

**Bachelor Project**



**Czech  
Technical  
University  
in Prague**

**F3**

**Faculty of Electrical Engineering  
Department of Cybernetics**

## **Rijke's Tube – An Experimental Platform for Modeling and Control in Thermoacoustics**

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**Field of study: Cybernetics and Robotics**

**Subfield: Robotics**

**May 2017**



## BACHELOR PROJECT ASSIGNMENT

**Student:** Lukáš Černý

**Study programme:** Cybernetics and Robotics

**Specialisation:** Robotics

**Title of Bachelor Project:** Rijke's Tube - An Experimental Platform for Modeling and Control in Thermoacoustics

### Guidelines:

The ultimate task is to build a laboratory experimental platform known as Rijke's tube, which is used for experiments in thermoacoustics. An inspiration can be taken from a description of one particular setup in [1]. The platform will be used for educational and research experiments in modeling, analysis and control of spatially distributed systems. The work will consist of selection of suitable components (glass tube, heat source, microphone as an acoustic pressure sensor, loudspeaker as an actuator, flow velocity sensor and perhaps some more), their assembly, design and realization of the electronics for data acquisition, and finally design and implementation of a feedback regulation loop. Finally, some experiments will be conducted and documented to demonstrate the functionality of the platform.

### Bibliography/Sources:

- [1] Epperlein, J.P., B. Bamieh, and K.J. Astrom: Thermoacoustics and the Rijke Tube: Experiments, Identification, and Modeling. IEEE Control Systems 35, no. 2 (April 2015): 57-77. doi:10.1109/MCS.2014.2384971.
- [2] Whitham, G. B.: Linear and Nonlinear Waves. 1 edition. New York, NY: Wiley-Interscience, 1999.

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Prague, January 26, 2017



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## Declaration

I declare that the presented work was developed independently and that I have listed all sources of information used within it in accordance with the methodical instructions for observing the ethical principles in the preparation of university theses.

Prague, date .....

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signature

## Abstract

This work outlines one particular design and assembly of Rijke's tube with introduced feedback in the form of a microphone and a loudspeaker. The feedback was used to control and stabilize the system; thus, loud humming that is typical for Rijke's tube was completely suppressed. Further, a few experiments are documented to show the basic properties of Rijke's tube. These are only a reconstruction of experiments presented in available literature.

**Keywords:** Rijke's tube, thermoacoustics, acoustics, experimental platform, dynamic system, feedback control, heating coil

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## Abstrakt

Tato práce popisuje návrh a sestavení Rijkeho trubice včetně zavedení zpětné vazby v podobě mikrofону a reproduktoru. Takto zavedená zpětná vazba byla použita k řízení a stabilizaci systému, což mělo za následek úplné potlačení hlasitého hučení, které je pro Rijkeho trubici typické. Dále jsou na několika experimentech ukázány základní vlastnosti Rijkeho trubice. Tyto jsou pouze rekonstrukcí experimentů popsáných v dostupné literatuře.

**Klíčová slova:** Rijkeho trubice, termoakustika, akustika, experimentální platforma, dynamický systém, zpětnovazební řízení, topná spirála

**Překlad názvu:** Rijkeho trubice – Experimentální zařízení pro modelování a řízení v termoakustice

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# Chapter 1

## Introduction

Rijke's tube, or the Rijke tube, is a vertically oriented tube open at both ends with a source of heat energy placed in the lower half. The transfer of heat energy from the source can, under the right circumstances, cause thermoacoustic instability. This is manifested as oscillations in pressure and velocity of the gas filling the tube (typically air) which is observed as loud humming. An important factor is the amount of heat energy transferred from the source to the gas. To initiate the instability, the source has to be able to provide enough energy. In this work, a heating coil connected to voltage source was used as the heating element. The power consumption of the source was around 380 W.

Rijke's tube is suitable to be used as an experimental platform for educational purposes. The coupling between heat transfer and acoustics can be subject to mathematical modeling of spatially distributed systems. Having placed a microphone at one end of the tube, one can easily investigate the humming resulting from the unstable coupling. Further, placing a loudspeaker at the other end makes it possible to add an external signal into the system. Moreover, the microphone as a sensor and the loudspeaker as an actuator may form a feedback loop that can even stabilize the system; thus, suppress the humming. The feedback loop can also serve to system analysis using closed-loop identification techniques.

Description of the construction and particular setup is given in chapter 2. Chapter 3 evaluates some of the performed experiments. It also documents system stabilization and identification. Finally, conclusion, final remarks, and possible directions for future research are mentioned in chapter 4.

## 1.1 Related Work

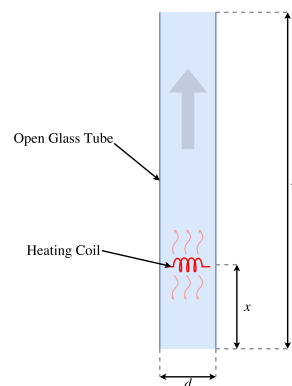
Rijke's tube was discovered by P. L. Rijke in 1859. As he states in [8], he used glass tube with a disc of wire-gauze inside. He used a burner to heat the disc up. After removing the burner, the tube started to hum loudly. The humming went away as the disc was cooling down. Author of [1] and [2] used electrically driven heating coil and feedback control techniques to stabilize the system. In fact, [1] was main inspiration when working on this project. Worth mentioning are also [7] and [5] where horizontally mounted tubes with blowers were used.

## 1.2 Mechanism of Rijke's Tube

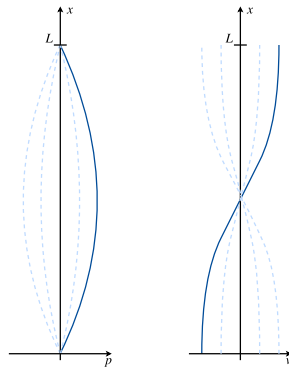
The presence of the source of heat energy creates an upward flow in the tube (in fig. 1.1 depicted as blue arrow). This flow increases as the heating element heats up. If the flow is fast enough, oscillations of acoustic velocity and pressure are excited. Usually, fundamental frequency is dominant in these oscillations. Fundamental frequency is given by the length of of the tube  $L$  and the speed of sound in the tube  $c$  (approximately  $343 \text{ m s}^{-1}$ ) as

$$f_0 = \frac{c}{2L}. \quad (1.1)$$

Actually, the oscillations form standing half-wave inside the tube as depicted in fig. 1.2.



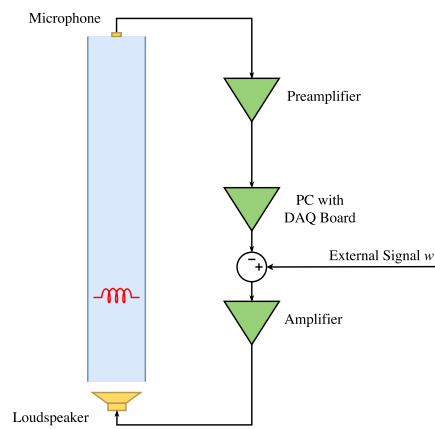
**Figure 1.1:** An illustration of Rijke's tube



**Figure 1.2:** Fundamental acoustic mode in Rijke's tube ( $p$  is pressure,  $v$  is acoustic velocity)

### 1.3 Feedback Control

Feedback loop consisting of a microphone and a loudspeaker (fig. 1.3) enables the system analysis and stabilization. The external signal  $w$  is used for system identification.



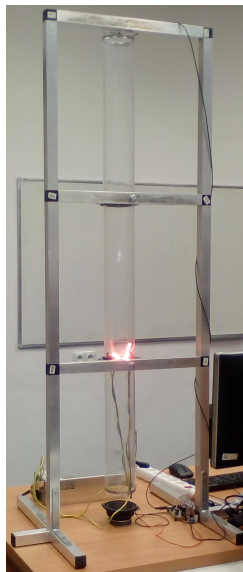
**Figure 1.3:** Feedback loop



## Chapter 2

### Construction of Rijke's Tube

This chapter describes selected components and the process of construction of Rijke's tube with feedback loop formed of an electret microphone, preamplifier, PC with data acquisition board, power amplifier, and a loudspeaker. The preamplifier is one of custom design built on a prototyping shield. Couple of off-the-shelf electret microphone modules with built-in preamplifiers were tried but all of them happened to either offset or distort measured signal. For this reason, custom preamplifier was designed which turned out to be a cheap and effective solution. The final setup of the apparatus is in fig. 2.1.



**Figure 2.1:** The constructed apparatus

## 2.1 Quartz Glass Tube

Assuming the heating element to reach high temperatures, quartz glass was selected as appropriate material for the tube. Quartz glass is fused quartz (or fused silica) without any additives that would lower the melt temperature. Its high melting point and low coefficient of thermal expansion make it resistant to thermal shocks. Actually, the tube did not break even if it came in contact with the heating element. The dimensions of the tube are

$$L = 1310 \text{ mm}, \quad (2.1)$$

$$d = 85 \text{ mm} \quad (2.2)$$

where  $L$  is its length and  $d$  is inner diameter.

To keep the tube in vertical position, custom stand was built. It is made of aluminium profiles and tube bushings that allow the tube to be fixed in different heights. This makes it easy to set a particular distance between the tube and the loudspeaker. Another bushings are used to hold the microphone and the heating element.

## 2.2 Heating element

Kanthal wire with diameter 0.64 mm rolled into a coil was used as the heating element. The length of the wire was approximately 1.4 m making its resistance at room temperature

$$R_0 = 6.09 \Omega. \quad (2.3)$$

The coil was connected to voltage source Mean Well SPV-1500-48 (see [6]) with output voltage  $U = 48 \text{ V}$ . The maximum current that can be drawn from the source is  $I_{\max} = 32 \text{ A}$ . Therefore, its maximum power consumption is more than 1.5 kW which sufficient to heat up the coil. In fact, the actual power consumption when the coil is not heated up yet can be calculated as

$$P = \frac{U^2}{R_0} = 378.3 \text{ W}. \quad (2.4)$$

After heating the coil up, its resistance increases a little (see section 3.4) lowering the power consumption to 373 W.



It is possible to make the the power consumption higher by making the wire shorter. Nonetheless, a thicker wire should be used because the one used here burned at around 500 W. (According to [4], the wire can operate at temperatures up to 1400 °C.)

## 2.3 Microphone

To measure sound signal at the top of the tube, an electret microphone MCE-100 with an internal FET amplifier was used. This microphone works as a capacitor that requires connecting to bias voltage. The sound signal is then measured as voltage changes around the bias voltage. Parameters of the microphone are in tab. 2.1.

The frequency range of the microphone is sufficient, since higher frequencies are not expected to be present in the spectrum of measured sound signal. The sensitivity is given by the manufacturer for frequency of 1 kHz only. For simplicity's sake, we can assume the frequency response to be flat throughout the frequency range. However, it would be preferable to measure the microphone frequency response so we could determine signal distortion.

The microphone was connected to the preamplifier via coaxial cable of length 1.5 m with audio jack connector at both ends.

Parameter	Value
Frequency range	50–10 000 Hz
Sensitivity	5 mV Pa <sup>-1</sup> at 1 kHz
Impedance	6 kΩ
Operating voltage	1.5–10 V
Dimensions	9.7 mm × 6.7 mm

**Table 2.1:** Electret microphone MCE-100 parameters



Name	Description
U1	5 V positive voltage regulator L78L05 (see [10])
U2	Operational amplifier OP07C (see [11])
R1	Resistor 220 $\Omega$
R2	Resistor 470 $\Omega$
R3, R8	Resistor 10 k $\Omega$
R4, R5, R7	Resistor 47 k $\Omega$
R6	Resistor 1 k $\Omega$
R9	Resistor 33 k $\Omega$
R10	Trimmer resistor (potentiometer) 5 k $\Omega$
C1, C3, C6, C7, C11	Ceramic capacitor 100 nF
C2	Electrolytic capacitor 2200 $\mu$ F, 16 V
C4, C5	Electrolytic capacitor 470 $\mu$ F, 16 V
C8	Electrolytic capacitor 100 $\mu$ F, 6 V
C9, C10	Electrolytic capacitor 1 $\mu$ F, 6 V
C12, C13	Electrolytic capacitor 10 $\mu$ F, 16 V
J1	Connector PC-GK2.1, connected to 12 V, 2 A DC power supply
J2	Jumper providing 12 V power supply
J3	Jack 3.5 mm mono connector for input signal
J4	Output signal jumper

**Table 2.2:** List of components used to assemble the preamplifier

The preamplifier is based on OP07C, a high precision op-amp (operational amplifier) with very low input offset voltage. The whole circuit is to be powered by single 12 V source connected to socket J1. The op-amp is wired as a non-inverting amplifier and its non-inverting input is biased toward one-half the source voltage by voltage divider formed of resistors R4 and R5. Capacitor C9 filters out the DC component of the input signal and the AC component is then superposed on the bias voltage of 6 V. As a result, the amplified op-amp output signal is also superposed on this bias voltage. Capacitor C12 passes only the AC component. That together with presence of the pull-down resistor R8 yield the output signal on jumper J4 without the DC component. In other words, the output signal is a waveform oscillating around 0 V.

Potentiometer R10 allows adjusting the gain. The minimum gain is

$$A_{\min} = 1 + \frac{R9}{R10} = 7.6 \quad (2.5)$$

and by adjusting the potentiometer we can set the gain arbitrarily high. However, the output signal is constrained by the supply voltage.

Resistors R6 and R7 together with capacitor C10 filter out the power supply noise by bypassing it to ground without affecting the input signal. To suppress the power supply noise even more, two RC circuits are used. One consisting of R1, C3, and C4. The other consisting of R2, C5, and C6. C1, C2, C7, C8, and C11 are bypassing or decoupling capacitors.

Voltage regulator U1 serves to provide 5 V power supply for the microphone. In-series resistor R3 is necessary to avoid damaging the internal FET amplifier.

## 2.5 Speaker and Amplifier

At bottom end of the tube, universal loudspeaker Monacor SP-10/4 was placed. Its parameters are in tab. 2.3. It is driven by audio power amplifier TDA2030A (see [9]) that was bought as off-the-shelf module. Its maximum output power is 18 W.

Parameter	Value
Impedance	4 $\Omega$
Wattage	15–25 W
Dimensions	105 mm $\times$ 55 mm

**Table 2.3:** Loudspeaker Monacor SP-10/4 parameters

## 2.6 Data Acquisition

To acquire data from the microphone and actuate the loudspeaker, a computer with data acquisition board MF624 by Hummusoft was used. It contains 8 channel fast 14 bit ADC (analog-to-digital converter) and 8 channel 14 bit DAC (Digital-to-analog converter.) The output of microphone preamplifier was connected to the board's ADC and the power audio amplifier was connected to board's DAC.

The board was used together with Simulink Real-Time Windows Target. The reading frequency was 10 kHz. Fig. 2.4 and 2.5 depict Simulink models that implement data collection and feedback loop control. To filter out high-frequency noise, low-pass filter with time constant  $\tau = 1$  ms was used in the

data acquisition. However, to eliminate time delay, it was not used in the feedback loop.

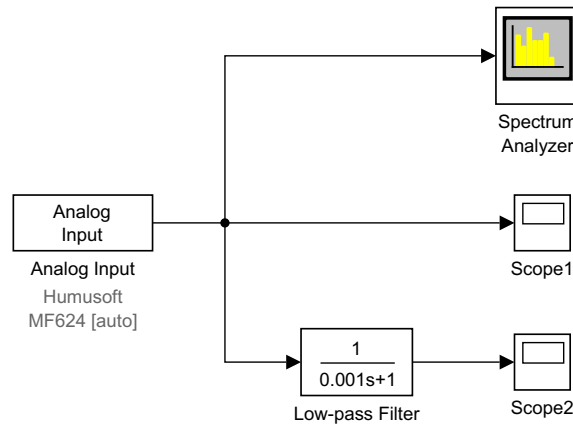


Figure 2.4: Simulink model implementing data acquisition

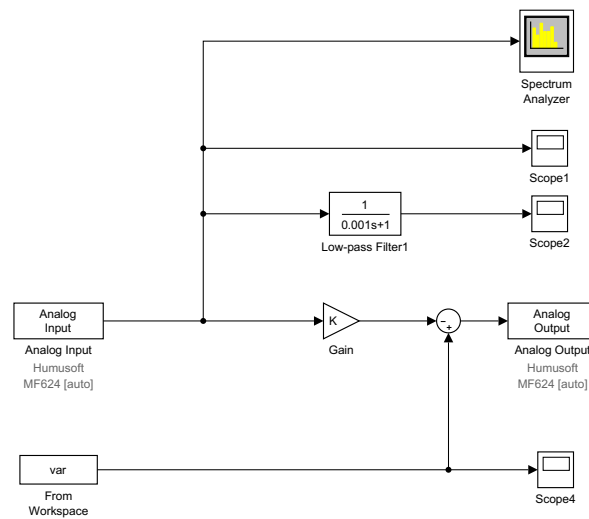


Figure 2.5: Simulink model implementing feedback loop



## Chapter 3

### Experimental Evaluation

This chapter gives an evaluation of performed experiments. Firstly, the instability of the system is observed. Secondly, feedback is introduced to stabilize the system. Then, system identification is done. And finally, temperature of the heating coil is estimated.

#### 3.1 Instability Observation

Fig. 3.1 shows the initial growth of oscillations of acoustic pressure in the tube. A close look at the oscillations is given in fig. 3.2 from which we can see dominant frequency of 131.6 Hz. This value is consistent with the expectation of standing half-wave in the tube and it satisfies (1.1). This frequency can also be seen in amplitude spectrum 3.3 of measured signal (for  $x = L/4$ ) together with the second harmonic of twice the frequency. As shown in spectrum 3.4, for heating coil position  $x = L/3$  the second harmonic is not that significant as for  $x = L/4$ .

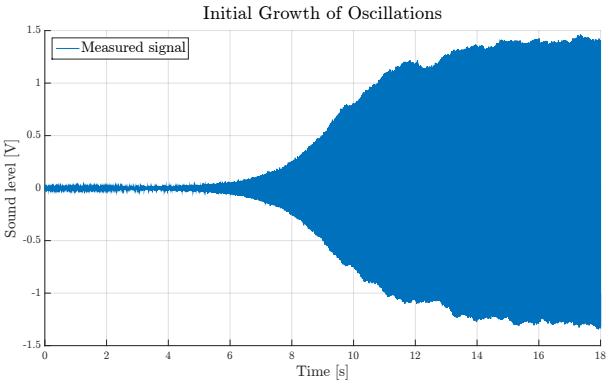


Figure 3.1: Initial growth of oscillations

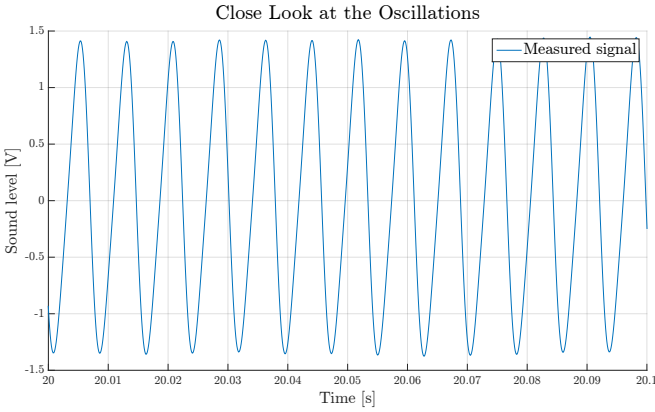


Figure 3.2: Close look at the oscillations

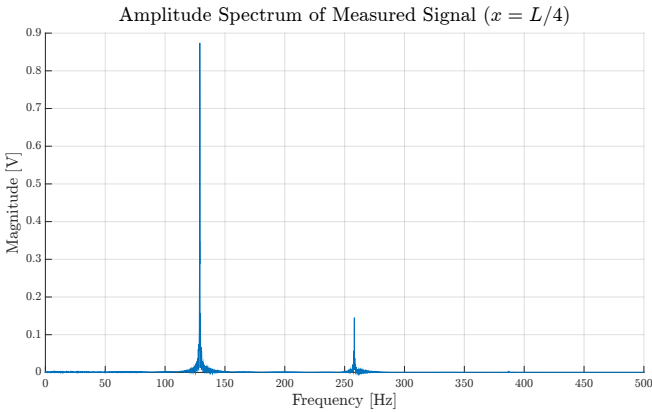
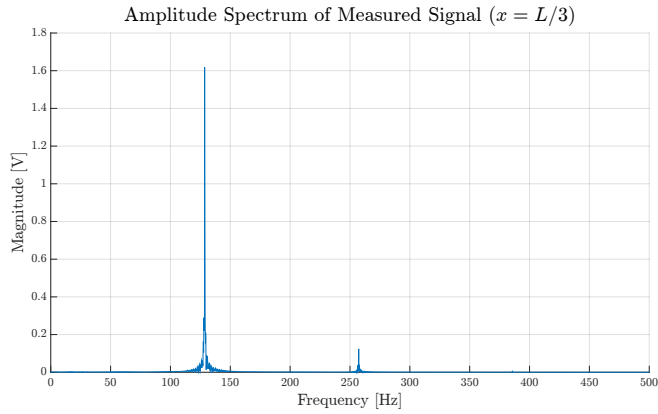


Figure 3.3: Single-sided amplitude spectrum of measured signal for  $x = L/4$





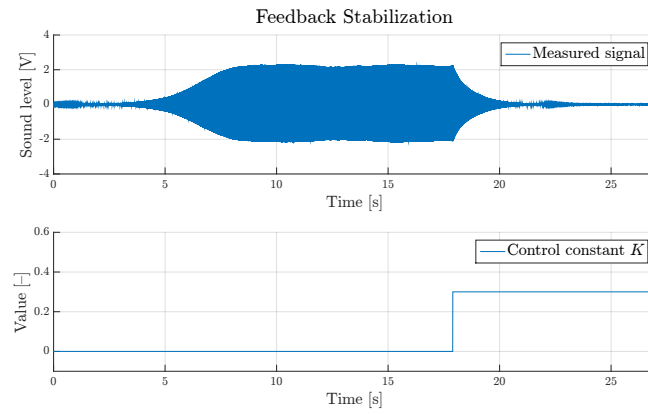
**Figure 3.4:** Single-sided amplitude spectrum of measured signal for  $x = L/3$

## 3.2 Feedback Control

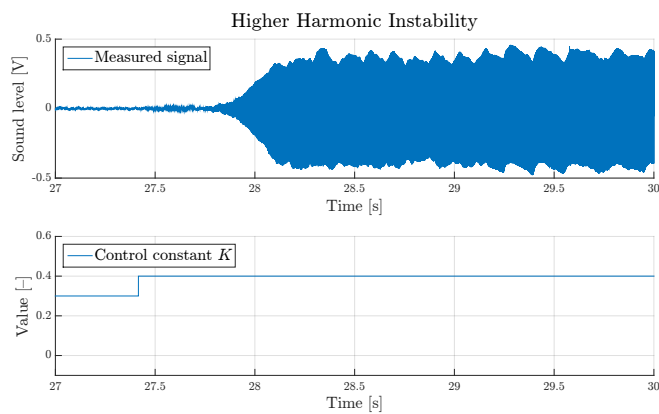
To stabilize the system, simple proportional regulator is sufficient for heating coil position  $x = L/3$ . For  $x = L/4$ , it turned out to be impossible. Values of control constant  $K$  around 0.3 stabilize the system making the humming go away. The result of regulation is shown in fig. 3.5 where  $K$  changes from 0 to 0.3. Since the the humming disappears and the regulation loop consists of proportional feedback only, resulting signal fed to the speaker is zero. This means that the regulator actually stabilizes the system. Hence, the process of stabilization should not be confused with active noise cancelation.

If the constant  $K$  is increased a little, the system becomes unstable again (fig. 3.6). This time a higher harmonic is dominant, specifically the 13th (fig. 3.7).

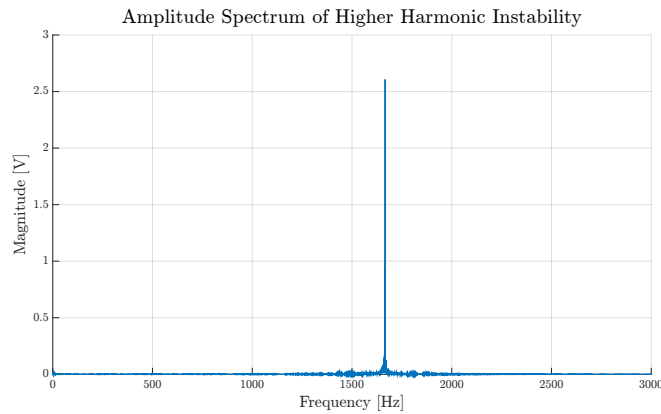
### 3. Experimental Evaluation



**Figure 3.5:** Stabilization of the system



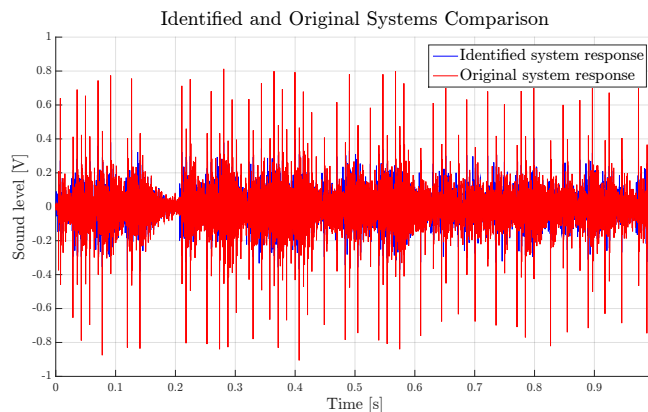
**Figure 3.6:** Higher harmonic instability



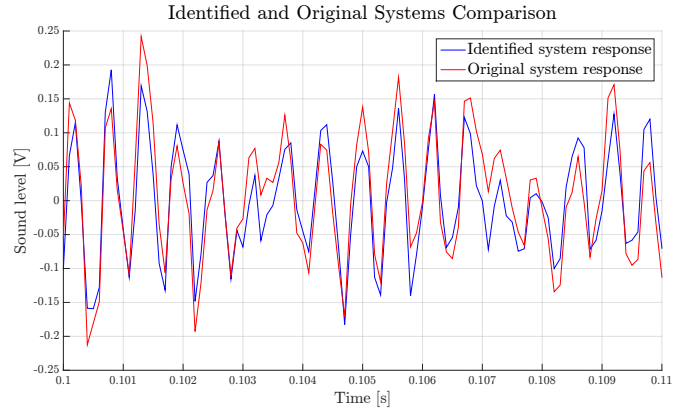
**Figure 3.7:** Single-sided amplitude spectrum of higher harmonic instability

### 3.3 System Identification

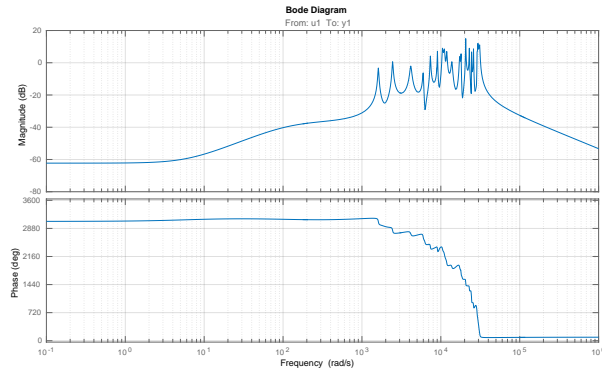
The stable closed-loop system was identified by adding external signal  $w$  into the regulation loop. For this signal, PRBS (pseudorandom binary sequence) of duration 20s was generated. The first ten seconds were used for identification and the other ten were used for verification of identified system. In order to be able to identify it, signal measured by the microphone and signal fed to the loudspeaker were collected. From these, the system was identified using parametric method known as *ARX*. The number of poles was set to 44 and the number of zeros was set to 22. This method gave relatively accurate (see fig. 3.8 and 3.9). Fig. 3.10 frequency response obtained from *ARX* method.



**Figure 3.8:** Comparison of responses to test signal



**Figure 3.9:** Comparison of responses to test signal (close look)



**Figure 3.10:** Frequency response from results of  $ARX$

### 3.4 Heating Coil Temperature

The heating coil temperature can be estimated by measuring increase of its resistance. Using approximation of linear temperature coefficient of resistance for kanthal wire (obtained from [4])

$$\alpha \approx 2 \times 10^{-5} \text{ K}^{-1} \quad (3.1)$$

we can estimate the wire temperature  $T$  as

$$T = T_0 + \frac{R - R_0}{\alpha R_0} \quad (3.2)$$

where  $R_0$  is the wire resistance at room temperature  $T_0$  and  $R$  is its resistance at temperature  $T$ . Resistance  $R_0$  is given by (2.3). Resistance  $R$  was determined from measured voltage of the power supply

$$U = 48.06 \text{ V} \quad (3.3)$$

and measured current running through the wire when heated up

$$I = 7.77 \text{ A}. \quad (3.4)$$

Applying Ohm's law we get

$$R = \frac{U}{I} = 6.19 \Omega. \quad (3.5)$$

Finally, by considering room temperature

$$T_0 \approx 20 \text{ }^\circ\text{C} \quad (3.6)$$

we obtain

$$T = 803 \text{ }^\circ\text{C}. \quad (3.7)$$

Nonetheless, this quantification is a very rough approximation only.





## Chapter 4

### Conclusion

In this thesis, the process of construction of Rijke's tube is described. It also documents a few experiments performed to illustrate basic properties of the tube. For instance, observations have shown that fundamental frequency of the tube dominates the spectrum of the oscillations which corresponds to standing half-wave inside the tube. Also, stabilization of the tube using proportional regulator was achieved. Finally the tube as a system was identified using parametrical methods.

All the performed experiments might be improved if frequency response of used microphone and preamplifier was available. Knowledge of the response would make it possible to determine the signal distortion.

Possible direction for future research is measuring the sound at different places and measuring the air flow in the tube. Another interesting observations might be done by changing the position of the heating coil and the loudspeaker.







## Appendix A

### Bibliography

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## Appendix B

### Abbreviations

AC	Alternating current
ADC	Analog-to-digital converter
DAC	Digital-to-analog converter
DAQ	Data acquisition
DC	Direct current
FET	Field-effect transistor
Op-amp	Operational amplifier
PC	Personal computer
PRBS	Pseudorandom binary sequence





## Appendix C

### CD Contents

The enclosed CD contains the following files.

- *Lukas\_Cerny\_BP\_May\_2017.pdf* – this bachelor thesis in *pdf* format
- *Lukas\_Cerny\_BP\_Assignment.pdf* – bachelor project assignment
- *MicrophonePreamplifier.pdf* – schematic of the designed preamplifier
- *libs/* – directory containing libraries required to use DAQ board
- *ReadData.slx* – Simulink model implementing data acquisition
- *FeedbackLoop.slx* – Simulink model implementing feedback loop
- *InitMF624.m* – MATLAB script initializing DAQ board