

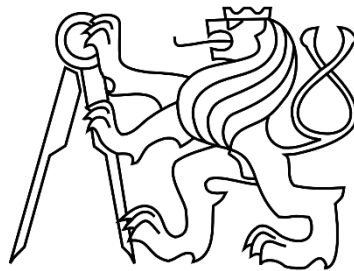
**Czech Technical University in Prague
Faculty of Mechanical Engineering
Department of Aerospace Engineering**

Master's Thesis

**A Conceptual Design of a Four-Seat
GA Aircraft**

Bc. Daniel Stastny

Prague 2017



Annotation List

Author: Bc. Daniel Stastny

Supervisor: Ing. Jiří Brabec, Ph.D.

Academic year 2017/2018

Name of Master's Thesis in english: A Conceptual Design of a Four-Seat GA Aircraft

Name of Master's Thesis in czech: Konceptní návrh čtyřmístného letounu kategorie GA

Bibliographic data:	Number of pages:	63
	Number of characters:	55 982

Keywords: Conceptual design, GA aircraft, constraint analysis, four seating configuration, initial design.

Abstract:

In this Thesis, a conceptual design is proposed for a four seat GA aircraft. The objective is to design an aircraft which has a cruising speed of 300 km/h and should have a range of 2000 km. The weight will be estimated according to the mission profile for our flight. All the aerodynamic characteristics of this aircraft such as lift and drag polar will be established, followed by the moment characteristics.

Declaration:

I declare that this thesis and the work presented in it are my own and has been generated by me as the result of my own original research.

In Prague / V Praze dne

Signature / podpis

Acknowledgment:

I would like to thank my supervisor Ing. Jiří Brabec, Ph.D. for his help and his guidance throughout my work.

Table of Contents

1	Introduction	1
2	Objective	1
3	Research	1
3.1	Setting the requirements	2
4	Constraint analysis	4
4.1	The method of constraining.....	4
4.2	Constraining for a level constant-velocity turn	5
4.3	Constraining for a desired rate of climb	6
4.4	Constraining for a desired take-off distance.....	6
4.5	Constraining for a desired cruise airspeed	7
4.6	Constraining for a desired service ceiling	7
4.7	Plotting the results.....	8
5	Selecting an appropriate engine	11
5.1	Engine choice	11
5.2	Effect of altitude on engine power	11
5.3	Engine power and the requirement for fuel consumption	13
6	Weight estimation	14
6.1	The mission profile.....	14
6.2	Component weight estimation.....	17
6.2.1	Installed Engine weight.....	18
6.2.2	Estimating the wing weight.....	18
6.2.3	Estimating the fuselage weight.....	18
6.2.4	Estimating the weight of the horizontal tail	19
6.2.5	Estimating the weight of the vertical tail.....	20
6.2.6	Component weight as a fraction of empty weight.....	20
7	Designing the aircraft components.....	21
7.1	Selection of the airfoil for the wing layout.....	21
7.2	Cockpit layout	26
7.3	Empennage	28
8	Weight analysis and the center of gravity	32
9	Aerodynamic characteristics	34
9.1	Neutral point.....	34
9.1.1	Shift of neutral point due to influence of fuselage	34
9.1.2	Shift of neutral point due to the influence of the horizontal tail.....	35
9.2	The Lift enhancement.....	36
9.2.1	Increase in lift due to flaps see ref [11] $\Delta c_{LW} = K_b \cdot \Delta c_l \cdot c_{l\alpha} w c_{l\alpha} \cdot a \delta c_{L\alpha} \delta c_l$ #9.12	36
9.2.2	Slope of the lift curve with flaps deflected ref. [11].....	37
9.2.3	Increase in c_{Lmax} due to the deflection of the flaps ref. [11]	37
9.2.4	Maximum coefficient of lift of the wing with flaps deflected ref. [11].....	37

9.3	The effect of the fuselage on the lifting curve.....	38
10	Polar of the Aircraft.....	40
10.1	The wing drag	40
10.2	Drag due to the Horizontal tail.....	40
10.3	Drag due to the Vertical tail	41
10.4	Drag due to the presence of the landing gear	41
10.5	Drag due to the engine installation.....	41
10.6	Total drag of the aircraft.....	41
11	Moment characteristics	43
11.1	Pitching moment curve without any high-lift-device.....	43
11.2	Pitching moment coefficient of wing with influence of the fuselage.....	44
11.3	Pitching moment coefficient at zero lift wing/fuselage.....	44
11.4	Setting up the horizontal tail	45
11.5	Coefficient of pitching moment at zero lift, whole aircraft.....	45
11.6	Slope of the moment curve of the aircraft.....	45
11.7	Final pitching moment curve.....	46
12	Required surface area for the Horizontal tail	47
12.1	Balancing at the forward limit.....	47
12.2	Balancing at aft limit.....	47
12.3	Determining the required horizontal tail area.....	48
13	Flight performances.....	49
13.1	Required thrust	49
13.2	Required propulsive power	50
14	Design speeds and envelope working loads	52
14.1	Design speeds.....	52
15	Conclusion.....	53
16	References	54

List of Tables

Table 3.1. Statistical Parameters of similar GA Aircraft.....	2
Table 5.1. Continental Motors, engine specification according to manufacturer ref. [7]	11
Table 5.2. Piston Engine Power dependency on Altitude	12
Table 5.3. Engine Power and Fuel consumption according to manual	13
Table 5.4. Fuel consumption according to engine manual, ref. [7].....	13
Table 6.1. Typical mission profile, ref [6].....	14
Table 6.2. Statistical method of estimating the empty mass ratio	17
Table 6.3. Estimation of empty weight build up.	17
Table 6.4. Component weight as a fraction of empty weight.....	20
Table 7.1. The lift coefficient vs alpha for our studied airfoils.	22
Table 8.1. Loading configurations and the forward and aft c.g limits.	32
Table 8.2. The Loading configurations for Ground and Inflight and their location on the MGC.	33
Table 12.1. forward and after limit for determining the required sizing of the horizontal tail.	48

List of Figures

Figure 3.1. Corvalis TTx, ref [13]	2
Figure 3.2. Cirrus SR22, ref [14].....	2
Figure 3.3. Mooney M20R, ref [14]	3
Figure 3.4. Pipistrel Panthera, ref. [13]	3
Figure 3.5. Solaris Aviation Sigma 310, ref [15]	3
Figure 4.1. Typical Constraint Diagram, ref. [2].....	4
Figure 4.2. Schema of ground run, lift-off speed and its graphical explanation, ref. [2]	7
Figure 4.3. Constrain diagram showing the Wing loading vs. Thrust loading.....	8
Figure 4.4. Plotted graph for various requirements showing the thrust to weight ratio converted from figure 3.3 to the power required at each altitude.	9
Figure 4.5. Required Power for the various requirements normalized to sea level conditions. From this figure, we now can select our engine.	9
Figure 6.1. True Drag Polar illustrating the simplified and the adjusted drag model, ref. [2] .	15
Figure 7.1. The spanwise lift distribution for the non-twisted wing design.	25
Figure 7.2. The lift distribution for a twisted wing design.	25
Figure 7.3. A schematic drawing of the twisted wing design.....	26
Figure 7.4. Recommended field of view for the pilot and minimum internal cockpit dimensions.	26
Figure 7.5. A typical 2+2 seating configuration cockpit layout. Image is showing a cabin of Pipistrel Pathera aircraft.	27
Figure 7.6. Field of view for the cockpit configuration layout.....	27
Figure 7.7. Main dimensions of our designed fuselage	28
Figure 7.8. Simplified model for determining the tail arm.....	29
Figure 7.9. Wetted area of tail vs tail arm	30
Figure 7.10. The optimum point for the empennage.	31
Figure 9.1. Lifting curve for the wing with flaps	38
Figure 9.2. Lifting curve of the wing with fuselage and lifting curve of the Aircraft.....	39
Figure 10.1.....	42
Figure 11.1. Moment characteristics for wing, wing/fuselage and aircraft.	46
Figure 13.1. Thrust required vs Thrust available.	50
Figure 13.2. Propeller efficiency graph for a constant speed propeller showing efficiency at V_{max}	50

1 Introduction

Since the invention of the first motorized airplane over one hundred years ago, much has changed in aviation. Now there exist aircraft of almost any imaginable shape and size mend for any kind of purpose. No matter how a design may look like, everything starts with a sophisticated research being done for the class of aircraft one wants to design. To minimize any confrontation with problems at such early stage regarding the capabilities of a design, one must study the Certification Specifications of the design class. For General Aviation Aircraft it is the CS-23 ref. [1] in which it is stated that the upper allowable takeoff limit weight should not exceed 5670 kg. A second limitation is the required minimum stall speed at landing configuration V_{S0} which must not exceed 113 km/h. Each seat occupied must be certified for an occupant of 77 kg, however for GA a value of 91 kg is recommended as stated in Table 12-14 in ref. [2]. On the other hand, such a high weight will decrease the useful load which is the sum of total occupants, fuel and payload. For this reason, a slightly smaller value of 86 kg, see ref [] per occupant will be chosen. Taking compromises and choosing a lower value then the recommend one saves a hypothetical weight of 20 kg which then could be used for extra fuel or payload.

Ideally the designer wants to have an aircraft of low empty weight, easy handling and inexpensive manufacture and maintenance as well as a good performance, etc. All those problems the designer must face and each of those requires an individual approach. As an example, if the designer's only intention is to have a low stalling speed and a long range, which means a lot of fuel capacity, then an excellent solution for this would be a large wing area. Larger wing area on the other hand means a higher structural weight, loss of performance and a significant increase in drag. Taking this into account, the designer is faced not only to balance but also to take compromises in his design. For this reason, trade studies are done. Companies spend a lot of money and time for this early stage to study and achieve the best possible solution.

2 Objective

The objective of this thesis is to come up with a conceptual design for a four-seating configuration GA aircraft. It should be capable of achieving a cruising speed of at least 300 km/h and have a range of 2000km. All the main aircraft parameters should be presented.

3 Research

After deciding what aircraft class and Certification Specification, it is then the next step to find some similar already existing aircraft and study their data. The best source for this is a series of books called: "*Jane's All the World's Aircraft*" in particular the Aircraft "*In Service, Production and Development*" where all important parameters are listed. The goal is to find similar aircraft regarding performance, endurance and size as our mission profile for our design.

3.1 Setting the requirements

Before starting any kind of research, design goals should be set. By setting some important parameters for our design such as range and cruising speed, we limit the search to some aircraft preferred which are listed in Table 2.1.

The minimum Design requirements for the Aircraft are as followed:

- Four-Seating configuration assuming occupant weight of 86 kg
- Luggage weight per occupant = 8 kg
- Cruising speed $V_C \geq 310$ km/h
- Stall speed $V_{S0} \leq 110$ km/h
- Service ceiling $h_{\text{ceiling}} \geq 6000$ m
- Rate of climb $ROC \geq 7$ m/s
- Takeoff ground run < 400 m
- Range = 2000 km

	Corvalis TTx	Cirrus SR22	Mooney M20R	Pipistrel Panthera	Solaris Aviation Sigma 310	Average Value
MTOW [kg]	1633	1542	1528	1315	1352	1474
Empty Mass [kg]	1134	1009	1009	815	850	963
Fuel Mass [kg]	278	250	242	350	250	274
Rated Power [kW]	231	231	224	194	231	222
Max. Wing Loading [kg/m ²]	125	111	94	117	105	110
Wing Area [m ²]	13,12	13,9	16,26	11,2	12,94	13,48
Wing Span [m]	10,97	11,68	11	10,86	9,5	10,80
Wing Aspect Ratio [-]	9,17	9,81	7,44	10,53	6,97	8,78
Wing Taper Ratio [-]	0,73	0,5	0,49	0,51	0,63	0,57
V_C [km/h]	332	343	356	367	320	344
$V_{y\text{max}}$ [m/s]	7,12	7,12	6,35	7,9	6,65	7,03
V_{S0} [km/h]	110	112	110	112	106	110
Range [km]	2315	1942	2370	1852	1555	2007
Takeoff distance [m]*	393	480	792	540	427	526

*Takeoff distance – Distance till aircraft reaches 15 m above ground (obstacle height)

Table 3.1. Statistical Parameters of similar GA Aircraft

Table 3.1. will serve as an orientation for initial design.



Figure 3.1. Corvalis TTx, ref [13]



Figure 3.2. Cirrus SR22, ref [14]



Figure 3.3. Mooney M20R, ref [14]



Figure 3.4. Pipistrel Panthera, ref. [13]



Figure 3.5. Solaris Aviation Sigma 310, ref [15]

4 Constraint analysis

4.1 The method of constraining

At this point we can start with the constraint analysis with the parameters we defined in the previous chapter. The main advantage of a Constraint Analysis is that it can be used to assess the optimum design point(s) required for the wing area and power plant design, such that it will meet all performance requirements.

The following general performance characteristics are used for this analysis:

- Level constant velocity turn
- Desired rate of climb (ROT)
- Desired T-O distance
- Desired cruise airspeed
- Service ceiling limitations
- Stalling speed requirement
- Desired ground run

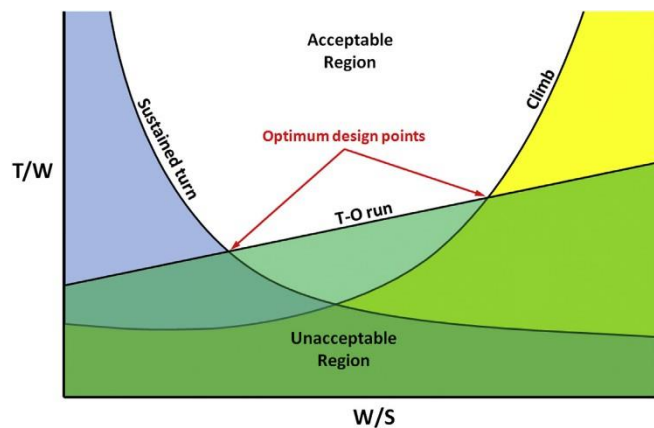


Figure 4.1. Typical Constraint Diagram, ref. [2]

In addition to this we also need to perform a first estimation of the gross weight. Since our design is intended for a four-seater aircraft and as required according to CS 23.25 weight limits – maximum weight, the weight should not be less than with:

(i)

- each seat occupied, assuming a weight of 77kg for each occupant;
- oil at full capacity;
- at least enough fuel for maximum continuous power operation of at least 30 minutes for day VFR approved airplanes and at least 45 minutes for night VFR and IFR approved airplanes;

(ii)

- the required minimum crew, and fuel and oil to full tank capacity.

According to ref. [3] the Author D. Stinton has an equation for the first estimation of gross weight (MTOW or MTOM).

$$\text{MTOM} = \text{factor} * \text{payload} \quad (4.1)$$

where the factor for small aircraft such as very light aircraft starts from 2,5. Aircraft designed for long range flights may have factors of 6 to 8 times the payload. Again, the statistical data can help to guess a factor value. In equation (4.1) the payload is referred to the weight of the occupants and the luggage weight. For the first estimation, a factor of 4 will be used. This blends in with our statistical data.

Now the total payload (occupants and luggage) can be calculated. The max. occupant weight was set to 86 kg and the max luggage weight to 8 kg, thus giving a total payload of 376 kg

According to the equation (3.1) the maximum take-off weight is as followed:

$$M_{TOW} = 4 \cdot 376 = 1504 \text{ kg}$$

Now a constraint analysis according to the set parameters can be performed. Some parameters such as the min. drag coefficient $C_{D \min}$, the estimated drag coefficient at takeoff configuration $C_{D TO}$ as well as the lift coefficient at takeoff configuration $C_{L TO}$ must be defined. At this point, it can only be estimated, therefore for a high performance GA Aircraft the values below are from Table 3-1 according to ref [2]. The aspect ratio is chosen according to statistical data.

- aspect ratio = 9 [-]
- minimum drag coefficient (estimated) = 0,025 [-]
- drag coefficient at takeoff configuration (estimated) = 0,035 [-]
- lift coefficient at takeoff configuration (estimated) = 0,7 [-]

In order to plot a graph as seen in Figure 3.1, equations which are a function of wing loading [kg/m^2] need to be set, from which is then calculated the required T/W ratio for each desired requirement.

4.2 Constraining for a level constant-velocity turn

For a level constant-velocity turn or in other words the trust required to maintain a specific banking load factor n at a specific airspeed and altitude, the following equation can be used:

$$\left(\frac{T}{W}\right)_1 = q_1 \left[\frac{C_{D \min}}{\frac{M}{S} g} + k \left(\frac{n}{q_1}\right)^2 \frac{M}{S} g \right] ; [-] \quad (4.2)$$

$$q_1 = \frac{1}{2} \rho_1 v_c^2 ; [\text{N}/\text{m}^2] \quad (4.3)$$

$$\rho_1 = \rho_0 \left(1 - \frac{0,0065}{288,15} h\right)^{4,256} ; [\text{kg}/\text{m}^3] \quad (4.4)$$

where q_1 is dynamic pressure at cruising altitude h , and cruising speed v_c and ρ_1 is density at cruising altitude h , where ρ_0 is density at sea level according to ISA. k is the lift induced drag constant which will be explained later in more detail. M/S is the wing loading, the dependent variable. It will be defined in a range from 20 to 200 kg/m^2 . The banking load factor n , for which a max. bank angle ϕ of 45° at cruising speed was set, leads to a value of $\sqrt{2} \approx 1,414$ according to equation:

$$n = \frac{1}{\cos \phi} ; [-] \quad (4.5)$$

4.3 Constraining for a desired rate of climb

The required thrust to weight ratio for a desired rate of climb can be found according to the following equation:

$$\left(\frac{T}{W}\right)_2 = \frac{v_v}{v_{climb}} + \frac{q_2}{\frac{M}{S} * g} * C_{D\ min} + \frac{k}{q_2} * \frac{M}{S} * g ; [-] \quad (4.6)$$

$$q_2 = \frac{1}{2} * \rho_0 * v_{climb}^2 ; [N/m^2] \quad (4.7)$$

where q_2 is dynamic pressure at sea level and climb speed v_{climb} . The climb speed is based on statistical data from which the average was taken. (Cirrus SR22 – 101 KTS; Pipistrel Panthera 80 KTS) see references [4], [5] respectively. The average of 90,5 KTS \approx 170 km/h rounded off to the nearest ten. v_v is the required rate of climb.

To calculate the lift induced drag constant k , the so-called Oswald's span efficiency for a straight wing must be estimated. This can be calculated according to the equation (3.8) and the lift induced drag according to the equation (3.9).

$$e = 1,78 * (1 - 0,045 * AR^{0,68}) - 0,64 \quad (4.8)$$

For the chosen aspect ratio, the equation (3.8) will give a span efficiency of 0,7831. This equation is defined by Raymer, ref. [6]. More sophisticated equations do exist, however, for their estimation more parameters are required which for our design are simply not available.

According to the wing theory, the induced drag coefficient of a 3D wing with an elliptical lift distribution equals the square of the lift coefficient divided by the product of aspect ratio and π . However, few wings actually have an elliptical lift distribution. The extra drag due to the nonelliptical lift distribution and the flow separation can be accounted for using "e".

The lift induced drag constant k can be found according to the following equation:

$$k = \frac{1}{\pi * AR * e} \quad (4.9)$$

4.4 Constraining for a desired take-off distance

The required thrust to weight ratio to achieve a desired take-off distance can be calculated according to the equation:

$$\left(\frac{T}{W}\right)_3 = \frac{v_{LOF}^2}{2 * g * S_G} + \frac{q_3 * C_{D\ TO}}{\frac{M}{S} * g} + \mu * \left(1 - \frac{q_3 * C_{L\ TO}}{\frac{M}{S} * g}\right) \quad (4.10)$$

$$q_3 = \frac{1}{2} * \rho_0 * \left(\frac{v_{LOF}}{\sqrt{2}}\right)^2 \quad (4.11)$$

where v_{LOF} is the lift-off speed at the point where the wheels lift of the ground, S_G is the ground run as can be seen in Figure 3.2. μ is the ground friction coefficient according to Table 17-3 *Ground Roll Friction Coefficients* as presented in ref [2]. For our design we will

assume a value of 0,04. The dynamic pressure q_3 is calculated at ground density ρ_0 and for $\frac{v_{LOF}}{\sqrt{2}}$, as is explained below.

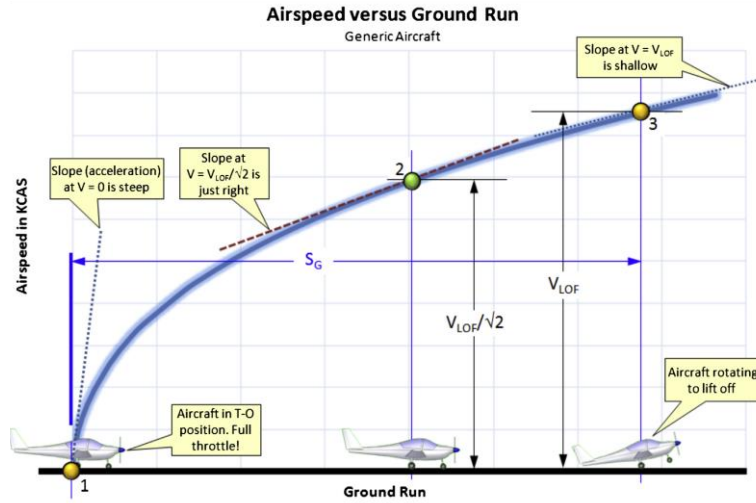


Figure 4.2. Schema of ground run, lift-off speed and its graphical explanation, ref. [2]

If we would consider the acceleration at point 1, where the acceleration is at its highest, the ground run would be much lower. The opposite is true at point 3 where the acceleration is very small and therefore, if point 3 would be considered, the ground run would be much larger. This implies that somewhere between these two points must exist a speed for which the ground run agrees well to experimental data. This speed is $\frac{v_{LOF}}{\sqrt{2}}$.

4.5 Constraining for a desired cruise airspeed

The required trust to weight ratio to achieve a certain speed, in this case the cruise speed, can be calculated according to the equation:

$$\left(\frac{T}{W}\right)_4 = q_1 * C_{D\ min} * \frac{1}{\frac{M}{S} * g} + \frac{k}{q_1} * \frac{M}{S} * g \quad (4.12)$$

where q_1 is the dynamic pressure according to the equation (3.3) where the cruise speed V_C is implemented.

4.6 Constraining for a desired service ceiling

The required trust to weight ratio to achieve a desired service ceiling, can be calculated according to the equation:

$$\left(\frac{T}{W}\right)_5 = \frac{v_v\ ceiling}{\sqrt{\frac{2}{\rho} * \frac{M}{S} * g * \sqrt{\frac{k}{3 * C_{D\ min}}}}} + 4 * \sqrt{\frac{k * C_{D\ min}}{3}} \quad (4.13)$$

$$\rho_2 = \rho_0 * \left(1 - \frac{0,0065}{288,15} * h_{ceiling}\right)^{4,256} \quad (4.14)$$

where $v_{v \text{ ceiling}}$ is the lowest acceptable rate of climb, usually 100 fpm = 0,508 m/s and ρ_2 is the density at service ceiling. However, many aircraft manufacturer certify their aircraft only up to a maximum certified ceiling of 7620 m since any higher would require a cabin pressurization. This means that these aircraft can reach higher ceilings as they are certified for and since they do not reach the minimum acceptable rate of climb at the certified altitude, they have a greater advantage over those who are certified truly to the absolute service ceiling.

For the constraint analysis, the software MATLAB was used. For every other calculation, Microsoft Office Excel.

4.7 Plotting the results

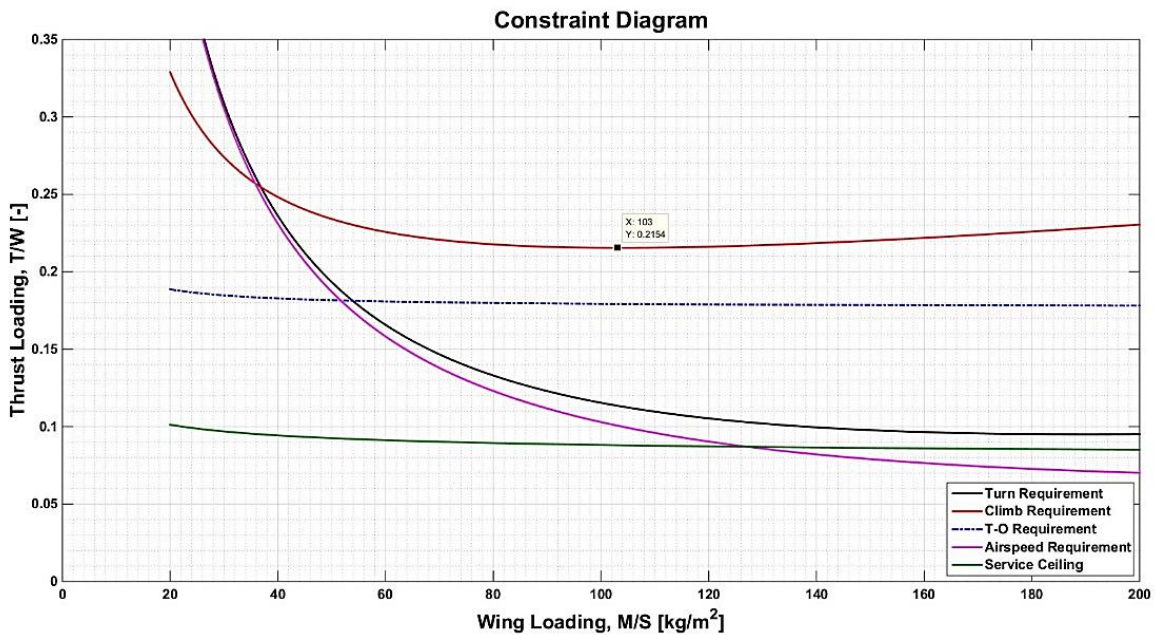


Figure 4.3. Constrain diagram showing the Wing loading vs. Thrust loading

We can see that everything above a wing loading of 37 kg/m² is influenced by the climb requirement only. The optimum point lies therefore at the minima of curve for the climb requirement as can be seen in the figure above. For a wing loading of 103 kg/m² the required thrust to weight ratio is 0,21544. Meaning that, if we take the estimated design mass M_{TOM} of 1504 kg, then this would require a wing area of 14,6 m². A rather large area compared to our statistical data. The required thrust as well as the Power for this point can be calculated according to the following equations:

$$T = \frac{T}{W} Mg ; [N] \quad (4.15)$$

$$P = \frac{T v_{climb}}{\eta 1000} ; [kW] \quad (4.16)$$

For our design, we estimate a propeller efficiency of 0,85. However, we must bear in mind that such can only be achieved with a perfect Constant-speed propeller. Thrust and Power required to fulfill the climb requirement therefore are:

$$T = 3177,6 \text{ N}$$

$$P = 176,53 \text{ kW}$$

This is however only true for the climb requirement, since it is not altitude dependent. All other requirements, except for the take-off requirement, are dependent on altitude and must be normalized to the performance which is needed at sea level.

It is now possible to plot the required Power as a function of wing loading as seen in the figure below.

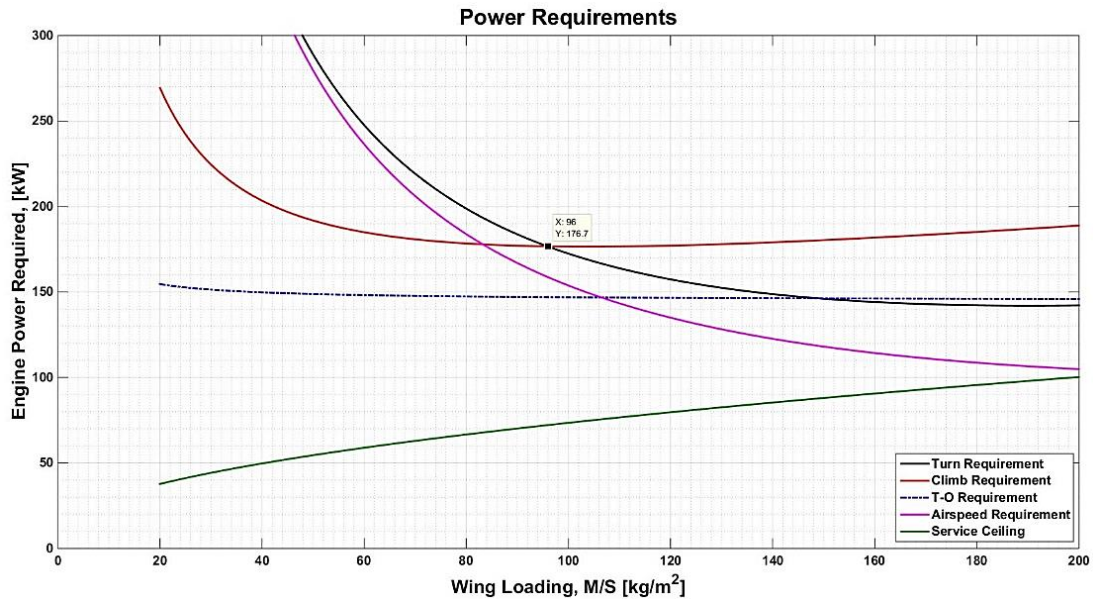


Figure 4.4. Plotted graph for various requirements showing the thrust to weight ratio converted from figure 3.3 to the power required at each altitude.

It is evident that the optimum point has shifted to a lower wing loading. However, the turn requirement is starting to dominate and is the most challenging to compute with as will be seen in the next figure.

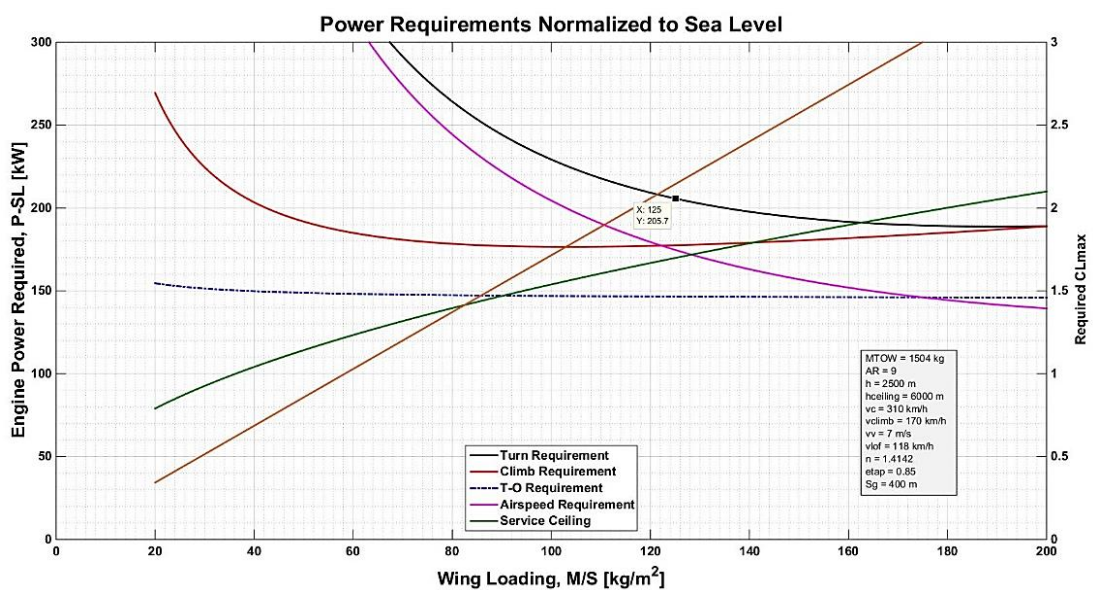


Figure 4.5. Required Power for the various requirements normalized to sea level conditions. From this figure, we now can select our engine.

After converting the required power for each requirement to sea level conditions, the so-called Power normalization, we can now choose an engine for which its power will fulfill all the requirements in this constraint analysis. However, we can no longer consider the optimum points from figure 3.3. and figure 3.4. since it would not fulfill the required C_{Lmax} . The new optimum point bounded by the service ceiling and the turn requirement has a wing loading of 163 kg/m^2 which is unrealistic for our design. It would more over require a C_{Lmax} of 2,79 at a wing area of $9,227 \text{ m}^2$ and a wingspan of 9,11 m. One must be careful not to come to any sudden conclusions. One must choose a point which lies in the boundary area as well as be realistically viable. From this new point, we can select the required engine. For the design, a wing loading of 125 kg/m^2 was chosen which gives as a C_{Lmax} of 2.14 that can be solved with an ordinary high-lift device.

5 Selecting an appropriate engine

5.1 Engine choice

From our design point in figure 3.5. we can see that for a wing loading of 125 kg/m^2 an engine of at least 206 kW rated power should be selected. Since most GA aircraft fly with piston engines it would be only wise to choose this one as well. The second thing to consider when selecting an engine is the type of fuel used. The chosen engine must be certified for the type of fuel one wants to use. The most important points to bear in mind is that the overall take-off weight as well as other parameters in our design may change. The M_{TOW} generally never stays as low as estimated and therefore we must take this potential weight into account. The propeller efficiency of 0,85 was chosen very optimistically and therefore we must reflect that the real value will probably be lower, somewhere around 0,8 and 0,85. Last but not least, the coefficient of lift and drag at take-off configuration as well as the minimum drag coefficient are only recommended values and may change slightly for our design, generally the minimum drag coefficient increases. Recalculating the constrain analysis and considering, since our calculations are set very optimistically, a structure that weights 50kg more, a guessed value, and a decreased propeller efficiency of 0,8 due to imperfections, then for the same wing loading we will get a required rated power at sea level conditions of 226 kW or 303 BHP. However, the maximum coefficient of lift $C_{L_{\text{max}}}$ also increases to 2,27.

Most common piston engines for high performance single engine aircraft are Lycoming or Continental Motors. Lycoming engines tend to weight little more than their competitors but it is lastly up to the manufacturer which of those he chooses. From our statistical data, four out of 5 use a Continental engine.

For our design, a Continental Motors 550series engine was chosen. In the table below are three variations of the same engine type, N, P and R. It is evident that the IO-550-P engine has the highest power to mass ratio and the lowest dry weight. This is the engine used for our design.

Continental Motors 550 Series												
#	Rated Power 100%		Fuel Consumption Rated Power 100%		Power at Cruise 85%		Fuel Consumption P. Cruise 85%		Power at Cruise 75%		Fuel Consumption P. Cruise 75%	
	[BHP]	[kW]	[lbs/h]	[kg/h]	[BHP]	[kW]	[lbs/h]	[kg/h]	[BHP]	[kW]	[lbs/h]	[kg/h]
IO-550-N	310	231,167	160	72,57	262	195,4	132	59,87	232	173,0	118	53,52
IO-550-P	310	231,167	160	72,57	262	195,4	132	59,87	232	173,0	118	53,52
IO-550-R	310	231,167	160	72,57	262	195,4	132	59,87	232	173,0	118	53,52

Continental Motors 550 Series						
Power at Cruise 65%	Fuel Consumption P. at Cruise 65%	RPM	Dry Weight [kg]	P/M [kW/kg]	SFC 100% [kg/kWh]	
	[lbs/h]					
201	149,9	108	48,99	2700	204,25	1,132
201	149,9	108	48,99	2700	194,59	1,188
201	149,9	108	48,99	2700	199,35	1,160

Table 5.1. Continental Motors, engine specification according to manufacturer ref. [7]

5.2 Effect of altitude on engine power

Since the power, for a piston engine, decreases with altitude we have to find the optimum power selection for our cruising altitude of 2500 m. Smaller GA aircraft generally fly between 1500 and 3500 m.

There are many models for how to calculate the power-altitude dependency. For our design, we are going to compare two models. A simple altitude-dependency model and the Gagg & Ferrar model, ref. [2].

Simple altitude-dependency model:

$$P = P_{SL} \left(\frac{\rho}{\rho_{SL}} \right) = P_{SL} \sigma ; [kW] \quad (5.1)$$

where P is the Power at altitude, P_{SL} is Power at sea level, ρ is the density at altitude and σ is the density ratio at altitude.

Gagg and Ferrar model:

$$P = P_{SL} \left[\sigma - \left(\frac{1 - \sigma}{7,55} \right) \right] ; [BHP] \quad (5.2)$$

where P_{SL} is the Power at altitude in BHP, therefore P, the power at altitude will also be in BHP. σ is the density ratio.

After calculating each model to our desired service ceiling, we will get the following dependency.

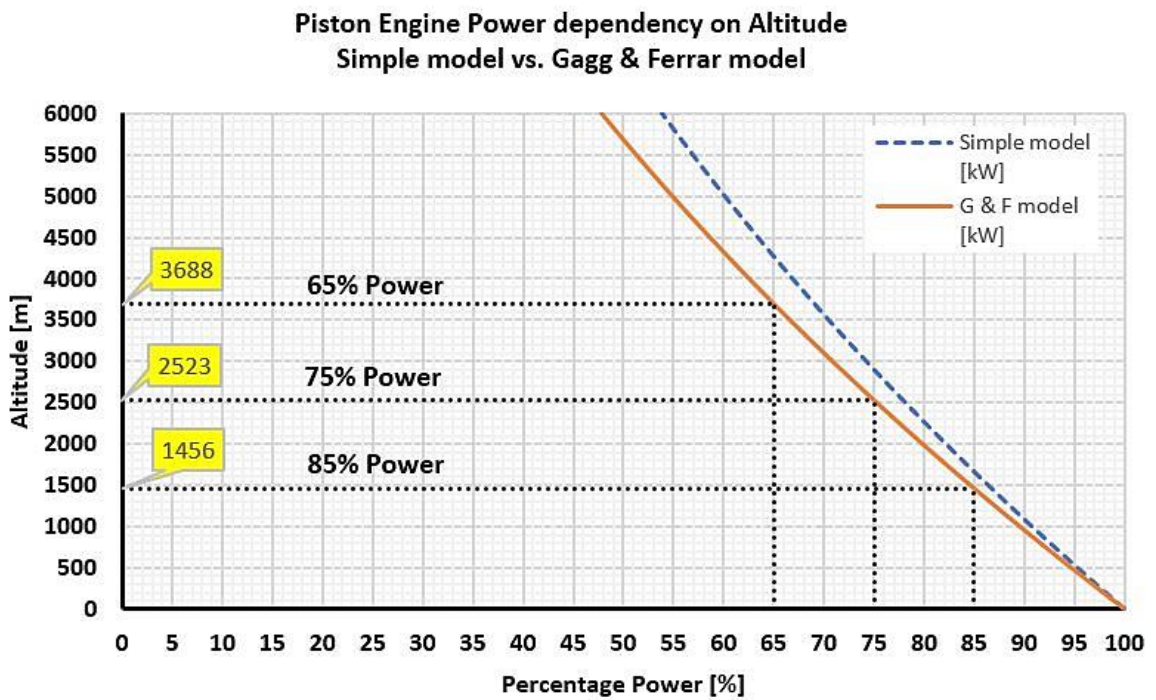


Table 5.2. Piston Engine Power dependency on Altitude

The Gagg and Ferrar model matches much better with the manufacturer data than the other model. This is why it is recommended to use for design work. Ref. [2].

5.3 Engine power and the requirement for fuel consumption

The rated fuel consumption for each cruising power can be found in the engines manual. This is found in table 2.7. IO-550-P Specification in ref. [7].

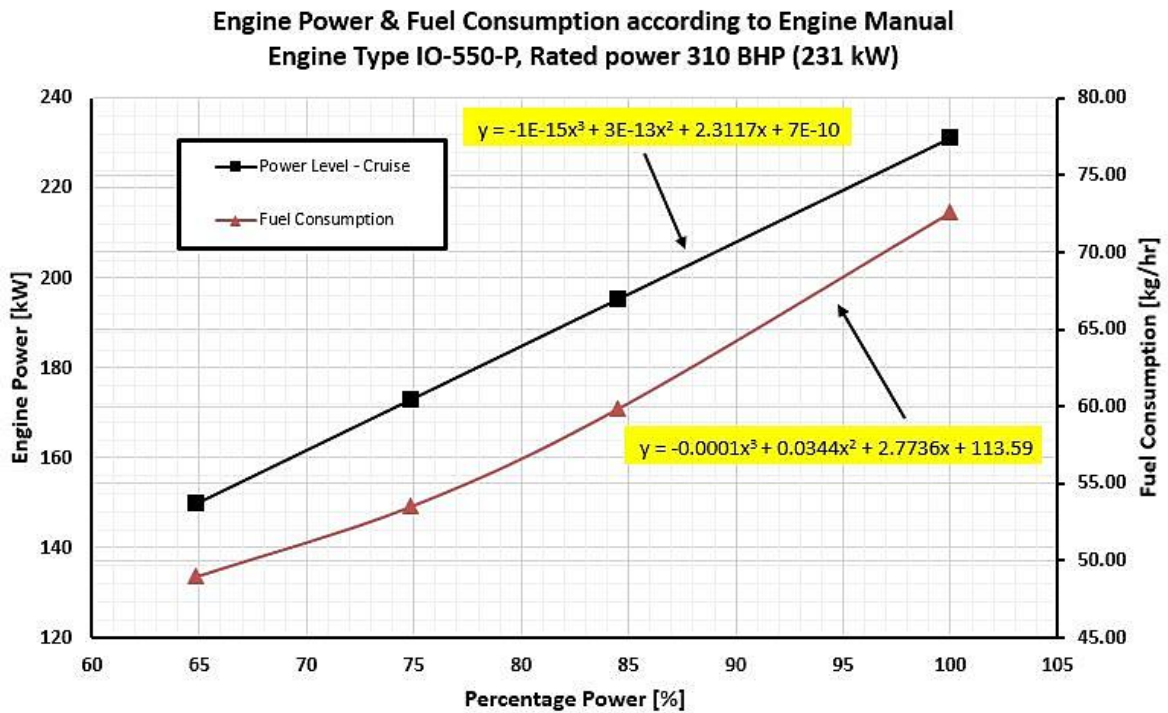


Table 5.3. Engine Power and Fuel consumption according to manual

Engine Manual Data				
Power Level - Cruise			Fuel Consumption	
[BHP]	[kW]	%	[lbs/hr]	[kg/hr]
310	231.17	100	160	72.57
262	195.37	85	132	59.87
232	173.00	75	118	53.52
201	149.89	65	108	48.99

Table 5.4. Fuel consumption according to engine manual, ref. [7]

On the right side are the fuel consumption for each power level. Note that these are the maximum values. For best power or best economy we must refer to the performance chart presented in the engine manual, see figure 2-34 ref. [7].

6 Weight estimation

6.1 The mission profile

Based on the target performance, various performance parameters were calculated and plotted to find the optimal design point. The next step is to estimate the initial empty and gross weight by calculating the weight ratios. Those Ratios can either be obtained from statistical data or by calculating them directly from the mission profile. How such a typical mission profile looks like, can be seen below. Ref.[6].

The mission profile is divided into 6 sections:

- 0 → 1: Warmup & Takeoff - $\frac{W_1}{W_0}$
- 1 → 2: Climb - $\frac{W_2}{W_1}$
- 2 → 3: Cruise - $\frac{W_3}{W_2}$
- 3 → 4: Loiter - $\frac{W_4}{W_3}$
- 4 → 5: Descent - $\frac{W_5}{W_4}$
- 5 → 6: Land - $\frac{W_6}{W_5}$

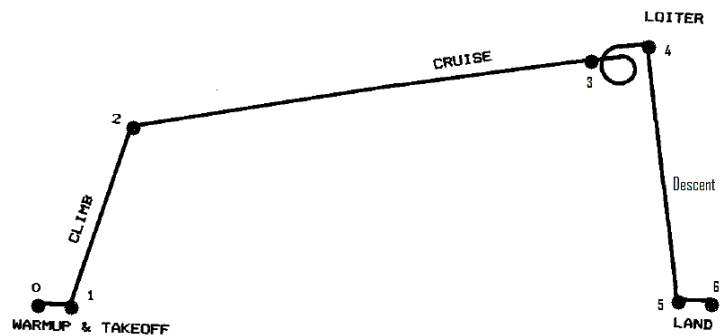


Table 6.1. Typical mission profile, ref [6]

Each section has its weight ratio based on its previous point. To find

the total mission weight ratio, we must take the product of all the sections, hence, we get $\frac{W_6}{W_0}$.

According to ref. [2], the weight ratios for warm-up, takeoff, climb and landing remain more or less constant for any particular class of aircraft, thus we will use the weight ratio for those section according to table 20-6 *Weight ratios for GA aircraft, Single-engine piston*, in ref. [2].

- Warm-up and takeoff = 0,995
- Climb = 0,988
- Descent = 0,997
- Landing = 0,995

The weight ratio for cruise and loiter must be calculated since they depend strongly on the cruise setting, specifically the fuel consumption, altitude, cruise speed, propeller efficiency, range, and last but not least the overall climate conditions. However, we are assuming ideal ISA conditions, a propeller efficiency of 0,85, a fuel consumption for best power at 45 kg/h for cruise and 54 kg/h at continuous power for loiter, see figure 2-34 in ref. [7] as well as a cruising speed of 330 km/h at 2500 m. Finally, for our design we want to achieve a range of at least 2000 km. Regarding the loiter time, fuel for maximum continuous power operation of at least 30 minutes for day VFR must be available.

The specific fuel consumption for the cruise and loiter section can be calculated as followed:

$$c_t = \frac{q_{fc,l} \cdot g}{P \cdot 1000 \cdot 3600 \cdot \eta} v_c ; [1/s] \quad (6.1)$$

where $q_{fc,L}$ is the mass fuel consumption per hour for cruise and loiter respectively. P is the power available at cruise altitude, η is the propeller efficiency and v_c is the cruising speed.

For cruise, $c_t = 7,64147 \cdot 10^{-5} [1/s]$

For loiter, $c_t = 9,16977 \cdot 10^{-5} [1/s]$

For a aspect ratio of 9,2 Oswald's span efficiency e is 0,7778 and therefore, induced drag constant k will be 0,0445.

$(C_L/C_D)_{max}$ or the smallest coefficient of drag at the greatest coefficient of lift which in fact represents the point where a line, whose origin is at the zero point, is tangent to the drag polar. However, at this moment we can only assume the point since we do not have enough data to create a drag polar. $(C_L/C_D)_{max}$ can be estimated by the following equation. However, assuming the simplified drag model:

$$LD_{max} = \left(\frac{C_L}{C_D}\right)_{max} = \frac{1}{\sqrt{4 \cdot C_{Dmin} \cdot k}} ; [-] \quad (6.2)$$

For cruise, $(C_L/C_D)_{max} = 14,993 [-]$

For loiter, see ref. [6] it is $0,866 \cdot (C_L/C_D)_{max}$ of cruise = 12,984 [-]

To make a better prediction of $(C_L/C_D)_{max}$ the adjusted drag model can be calculated by the following equation:

$$LD_{max} = \left(\frac{C_L}{C_D}\right)_{max} = \frac{1}{\sqrt{4 \cdot C_{Dmin} \cdot k + (2k \cdot C_{LminD})^2 - 2k \cdot C_{LminD}}} ; [-] \quad (6.3)$$

However, we are not going to use this equation at this point since we are not able to make a prediction for C_{LminD} , the lift at C_{Dmin} . In the illustration below are both models represented.

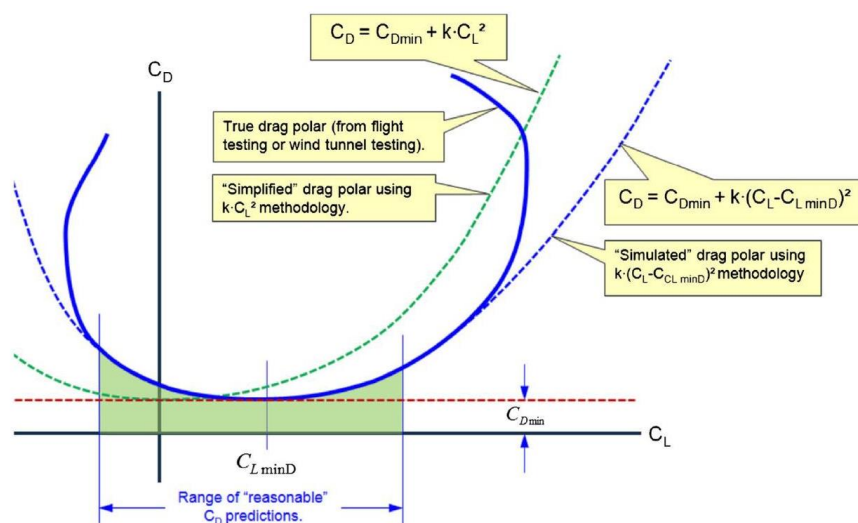


Figure 6.1. True Drag Polar illustrating the simplified and the adjusted drag model, ref. [2]

The total mission weight ratio:

$$\frac{W_6}{W_0} = \frac{W_1}{W_0} \frac{W_2}{W_1} \frac{W_3}{W_2} \frac{W_4}{W_3} \frac{W_5}{W_4} \frac{W_6}{W_5} ; [-] \quad (6.4)$$

If the mass before and after the cruise sections is known, the cruising range can be determined, ref. [6].

$$R = \frac{v_c}{c_t} \frac{L}{D} \ln \frac{W_{i-1}}{W_i} ; [km] \quad (6.5)$$

then:

$$\frac{W_i}{W_{i-1}} = e^{\left(\frac{-R \cdot c_t}{v_c \cdot \frac{L}{D}} \right)} ; [-] \quad (6.6)$$

The same is valid for the Endurance at loiter. If 30 minutes for day VFR are required the equation for endurance is as followed, ref. [6]:

$$E = \frac{L/D}{c_t} \ln \frac{W_{i-1}}{W_i} ; [minutes, hours] \quad (6.7)$$

then:

$$\frac{W_i}{W_{i-1}} = e^{\left(\frac{-E \cdot c_t}{L/D} \right)} ; [-] \quad (6.8)$$

Therefore, the weight ratio for the cruise section is:

$$\frac{W_3}{W_2} = 0,8948 [-]$$

and the weight ratio for the loiter section is:

$$\frac{W_4}{W_3} = 0,9874 [-]$$

The total mission weight ratio according to equation (5.4.) therefore is:

$$\frac{W_6}{W_0} = 0,8616 [-]$$

According to Raymer in equation 3.11 ref. [6], the total fuel fraction, assuming 6% for reserve and trapped fuel, can be calculated as followed:

$$\frac{W_f}{W_0} = 1,06 \left(1 - \frac{W_x}{W_0} \right) ; [-] \quad (6.9)$$

where the fraction W_x/W_0 represents the total mission weight ratio.

Hence, the fuel fraction ratio for our mission is:

$$\frac{W_f}{W_0} = 0,1468 [-]$$

Equation for estimating the gross weight, according to ref. [6] equation (6.1)

$$W_0 = \frac{W_{Crew} + W_{Luggage}}{1 - \frac{W_f}{W_0} - \frac{W_e}{W_0}} \quad (6.10)$$

where $W_{Occupants}$ is the total mass of passengers, including the pilot. $W_{Luggage}$ is the total mass of the luggage.

From ref. [2] equation 6-14, we can estimate the empty fraction for our design where W_0 is in kg. This equation is based on statistical data, see figure 6-2 in same reference.

$$\frac{W_e}{W_0} = 0,8578 - 0,0333 \cdot \ln W_{0,guess} [-] \quad (6.11)$$

After substituting equation (5.11) into (5.10) we can get, after interpolation the estimated takeoff weight for our design.

Estimated $M_{TOW} = 1565$ kg

Hence, fuel mass $M_f = 230$ kg and the empty mass ratio of $0,6129 \rightarrow M_e = 959$ kg.

Raymer, ref. [6], is describing a more complex statistical method for estimating the empty mass ratio which is a function of aspect ratio, v_{max} , wing loading, power loading, gross weight and the constants as can be seen below.

Table 6.2 Empty weight fraction vs W_0 , A, hp/ W_0 , W_0/S , and V_{max} (mph)

$W_e/W_0 = a + b W_0^{C1} A^{C2} (hp/W_0)^{C3} (W_0/S)^{C4} V_{max}^{C5}$							
	<i>a</i>	<i>b</i>	<i>C1</i>	<i>C2</i>	<i>C3</i>	<i>C4</i>	<i>C5</i>
General aviation—single engine	-0.25	1.14	-0.20	0.08	0.05	-0.05	0.27

Table 6.2. Statistical method of estimating the empty mass ratio

Since at this point, the maximum speed for our aircraft is unknown, we may not use this equation.

6.2 Component weight estimation

Raymer, ref. [6] table15.2, gives a first approximation for the empty weight build up.

Item	Fraction	Multiplier	C.G Approx Location	Fraction of empty weight		Comment		
				lbf	kg			
Wing	2.5	$S_{exposed\ planform}$ [ft ²]	134.76	40 % MAC	337	153		
HT	2	$S_{exposed\ planform}$ [ft ²]	30.991	40 % MAC	62	28		
VT	2.5	$S_{exposed\ planform}$ [ft ²]	16.035	40 % MAC	40	18		
Fuselage	1.4	$S_{wetter\ area}$ [ft ²]	166.178	40-50% length	233	106		
Installed Engine	1.601	Engine weight [lb]	429	-	687	312		
Landing Gear	0.057	TOGW [lb]	3450	-	167	76	Main Gear 85%	
				-	29	13	Nose Gear 15%	
All else empty	0.162	TOGW [lb]	3450	40-50% length	559	254		
					SUM	2114	959	
					Target empty weight	2115	959	

Table 6.3. Estimation of empty weight build up.

Since the weight for our engine is known the fraction for “Installed Engine” was changed from originally 1.4 to 1.601 to reflect the true weight. Also, the fraction for all “else empty” was changed from originally 0.1 to 0.162 in order to fulfill our estimated empty weight of 959 kg.

6.2.1 Installed Engine weight

In order to estimate the installed weight for our aircraft, we will use the following approximations from ref. [6].

$$W_{EI} = 2,575 \cdot W_{ENG}^{0,922} \cdot N_{ENG} \quad (6.12)$$

where W_{ENG} is the uninstalled weight of the engine in lbf and N_{ENG} is the number of the engines. Thus, the installed engine weight is 688,45 lbf or 312,3 kg.

6.2.2 Estimating the wing weight

The equation for estimating the weight of the wing, according to Raymer:

$$W_W = 0,036 \cdot S_W^{0,758} W_{FW}^{0,0035} \left(\frac{AR_W}{\cos^2 \Lambda_c} \right)^{0,6} q^{0,006} \lambda^{0,04} \left(\frac{100 \cdot \frac{t}{c}}{\cos \Lambda_c} \right)^{-0,3} (n_Z W_0)^{0,49} \quad (6.13)$$

where S_W is the wing area in ft^2 , W_{FW} is the fuel weight in the wing, however for estimating the weight of the wing structure only $W_{FW}^{0,0035} = 1$, AR_W is the wings aspect ratio. Λ_c is the wing sweep at 25% MGC in radians, q is the dynamic pressure at cruise in lbf/ft^2 , λ is the taper ratio of the wing, t/c is the wing thickness to chord ratio, n_Z is the ultimate load factor described by $1.5 \cdot n_{\max}$ and W_0 is the estimated gross weight.

For a airfoil thickness of 0,131, such as the airfoil MS (1)-0313, a wing area of $12,52\text{m}^2$ or $134,76\text{ft}^2$, a wing sweep at the quarter chord line of $1,59^\circ$, a dynamic pressure of $84\text{lbf}/\text{ft}^2$, ultimate load factor of 5.7, cruise density at 2500m of $0,001857\text{slugs}/\text{ft}^3$ as well as for a gross weight of 1565 kg or 3450 lbf, the weight of the wing was estimated as 150,32 kg.

6.2.3 Estimating the fuselage weight

Raymer, ref. [6], is describing a equation for estimating the fuselage weight. For main fuselage dimensions, revere to chapter 6.

$$W_{HT} = 0,052 \cdot S_{FUS}^{1,086} (n_Z W_0)^{0,177} l_{HT}^{-0,051} \left(\frac{l_{FS}}{d_{FS}} \right)^{-0,072} q^{0,241} + 11,9 (V_P \Delta P)^{0,271} \quad (6.14)$$

where S_{FUS} is the wetted fuselage area in ft^2 measured directly from the 3D cad model as $18,966\text{m}^2$ or $204,148\text{ft}^2$, l_{HT} is the tail arm, l_{FS} is the fuselage length, d_{FS} is the depth of the fuselage, q is the dynamic pressure at cruise altitude, V_P is the volume of the cabin $\approx 2,562\text{m}^3$ or $\approx 90,47\text{ft}^3$ and ΔP is the cabin differential pressure, typically 8 psi.

Thus, the estimated weight for the fuselage is 107,15 kg.

On the other hand, Nicolai, see also ref. [2], describes this estimation as followed:

$$W_{HT} = 200 \cdot \left[\left(\frac{n_Z W_0}{10^5} \right)^{0,286} \left(\frac{l_F}{10} \right)^{0,857} \left(\frac{\omega_F + d_F}{10} \right) \left(\frac{V_H}{100} \right)^{0,338} \right]^{1,1} \quad (6.15)$$

where l_F is the length of the fuselage, ω_F is the max width of the fuselage, d_F is the max depth of the fuselage and V_H is the maximum level airspeed at sea level in knots. This value can be estimated according to the simplified drag model. The equation for determining V_{max} or also written as V_H and described as the maximum level airspeed, is a very important parameter for any aircraft manufacturer since it has a great marketing appeal.

Thus, the equation for V_{max} , see ref. [2], chapter 19, is:

$$V_{max} = \sqrt{\frac{T_{max} + \sqrt{T_{max}^2 - 4C_{Dmin}kW^2}}{\rho S C_{Dmin}}} ; \left[\frac{m}{s} \right] \quad (6.16)$$

However, since Thrust is a function of airspeed itself, it is almost impossible that thrust is known at v_{max} . Since $T = P/\nu$ we can substitute this into equation 5.16.

Hence, giving, after manipulation:

$$\rho S C_{Dmin} V_{max}^3 = \eta_p P + \sqrt{(\eta_p P)^2 - 4W^2 V_{max}^2 C_{Dmin} k} \quad (6.17)$$

V_{max} can be found by iteration and by guessing a first possible value.

However, this can be a very time-consuming process. This is why a different method was used, the so-called bisection method. See ref. [2] and the VB code in the attachments.

Therefore, after solving, V_{max} leads to 354 km/h or 191 knots.

Hence, the weight estimation according to Nicolai is 116,04 kg.

6.2.4 Estimating the weight of the horizontal tail

According to Raymer, the HT can be estimated as:

$$W_{HT} = 0,016(n_Z W_0)^{0,414} q^{0,168} S_{HT}^{0,896} \left(\frac{100 \cdot \frac{t}{c}}{\cos \Lambda_{HT}} \right)^{-0,12} \left(\frac{AR_{HT}}{\cos^2 \Lambda_{HT}} \right)^{0,043} \lambda_{HT}^{-0,02} \quad (6.18)$$

where q is the dynamic pressure at cruise altitude, S_{HT} is the horizontal tail area, Λ_{HT} is the HT sweep angle at 25% MGC in radians.

Thus, the estimated weight for the HT is 16,03 kg. However, since this estimated weight turned out lower, then the estimated weight in table 5.3, the following equation according to Nicolai, see also ref. [2] for estimating the HT will be used.

$$W_{HT} = 127 \cdot \left[\left(\frac{n_Z W_0}{10^5} \right)^{0,87} \left(\frac{S_{HT}}{100} \right)^{1,2} \left(\frac{l_{HT}}{10} \right)^{0,483} \sqrt{\frac{b_{HT}}{t_{HT \max}}} \right]^{0,458} \quad (6.19)$$

where b_{HT} is the HT span in feet, $t_{HT \max}$ is the HT max root chord thickness in inches.

Thus, giving a closer estimation of the HT weight of 21,41 kg.

6.2.5 Estimating the weight of the vertical tail

According to Raymer, the equation for estimating the weight of the VT is:

$$W_{HT} = 0,073(1 + 0,2F_{tail})(n_Z W_0)^{0,376} q^{0,122} S_{VT}^{0,873} \left(\frac{100 \cdot t}{c} \right)^{-0,49} \left(\frac{AR_{VT}}{\cos^2 \Lambda_{VT}} \right)^{0,357} \lambda_{VT}^{0,039} \quad (6.20)$$

where for a conventional tail, $F_{tail} = 0$, S_{VT} is the vertical tail area in feet.

Thus, for a NACA 0012 profile, the estimated weight for the vertical weight is 8,33 kg. This estimation is unrealistically low, considering a vertical area of 1,49 m². Multiplying this value by 2 will come the result estimated in table 5.3. closer. Other components such as the landing gear will be estimated according to table 5.3. However, the estimated 76 kg for a retractable main landing gear seems to be too much. This would otherwise add an equivalent of half of the wing weight.

6.2.6 Component weight as a fraction of empty weight

The purpose of this section is to find out the weight distribution as a fraction of the total empty weight, thus making it possible for a designer to design parts with emphasis on weight and to locate any specific components or locations on the aircraft which need to be reduced in weight. This is the so-called weight budgeting.

This is especially important for estimating the center of gravity, thus making it possible to create the so-called C.G. Envelope under various loading scenarios.

	% of Empty weight	weight [kg]	Wcomp/Wmetal	
Engine Casing	0.74	7.10	1	
Fuselage	12.10	116.04	estimation	
Cockpit Windows	2.68	25.70	1	
Wing	15.67	150.32	estimation	
Engine	32.57	312.30	estimation	
Engine Mounting	1.48	14.19	1	
Propeller	3.44	33	estimation	
Main landing Gear	6	50.64	0.88	
Front landing Gear	2.4	20.25	0.88	
Control wing	1.52	14.58	1	
Control tail	1.15	11.03	1	
Cockpit	1.88	18.03	1	
Avionics	1.3	12.47	1	
Batterie	3	28.77	1	
VT	1.74	16.66	estimation	
HT	2.233	21.41	estimation	
Balance weight	19.5 kg	19.5		
% of Me	89.90	871.98	959	Empty Mass [kg]
PAX	86kg	344		
Luggage	8 kg	32	87.02	weight saved [kg]
Fuel	230 kg	230		
		1477.98	1565	MTOM [kg]

Table 6.4. Component weight as a fraction of empty weight.

As can be seen from table 6.4 our empty mass target of 959 kg was not reached, saving 87 kg of weight. However, at this moment it is only an estimation. A more detailed direct weight

estimation would very likely lead to a different result. The weight reduction factor $W_{\text{comp}}/W_{\text{metal}}$, ref. [8] is a factor used when converting from a metal aircraft or part to a aircraft or part made from composite.

7 Designing the aircraft components

7.1 Selection of the airfoil for the wing layout

Many GA aircraft of the past use a four up to a six digit NACA airfoil or some kind of modification of its original geometry. Many tunnel data experiments are published and it can be sad that they are well studied. Many aircraft designers used therefore those airfoils. For a modern GA aircraft, as well as to keep up with the competitors of today, designers switch their choice to a more modern and advanced airfoil such as the Natural laminar flow airfoils (NLF), thus making it possible to sustain a laminar flow as far as 50% of the chord. Such airfoils generate less drag and tend to have a extended drag bucket. A region where the usual drag polar drops to a lower drag coefficient. This region is in big favor for takeoff and climb and many NLF airfoils are designed in such a way, that the drag bucket region extends greatly for low up to medium coefficients of lift. However, besides sustaining a longer laminar flow, NLF airfoils tend to be very sensitive when it comes to imperfections such as insects, icing or dirt. This imperfection may cause an imbalance between the left and right wing and may lead to unpredictable stall handling. This is why, when choosing an airfoil, a high coefficient of lift and a low coefficient of drag are not the only criteria. Besides the type of stall characteristic, choosing an adequate moment coefficient will reduce the trim drag. However, there is no need to focus too much on c_{m0} and weigh it more than the other two. A proper empennage design will accommodate this problem.

To find the different characteristics of such airfoils, a comparison between 5 of such airfoils was made and will be discussed.

The NLF (1)-0115 has a thickness to chord ratio of 15% at 44,1% and a maximum camber of 1,8% at 30% chord. It was designed to achieve a $C_{l_{\text{max}}}$ of 1,5 at $Re = 2,6 \cdot 10^6$ with no flaps. The moment coefficient at zero angle of attack, c_{m0} is around -0,055. The hinge moment c_H for a 20% aileron is same as for a NACA 63₂-215. The loss in $C_{l_{\text{max}}}$ due to edge roughness, rain or insects at $Re=2,6 \cdot 10^6$ is 11%. However, this airfoil was primarily designed for a wing loading of 719 to 958 N/m² or 73-97 kg/m². Since our expected wing loading is going to be higher, the airfoil becomes unusable for our design.

Other analyzed airfoils were NLF (1)-0416, NLF (1)-0215F, NLF 0414F and MS (1)-0313 as can be seen in table 6.1 and 6.2. It is clear that airfoil NLF-0215F has the highest pitching moment coefficient, however it also has the best stall characteristic out of the airfoils as well as the second highest $C_{l_{\text{max}}}$ of 1,846 at $\alpha_{\text{cr}} \approx 18^\circ$. Thus, the average moment coefficient for the linear part is -0,147. This said, we cannot blindly believe that these values are exact. From experience, it is known that the software *xflr5*, which is just a more user-friendly version of *xfoil*, overestimates and sometimes even underestimates certain values. Measurements from a wind tunnel, if available, should always be rated above any simulated data. The measured $C_{l_{\text{max}}}$ of the wind tunnel was 1,56 at $\alpha_{\text{cr}} = 11,2^\circ$. Thus, the measured $C_{l_{\text{max}}}$ is 18,3% lower and α_{cr} is as much as 61% lower. see ref. [9]

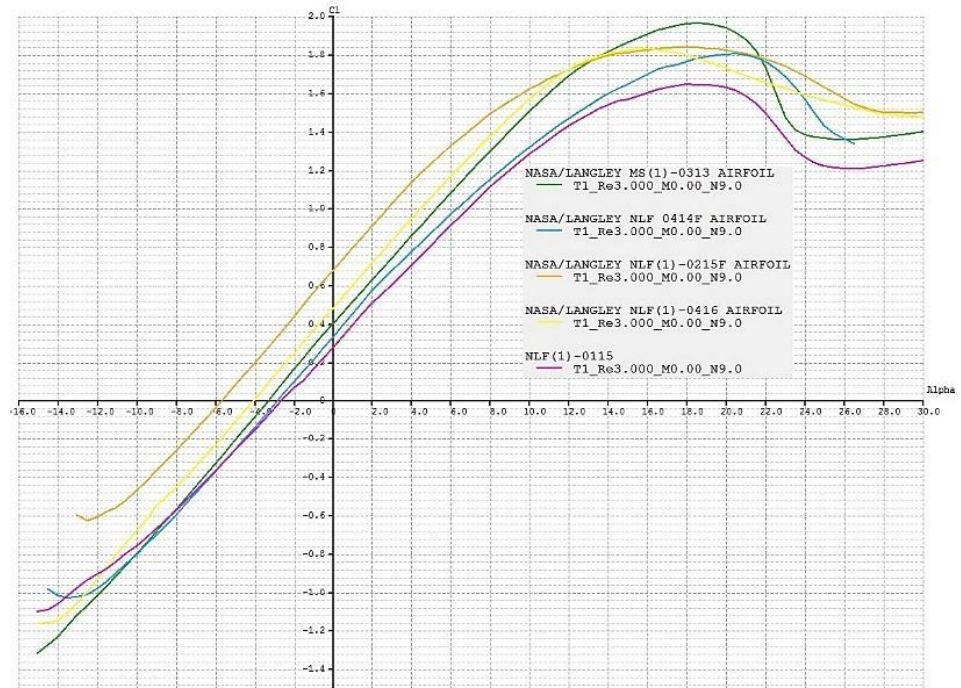


Table 7.1. The lift coefficient vs alpha for our studied airfoils.

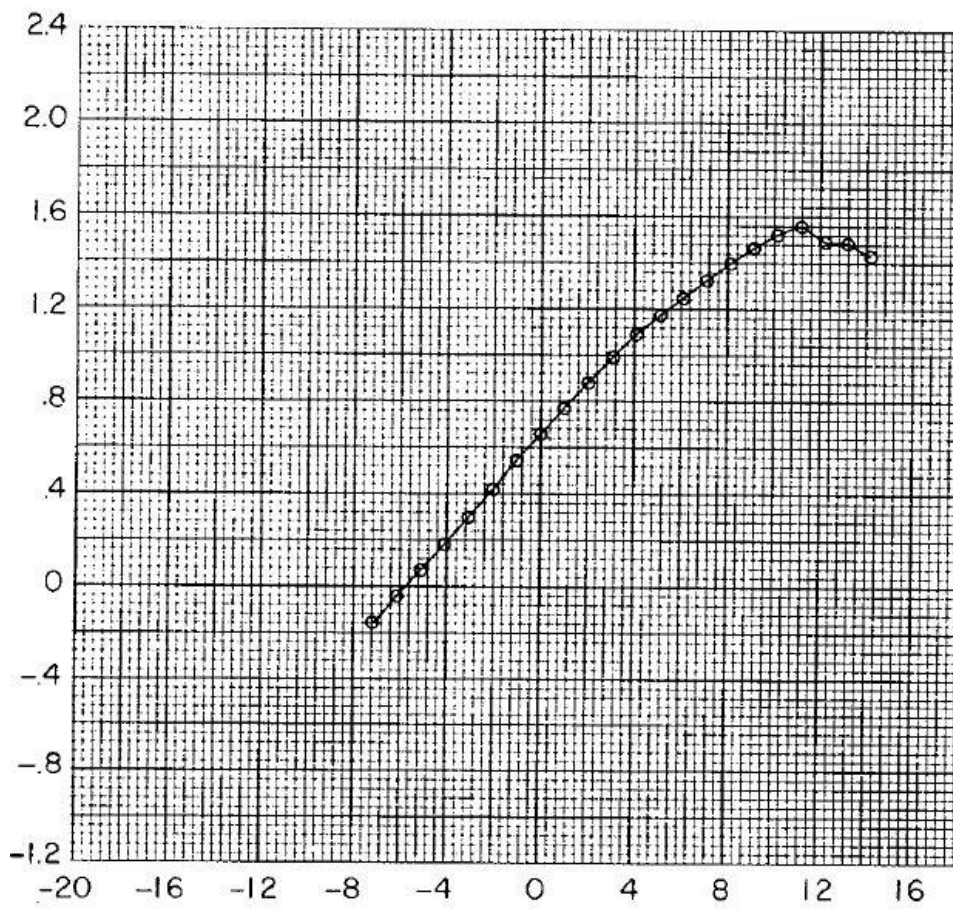


Figure 7.2. Tunnel measurements for a NLF 0215F at $Re=3 \cdot 10^6$ and $M=0,1$. ref. [9]

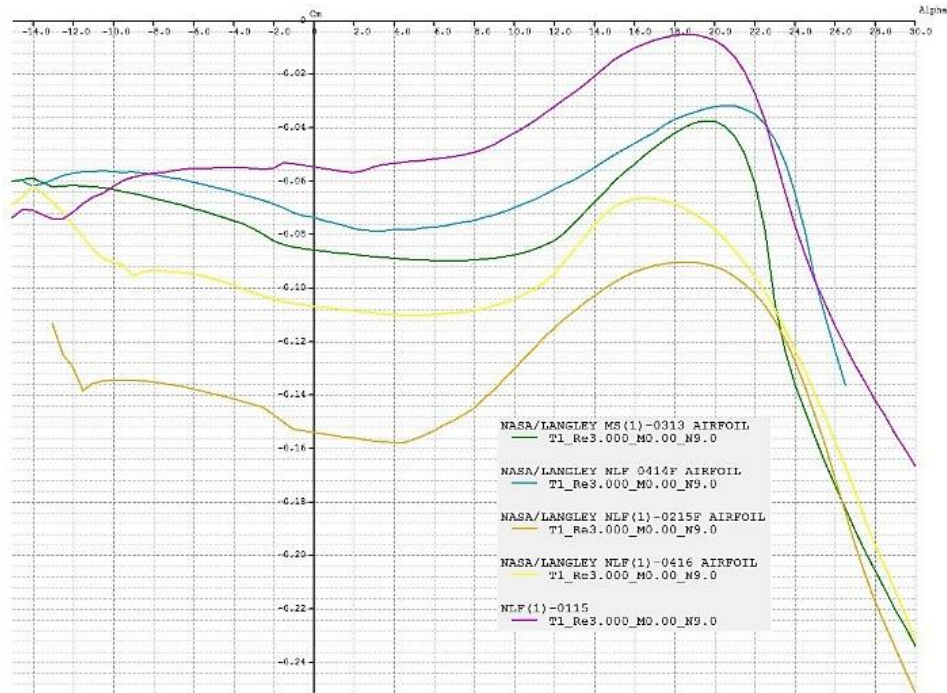


Table 7.3. Pitching moment coefficient dependent on alpha for various airfoils

For our design, the airfoil MS (1)-0313 will be used since it has the highest coefficient of lift and a moderate moment coefficient.

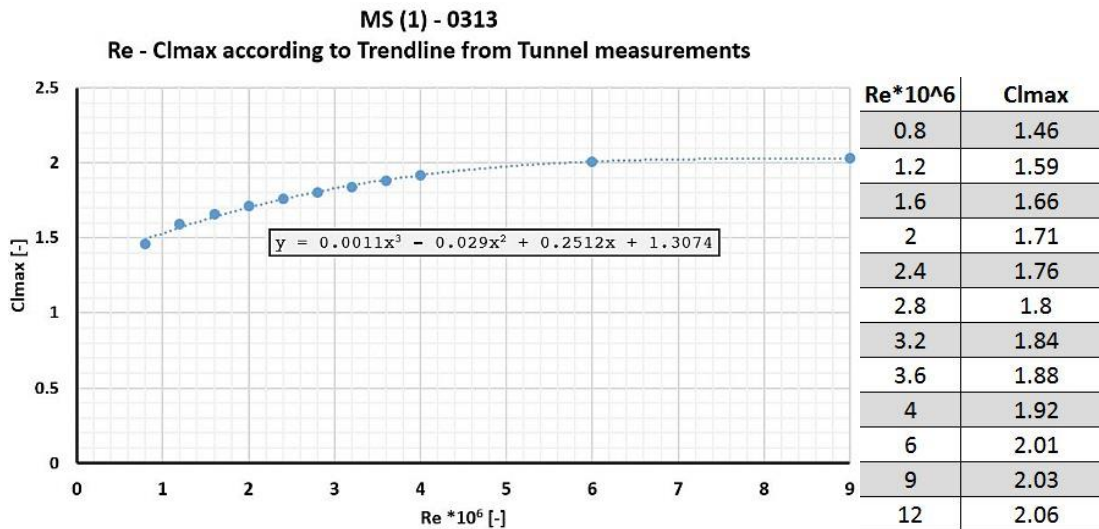


Table 7.4. Tunnel data for the airfoil MS (1)-0313.

However real wind tunnel data will also be used. See ref. [10]

The following measurements are available for the airfoil MS (1)-0313

Thus, for a first estimation of the Reynolds number and for a root chord of 1,5 m as well as the stall speed of 107 km/h will give $Re \approx 3 \cdot 10^6$.

$$Re = \frac{\rho VC}{\mu} \approx 68500VC ; [-] \quad (7.1)$$

where μ is air viscosity, ρ is air density, V is reference speed and C is the reference chord.

For a trapezoidal wing configuration, the following equation can be used to calculate the coefficient of lift for the wing.

$$C_{Lmax} = 0,95C_{lmax} + 1,1 \frac{bflaps}{b/2} ; [-] \quad (7.2)$$

where $\frac{bflaps}{b/2}$ was chosen as 0,5.

Then, and by using the equation of the trend line, we get a C_{lmax} of 1,83.

Hence, C_{Lmax} is therefore 2,288.

Then the required wing area should be according to equation:

$$S = \frac{2Mg}{\rho v_s^2 C_{Lmax}} ; [m^2] \quad (7.3)$$

Since a first estimation of the aspect ratio for the wing, see constraint analyzes was set, we now can calculate the required wing span as:

$$b = \sqrt{AR \cdot S} ; [m] \quad (7.4)$$

Thus, the root chord can be calculated as:

$$C_R = \frac{2S}{b(1 + \lambda)} ; [m] \quad (7.5)$$

Hence, we get a first estimation of $S = 11.71$ m, $b = 10.38$ m and $C_R = 1,4$ m. Its Re now will be different as the estimated reference chord has changed. After iteration by changing the estimated Re of $3 \cdot 10^6$ to $2.88 \cdot 10^6$ will lead to a close match with the new estimated value. Hence, the new values then are C_{Lmax} is 2,276 for a wing span S of 11,775 m and a root chord length of 1,414 m.

The slope of the lifting curve C_l^α according to the new Reynolds number will be $0,1178^\circ$ or 6,7494 radians.

In order to find the lift distribution as well as the spanwise location where the wing will stall first. we will use the software “*Glauert III v1.00*”. From which leads that the C_{Lmax} of the wing is 1,646, and with flaps 2,114. Therefore, the new required wing area will be 12,678m.

From this newly completed wing area we will determine the final wing span for our design and all other necessary dimensions. We also need to check if the 5% margin of the total sectional lift at the root location of the aileron is available.

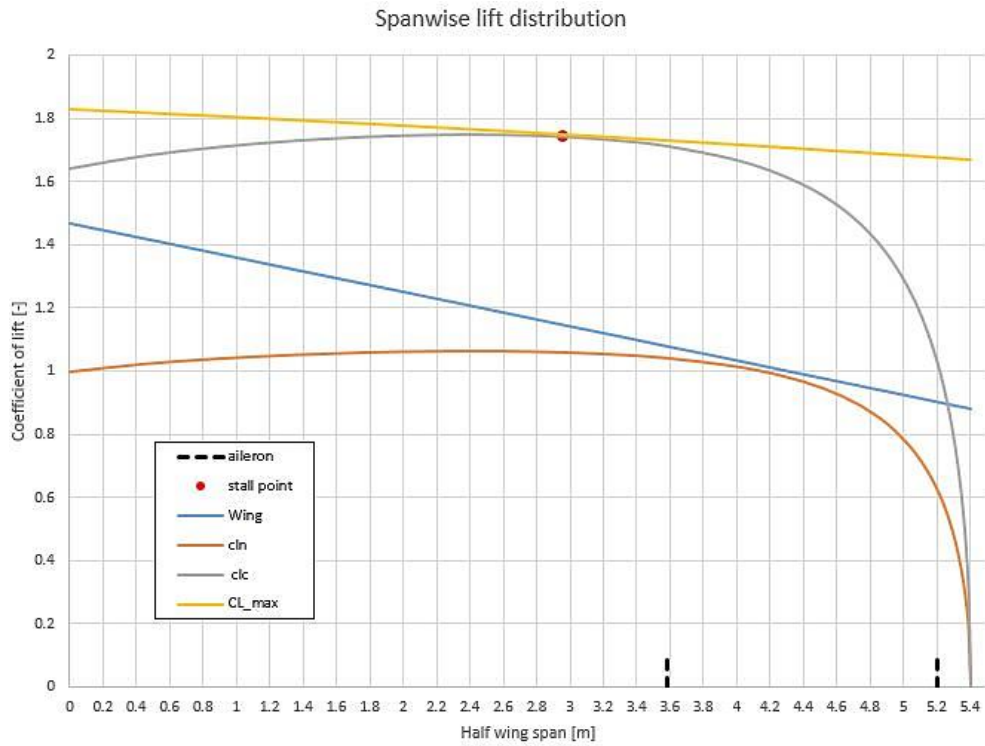


Figure 7.1. The spanwise lift distribution for the non-twisted wing design.

After applying adequate twist to the root and tip chord, till the required 5% at the root of the aileron is fulfilled.

Thus, giving the following lift distribution.

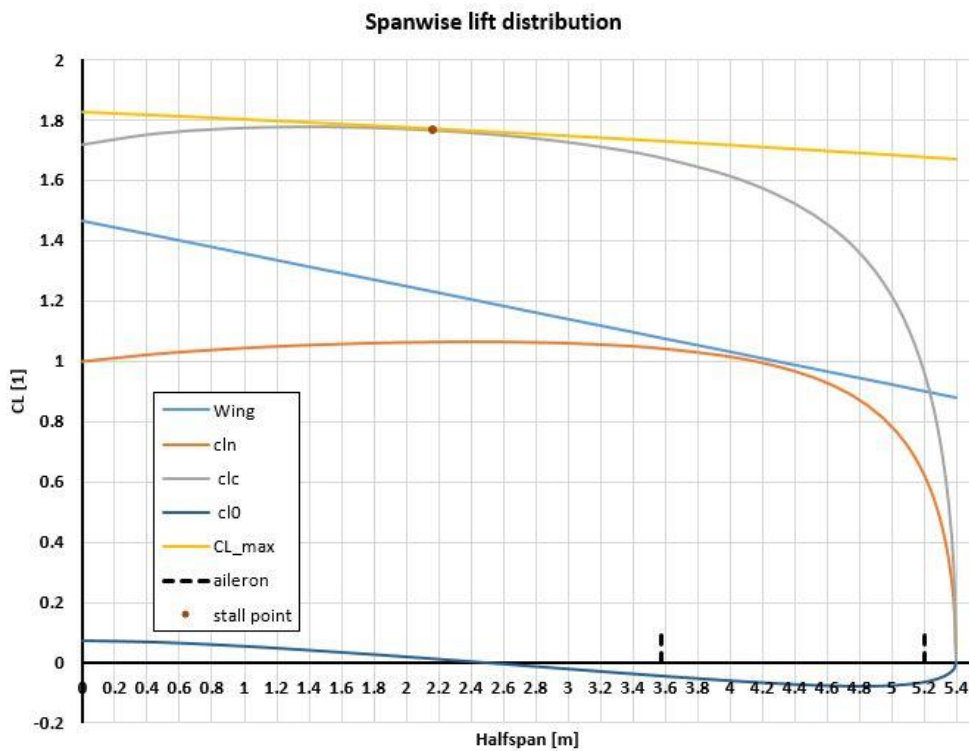


Figure 7.2. The lift distribution for a twisted wing design.

The final wing layout will can be seen below.

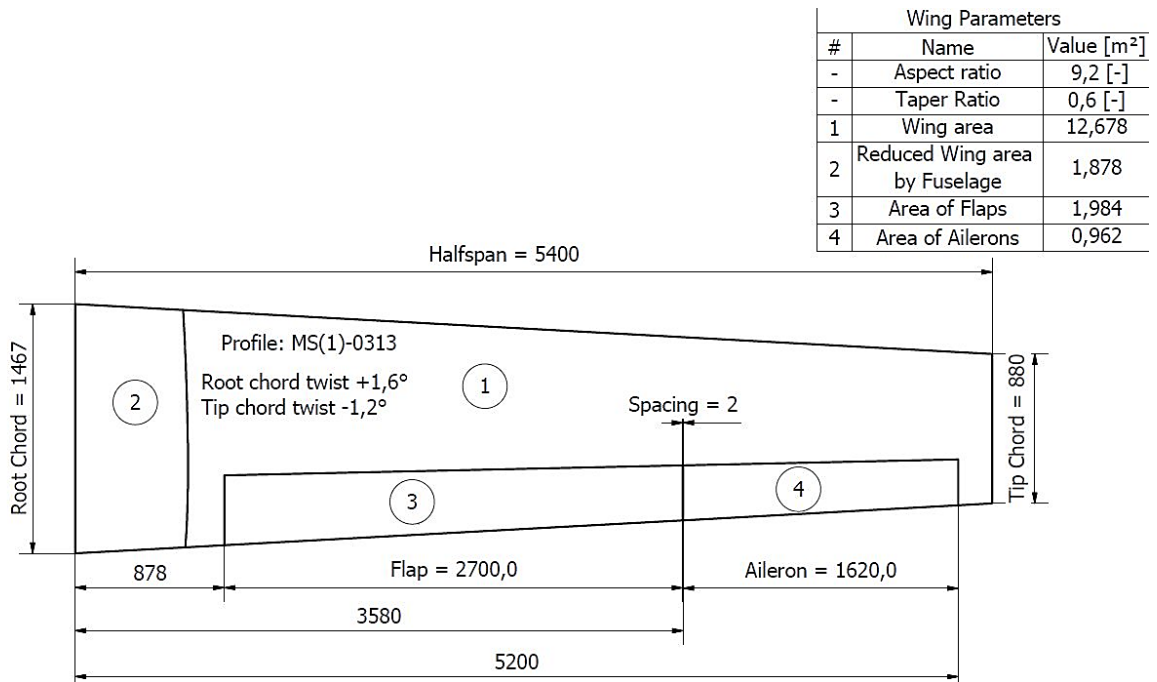


Figure 7.3. A schematic drawing of the twisted wing design.

7.2 Cockpit layout

A modern GA aircraft should be able to accommodate a 95th percentile male. Meaning, that 95 % of all people are below or as tall as that. Thus, the cockpit should be sized so that a 186,7 cm tall male could sit inside without any problem.

Since our aircraft is going to have a standard 2 + 2 seating configuration, meaning that 2 seats in front and 2 in the back, our cockpit layout should be such that nor the passenger sitting next to the pilot nor the people behind him should limit the pilot's field of view in any way. Such a recommended field of view as well as a min. internal width and height of the cabin can be seen in the figure below.

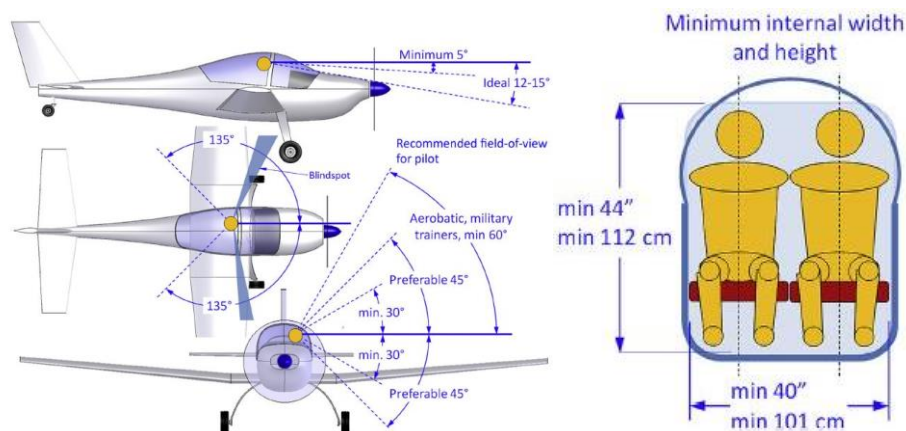


Figure 7.4. Recommended field of view for the pilot and minimum internal cockpit dimensions.

A typical modern cockpit may look like as seen in figure 6.2.



Figure 7.5. A typical 2+2 seating configuration cockpit layout. Image is showing a cabin of Pipistrel Pathera aircraft.

For our design, such a cockpit layout was chosen, so it fulfills the design requirements for a modern GA aircraft.

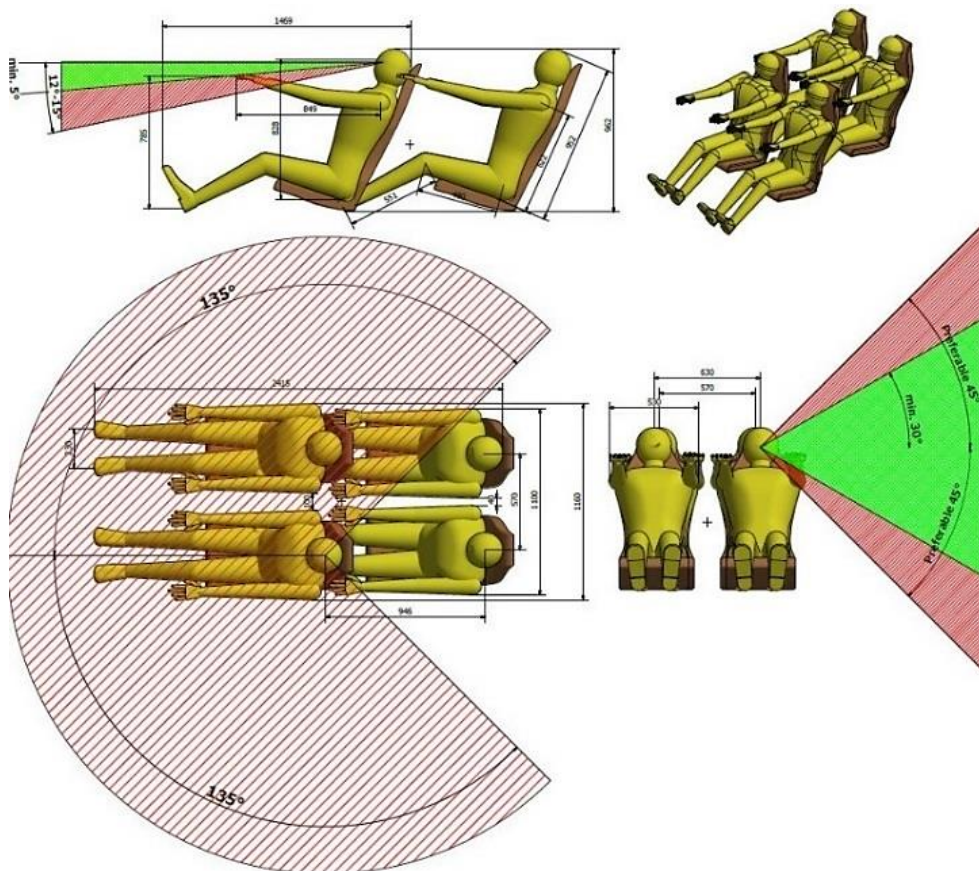


Figure 7.6. Field of view for the cockpit configuration layout.

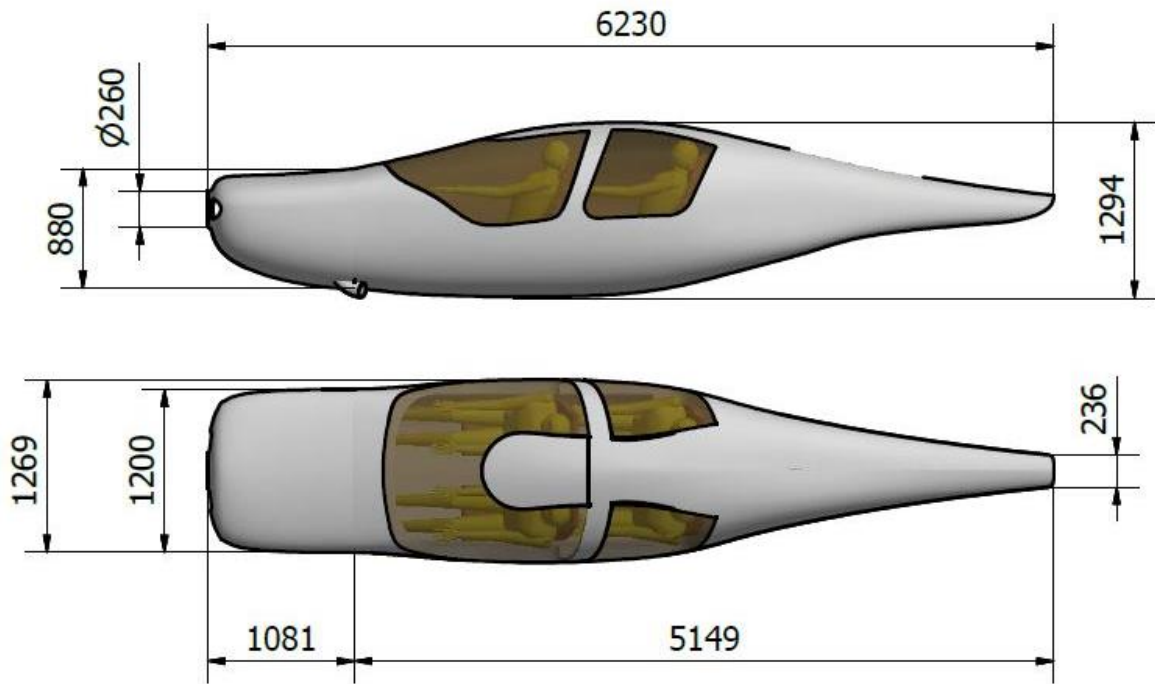


Figure 7.7. Main dimensions of our designed fuselage

7.3 Empennage

Sizing the tail surfaces correctly and determining the required tail arm can be a tricky task. Therefore, statistical data will help to find a first estimation of the tail volume, aspect as well as taper ratio.

Empennage Statistics				
	Corvalis TTx	Cirrus SR22	Pipistrel Panthera	AVG
HT Volume [-]	0.774	0.637	0.588	0.666
VT Volume [-]	0.042	0.033	0.040	0.038
AR _{HT} [-]	5.272	6.110	6.614	5.999
AR _{VT} [-]	1.459	1.614	1.384	1.486
λ _{HT} [-]	0.694	0.613	0.468	0.592
λ _{VT} [-]	0.274	0.502	0.549	0.442
S _{HT} [m ²]	3.156	2.510	1.474	2.380
S _{VT} [m ²]	1.615	1.335	0.964	1.305
b _{HT} [m]	4.079	3.916	3.122	3.706
b _{VT} [m]	1.535	1.468	1.155	1.386

Table 7.8. Main parameters of the empennage

Since most GA aircraft tail surfaces tend to have the center of mass very close to each other, it is possible to use the same tail arm for both, the horizontal as well as the vertical tail. This makes it easier for an optimization of the tail surfaces.

To study the dependency of the tail arm on surface area, we will assume the following simplified model, see ref. [2]

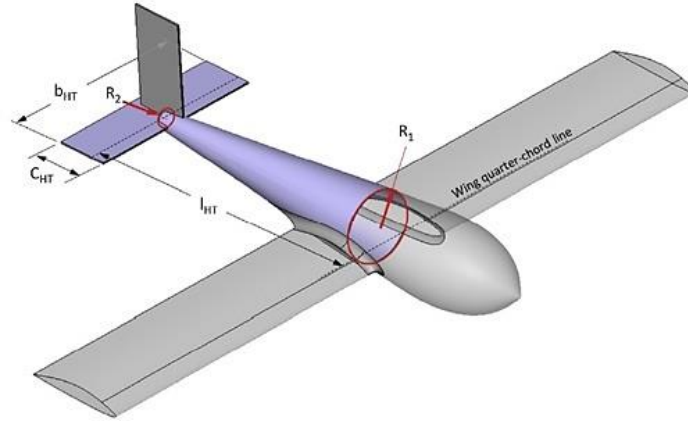


Figure 7.8. Simplified model for determining the tail arm.

It is assumed that the fuselage between the wing quarter chord line and the tail quarter chord line has a conical shape. Further, the horizontal and vertical tail quarter chord lines are equally located.

Thus, the surface area of the frustum is:

$$S_F = \pi(R_1 + R_2) \cdot l_T \quad (7.6)$$

where R_1 and R_2 is the start and end cone radius and l_T is the tail arm.

The total wetted area is therefore:

$$S_{wet} \approx S_F + 2S_{HT} + S_{VT} \quad (7.7)$$

where S_{HT} and S_{VT} is the surface area of horizontal and vertical tail respectively.

Thus, S_{HT} and S_{VT} is calculated as:

$$S_{HT} = \frac{V_{HT} \cdot S_{ref} \cdot c_{ref}}{l_T} \quad (7.8)$$

$$S_{VT} = \frac{V_{VT} \cdot S_{ref} \cdot b_{ref}}{l_T} \quad (7.9)$$

Where S_{ref} , c_{ref} and b_{ref} are reference dimensions taken from the wing.

Thus, the surface area dependency for HT vs tail arm will be

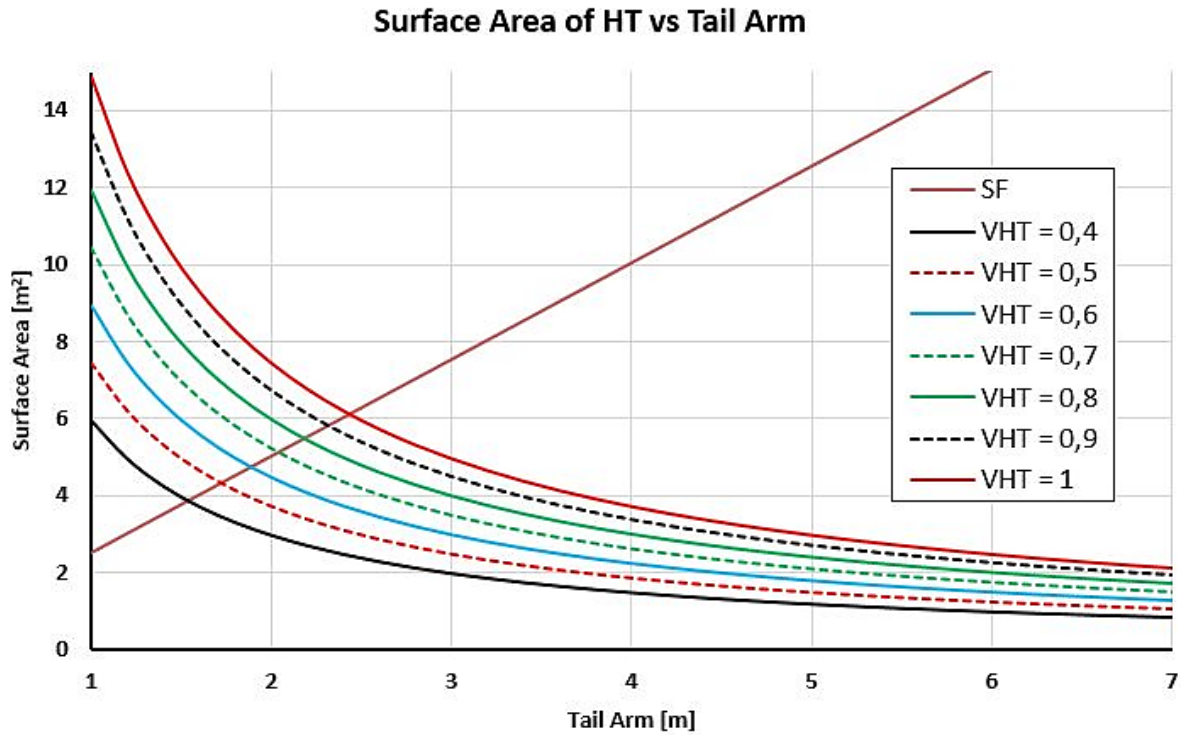


Figure 7.2. The dependency of HT Tail volume on tail arm

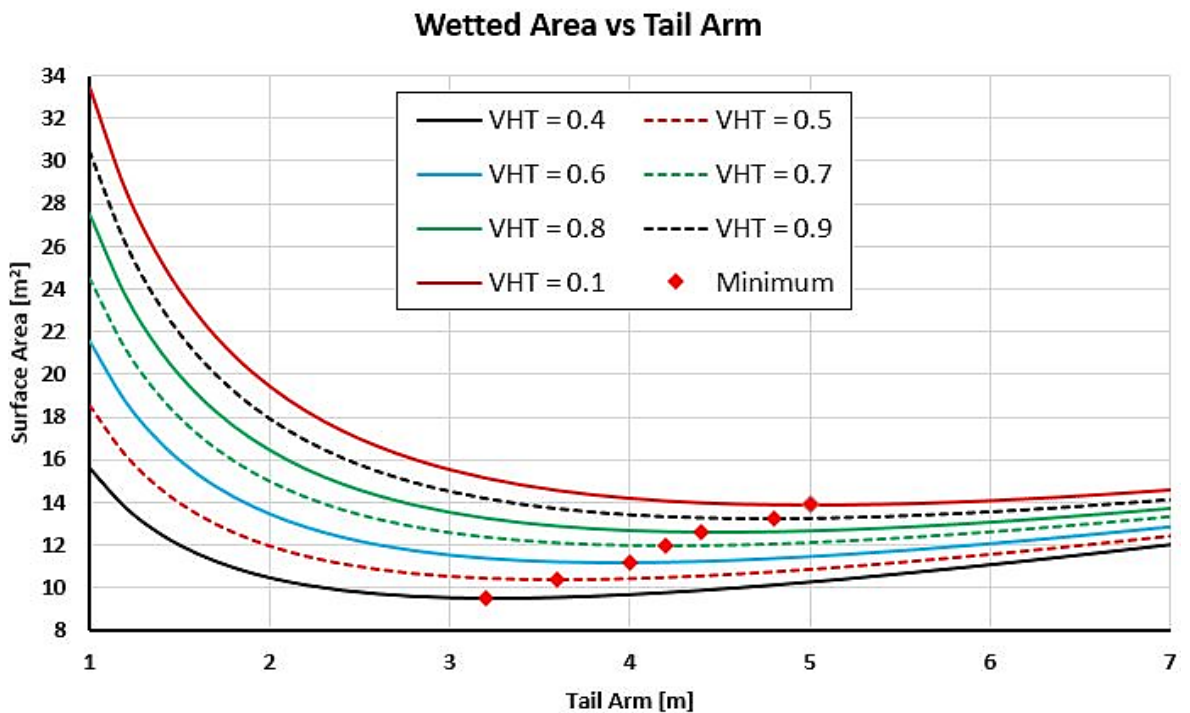


Figure 7.9. Wetted area of tail vs tail arm

To find the min. wetted area and therefore the optimum tail arm, the VT is considered as well.

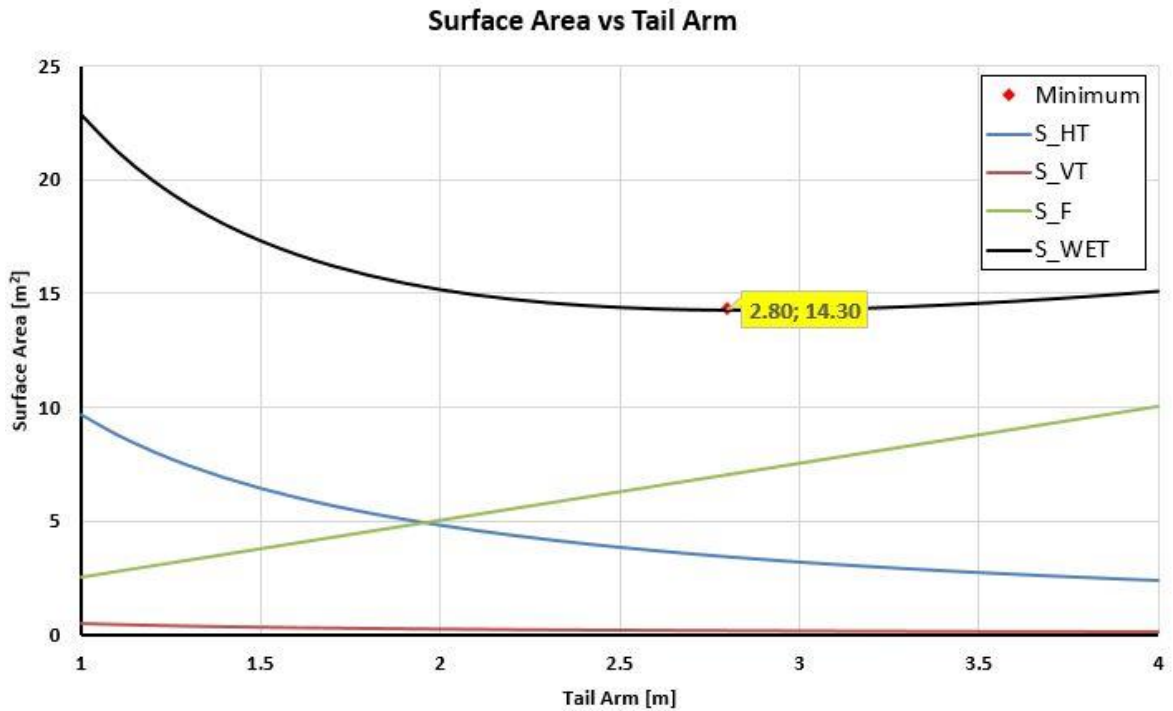


Figure 7.10. The optimum point for the empennage.

However, in order to achieve an optimum design, hence a minimum wetted area for the tail, a tail arm of 2,8 length would be required, thus unbalancing the aircraft and therefore tending to have an unstable design.

The initial tail sizing, considering the horizontal and vertical tail is:

$$l_T = \sqrt{\frac{2S_{ref}(V_{HT} \cdot c_{ref} + V_{VT} \cdot b_{ref})}{\pi(R_1 + R_2)}} \quad (7.10)$$

Thus, for our design the initial tail surfaces will have the following parameters

Horizontal Tail		Vertical Tail	
AR_{HT}	5.6907	AR_{VT}	1.485
V_{HT}	0.6369	V_{VT}	0.0335
TR_{HT}	0.61566	TR_{VT}	0.41966
C_{avg HT}	0.7102	C_{avg VT}	0.957
S_{HT}	2.8700	S_{VT}	1.360
b_{HT}	4.0413	b_{VT}	1.421
Cr_{HT}	0.8791	Cr_{VT}	1.348
Ct_{HT}	0.5412	Ct_{VT}	0.566
MGC_{HT}	0.7236	MGC_{VT}	1.010
L_{Tail}	3.3714		

Table 7.4. Initial tail dimensions

8 Weight analysis and the center of gravity

To calculate the center of gravity for our design, we will use table 5.4 and record each components center of gravity.

Thus C.G. can be calculates as followed

$$x_{CG} = \sum_i^n \frac{x_{t_i} \cdot m_i}{m_i}$$

$$y_{CG} = \sum_i^n \frac{y_{t_i} \cdot m_i}{m_i}$$

$$z_{CG} = \sum_i^n \frac{z_{t_i} \cdot m_i}{m_i}$$

From it, the possible weight configurations are calculated from which the C.G. envelope follows.

Weight Configuaraion	Weight [kg]	x _{C.G.} [mm]	location of C.G [%] C _{MGC}	x _{C.G.} of C _{MGC} [mm]
EW	872.0	2298.46	12.67	150.81
EW + minF	885.8	2304.00	13.13	156.34
EW + 1/2F	987.0	2339.87	16.15	192.20
EW + FF	1102.0	2372.64	18.90	224.96
EW + P + 20%RF + 34kgB	1011.2	2386.08	20.03	238.39
EW + P + FF + 34kgB	1222.0	2432.28	23.91	284.58
EW + P + OF + 20%RF + 25kgL/B	1082.78	2385.87	20.01	238.18
EW + P + OB + 20%RF	1057.78	2424.28	23.23	276.58
EW + P + OB + 20%RF + 2.L	1073.78	2447.98	25.22	300.27
EW + P + OB + 1/2F	1158.98	2444.33	24.92	296.62
EW + P + OB + 1/2F + 2.L	1174.98	2465.72	26.71	318.00
EW + P + OB + FF	1273.98	2463.25	26.51	315.53
EW + P + OB + FF + 2.L	1289.98	2482.49	28.12	334.77
EW + P + OF + OB + 1/2F	1244.98	2453.06	25.65	305.35
EW + P + OF + OB + 1/2F + 1.L	1252.98	2463.03	26.49	315.31
EW + P + OF + OB + 1/2F + 2.L	1260.98	2472.88	27.31	325.16
EW + P + OF + OB + 1/2F + 3.L	1268.98	2482.60	28.13	334.88
EW + P + OF + OB + FF	1359.98	2470.04	27.08	322.32
EW + P + OF + OB + FF + 1.L	1367.98	2479.08	27.84	331.35
EW + P + OF + OB + FF + 2.L	1375.98	2488.01	28.59	340.28
EW + P + OF + OB + FF + 3.L	1383.98	2496.83	29.33	349.10
EW + P + OF + 2.OB + 1/2F	1330.98	2521.79	31.42	374.05
EW + P + OF + 2.OB + 1/2F + 1.L	1338.98	2530.71	32.17	382.96
EW + P + OF + 2.OB + 1/2F + 2.L	1346.98	2539.52	32.91	391.78
EW + P + OF + 2.OB + 1/2F + 3.L	1354.98	2548.24	33.64	400.48
EW + P + OF + 2.OB + 1/2F + 4.L	1362.98	2556.84	34.37	409.09
EW + P + OF + 2.OB + FF	1445.98	2532.29	32.30	384.55
EW + P + OF + 2.OB + FF + 1.L	1453.98	2540.45	32.99	392.70
EW + P + OF + 2.OB + FF + 2.L	1461.98	2548.52	33.67	400.77
EW + P + OF + 2.OB + FF + 3.L	1469.98	2556.50	34.34	408.75
MTOW	1477.98	2564.40	35.00	416.64
			MIN C_MGC [%]	20.01
			MAX C_MGC [%]	35.00

Table 8.1. Loading configurations and the forward and aft c.g limits.

From the CG envelope, we can see that the center of gravity in the flight configuration moves between 20 and 35 % of the mean geometric chord.

CG-ENVELOPE

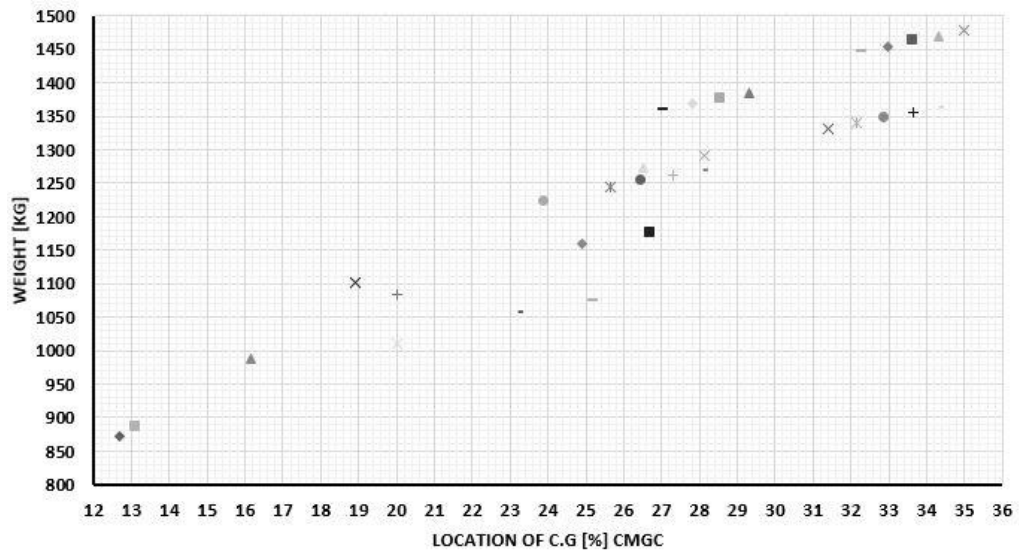


Table 8.2. The Loading configurations for Ground and Inflight and their location on the MGC.

9 Aerodynamic characteristics

9.1 Neutral point

9.1.1 Shift of neutral point due to influence of fuselage

It is a point on the aircraft where the pitching moment is constant when the aerodynamic forces act on it. It plays a great part in the longitudinal static stability of the aircraft.

To fulfill the requirements for longitudinal static stability, the CG must be in front of the neutral point. For this case, the recommendations for the location of CG are: (20-35) % MGC.

To find the neutral point on the aircraft we have to shift the aerodynamic center of the wing. This shift is the cause of the interaction of the tail and the wing. The tail has a positive effect for the neutral point which shifts the location of the neutral point into the positive direction of x axis, whereas the fuselage has the opposite effect.

The fuselage has a destabilizing effect on to the neutral point which moves the aerodynamic center of the wing lying at 25 % of MGC into the negative direction of the x axis, denoted by equation

$$\Delta \bar{x}_{NBf} = -k_{tr} \cdot \frac{1}{c_{Lwing}^\alpha} \cdot \frac{S_{tr}}{S} \cdot \frac{l_{tr}}{c_{MGC}} \quad (9.1)$$

The coefficient K_{tr} can be read of from ref. [16]

The slenderness ratio of the fuselage λ_{tr} as well as the location of the c.g. on the fuselage with origin point being the tip of the aircraft is chosen together with the length l_{tr} and width c_{tr} of the fuselage.

$$\lambda_f = \frac{l_{tr}}{c_{tr}} = \frac{6.695}{1.269} = 5,28 [-] \quad (9.2)$$

$$\frac{x_{tr}}{l_{tr}} = \frac{2,562}{6,695} = 0,38 [-] \quad (9.3)$$

Then from the graph in the attachments from ref. [19] we can read:

$$K_{tr} = 0,0026 [-]$$

$c_{Lwing}^\alpha = 5,3814 \text{ rad}^{-1}$ or $0,0939 \text{ 1/deg}$ as was calculated by Glauert for our twisted wing design.

$$\Delta \bar{x}_{NBf} = -k_{tr} \cdot \frac{1}{c_{Lwing}^\alpha} \cdot \frac{S_{tr}}{S} \cdot \frac{l_{tr}}{c_{MGC}} = -0,0026 \cdot \frac{1}{0,0939} \cdot \frac{6,695 \cdot 1,269}{12,678} \cdot \frac{6,695}{1,198} = -10,37 \%$$

This rather large shift due to the fuselage can be partially explained by looking at the cad model and how the fuselage was modeled. By analyzing the cad model, it can be seen that a rather large portion of the total volume of the fuselage lies near the tail section due to incorrect modeling. This shifts the center of gravity much further back. Unfortunately, it was not considered at the time of modeling.

9.1.2 Shift of neutral point due to the influence of the horizontal tail

As has already been mentioned before the tail surfaces have a positive effect on the neutral point which shift it into the positive direction of the x axis. $c_{LHT}^\alpha = 0,07496$ 1/deg (Glauert).

$$\bar{x}_{NBHT} = c_{LHT}^\alpha \cdot \left(\frac{1}{c_{Lwing}^\alpha} - D \right) \cdot V_{HT} \cdot k_{HT} \quad (9.4)$$

$$D = \frac{d\varepsilon}{d_{CLW}} \quad (9.5)$$

D can be alternatively calculated, see ref [19]

$$D_{wHT} = \frac{46,2}{AR} x_1 x_2 x_3 \quad (9.6)$$

where D is the derivative of the downwash.

$$D_{wHT} = \frac{46,2}{AR} x_1 x_2 x_3 = \frac{46,2}{9,2} \cdot 0,92 \cdot 1,16 \cdot 0,88 = 4,716 [-]$$

S_1 is the reduced area of the HT by the fuselage. It is measured from the cad model as 0,343 m².

then $S_1/S_{HT} = 0,12$ [-]

therefore $k_{HT} = 0,9$

$$\bar{x}_{NBHT} = 0,07496 \cdot \left(\frac{1}{0,0939} - 4,716 \right) \cdot 0,6369 \cdot 0,9 = 25,5\%$$

Position of the neutral point is given according to the equation

$$\bar{x}_{NB} = \bar{x}_{AC} + \bar{x}_{NBtr} + \bar{x}_{NBVOP} \quad (9.7)$$

then $\bar{x}_{NB} = 0,25 - 0,1037 + 0,255 = 0,401 = 40,13\%$

The margin of static stability is therefore:

$$Stab = \bar{x}_{NB} - \bar{x}_{cg} \quad (9.8)$$

$$\bar{x}_{NB} - \bar{x}_{t\ddot{z}} = 0,401 - 0,35 = 0,051 = 5,1 \% \text{ of } c_{MGC}$$

A reasonable static lower margin should not be below 5%. See ref [16]. The calculated static stability margin is just above the limit. However, future work on this design could be to improve this value.

9.2 The Lift enhancement

For our design, a plain flap of 30% was chosen. The maximum deflection is 20°. In order to find the change of the slope of the airfoil due to flap deployment, we need to calculate the change in lift caused by the deployment. See ref. [11]. The incremental lift coefficient due to a plain flap deflection with a sealed gap is given by:

$$\Delta c_l = (\delta_f) \cdot \left(\frac{(c_{l\delta})}{(c_{l\delta})_{\text{theory}}} \right) \cdot (c_{l\delta})_{\text{theory}} \cdot k' \quad (9.9)$$

$$\Delta c_{l_{\text{max}}} = k_1 \cdot k_2 \cdot k_3 \cdot (\Delta c_{l_{\text{max}}})_{\text{base}} \quad (9.10)$$

$$(c_{l\alpha})_{\delta} = \left(\frac{c'}{c} \right) \cdot c_{l\alpha} \quad (9.11)$$

k'	0,62	correction factor
$(c_{l\delta})_{\text{theory}}$	4,55	correction factor (see attachment)
$\left(\frac{(c_{l\delta})}{(c_{l\delta})_{\text{theory}}} \right)$	1	
δ_f	30 [deg]	flap deflection angle
k_1	1,05	correction factor
k_2	0,78	correction factor
k_3	1	correction factor
$(\Delta c_{l_{\text{max}}})_{\text{base}}$	0,98	(see attachment for a profile thickness of 12%)

Hence,

$$\Delta c_l = 30 \cdot \pi/180 \cdot 1 \cdot 4,55 \cdot 0,62 = 1,4771 [-]$$

$$\Delta c_{l_{\text{max}}} = 1,05 \cdot 0,78 \cdot 1 \cdot 0,98 = 0,8026 [-]$$

$$(c_{l\alpha})_{\delta} = 1 \cdot 5,959 = 5,959 [\text{rad}^{-1}]$$

9.2.1 Increase in lift due to flaps see ref [11]

$$\Delta c_{L_w} = K_b \cdot (\Delta c_l) \cdot \left(\frac{c_{l\alpha w}}{c_{l\alpha}} \right) \cdot \left[\frac{(a_{\delta})_{c_L}}{(a_{\delta})_{c_l}} \right] \quad (9.12)$$

where:

K_b	0,57 [1]	flap span factor (Fig. 8.52)
$c_{l\alpha w}$	5,3814 [1/rad]	wing lift curve slope
$c_{l\alpha}$	6,749 [1/rad]	profile lift curve slope
$\frac{(a_{\delta})_{c_L}}{(a_{\delta})_{c_l}}$	1,04 [1]	ratio of the 3D to the 2D flap effectiveness parameter

$$\Delta c_{L_w} = 0,57 \cdot 1,4771 \cdot \left(\frac{5,3814}{6,749} \right) \cdot 1,04 = 0,6981 [-]$$

9.2.2 Slope of the lift curve with flaps deflected ref. [11]

$$(c_{L\alpha w})_{\delta} = c_{L\alpha w} \cdot \left[1 + \left(\frac{c'}{c} - 1 \right) \cdot \left(\frac{S_{wf}}{S} \right) \right] \quad (9.13)$$

$\left(\frac{c'}{c} \right)$ ratio of extended wing chord to the flaps-up chord (for plain flaps =1)

$\left(\frac{S_{wf}}{S} \right)$ ratio of the flapped wing area to the total reference area (wing)

$$\frac{S_{wf}}{S} = \frac{6,316}{12,678} = 0,5218 [-] \quad (9.14)$$

$$(c_{L\alpha w})_{\delta} = 5,3814 \cdot [1 + (1 - 1) \cdot 0,52391] = 5,3814 \text{ rad}^{-1}$$

9.2.3 Increase in c_{Lmax} due to the deflection of the flaps ref. [11]

$$\Delta c_{LmaxW} = (\Delta c_{lmax}) \cdot \left(\frac{S_{wf}}{S} \right) \cdot K_{\Delta} \quad (9.15)$$

Δc_{lmax}	0,5968	the change in maximum lift coefficient of the profile
K_{Δ}	0,92	correction factor for the swept wing configuration

$$\Delta c_{LmaxW} = 0,8026 \cdot 0,5218 \cdot 0,92 = 0,3853 [-]$$

9.2.4 Maximum coefficient of lift of the wing with flaps deflected ref. [11]

$$c_{LWmax\delta} = c_{Lw} + \Delta c_{LmaxW} = 1,6459 + 0,3853 = 2,0312 [-]$$

Now it is possible to construct the lifting curve for the wing with extended flaps. From the graph 2.1, even though the change in lift is very small, the angle of zero lift increased greatly.

Then α_0 for flaps deployed can be calculated according to:

$$\alpha_0 = - \frac{C_{L_0} + C_{LW}}{C_{LW}^{\alpha}} \cdot \frac{180}{\pi} \quad (9.16)$$

$$\alpha_0 = - \frac{0,316 + 0,6981}{5,3814} \cdot \frac{180}{\pi} = -10,8 [deg]$$

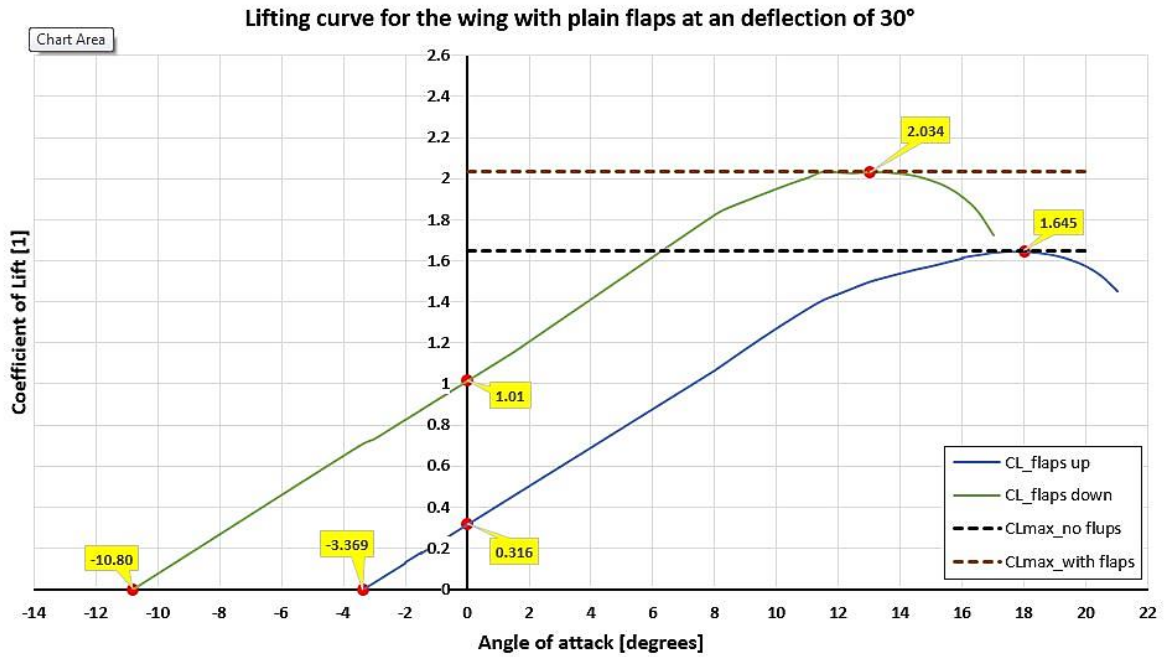


Figure 9.1. Lifting curve for the wing with flaps

9.3 The effect of the fuselage on the lifting curve

In this section, see ref. [12] can be seen what effect the fuselage will have on the lifting curve of the wing, given by the following equations:

$$d_f = \sqrt{\frac{4 \cdot S_f}{\pi}} = \sqrt{\frac{4 \cdot 1,39607}{\pi}} = 1,3332 \text{ m} \quad (9.17)$$

where S_f is the cross-section of the fuselage at its maxim width and height. d_f is therefore the calculated diameter of this section.

$$K_{wf} = 1 + 0,025 \cdot \left(\frac{d_f}{b}\right) + 0,25 \cdot \left(\frac{d_f}{b}\right)^2 \quad (9.18)$$

K_{wf} is a correction factor taking into account the calculated diameter and wingspan.

Then $K_{wf} = 0,9993$ [-]

$$C_{L\alpha wf} = C_{L\alpha w} \cdot K_{wf} \quad (9.19)$$

This correction factor decreases in this case the lift slope of the wing.

$$C_{L\alpha wf} = 5,3775 \text{ [1/rad]}$$

$$r = \frac{2l_{vop}}{b} = 0,6243 \text{ [-]} \quad (9.20)$$

$$m = \frac{2y_{vop}}{b} = 0,1054 \text{ [-]} \quad (9.21)$$

The downwash derivative can be calculated per equation:

$$\frac{d\varepsilon}{d\alpha} = 1,75 \cdot \frac{C_{L\alpha w}}{\pi \cdot (\lambda \cdot r)^{0,25} (1 + m)} = 0,3768 [-] \quad (9.22)$$

Thus giving the lifting slope of the aircraft

$$C_{L\alpha} = C_{L\alpha wf} + C_{L\alpha h} \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right) \cdot \frac{S_h}{S} \cdot \frac{q_h}{q} \quad (9.23)$$

$\frac{q_h}{q}$ is the ratio of dynamic pressure on the horizontal tail and the wing.

Thus the lift at zero angle of attack of the wing together with the fuselage is:

$$C_{L0wf} = -\alpha_0 \cdot C_{L\alpha wf} = -3,3694 \cdot 5,3775 = 0,3162 \quad (9.24)$$

$$\varepsilon_{oh} = \frac{d\varepsilon}{d\alpha} \cdot \alpha = 0,0222 [-] \quad (9.25)$$

$$C_{L0} = C_{L0wf} + C_{L\alpha h} \cdot \frac{S_h}{S} \cdot \frac{q_h}{q} \cdot (i_h - \varepsilon_{oh}) \quad (9.26)$$

$$\alpha = -\frac{C_{L0}}{C_{L\alpha}} = -\frac{0,2979}{5,8926} \cdot \frac{180}{\pi} = -2,9^\circ \quad (9.27)$$

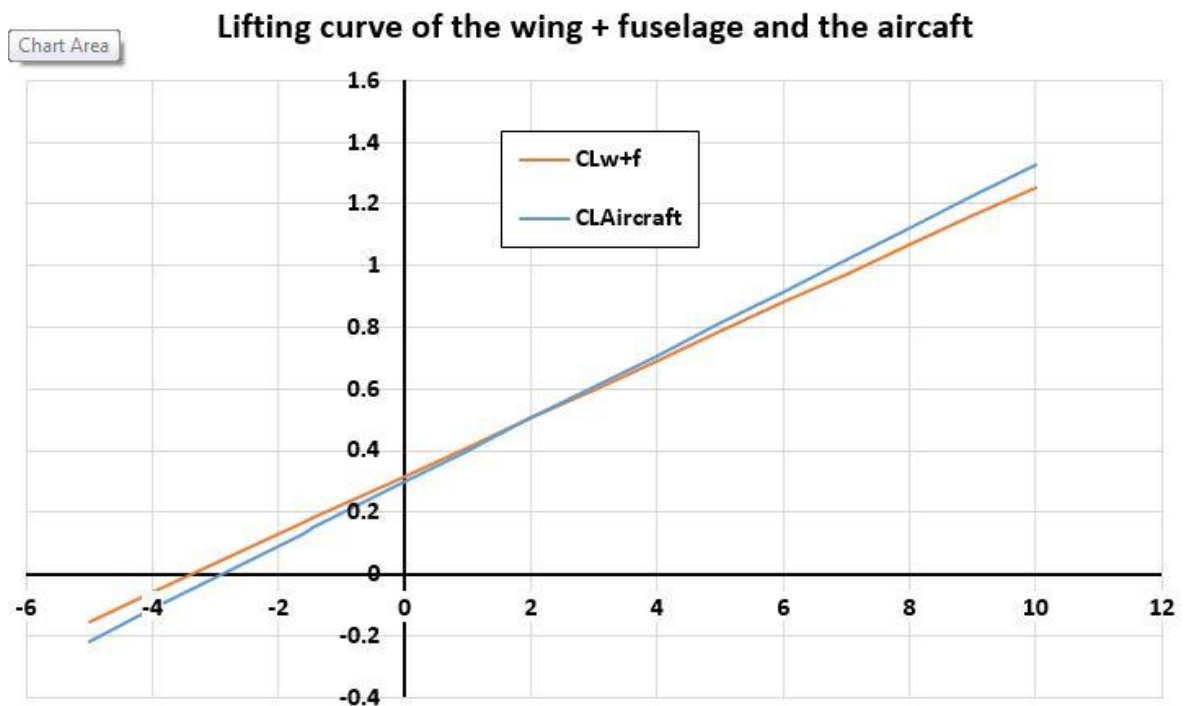


Figure 9.2. Lifting curve of the wing with fuselage and lifting curve of the Aircraft

10 Polar of the Aircraft

The total drag polar of the aircraft is constructed by calculating the drag coefficient for the wing, fuselage, tail surfaces, landing gear and last but not least the engine installation first and adding them up.

10.1 The wing drag

The drag due to the wing **only**, can be calculated according to the following equations:

$$c_{D_{wing}} = c_{D_p} + c_{D_i} [-] \quad (10.1)$$

c_{D_p} profilový odpor určen dle vztahu:

$$c_{D_p} = c_{D_{p_{min}}} \cdot \frac{S_{wet}}{S} + 0,75 \cdot \Delta c_{D_p} \cdot \left(\frac{C_L - C_{L_i}}{C_{L_{max}} - C_{L_i}} \right)^2 [-] \quad (10.2)$$

$c_{D_{p_{min}}}$	0,0075 [1]	minimum drag of profile
S_{wet}	10,383 [m ²]	wetted wing surface, (wing area without fuselage)
C_{L_i}	0,3 [1]	coefficient of lift at $c_{D_{p_{min}}}$
$C_{L_{max}}$	1,4576 [1]	maximum coefficient of lift without flaps

$$\Delta c_{D_p} = \frac{67 \cdot C_{L_{max}}}{(\log_{10} Re)^{4,5}} - 0,0046 \cdot \left(1 + 2,75 \cdot \frac{t}{c} \right) = 0,01274 \quad (10.3)$$

c_{D_i} induced drag coefficient can be calculated per equation:

$$c_{D_i} = \frac{C_L^2}{\pi \cdot \lambda} \cdot (1 + \delta) = 0,065699 \cdot C_L^2 \quad (10.4)$$

Then the final expression for the wing drag will be:

$$c_{D_{wing}} = 0,072075 \cdot C_L^2 - 0,00383 \cdot C_L + 0,007007$$

10.2 Drag due to the Horizontal tail

The drag coefficient for the Horizontal tail **only**, can be calculated per equation:

$$C_{D_H} = R_{wf} R_{LS} C_{f_{emp}} \left(1 + L \left(\frac{t}{c} \right) + 100 \cdot \left(\frac{t}{c} \right)^4 \right) \frac{S_{wet_H}}{S} \quad (10.5)$$

10.3 Drag due to the Vertical tail

The drag coefficient for the vertical tail only, can be calculated in a similar way as for the horizontal tail:

$$C_{D_V} = R_{wf} R_{LS} C_{f_{emp}} \left(1 + L \left(\frac{t}{c} \right) + 100 \cdot \left(\frac{t}{c} \right)^4 \right) \frac{S_{wetV}}{S} \quad (10.6)$$

10.4 Drag due to the presence of the landing gear

The drag coefficient for the landing gear only, can be calculated per equation:

$$C_{D_{gear}} = \sum_{i=1}^n \left[\left\{ (C_{D_{gear_{c_L=0}}})_i + p_i \cdot c_L \right\} \cdot (S_{gear})_i / S \right] \quad (10.7)$$

$C_{D_{gear_{c_L=0}}}$ drag coefficient of the landing gear at zero lift
 S_{gear} reference area of the landing gear
 p_i coefficient due to lift on the landing gear

Values for the main landing gear:

$C_{D_{gear_{c_L=0}}}$ 0,69 [1]
 S_{gear} 0,0385 [m²]
 p_i -0,226 [1]

Values for the nose landing gear:

$C_{D_{gear_{c_L=0}}}$ 0,565 [1]
 S_{gear} 0,0385 [m²]
 p_i 0 [1]

10.5 Drag due to the engine installation

The drag coefficient for the engine installation only, can be calculated per equation:

$$C_{D_{eng}} = \frac{0,015 \cdot b_f \cdot h_f}{S} \quad (10.8)$$

b_f 0,909 [m] maximum width of fuselage
 h_f 1,368 [m] maximum height of fuselage
 S 12,105 [m²] wing area

10.6 Total drag of the aircraft

The total drag of the aircraft is the sum of the hear calculated components:

$$C_D = C_{D_{wing}} + C_{D_{fus}} + C_{D_H} + C_{D_V} + C_{D_{gear}} + C_{D_{eng}} \quad (10.9)$$

In the next graph, it can be seen, as more and more components are added to the aircraft, the polar moves more and more right, having the effect of an increasing $C_{D_{min}}$.

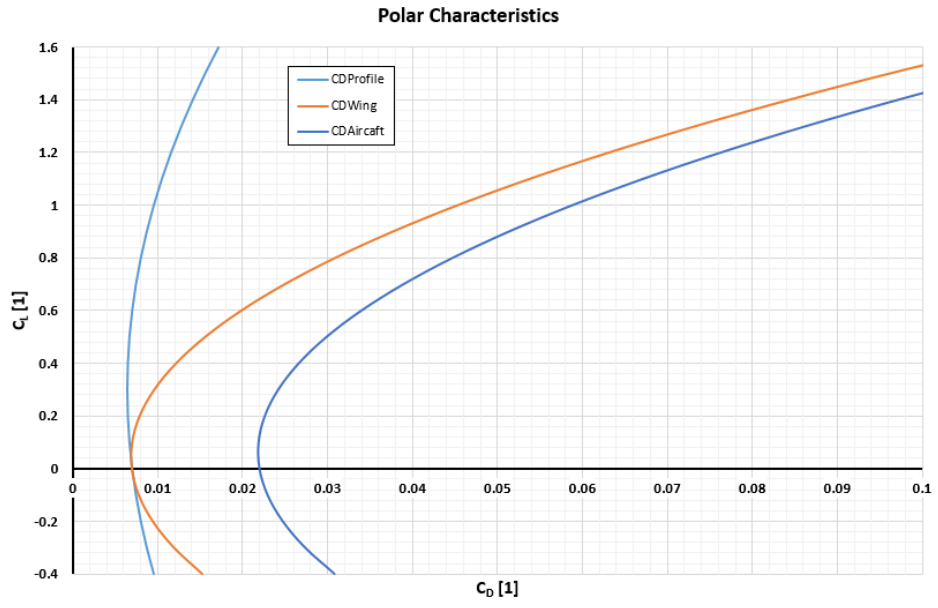


Figure 10.1.

11 Moment characteristics

To find the pitching moment characteristics of the aircraft, we first need to find the moment characteristics for the wing itself, then the wing together with the fuselage and finally the characteristics from the tail surfaces are added.

11.1 Pitching moment curve without any high-lift-device

In this section the pitching moment characteristics for the wing are calculated. First the pitching moment at zero lift is calculated per equation:

$$c_{m0} = \left[\frac{(\lambda \cdot \cos^2 \chi)}{(\lambda + 2 \cos \chi)} \right] \cdot \frac{(c_{m0r} + c_{m0t})}{2} = -0,0575 [-] \quad (11.1)$$

$$\Delta_{\varepsilon} c_{m_{ac}} = \frac{\Delta c_{m0}}{\varepsilon_t} \cdot \varepsilon_t \quad (11.2)$$

λ	9,2	[1]	aspect ratio
$\chi_{c/4}$	0	[1]	sweep angle of the wing to the quarter chord point
ε_t	-1.2	[deg]	twist angle at the tip of the wing
c_{m0t}	-0,07	[1]	pitching moment coefficient for the tip profile
c_{m0r}	-0,0753	[1]	pitching moment coefficient for the root profile

$$\Delta_{\varepsilon} c_{m_{ac}} = \frac{-0,0012}{-1} \cdot (-1) = -0,0012$$

$$c_{m0w} = c_{m0} + \Delta_{\varepsilon} c_{m_{ac}} = -0,0575 - 0,0012 = -0,0587$$

The slope of the pitching curve can be found per equation:

$$\left(\frac{dc_m}{dc_l} \right)_w = x_{ref} - x_{ac} \quad (11.3)$$

x_{ref}	reference point (usually C.G.)
x_{ac}	aerodynamic center of the wing at 25% MAC

The slope of the moment curve of the wing is calculated by using the weight configuration, by using forward center of gravity of 25% MAC.

11.2 Pitching moment coefficient of wing with influence of the fuselage

The gain in lift caused by fuselage compared to the lift of the entire aircraft is very small and therefore the following can be assumed: $C_{L_{wf}} = C_{L_w}$

11.3 Pitching moment coefficient at zero lift wing/fuselage

The pitching moment coefficient of the wing and its increase due to the fuselage with respect to the aerodynamic chord can be calculated per equation:

$$(c_{mac})_{wf} = (c_{mac})_w + \Delta_f c_{mac} \quad (11.4)$$

The contribution due to the fuselage can be calculated per equation:

$$\Delta_f c_{mac} = -1,8 \cdot \left(1 - \frac{2,5 \cdot b_f}{l_f}\right) \cdot \frac{\pi}{4} \cdot \frac{b_f \cdot h_f \cdot l_f}{S \cdot \bar{c}} \cdot \frac{C_{L_0}}{(C_{L\alpha})_{wf}} \quad (11.5)$$

C_{L_0}	0,298 [1]	coefficient of lift for $\alpha_f = 0$
S	12,677 [m ²]	wing surface area [m ²]
\bar{c}	1,198 [m]	mean aerodynamic chord
b_f	1,269 [m]	maximum width of fuselage
h_f	1,294 [m]	maximum height of fuselage
l_f	6,23 [m]	length of the fuselage
$(C_{L\alpha})_{wf}$	5,3775 [1/rad]	slope of the lifting curve of the wing/fuselage

Pitching moment coefficient of the wing and fuselage:

$$(c_{mac})_{wf} = c_{m0w} + \Delta_f c_{mac} \quad (11.6)$$

The slope for the moment curve for the wing/fuselage is calculated in a similar way as for the wing.

$$\left(\frac{dc_m}{dc_l}\right)_{wf} = x_{ref} - x_{ac} \quad (11.7)$$

x_{ref}	reference point (usually C.G.)
x_{ac}	aerodynamic center of the wing and fuselage

The aerodynamic center for the wing/fuselage can be calculated per equation:

$$\left(\frac{x_{ac}}{\bar{c}}\right)_{wf} = \left(\frac{x_{ac}}{\bar{c}}\right)_w + \frac{\Delta_{f1} x_{ac}}{\bar{c}} + \frac{\Delta_{f2} x_{ac}}{\bar{c}} \quad (11.8)$$

$\frac{\Delta_{f1} x_{ac}}{\bar{c}}$ correction of the forward shift due to fuselage ref. [12]

$$\frac{\Delta_{f1} x_{ac}}{\bar{c}} = -\frac{1,8}{(C_{L\alpha})_{wf}} \cdot \frac{b_f \cdot h_f \cdot l_{fm}}{S \cdot \bar{c}} = -0,064 [-] \quad (11.9)$$

Then the new aerodynamic center for the wing/fuselage will be:

$$\left(\frac{x_{ac}}{\bar{c}}\right)_{wf} = 0,25 - 0,0619 = 0,186$$

The slope of the moment curve for the wing/fuselage will be:

$$\left(\frac{dc_m}{dc_l}\right)_{wf} = 0,2512 - 0,1881 = 0,164 \left[\frac{1}{rad}\right]$$

11.4 Setting up the horizontal tail

The required angle of incidence relative to the fuselage datum line is:

$$(i_h)_f = \frac{c_{m_{owf}} + c_{L0} \frac{x_{cg} - x_{ac}}{\bar{c}}}{c_{lh\alpha} \cdot \frac{S_h \cdot l_h}{S \cdot \bar{c}} \cdot \frac{q_h}{q}} + \frac{\frac{d\varepsilon_h}{d\alpha}}{(c_{l\alpha})_w} \cdot c_{L0} \quad (11.10)$$

$c_{lh\alpha}$	3,314 [1/rad]	slope of the lifting curve of the horizontal tail
S_h	2,11 [m ²]	surface area of the horizontal tail
l_h	4,01 [m]	horizontal tail arm
$\frac{q_h}{q}$	0,85	the decrease in dynamic pressure at the horizontal tail
$\frac{d\varepsilon_h}{d\alpha}$	0,37847	derivation of the downwash angle of the horizontal tail
C_{L0}	0,1139	coefficient of lift where the angle of attack equals zero
$(c_{l\alpha})_w$	4,2474	slope of the lifting curve of the wing

$$i_h = (i_h)_f - \frac{C_{L0}}{C_{L\alpha}} = -0,0320 \text{ rad} \quad (11.11)$$

11.5 Coefficient of pitching moment at zero lift, whole aircraft

The pitching moment coefficient at zero lift for the aircraft can be found per equation:

$$\begin{aligned} c_{m_0} &= (c_{m_{ac}})_{wf} - c_{lh\alpha} \cdot i_h \cdot \frac{S_h \cdot l_h}{S \cdot \bar{c}} \cdot \frac{q_h}{q} \\ &= 0,0162 \end{aligned} \quad (11.12)$$

11.6 Slope of the moment curve of the aircraft

To find the slope of the moment curve, the neutral point must be calculated first according to equation:

$$\left(\frac{x_m}{\bar{c}}\right) = \left(\frac{x_{ac}}{\bar{c}}\right)_{wf} + \frac{c_{lh\alpha}}{c_{l\alpha}} \cdot \left(1 - \frac{d\varepsilon_h}{d\alpha}\right) \cdot \frac{S_h \cdot l_h}{S \cdot \bar{c}} \cdot \frac{q_h}{q} \quad (11.13)$$

= 0,3696

The slope of the moment curve for the aircraft can be calculated per equation:

$$\left(\frac{dc_m}{dc_l}\right) = x_T - x_m = 0,2512 - 0,3696 = -0,1184 \left[\frac{1}{\text{rad}}\right] \quad (11.14)$$

11.7 Final pitching moment curve

Since all the relevant pitching moment characteristics are known, we now can plot the graph for the slope of the wing, wing/fuselage and the whole aircraft.

The line equation for the pitching moment for the aircraft:

$$c_m = c_{m0} + c_L \cdot \frac{dc_m}{dc_L} = 0,0162 - 0,1184 \cdot c_L \quad (11.15)$$

This equation is valid only for a fixed stick setting and the forward C.G.

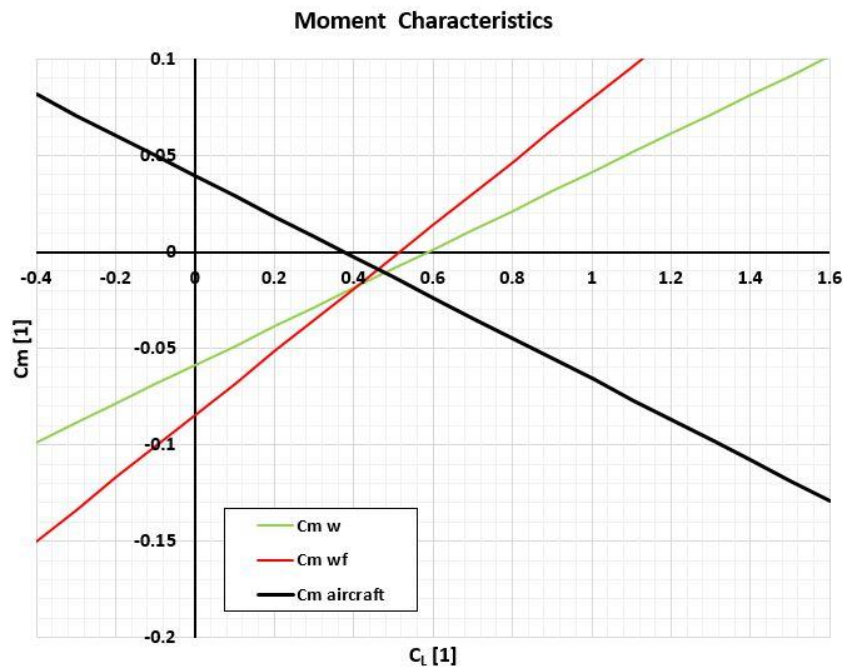


Figure 11.1. Moment characteristics for wing, wing/fuselage and aircraft.

12 Required surface area for the Horizontal tail

The required surface area for the horizontal tail will be calculated for the forward and aft limit, where the last will have a reserve of $\Delta x_{sm} = 0,015 l_{VOP}$.

12.1 Balancing at the forward limit

First the stability limit of 1,5% will be defined according to the requirements:

$$\Delta x_{sm} = 0,015 \cdot \frac{l_h}{\bar{c}} = 0,0422 \quad (12.1)$$

l_h 4,01 [m] horizontal tail arm

Then the pitching moment coefficient of the aircraft with respect to the aircrafts A.C.:

$$c_{mac} = -0,3 \cdot \Delta c_{Lw} = -0,3 \cdot 1,3589 = -0,44312 \quad (12.2)$$

Δc_{Lw} increase in the coefficient of lift previously calculated

Not it is possible to calculate the coefficient of lift for the horizontal tail per equation:

$$c_{lh} = -0,35 \cdot A_h^{1/3} = -0,35 \cdot 4^{1/3} = -0,62487 \quad (12.3)$$

A_h aspect ratio of the horizontal tail

Finally, the equation for the required horizontal tail:

$$\frac{S_h}{S} = \frac{-\frac{c_{mac}}{c_{Lmax}} + \Delta x_{cg} + \Delta x_{ac} + \Delta x_{sm}}{\left\{ \left(1 - \frac{d\varepsilon}{d\alpha}\right) \cdot \frac{c_{Lh\alpha}}{c_{L\alpha}} - \frac{c_{Lh}}{c_{Lmax}} \right\} \cdot \frac{q_h}{q}} \cdot \frac{\bar{c}}{l_h} = 0,268 \quad (12.4)$$

12.2 Balancing at aft limit

$$\left(\frac{x_h}{\bar{c}}\right) = \left(\frac{x_{ac}}{\bar{c}}\right)_{wf} + \frac{c_{lh\alpha}}{c_{l\alpha}} \cdot \left(1 - \frac{d\varepsilon_h}{d\alpha}\right) \cdot \frac{S_h \cdot l_h}{S \cdot \bar{c}} \cdot \frac{q_h}{q} - \Delta x_{sm} \quad (12.5)$$

After rearranging:

$$\frac{S_h}{S} = \frac{\left(\frac{x_h}{\bar{c}}\right) - \left(\frac{x_{ac}}{\bar{c}}\right)_{wf} + \Delta x_{sm}}{\frac{c_{lh\alpha}}{c_{l\alpha}} \cdot \left(1 - \frac{d\varepsilon_h}{d\alpha}\right) \cdot \frac{l_h}{\bar{c}} \cdot \frac{q_h}{q}} = 0,207 \quad (12.6)$$

12.3 Determining the required horizontal tail area

The required HT area is chosen as the biggest value. $\frac{S_h}{S} = 0,2067 [-]$

$$S_h = 0,2067 \cdot S = 0,2067 \cdot 12,677 = 2,62 \text{ m}^2$$

In the graph below it can be seen the dependency of the location of the C.G. and the requirements for the size of the horizontal tail, taking in to account the forward and aft C.G. limits.

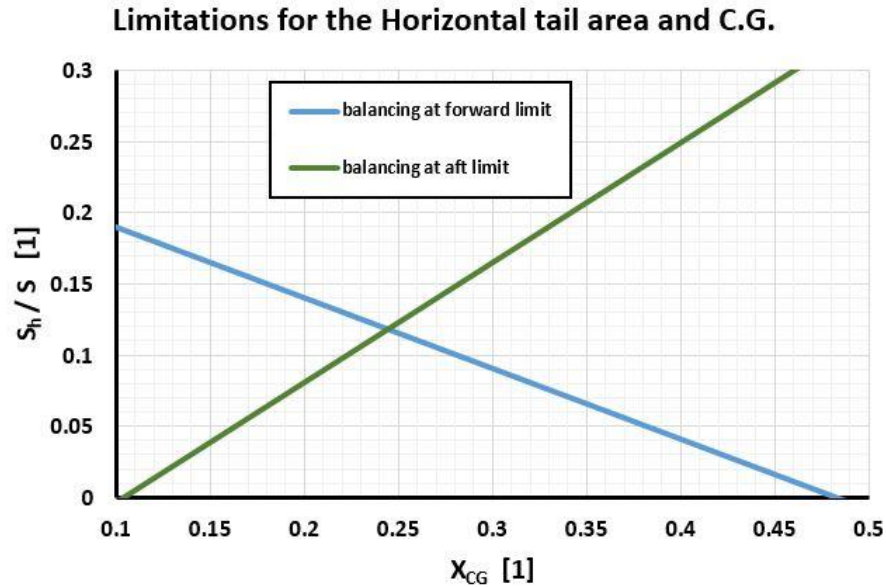


Table 12.1. forward and after limit for determining the required sizing of the horizontal tail.

13 Flight performances

13.1 Required thrust

Thrust was determined, taking into consideration a steady uniform flight $n = 1$, where drag force is equal to the thrust force.

$$D = T_p = \frac{1}{2} \cdot c_{Di} \cdot \rho \cdot v_i^2 \cdot S \quad (13.1)$$

$$T_{\text{STATIC}} = 0.85P^{\frac{2}{3}}(2\rho A_2)^{\frac{1}{3}} \left(1 - \frac{A_{\text{spinner}}}{A_2}\right) \quad (13.2)$$

Where A_{spinner} is the propeller spinner area and A_2 is the propeller disc area, see ref [2]

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ V_C^3 & V_C^2 & V_C & 1 \\ 3V_C^2 & 2V_C & 1 & 0 \\ V_H^3 & V_H^2 & V_H & 1 \end{bmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} T_{\text{STATIC}} \\ T_C \\ -\eta_p \cdot 325.8 \cdot P/V_C^2 \\ T_H \end{pmatrix}$$

To determine the required thrust we need to know the coefficient of drag as a function of coefficient of lift for different speed, from the min. value till max. value. The coefficient of drag for a given coefficient of lift we can read of the drag polar, and from the equation we can find various C_L as a function of speed.

$$c_{Li} = \frac{G}{\frac{1}{2} \cdot \rho \cdot v_i^2 \cdot S} \quad (13.2)$$

Optimum speed can be found at the minimum of the required thrust curve or at the maximum c_L/c_D

Maximum speed V_{max} can be found where the Thrust required and the available Thrust cross.

13.2 Required propulsive power

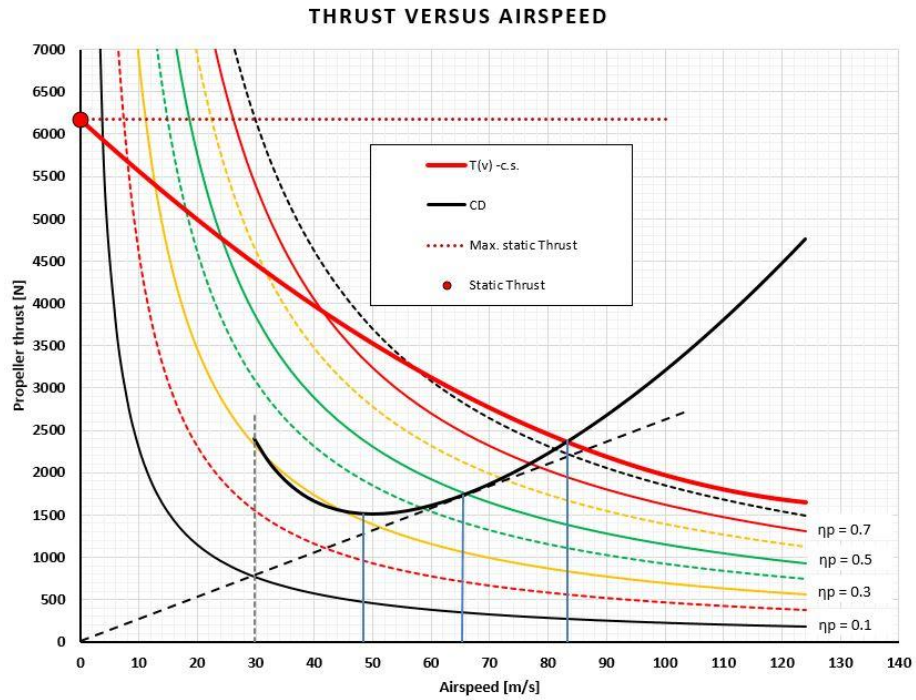


Figure 13.1. Thrust required vs Thrust available.

$$V_{\max} = 302,4 \text{ km/h}$$

$$V_{LD\max} = 172,8 \text{ km/h}$$

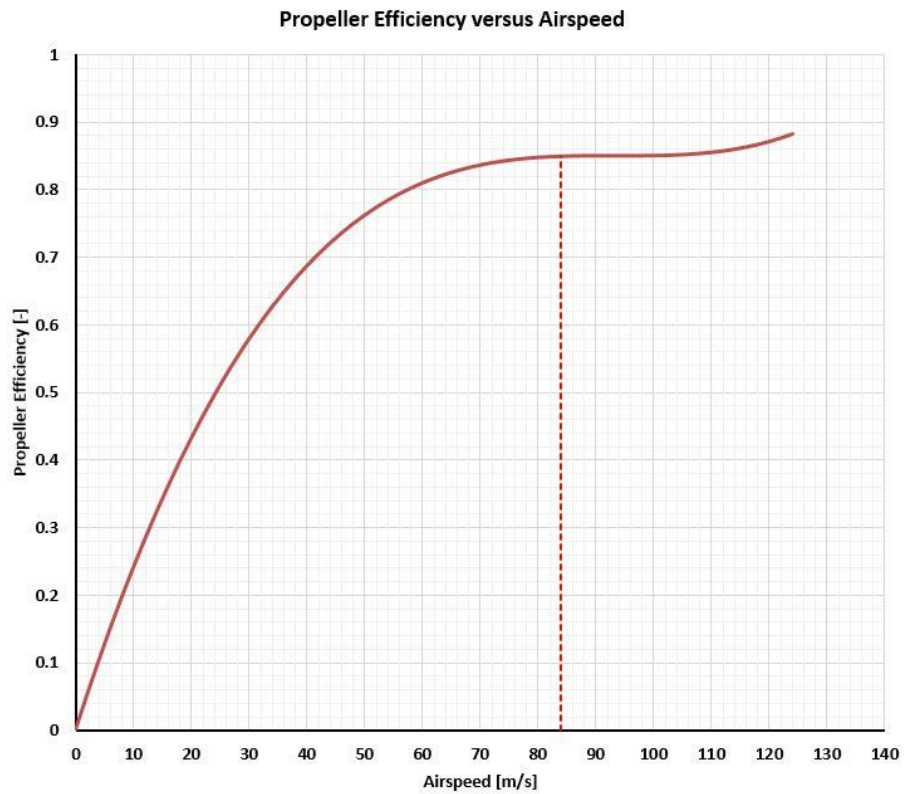


Figure 13.2. Propeller efficiency graph for a constant speed propeller showing efficiency at V_{\max} .

Maximum range

$$s_{\max} = \frac{\eta_p}{g \cdot q_f} \cdot \left(\frac{c_L}{c_D}\right)_{\max} \cdot \ln\left(\frac{1}{1 - \frac{m_f}{m}}\right) \quad (13.3)$$

$$s_{\max} = \frac{0,8}{9,81 \cdot 54} \cdot 14,993 \cdot \ln\left(\frac{1}{1 - \frac{230}{1478}}\right) = 2239 \text{ km}$$

14 Design speeds and envelope working loads

14.1 Design speeds

Cruising speed can be calculated per:

$$v_c \geq 2,4 \cdot \sqrt{\frac{G}{S}} = 292,28 \frac{\text{km}}{\text{h}} \quad (14.1)$$

Dive speed can be calculated per:

$$v_D \geq 1,25 \cdot v_c = 365,34 \text{ km/h} \quad (14.2)$$

Stall speed at flight configuration is:

$$v_s = \sqrt{\frac{2}{\rho \cdot C_{L\max}} \cdot \frac{G}{S}} = 104,6 \frac{\text{km}}{\text{h}} \quad (14.3)$$

Maneuvering speed till a maximum bank angle of 40° is:

$$v_a \geq v_s \cdot \sqrt{n} = 124,21 \frac{\text{km}}{\text{h}} \quad (14.4)$$

Max. speed at flaps (30°) must not exceed load factor 2 at $U_{de}=7,62$ m/s.

$$v_f \geq 1,4 \cdot v_s \geq 1,4 \cdot 104,6 = 146,4 \frac{\text{km}}{\text{h}} \quad (14.5)$$

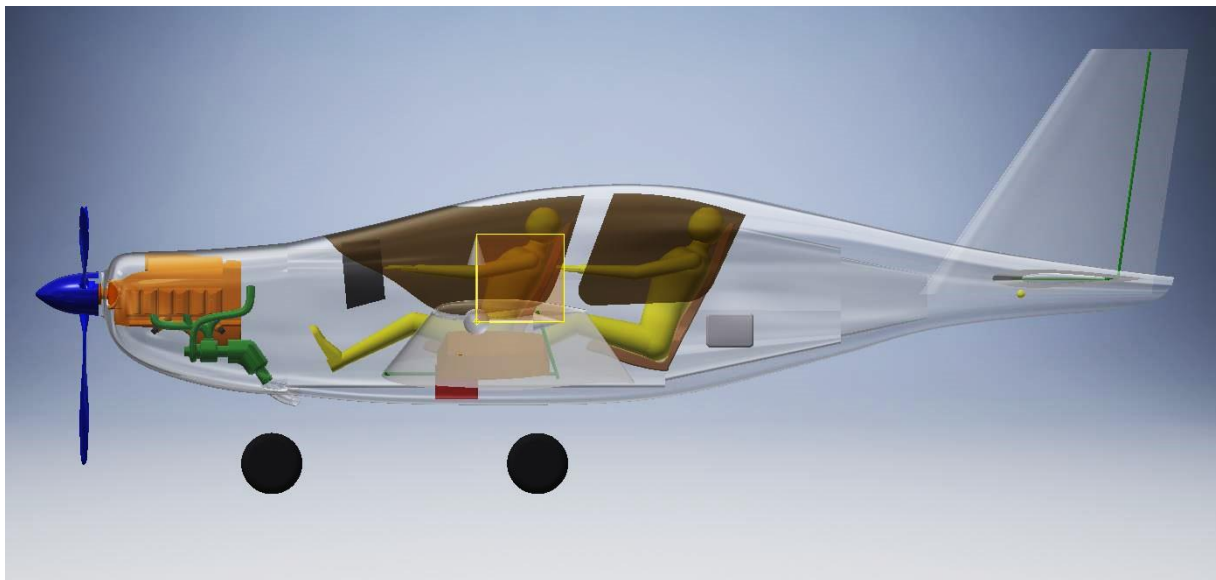
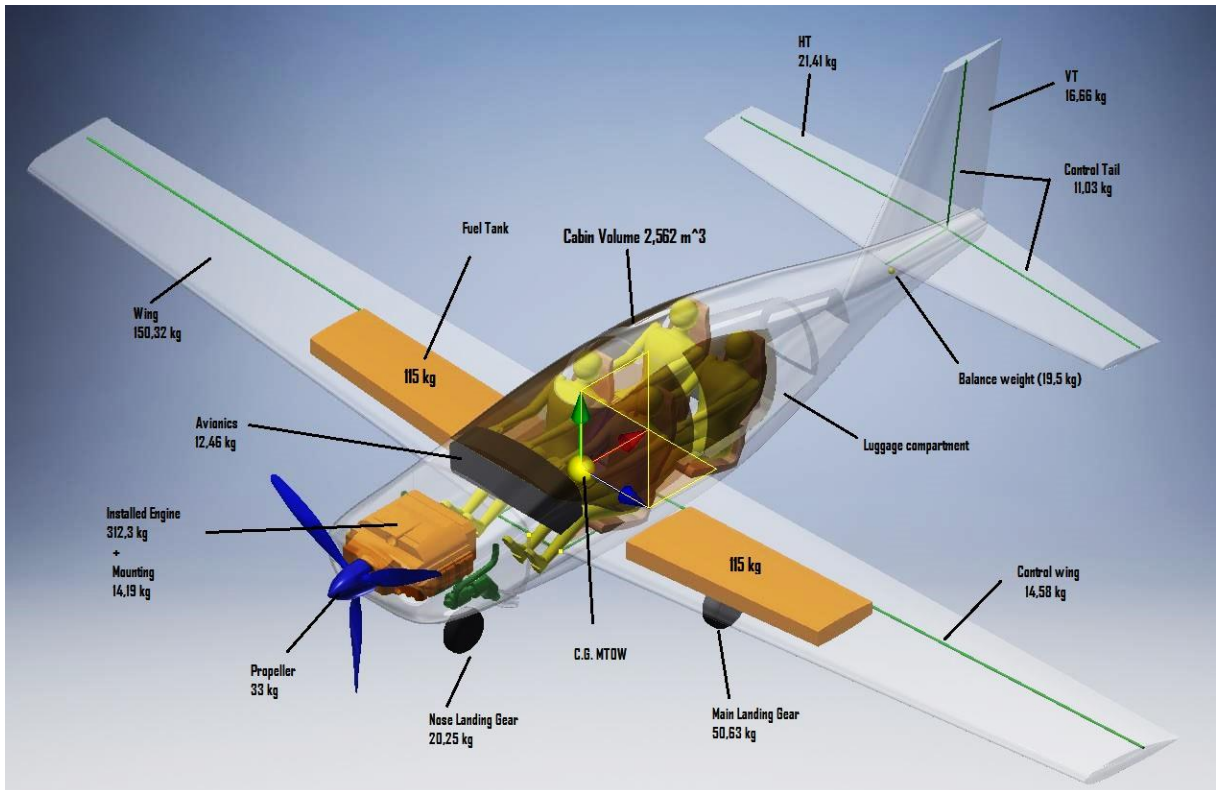
15 Conclusion

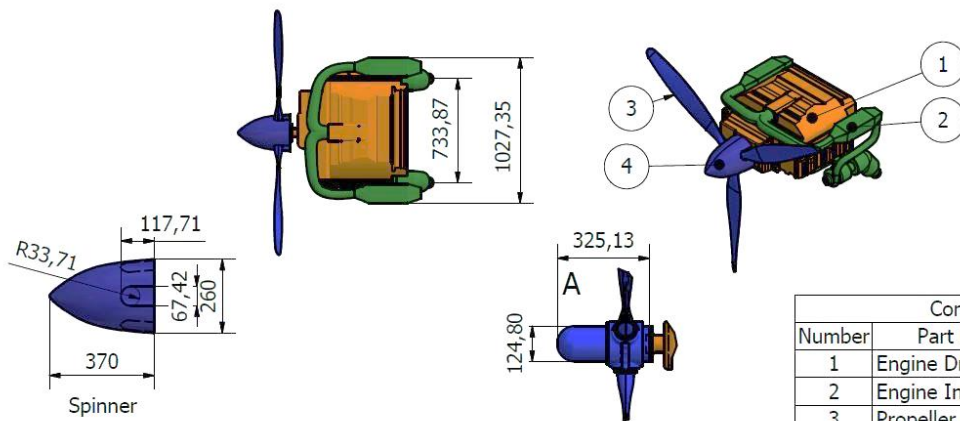
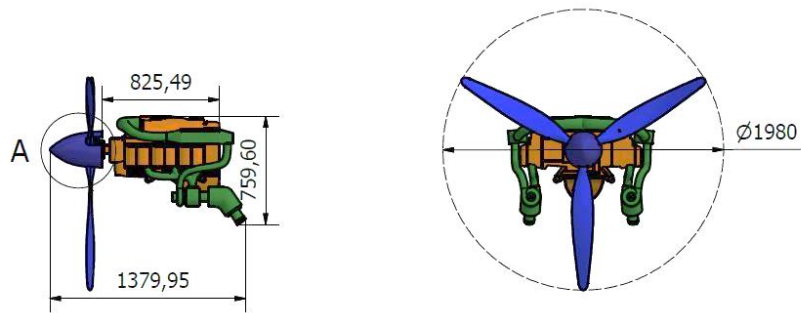
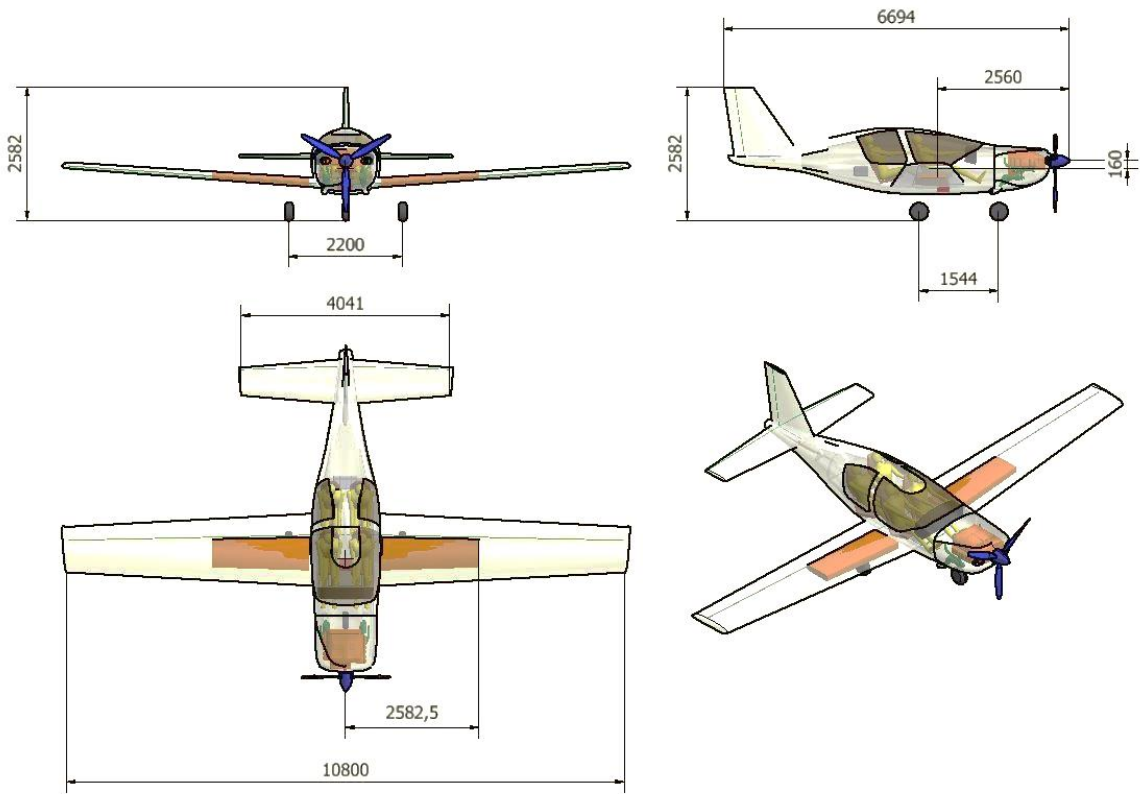
A conceptual design of a four seat GA aircraft was proposed. After comparing it to some existing aircraft and studied their statistical data. The aircraft was modeled using Autodesk Inventor. A constraint analysis showed the required thrust to weight ratios for each requirement we have set in order to fulfill the mission profile. We have estimated the Maximum takeoff weight as 1565 kg however after a estimated component weight evaluation the MTOM was reduced to 1478 kg and the empty mass from before 959 kg to 872 kg. However, the weight estimation is based on statistical data and trendlines. To estimate the components with more detail, a direct weight estimation method would be a better choice. This consists for example for the wing, by evaluating the skin, ribs and the beam weight separately. For the calculations, the simplified drag model was considered. This however considers only part of the drag polar and is to be used only for initial design. The aircraft was modeled using Autodesk Inventor Professional 2017. The code for the constraint analysis was written in Matlab R2014b. For finding the lift distribution the software Glauert III v1.00 was used. All other calculations were done in Microsoft Excel.

16 References

- [1] (EASA) European Aviation Safety Agency, 'Certification Specifications for Normal, Utility, Aerobatic, and Commuter Category Aeroplanes CS-23', vol. 36, no. Amendment 3, 2009.
- [2] S. Gudmundsson, *General Aviation Aircraft Design : Applied Methods*. 2014.
- [3] D. Stinton, 'Design of the Airplane'. 1987.
- [4] M. Range, 'Cirrus SR22 G2 GTS Specifications and Performance'.
- [5] C. E. O. I. Boscarol, 'Pipistrel Panthera', 2016.
- [6] D. Raymer, *Aircraft Design : A Conceptual Approach*. 1992.
- [7] C. Motors, 'Continental ® Aircraft Engine Maintenance Manual', 2011.
- [8] J. Roskam, 'I Preliminary Sizing of Airplanes'. 1985.
- [9] D. M. Somers, 'Design and Experimental Results for a Flapped Natural-Laminar-Flow Airfoil for General Aviation Applications', *NASA Tech. Pap.*, 1981.
- [10] R. J. McGhee and W. D. Beaseley, 'Low-speed Aerodynamic Characteristics of a 17-percent-thick Medium Speed Airfoil Designed for General Aviation Applications', no. TN D-7428, 1973.
- [11] J. Roskam, 'Airplane Design, Vol. VI, Prelim Calc of Aerodynamic Thrust & Power'. 1985.
- [12] Torenbeek E., *Synthesis of subsonic airplane design*. Delft University Press, 1976.
- [13] <http://www.panthera-aircraft.com/technical-data>
- [14] <https://www.flyer.co.uk/wp-content/uploads/2017/12/Pipistrel-Panthera-320x180.jpg>
- [15] <https://goo.gl/SGsYfY>
- [16] <https://www.flyhpa.com/files/2016/01/2016.02.01-01.24-flyhpa-56aeb3bb590e3-600x338.jpg>
- [17] <http://www.cirrusaircraft.co.uk/files/6214/8508/0551/22.jpg>
- [18] https://abpic.co.uk/pictures/full_size_069/1104015-large.jpg
- [19] D. Danek, 'Mechanika Letu', VUT Brno

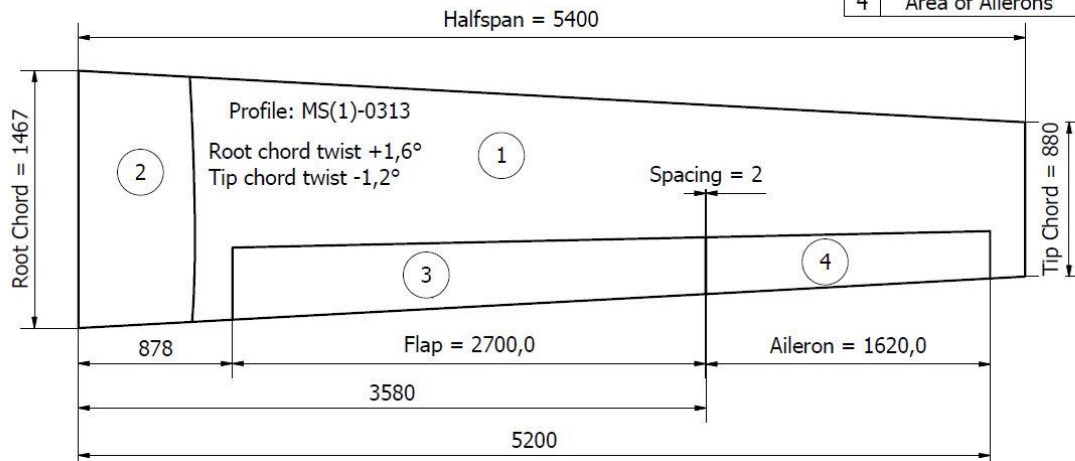
Attachments





Components		
Number	Part Name	Mass
1	Engine Dry	194,591 kg
2	Engine Installations	-
3	Propeller	33 kg

Wing Parameters		
#	Name	Value [m ²]
-	Aspect ratio	9,2 [-]
-	Taper Ratio	0,6 [-]
1	Wing area	12,678
2	Reduced Wing area by Fuselage	1,878
3	Area of Flaps	1,984
4	Area of Ailerons	0,962



Matlab code used in constraint analysis

```

%% Constraint Analysis
%%
%%
clear global
close all
clc

%% Requirements
%%
M=1565; % [kg]
MtoS=20:200; % [kg/m^2] Weight to wing area ratio range
239.4015:2777.0574
AR=9.2; % [-] Aspect Ratio
n=1/cos(45*(pi/180)); % [-] Load factor = 1/cos(phi)
h=2500; % [m] Flight Level (Cruise)
hceiling=6000; % [m] Service Ceiling
etap=0.85; % [-] Propeller efficiency

%%
%Lift and Drag coefficients from Table 3-1 (Typical Aerodynamic Characteristics of
selected Aircraft)
CDmin=0.025; % [-] Minimum drag coefficient
CDTO=0.035; % [-] Drag coefficient during T-O run
CLTO=0.7; % [-] Lift coefficient during T-O run

%%
%Aircraft speed and ground run performance Characteristics
vc=320/3.6; % [m/s] Cruise Speed
vclimb=170/3.6; % [m/s] Climb Speed
vv=7; % [m/s] Rate of climb (ROC)
vs=107/3.6; % [m/s] Stall Speed
vlof=118/3.6; % [m/s] Lift-off Speed (vs+10%)
Sg=400; % [m] Ground run

%%
%Constant values

```

```

rho0=1.225; % [kg/m^3] Air density
g=9.80665; % [m/s^2] Gravity
mu=0.04; % [-] Ground friction coefficient
vvceiling=0.508; % [m/s] Minimum acceptable Rate of climb(ROC)

%% Step 1: Estimation of Oswald's Span efficiency(straight wings) "e"
e=1.78*(1-0.045*AR^0.68)-0.64; % [-] %Oswald's Span efficiency

%% Step 2: Lift-induced drag constant "k"
k=1/(pi*AR*e); % [-] %Lift-induced drag constant

%% Step 3: T/W for Level Constant-velocity Turn
rho1=rho0*(1-(0.0065/288.15)*h)^4.256; % [kg/m^3]
q1=0.5*rho1*vc^2; % [N/m^2]
TtoW=q1*((CDmin./(MtoS*g))+k*(n/q1)^2).*(MtoS*g); % [-]

%% Step 4: T/W for a Desired Rate of Climb
q2=0.5*rho0*vclimb^2; % [N/m^2]
TtoW2=(vv/vclimb)+(q2./(MtoS*g))*CDmin+(k/q2).*(MtoS*g); % [-]

%% Step 5: T/W for a Desired T-O Distance
q3=0.5*rho0*(vlof/sqrt(2))^2; % [N/m^2]
TtoW3=(vlof^2/(2*g*Sg))+((q3*CDTO)./(MtoS*g))+((mu*(1-((q3*CLTO)./(MtoS*g))))); % [-]

%% Step 6: T/W for a Desired Cruise Airspeed
TtoW4=q1*CDmin*(1./(MtoS*g))+k*(1/q1).*(MtoS*g); % [-]

%% Step 7: T/W for a Service Ceiling(ROC=100fpm)
rho2=rho0*(1-(0.0065/288.15)*hceiling)^4.256; % [kg/m^3]
TtoW5=(1./sqrt(MtoS*g))*(vvceiling/(sqrt((2/rho2)*(sqrt(k/(3*CDmin))))))+4*sqrt((k*CDmin)/3); % [-]

%% Step 8: Plotting W/S vs. T/W
figure('Name','Wing Loading vs. Thrust Loading','units','normalized','position',[.25 .52 .4 .4]);
plot(MtoS,TtoW,'k'); hold on;
plot(MtoS,TtoW2,'r');
plot(MtoS,TtoW3,'-.b');
plot(MtoS,TtoW4,'m');
plot(MtoS,TtoW5,'b');
axis([0 200 0 0.35])
grid
grid minor
title('Constraint Diagram')
xlabel('Wing Loading, M/S [kg/m^2]')
ylabel('Thrust Loading, T/W [-]')
legend('Turn Requirement','Climb Requirement','T-O Requirement',...
'Airspeed Requirement','Service Ceiling','Location','north')

%% Step 9.1: Required Engine Power(kW)
T=TtoW*M*g; % [N]
T2=TtoW2*M*g; % [N]
T3=TtoW3*M*g; % [N]
T4=TtoW4*M*g; % [N]
T5=TtoW5*M*g; % [N]

P=((T*vc)/(etap))/1000; % [kW]
P2=((T2*vclimb)/(etap))/1000; % [kW]
P3=((T3*vclimb)/(etap))/1000; % [kW]
P4=((T4*vc)/(etap))/1000; % [kW]
P5=((T5*(sqrt((2/rho2)*(sqrt(k/(3*CDmin))))))/(etap))/1000; % [kW]

figure('Name','Wing Loading vs. Power Requirements','units','normalized','position',[.46 .04 .4 .4]);
plot(MtoS,P,'k'); hold on;
plot(MtoS,P2,'r');
plot(MtoS,P3,'-.b');
plot(MtoS,P4,'m');

```

```

plot(MtoS,P5,'b');
axis([0 200 0 300])
grid
grid minor
title('Power Requirements')
xlabel('Wing Loading, M/S [kg/m^2]')
ylabel('Engine Power Required, [kW]')
legend('Turn Requirement','Climb Requirement','T-O Requirement',...
       'Airspeed Requirement','Service Ceiling','Location','northeast')

%% Step 9.2: Required Engine Power(BHP)-Normalized to SL conditions
sigma=rho1/rho0;
sigma2=rho0/rho0;
sigma3=rho0/rho0;
sigma4=rho1/rho0;
sigma5=rho2/rho0;
P_SL=P/(1.132*sigma-0.132);      % [kW]
P_SL2=P2/(1.132*sigma2-0.132);  % [kW]
P_SL3=P3/(1.132*sigma3-0.132);  % [kW]
P_SL4=P4/(1.132*sigma4-0.132);  % [kW]
P_SL5=P5/(1.132*sigma5-0.132);  % [kW]

% figure('Name','Wing Loading vs. Power at Sea Level');
% plot(MtoS,P_SL,'k'); hold on;
% plot(MtoS,P_SL2,'r');
% plot(MtoS,P_SL3,'-.b')
% plot(MtoS,P_SL4,'m');
% plot(MtoS,P_SL5,'b');
% axis([0 200 0 300])
% grid
% grid minor
% title('Power Requirements Normalized to Sea Level')
% xlabel('Wing Loading, M/S [kg/m^2]')
% ylabel('Engine Power Required, [kW]')
% legend('Turn Requirement','Climb Requirement','T-O Requirement',...
%       'Airspeed Requirement','Service Ceiling','Location','south')
% hold off

%% Step 10: Adding Stall Speed Limits
qstall=(0.5*rho0*(vs^2));
CLmax=(1./qstall).*(MtoS)*g);

%%
figure('Name','Wing Loading vs. Power at Sea
Level','units','normalized','position',[.052 .04 .4 .4]);
hold on
plot(MtoS,P_SL,'k',MtoS,P_SL2,'r',MtoS,P_SL3,'-.b',MtoS,P_SL4,'m', ...
     MtoS,P_SL5,'b');
axis([0 200 0 300])
ax = plotyy(MtoS, P_SL, MtoS, CLmax);
ylim([0 300])
set(gca,'yTick',0:50:300)
set(ax(2),'YLim', [0 3]);
set(ax(2),'yTick', 0:0.5:3);
grid on
grid minor
title('Power Requirements Normalized to Sea Level')
xlabel('Wing Loading, M/S [kg/m^2]')
ylabel('Engine Power Required, P-SL [kW]')
legend('Turn Requirement','Climb Requirement','T-O Requirement',...
       'Airspeed Requirement','Service Ceiling','Location','south')

vc_km=vc*3.6;
vclimb_km=vclimb*3.6;
vlof_km=vlof*3.6;

annotation('textbox',...
          [0.76 0.3 0.3 0.12],...

```

```
'FitBoxToText','on',...
'String',{['MTOW = ' num2str(M) ' kg']...
['AR = ' num2str(AR)]...
['h = ' num2str(h) ' m']...
['hceiling = ' num2str(hceiling) ' m']...
['vc = ' num2str(vc_km) ' km/h']...
['vclimb = ' num2str(vclimb_km) ' km/h']...
['vv = ' num2str(vv) ' m/s']...
['vlof = ' num2str(vlof_km) ' km/h']...
['n = ' num2str(n)]...
['etap = ' num2str(etap)]...
['Sg = ' num2str(Sg) ' m']},...
'FontSize',11,...
'BackgroundColor',[.95 .95 .95],...
'Color',[0 0 0]);
```