V PRAZE

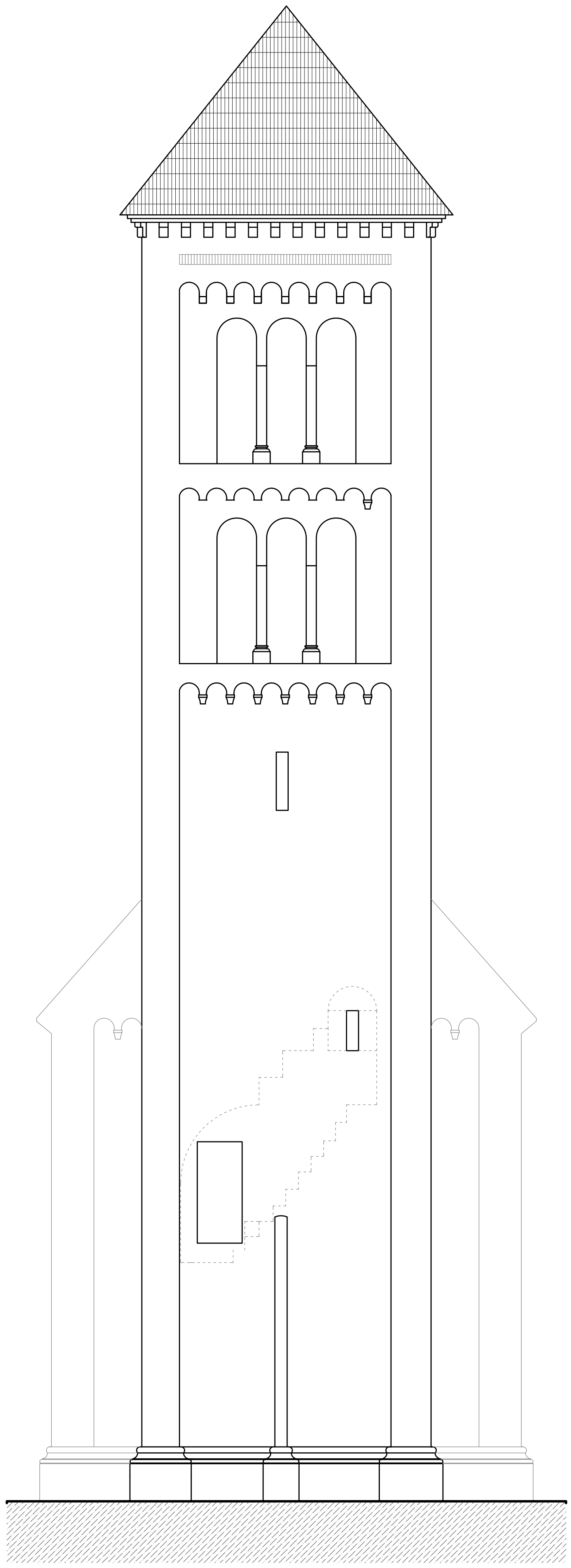
SAHC

W — E

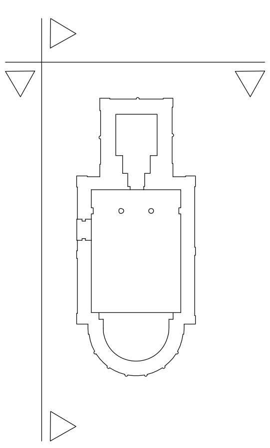
N — S



West elevation



South elevation



Project	Unit	Author
St. Jakub's Church in Cirkvice	SA8 - SAHC dissertation	Pablo Bañasco
Sheet	Scale	
5. Sections & Elevations 3	1:100	