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Department of Economics, Management and Humanities

Economical Operation of Transformer

Master Thesis

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Název tématu: Hospodárný provoz transformátoru

Pokyny pro vypracování:

- transformátor jako prvek v dopravě elektřiny
- hospodárná velikost při návrhu transformátoru
- sestavení technickoekonomického modelu pro výběr optimální velikosti transformátoru
- hospodárný provoz transformátorů

Seznam odborné literatury:

Toman, P., Drápela, J., Mišák, S., Orságová, J., Paar, Mr., Topolánek, D. a kol.: Provoz distribučních soustav, ČVUT, Praha 2011, ISBN 978-80-01-14935-8
Voženílek, P., Novotný, V., Mindl, P.: Elektromechanické měniče, ČVUT, Praha, 2007

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In Prague on 4.5.2015 Bc. Martin Pilous

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1 Table of content

2 Abstract and keywords

Transformer is electrical stationary machine which is primarily used for changing of voltage. It consist of two or more electrical windings situated around conjoint magnetic circuit. By regular diagnostics we are able to reveal emerging errors (failures) which help us to prevent overall destruction of the machine. Transformers are known for their high level efficiency and yet energy losses are representing significant part of overall costs. Parallel running of two or more machines can be helpful in terms of energy loss reduction. The initial costs will be logically bigger due to the need of more transformer purchases.

Transformátor je elektrický netočivý stroj, který se užívá především ke změně napětí. Sestává ze dvou a více elektrických vinutí kolem společného magnetického obvodu. Pravidelnou diagnostikou lze odhalit vznikající vady a předejít tím celkovému znehodnocení stroje. Transformátory mají vysokou účinnost, přesto však ztráty energie tvoří velkou část nákladů. Paralelní chod dvou a více strojů může tyto ztráty zredukovat, ale za cenu pořízení dalších strojů.

Keywords: Transformer, construction, losses, diagnostics, net present value, costs, economy, parallel operation

Klíčová slova: Transformátor, konstrukce, ztráty, diagnostika, NPV, náklady, hospodárnost, paralelní chod

3 Introduction

Transformers are important component in our current life. They are electrical machines with ability of voltage change in alternating current. Without them the current electricity system could not exist. We can find them in every household or factory. They are produced in various sizes and powers. In this paper I examine technical realization, diagnostics and economic model of the distribution transformer.

Basically transformer is nothing more than two coils with magnetic bond. However demands for greater performance, reliability and efficiency increased over time especially after their wider deployment. Nowadays transformers are complex machines with optimized construction using advanced materials and precise manufacturing. The final result is high efficiency and long service life of the device.

I decided to research transformers because I deal with this issue for a couple of years now. I wanted to introduce transformer as an electrical machine, explain its work principle and construction. I also described some selected methods of diagnostics which can help us to find out minor malfunctions before they can destroy whole machine.

No device can work without losses (or own consumption if you want) and transformer is no exception. Actually losses account for a significant portion of the operational costs. They can be reduced by purchasing higher quality machine or by parallel connection but in all cases that means increase of the investment costs. Because nobody wants to pay more than necessary balancing and minimization of costs was my task in the economical part where I explained economical model of transformer and showed possible usage in examples.

4 General description of transformers

Transformer is one of base electrical machines. It is specific by its ability to change voltage of electrical power. Like every electrical machine, transformer also can transfer energy in both directions.

Transformer is static AC to AC converter, which primarily increases or decreases voltage and non-conducingly links together various electrical circuits or networks. In some special applications it also can change number of phases or frequency. Transformer is the foundation of electrical network as we know it today. They are the major reason, why we prefer alternating current for power usage. Their size and power is very wide ranged - from the smallest used in electronics and with power of fractions of volt-amperes, to huge ones working with voltage of hundreds kilovolts and transferring electricity on long distances.

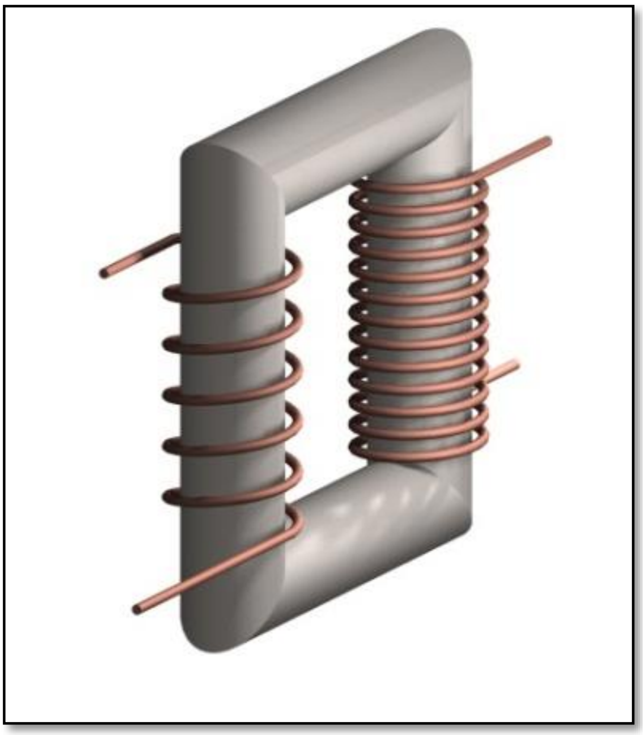

Figure 1. Model of single-phase transformer

Conversion of energies is ongoing in active part of machine, which we can split into two basic parts: electrical and magnetic. Electrical part consists of windings of coils. In case of transformers, these windings are stationary, in contrast with other electrical machines. Magnetic part is represented by magnetic circuit formed from ferromagnetic. Its purpose is to focus and conduct magnetic flow created by electrical part.

4.1 Types of transformers

Transformers serve for many applications and they have different forms. By their usage we can categorize them for example like in following list:

- Power transformers
  - Transferring energy between high voltage and very high voltage systems, means between generators and transmission systems and between transmission systems and distribution systems.
  - Primary voltage usually of 220 kV and above.
  - Nominal power usually of 500 kVA and above.
- Distribution transformers
  - Transferring energy between distribution and low voltage systems.
  - Decreasing voltage on 400 V or costumer’s service circuit voltage.
  - Usually 5 to 500 kVA.
- Phase-shifting transformers
  - Regulation of transferred active and reactive power.
  - Usually connecting different very high voltage systems for example different national systems.
  - Control of power flow and grid stabilization.
- Rectifier transformers
  - Containing rectifier circuit (thyristors / diodes) inside the tank.
  - Input is three phase AC, output is DC.
- Instrument transformers
  - Allowing measurements of high voltage circuits.
  - Secondary side for measuring devices.
  - Divided on voltage transformers (VT) and current transformers (CT).
- Constant voltage transformers
  - Ferroresonant transformer
  - Type of saturating transformer used as voltage regulator
- Autotransformers
  - Special type of transformer with only one winding.
  - Do not galvanically separate primal and seconder circuit.
  - Lighter, smaller, cheaper and with lower losses.

In this work I will examine only power and distribution transformers. However a lot of characteristics are similar or same to all types noted above. In technical aspects they are not different. Physical principles or used materials are same so are possibilities of isolation and constructions are also not too wide. They are not identical but they are still the same kind of electrical machine.

4.2 Single-phase transformer

The simplest type of transformer is single-phase one. For simplifying of the description I created the model of it (see Figure 1). It is formed from simple magnetic core and two coils with different number of turns. One side is called primary and the second is secondary. For explanation of physical principles I needed a scheme and it can be seen in Figure 2.

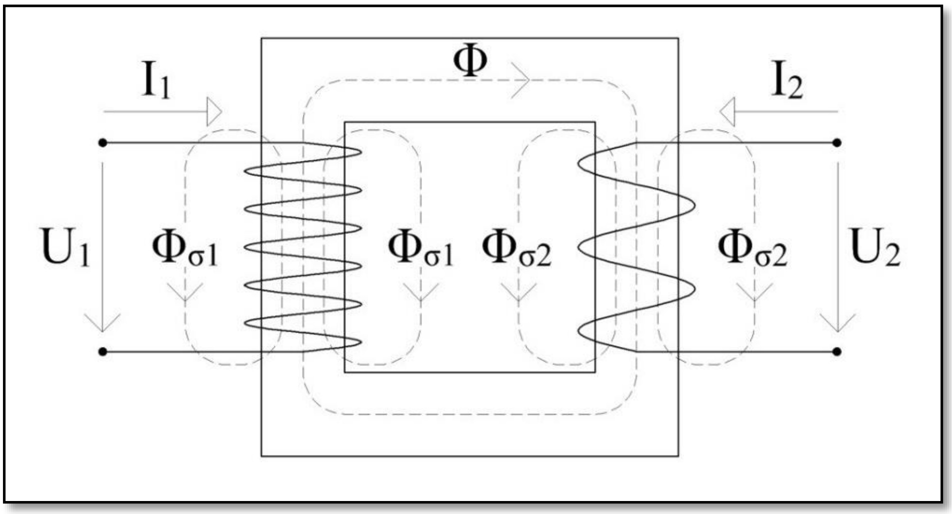

Figure 2. Scheme of single-phase transformer

The figure also contains markings of some base physical quantities. Description is following:

I	*[A]*	Electric current – effective value flowing through windings
U	*[V]*	Voltage – effective value of terminal voltage on windings
Φ	*[Wb]*	Main magnetic flux – it is focused by magnetic circuit and create magnetic coupling between both windings
Φσ	*[Wb]*	Stray magnetic flux – part of magnetic flux closed only around winding and which not affect the second one

Simplifying a little I can describe principles of its function like this. Alternating current in primary winding exited by ac voltage generates alternating magnetic flux in the core. The core – magnetic circuit – is here for conducting and bounding flux. And this flux subsequently inducts voltage into secondary winding. Turn ratio (*a*) of primary and secondary voltage (U_P, U_S) is same like ratio of coil turns on primary and secondary side (N_P, N_S).

$$\mathrm{a} = \frac{U_P}{U_S} = \frac{N_P}{N_S} \tag{1}$$

In generalized mathematical formulas I describe whole work principle in differential form of following Maxwell's equations:

$$\nabla \times \vec{H} = J + \frac{\partial \vec{D}}{\partial t} \tag{2}$$

Rotation of magnetic field intensity vector H is equal to free current density J and electric displacement current $\frac{\partial \vec{D}}{\partial t}$.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3}$$

Rotation of electric field intensity vector *E* is equal to negatively taken time evolution of magnetic field *B*.

4.3 Three-phase transformer

Function principle for single-phase transformer is the same for three-phase. The main difference between them is obviously in number of phases. For transferring energy we use three-phase system. That means three wires conducting electrical potential of sinusoidal character. And all that three sinusoids are shifted by 120° against each other. This system was introduced because of its capability to create rotating magnetic field. For energy transmission we use them more often than one-phased ones. But using of three single-phase instead of one three-phase is possible and equivalent. For example it is common in cases of highest power, where the three-phase one is too big for transportation.

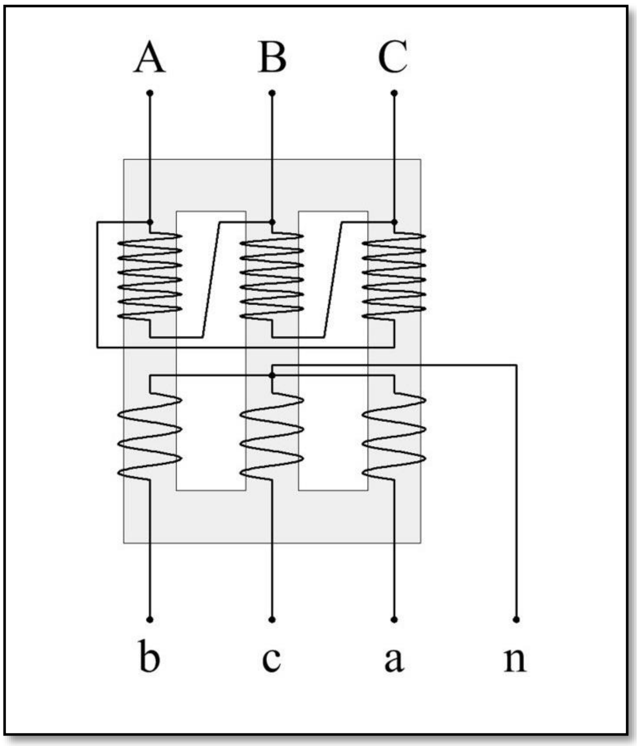

Figure 3. Scheme of three-phase transformer

So when we can use three single-phase transformers and why we should choose the three-phased one instead? Due to united magnetic core which is used for all three phases at once its production is much cheaper and because that core needs less material then three suitable cores together. This saving is visible on the first look in Figure 3. Magnetic core there has three limbs every for each one phase. This construction is posible thanks to the fact that sume of magnetic fluxes is zero:

$$\Phi_A + \Phi_B + \Phi_C = 0 \qquad (4)$$

Transformer is possible to be connected in different ways. That way is defined by code identification. For example many distribution transformers use connection Dyn1 also showed in Figure 3. What does it mean? First upper case letter represent primary high-voltage winding, second lower case letter secondary low-voltage winding. Letter *n* inform us about grounding and number indicates phase displacement.

D or d	Delta winding – triangular configuration
Y or y	Wye winding – star configuration
Z or z	Zigzag winding – interconnected star configuration
N or n	Indicates that system neutral is connected
1÷12	Phase displacement – number is in unit of 30 degrees

4.4 Losses of transformers

Transformer is simple machine only by its principle of working but there is complicated issue of calculations and constructions in behind. And in the beginning I will describe typical problems with transformers because used constructions were created to eliminate or decrease them.

Like every existing machine, also transformers cannot work without energy losses. We desire to reduce losses and improve efficiency as much as possible because transformers are machines which will work for decades and every tenth of percent in efficiency represent considerable financial cost. These losses are transformed into form of waste heat which we need to somehow drain out to prevent fatal damage caused by overheating. So our goal is also to decrease amount of waste heat and implement good cooling system. Losses have several sources.

Main problem are the losses in core. They are caused by hysteresis and eddy losses. Magnetic hysteresis is phenomena linked with all ferromagnetic materials. When some ferromagnetic was used for conduction of magnetic flux material becomes magnetized by retentive magnetism. Every time we change polarity of magnetic field this retentive force must be surmounted which leads to another loss. We can reduce this problem only by choose of appropriate steel. About the eddy losses it is clear that electrical current in secondary winding is made by magnetic induction. However by same effect are made unwanted current loops right in the magnetic core. They are called eddy currents. To inhibit them we assemble magnetic circuit from metal sheets as thin as possible because eddy losses are dependent on square of width of the core lamination material so this solution radically reduced these currents and thus losses. These sheets must be isolated from each other. The second way is create sheets from steel with higher electric resistivity but their function is only supportive.

It is technically possible to create transformer with magnetic circuit from one piece but it is very unpractical and whole construction would be too complicated. It follows a usage of a divided core but with it we have new factor of how to connect those parts together? On first look it is a simple question but many different variables are coming into play. The solution should calculate with edge of core, materials, type of connection between pieces, mechanical strength and difficulty of composing. Each connection creates new air gap and as it is known air has much worse magnetic conductivity which leads to another losses.

Last substantial thing is electrical resistivity of winding. It is dependent on material used and also wires diameter. As material only copper and aluminum come into consideration. Copper is used more often because over a similar price the copper winding is easier to handle and smaller in size. Several cross sectional shapes of conductor and structures of coil are possible. Specific shape is compromise between claims on conductivity and coil filling factor on one side and price and mounting difficulty on the other side.

5 Construction of transformer

Transformer is not a cheap machine and we usually suppose it will work for 25 to 45 years. Also during its running it generates additional costs in form of losses. Because of this, it is important to have good quality of manufactory. In this part I focused on description of the construction and procedures of how transformers are made.

Construction of transformers is quite diverse. There are several possible configurations of core and coils. Except of sorting on three-phase and one-phase there is also sorting based on shell and core type. Principle of function remains unchanged, but there are small differences. Shell type has magnetic side limbs around windings which is good for reducing of stray magnetic flux and therefore there is better magnetic coupling and the losses are lower. On the other hand transformers of core type are smaller, lighter and need less material so they are also cheaper. Differences in construction can be seen in Figure 4.

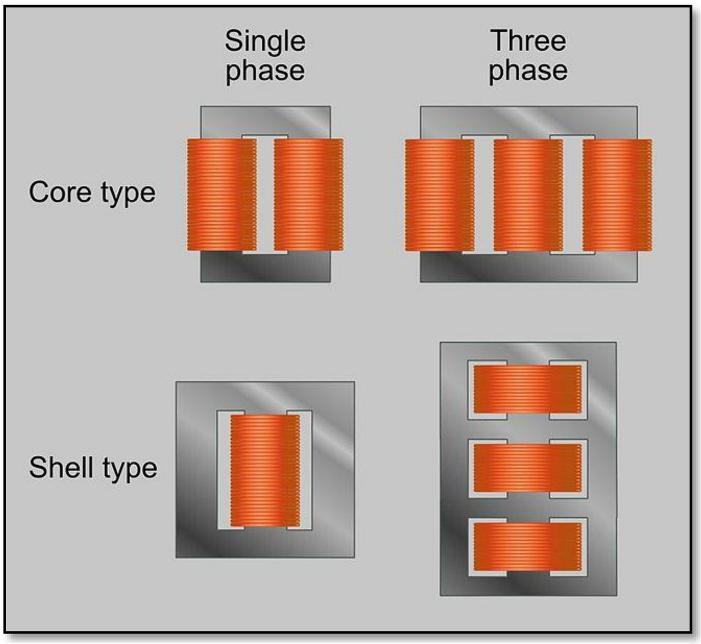

Figure 4. Examples of core and shell types of transformers [Transformers. In: Wikipedia [online]. 2012 [of 2014-04-17]. Available from: en.wikipedia.org/wiki/transformer

Core section can be circular or rectangular. In dependence on core section the shape of windings around is determined. Rectangular cross section is easier in terms of production and is better in usage of space. On the other hand circular one has especially superior mechanical stability under short circuit conditions but also it needs less material. Because of it power transformers have always round core section. But for small transformers rectangular shape is standard because of its easy mass production. In case of distribution ones there can be used both models.

5.1 Magnetic core

Chosen materials play the main role in this case. When we talk about power transformers the losses in core are not only an economical issue but also a serious technical problem because losses can create excessively hot zones which can destroy whole machine in short time. New discoveries in field of electric steel over past decades have been main contributor to more effective and smaller transformers. Losses in material are expressed by W/kg (watts per kilogram) for real power loss and VA/kg (volt-amperes per kilogram) for apparent power loss. These values are dependent on used frequency and flux density.

There are several types of commonly used constructions of magnetic cores. They are different in dependence on power of transformer and used material for core of course but it is also different from country to country because there are some traditionally used types.

5.1.1 Grain oriented steel

Most often used material for power and distribution transformers in these days is grain oriented (GO) silicon steel. It may be referred to in other various ways. This kind of steel has rearranged directions of grain formations inside the steel strips and it radically improves magnetic properties in the rolling direction. Power losses are significantly lower in right direction but they rise with changing of angle and in perpendicular direction can be thrice or more higher. This is reason why it is necessary to mount core of bigger transformers divided into individual limbs and yokes. Silicon addition increase permeability and resistivity. Saturation induction of this material is 1.6 to 1.8 T. In comparison with classical hot rolled steel where this value is around 1.35 T there is big difference and it resulted into new generation of transformers with distinctly reduced volume. Higher resistivity means less losses by eddy currents. Steel has the highest resistivity with silicon content around 11 % but with content this high it is too brittle and hard and unusable for handling. So the used content of silicon in steel for transformers is normally around 3 to 4 %. Common values of strip thickness are 0.2; 0.23; 0.27; 0.3 and 0.35 mm but it is not impossible to buy also strips of thickness 0.15. Losses in material are roughly 0.4 W/kg.

Magnetic core of transformer is mounted from these steel laminations. Because of elimination of eddy currents the luminous must be electrically separated one from another. This is achieved by thin coating of isolation material on every lamination. Such modified they are stacked and bound together and often also with insulated core clamping bolts going through them all and pressing them together to improve their mechanical stability. The core cross section of power transformers is circular so for achieving of the circular shape the width of laminations during stacking is variable. This cylindrical formation represents one of core limbs. Because of advantage in magnetic orientation of luminous whole core cannot be made from one or two pieces and all straight sections must be assembled separately. So these sections are created from different parts but they are usually mounted all at once to ensure all parts fit together. Cross section area of yoke can be little increased for reducing of the yoke portion of core loss and it can have different cross section shape (usually replaced by rectangular one) because around it there is no more coil. For additional solidity of construction the shapes of laminations in different layer can overlap other neighbors. Cross section of straight parts and overlapping in classical core construction is shown in Figure 5 below.

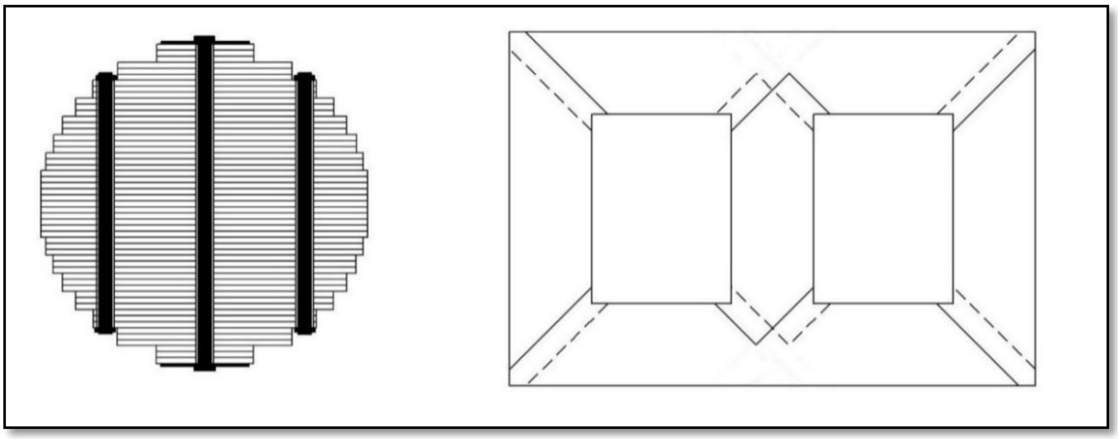

Figure 5. Cross section and overlapping of magnetic core

After the composing the top yoke is removed (composed) and prepared windings are pushed on the vertical limbs. Now the yoke is connected again and whole construction is finally bound together fixedly by clamping structure.

Construction of distribution transformer in Europe looks like smaller version of power transformers, but in the United States of America another type of construction is popular and used. Instead of one three-phase transformer they prefer one-phased ones usually serving a single residence or small group depending on the load. In this case it is the shell type so the core is mounted around the coil. Shell type is more common for all one-phase transformers. Magnetic circuit is again from grain oriented steel laminations but now they are curved into C-shapes and stacked like in Figure 6 so the cross section is rectangular. The cross section area of side limb and yoke is half that of the central limb because flux from central limb is split.

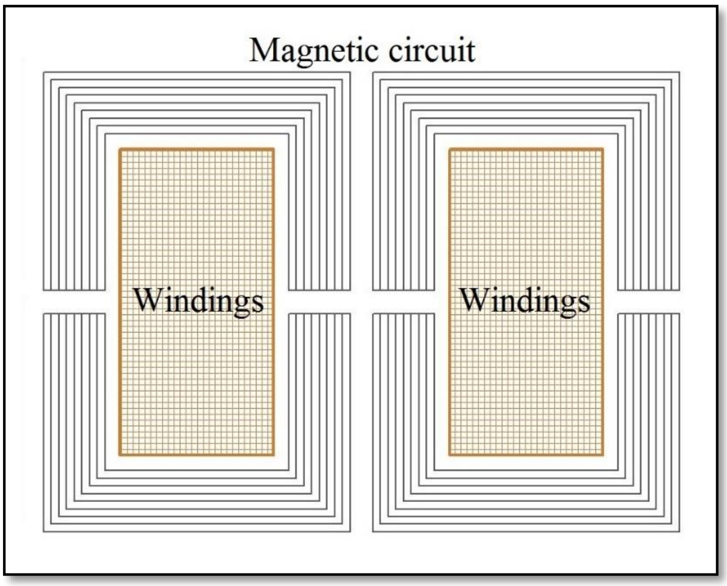

Figure 6. Construction of one-phase shell transformer

5.1.2 Non-oriented steel

Application of grain oriented steel on small low-voltage transformers is too expensive so for those transformers we use non-oriented steel which is cheaper but less effective. Advantage of this material is its isotropy (same properties in all directions) so production is easier and cheaper. Since the losses and efficiency do not have so high priority for mass produced small transformers the laminations are thicker than in case of power transformers and have they rectangular cross section. This kind of steel has similar composition like oriented one which means low carbon content and addition of silicon. Because of anisotropy it is possible to stack whole magnetic core from one-piece layers but more practical are two-piece ones. Typical example is E-I laminations like in Figure 7.

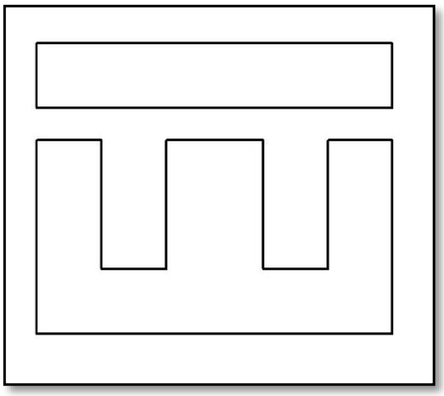
Figure 7. E-I laminations

5.1.3 Amorphous metal ribbon

Amorphous metal also known like metal glass is unique metallic material without atomic-scale crystal structure. It is usually manufactured by extremely rapid cooling called melt-spinning where melted metal is poured in very thin layer on rotating drum cooled by some liquid gas. For achieving of sufficient temperature gradient the material can be formed only in thin layers. Because of this final thickness of ribbon is approximately only 30 µm. For transformers alloy of iron with addition of silicon, boron and phosphor are used. This material is characterized by high permeability, flexibility, strength, hardness and unfortunately also by its high price. Magnetic losses in this case are extremely low, only around 0.1 W/kg. Resistivity is roughly thrice higher than normally used silicon steel and with combination of ribbon thickness it decreases losses via eddy current very well. To avoid electrical contact between layers the ribbon is coated by isolation material typically MgO. Amorphous metal is applied on magnetic core only for distribution and small power transformers because it has quite low saturation induction.

5.2 Windings

Windings are the most complicated part of transformers. Their manufacturing is technologically and economically complicated. Whole unit must satisfy conditions on mechanical and electrical toughness, temperature endurance, facility of cooling, low electrical losses and it must be as cheap as possible. There are several possibilities of how to arrange coils around core limbs. Main limiting factor is always maximal voltage between each turns. It is necessary to stack turns every time in such a way that there are no opportunity for short circuit which could possibly lead to decommission or to destroying of whole device.

There are some differences between construction of oil and dry transformers. Because dry transformers are in minority I am going to describe mainly oil ones in this part and I will explain some specific differences of dry transformers in part about dry isolation.

5.2.1 Materials

As a material for windings, there are only two possibilities. Electrical copper and aluminum. Choice between them is not always clear. There are many aspects which can influence our decision. So for better understanding I prepared this short summarization of their advantages. Impacts on transformer's quality are not so significant but it is useful to know the differences.

5.2.1.1 Copper

- Electrical conductivity
  - Copper has nearly twice higher conductivity than aluminum.
  - Resistivity: $\rho_{Cu} = 16\, n\Omega \cdot m$ - $\rho_{Al} = 30\, n\Omega \cdot m$
  - Copper conductors are nearly half the cross section size (with same resistance) and that mean also smaller and have shorter windings.
  - Thanks to the previous point magnetic core can be also smaller and smaller are proportions of whole transformer. That means another savings in terms of materials.
  - This small resistivity is achieved only with very pure copper, above 99.9 %. It is similar also for aluminum but there is recommended limit only 99.5 %.
- Tensile strength
  - Copper in general is more mechanically durable than aluminum. This material is also easier on handling and installation and that means savings on work and mounting devices.
  - Aluminum need special joining. There were cases of fires in past because of this.
- Thermal conductivity
  - Another important attribute is thermal conductivity especially in bigger transformers.
  - It is often improved by addition of silver (0.01 %) which helps to improve thermal conductivity without decreasing electrical one.

5.2.1.2 Aluminum

- Price
  - Actual price of aluminum on world markets is around 1 300 EUR/ton while price of copper is around 5 000 EUR/ton. Prices are changing but there are predictions that the ratio of prices will be less or more same or rising in favor of aluminum in future.
- Mass density
  - Mass density of aluminum is much lower than density of copper.
  - $D_{Al} = 2{,}70\ g/cm^3$ - $D_{Cu} = 8{,}89\ g/cm^3$
  - It is true that windings from aluminum need more material but only in the volume. Due to much lover mass density winding from aluminum is distinctly lighter notwithstanding it is bigger.
  - Since the price of material is dependent on the weigh and weight of windings from aluminum are approximately 1,7 times lighter and the whole winding from aluminum is roughly six times cheaper than winding from copper with same resistance.
- Heat capacity
  - Specific heat capacity of aluminum is much higher than copper have.
  - $c_{Al} = 896\ {}^{J}/_{kg \cdot K}$ - $c_{Cu} = 383\ {}^{J}/_{kg \cdot K}$
  - This ratio is decreased by different mass density and weight of whole windings but aluminum still has this advantage. Heat capacity of whole aluminum winding is approximately 1.4 times higher than copper one have.
  - It is an important parameter for short time overloading.
- Oxidation
  - Aluminum and copper both react with oxygen.
  - Aluminum creates only thin oxide layer on the surface which serves also as good insulator.
  - On the other hand copper oxides constantly. The result of this difference is that the winding from aluminum has a little longer lifespan.
- Thermal expansion
  - This is not so important parameter for oil transformers but it is very important in case of dry ones.

- Aluminum has very similar thermal expansion value like epoxy resin which is used as isolation in dry transformers. Windings in this isolation create solid form and mechanical stresses inside incurred due to different thermal expansion and small elasticity of resin can impair compactness of isolation and create cracks.
- It is possible to use copper conductors for dry isolation but in that case epoxy resin must be improved with fiberglass and some next additives and even in that case the final result is still not perfect.

Technical possibilities of usage in different kinds of transformers are very similar for both of these materials and because of it the most important parameter is usually the final price. That depends mainly on prices of materials and also on production costs which means work, special devices requirement, know-how etc. In order to market success the price must be competitive and warrantable. Based on mathematical model results of technical and economical analyze [14] for transformers over 190 kVA it is better to use copper windings and for smaller powers is preferable to use aluminum. [14]

5.2.2 Cross sections of conductors

Normal conductors have circular cross section but unfortunately this shape is usually not acceptable for transformers or electrical machines in general but the smallest powers. There are more reasons. Coil stacked from round wire has bad filling. That means that there is a lot of free space between the wires so the winding must be bigger. In case of layer separation by paper insulation it is necessary to use some adhesive to improve cohesion of it because layer of coils from round conductors has only a small contact area on surface. Round wire also always tends to bog down to lower layer of coil rounds. But this type of cross section has also one advantage. It is its price and with it also availability because manufacturing of this wire is much simpler and more massive than another cross sections. Hence can be usage of round wire economically preferable for small power distribution transformers. To reduce disadvantages of round wire it is often flatted into an oval shape.

For distribution transformers foil conductors for low voltage winding are often used. It is very often aluminum foil but copper one is also possible. Foil is wide nearly like whole winding but on the edges there must be enough space for overlapping of isolation layers. Edge of foil is round to eliminate the point of discharge risk. During manufacturing the insulator (typically cellulose paper which is also in form of foil like conductor) is attached on carrier construction and by rotating of the carrier the first layer is created. Now the conductor is connected. The conductor foil needs to have some pins for following connection on both ends. Normally it is cold pressed down strip. These pins are also isolated. Further rotating of carrier reel both layers and this process continues until the whole winding is created. On the end of the conductor is added pin again and overlapping of insulator is affixed. In case of transformer for higher power during the reeling are often added pads with spacers after several layers in order to create axial cooling path. This type of winding is cheap, quite easy and fast to manufacture and is often used. Another great feature is in its ability to handle short circuit currents because current in foil is spread over whole surface of the foil and there are minimal axial forces present. Final winding can have both possible shapes – circular or rectangular.

For power transformers rectangle cross section of conductor is always used because this shape allows best filing ratio in coil and that mean savings in terms of size and material needed for core. Unfortunately necessary cross section area of conductors in power transformers is usually too large for one wire. We can use single wire but with rising power the area is rising too. And after we reach some size the conductor is so difficult to form in required shape it is inapplicable. Hence we use stranded conductor formed from multiple strands. In this way constructed conductor is much easier on bending. Strands are joined together at ends of windings. Transposition of straps is necessary or circulating currents will be created. Without transposition, straps have different flux fields and thus also inducted voltage. So at the ends of straps arises different

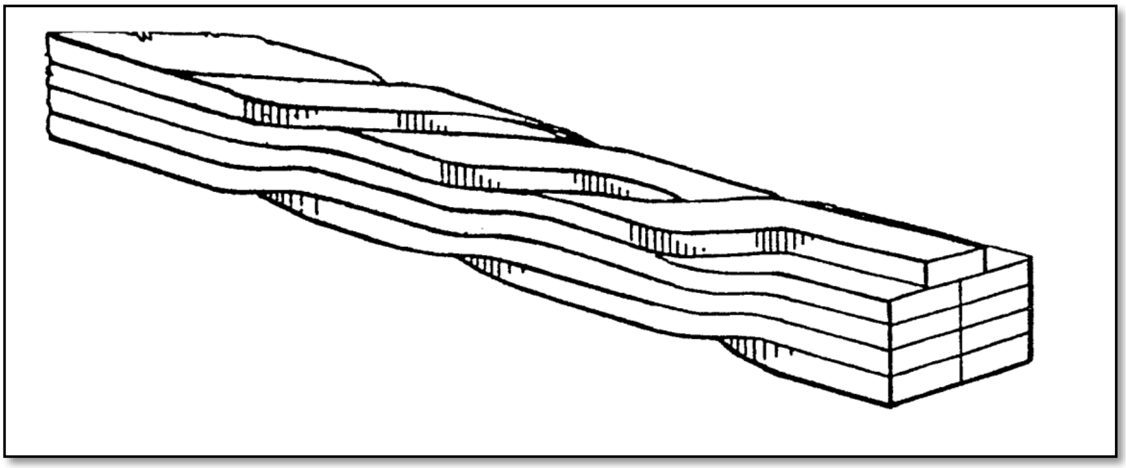

Figure 8.Continuously transposed cable [12]

potentials and they generate circulating currents. That results into another losses and heat waste. Transposition can be made in several ways but for best results is commonly used continuously transposed cable (CTC) like in Figure 8 which can be stranded from only few to several tents of strands. Stranding of conductor can be also applied on round wires where the transposition is resolved by twisting of wires into a rope.

5.2.3 Arrangement of windings

Windings can be classified as concentric or sandwich. These terms describe positions of primary and secondary coils eventually tertiary one if there is any. Differences between these two possible arrangements you can see in Figure 9. Because of easier isolation the low voltage parts are placed first over the core. Between windings there is a free space created by spacers for free oil flow to provide cooling and sufficient electrical isolation. Sandwich arrangement is almost exclusively used in shell type transformers.

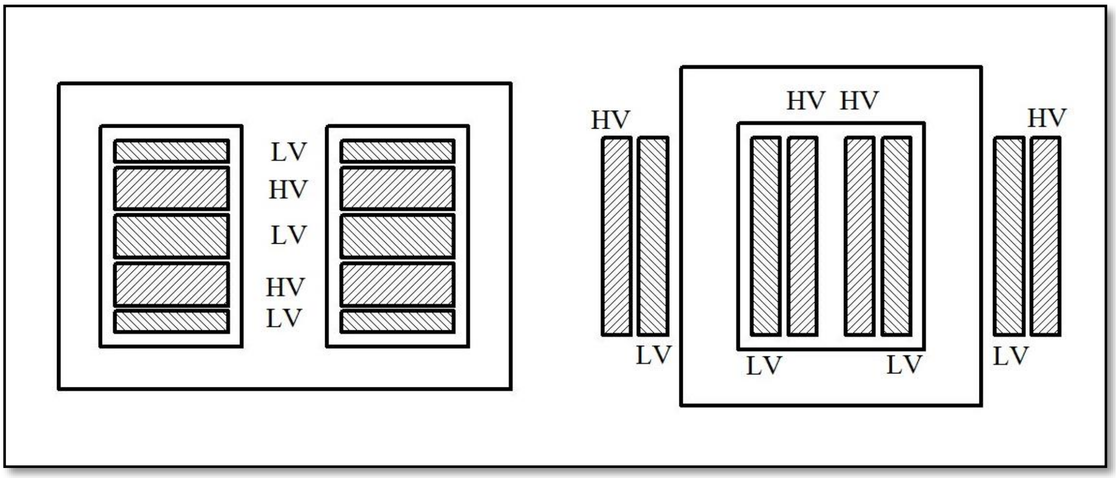

Figure 9. Concentric and sandwich arrangement

5.2.4 Structure of windings

There are many types so I only describe few selected which are most commonly used. Most important for concentric coils are the following: cylindrical winding, helical winding, cross over winding and continuous disc winding.

5.2.4.1 Cylindrical winding

This is also known as layer or barrel winding. It is the simplest variant of structure and insulated conductors are wound directly next to each other around carrier cylinder with spacers. On one layer of wounded conductors is wound to the next one and they are separated by solid insulation. In case when multi-core conductor is required two or more of them can be wound in parallel. Disadvantage of this variant is small mechanical strength against axial forces. All cross sections of conductors are common here.

"Variations of this winding are often used for applications such as tap windings used in load-tap-changing (LTC) transformers and for tertiary windings used for, among other things, third harmonic suppression." [12]

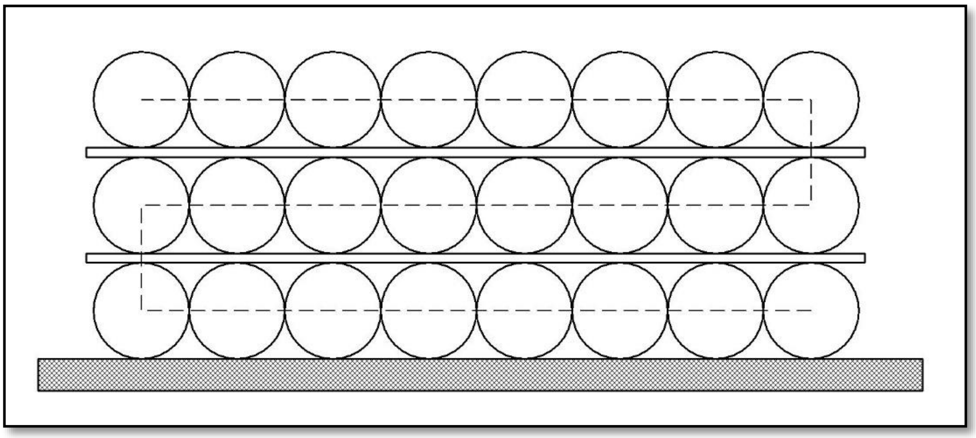

Figure 10. Cylindrical winding

5.2.4.2 Helical winding

This variant is usually selected for highest currents thus low voltage windings of middle and high capacity power transformers. Rectangular conductor from many strands is used here usually in flat form and touching in the radial direction. Process of winding is similar like in cylindrical winding but here are spacers between each turns mainly for improving of its cooling ability which is necessary here because of high currents and high amount of wasted heat.

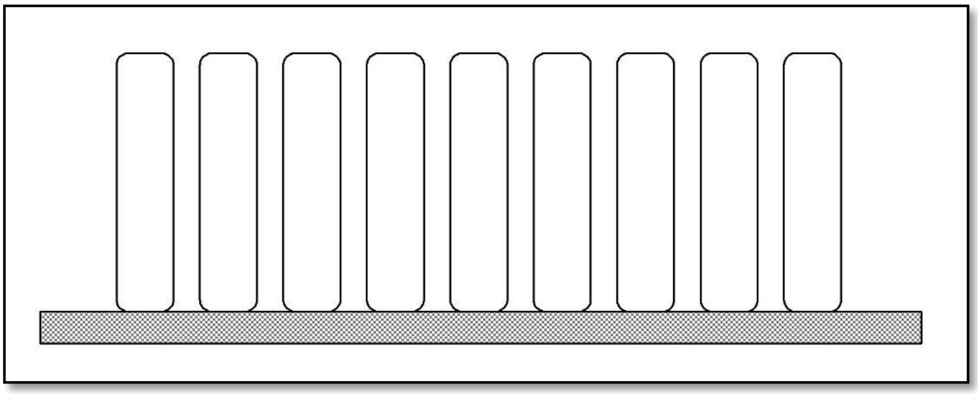

Figure 11. Helical winding

5.2.4.3 Cross over winding

Is also called pancake winding. Commonly used for high voltage windings of low rating transformers where current in conductor does not exceed around 25A. Typical I its use in sandwich coils.

"The whole winding is divided into a number of coils depending upon the voltage rating. Each coil is wound over former usual U pieces, with several numbers of layers and many turns per layer. The coil ends, one from inside and one from outside are joined to other similar coils in series. The various coils are separated by blocks of insulating material to allow of free oil circulation." [16]

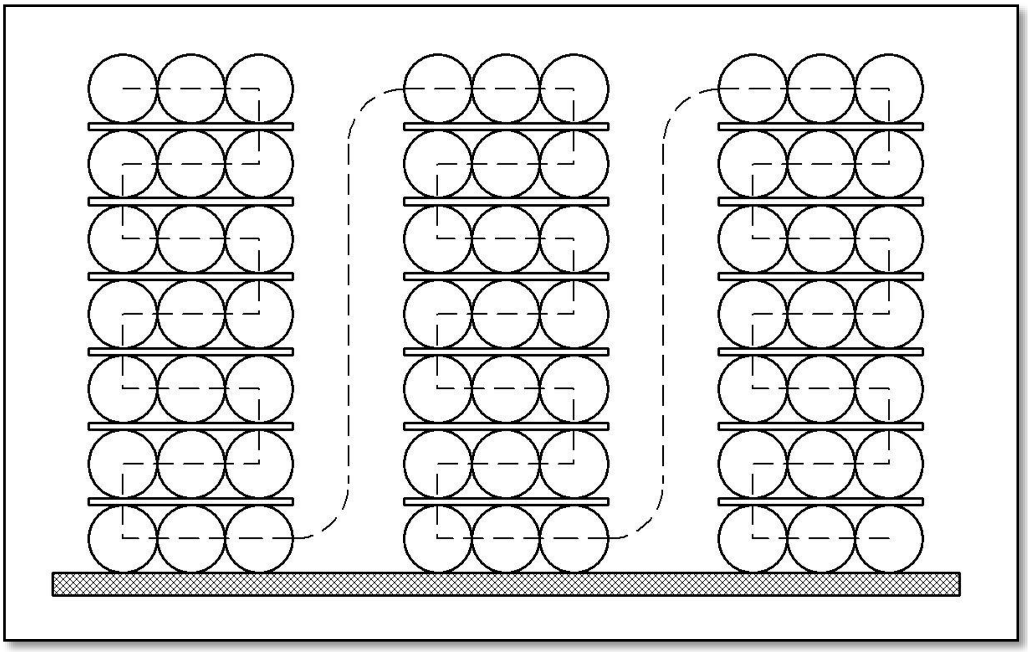

Figure 12. Cross over winding

5.2.4.4 Continuous disc winding

This type is very often in core-form transformers above 25 kV. Winding consist of separated discs similar to previous one. Each disc is created from one or more straps oriented in width of insulated conductor and stacked in layers of one round to a height. Discs are separated by spacers a connection between discs is alternating between inside and outside. Creating of this type winding is complicated because of that exact alternating connection.

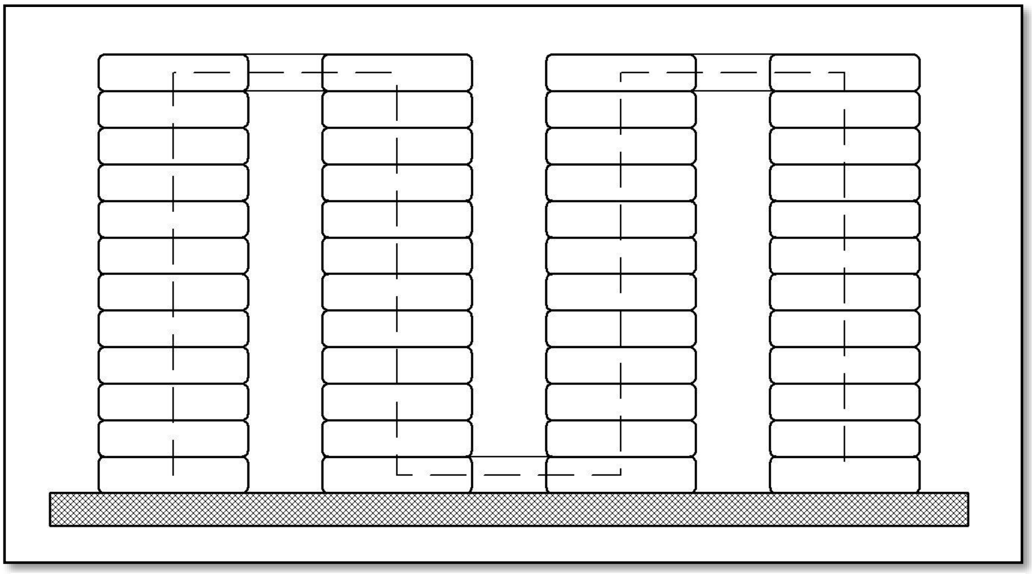

Figure 13. Continuous disc winding

5.3 Isolation systems

There are two main categories of isolation systems in transformers. Oil isolation and dry isolation. When we are talking about distribution transformers oil based isolation systems are used in most cases but dry isolation is also a possibility here. On the other hand in cases of power transformers there can be used only oil ones because dry isolation has not high enough electrical strength potential like the oil. Maximum voltage, for which are dry transformers manufactured is 35 kV.

5.3.1 Oil transformer construction

Transformer is inserted into tank which also forms container and it is filled by transformer oil, which serves like as an isolator as well as cooling medium. Oils used with these machines should chiefly have high enough electrical strength and low viscosity for their facility to penetrate whole machine and all pores. Essential here is dependence of viscosity on temperature. But there are also several next important parameters like thermal stability, oxidizing properties, combustibility or perhaps biodegradability. By source we can categorize transformer oils into three species:

- Mineral oils
    - Mixture of different hydrocarbons.
    - Obtained from petroleum refining.
    - The most widely used because it is most economical.
    - Faster aging antioxidant addition is recommended.

- Synthetic oils
    - Precisely defined composition.
    - Based on silicone fluids, polybutenes, organic esters, fluorocarbons or chlorinated hydrocarbons.
    - Excellent physical and chemical qualities.
    - Great oxidation properties, incombustibility and thermal stability.
    - Expensive.
- Vegetable oils
    - High biodegradability and nontoxicity.
    - Best mixtures have parameters comparable with other kinds of oils.
    - Important is balance between saturated and unsaturated fatty acids.
    - Lowest coefficient of thermal expansion

The oil is used very often in combination with electrical cellulose paper. This paper soaked by transformer oil creates a great electrical barrier and electrical strength of this combination is much higher than just sum of their values. Paper can be wrapped around conductors in windings and inserted between the layers of wires in coils. Inasmuch paper is not ideally homogeneous several layers are frequently used. If we use winding from thin and round wires is for isolation of turns on coil from each other usually used varnish which is applied on conductor already in production. It is enough because this electrical barrier must endure only different potentials in between near turns and it is not so high because voltage in coil is rising gradually. In this case the paper is used just for separation of places with higher voltage different typically between individual layers of coils. For this application is used paper in form of wider band. The paper is also used for isolation in foil windings. When we produce winding from thicker, compound conductor the paper is wrapped around it directly. The paper used here is in form of tape and it is wrapped around conductor piece by piece so as at each point were minimal prescribed number of layers. This process is difficult and each mistake can cause a crash or potential failure.

Oil is liquid so by its very nature it required to be in container. It is made from metal plate and transformer core with winding is inserted and strongly mounted inside. This container is sealed by the lid, from which bushings protrude. Problem with oil is its thermal expansion. Flexible walls of container are popular solution for small distribution transformers. This is so called hermetic design. Unfortunately, it is not possible to use this type of container for bigger transformers, so special external expansion reservoir is required for them. [17]

5.3.2 Dry transformer construction

Isolation material in dry transformers is epoxide resin. It is not so often used variant and highest possible voltage for this kind of transformer is 35 kV, so it is usable only for distribution transformers. Resin have isolation and supporting function. Wounded and prepared windings are inserted into form and suffused by epoxy. Windings are normally from round wires or foil and they already have thin isolation layer of varnish. The process of sufficing is necessary to do in vacuum to avoid air bubbles in finished winding. Form is commonly simple smooth cylinder, but is also possible to find windings with form of two concentric cylinders with gap between them, where in each cylinder is one winding. This version is smaller, because normal size of windings is bigger than necessary by electrical parameters. It is because of bad cooling.

Windings of these transformers were for long time made exclusively from aluminum. It is because of relatively low mechanical strength of epoxy. Resin itself have similar coefficient of thermal expansion like aluminum and in this combination the strength is satisfactory. In combination with copper conductor, mechanical stresses coming with changes of temperature were too high it tear the resin. But nowadays is here possibility to improve mechanical strength of epoxy by fiberglass and with it is possible to use also copper windings. It is not ideal, but the resin is now strong enough to hold out these stresses.

In contrast with oil transformers, dry ones do not need any container. They are often placed inside buildings, out of weather effects, and there is not required any cover to protect them. But it is not problem to add some, usually in form of tin box, which is possible to buy as accessory, if the placement of transformer require it.

5.3.3 Comparison

Both types of transformers have same advantages against other one. I compare them in selected criteria. It is obvious I can compare only comparable transformers, so comparisons are valid only for same powers, types of core, quality, etc.

5.3.3.1 *Advantages of oil transformers*

- Purchase price
  - It is depend many factors, but oil transformers are in generous cheaper than dry ones. Roughly 20 to 30 %.
- Operational costs
  - From the viewpoint of losses, oil transformers are better, because have lower losses. And saved energy mean saved money.
- Weather resistant
  - Oil transformers are normally placed outside, because they well tolerate weather conditions and usually do not need any special protection. On the other side, dry transformer for outside use require special covering boxes.
  - Dry transformers are quite limited by operational temperature of environment. They commonly cannot work in temperatures under -5°C. Oil types endure much more.
- Noisiness
  - Oil transformers are significantly quitter than dry ones.
- Size
  - Due to a worse cooling, dry transformers need bigger windings, so whole construction is little bigger.
- Service life
  - Oil transformers have usually higher durability and lifetime.

5.3.3.2 *Advantages of dry transformers*

- Safety
  - Dry transformers are difficult to ignite and usually self-extinguishing. Fires of oil transformers are not rare as they reach high temperatures and release hazardous substances. The risk of fire or its dangerousness can be reduced by using of special oils.
  - They have minimal ecological hazard. There are no problems with leakage of oil and they do not need special catch pit for like the oil transformers. Also their ecological destruction is easier.
  - Special construction works like fire walls or catch pits are not required.
  - Dry transformers are more resistant against vibrations and some chemicals.

    - Due to these facts dry transformers are predestined for placement in risky areas like buildings underground, natural parks, ships, etc.
- Maintenance
    - Oil machines require much more maintenance. While dry transformers are nearly maintenance-free, oil ones (except hermetic type) needs more upkeep.
- Electrical properties
    - Thanks to bigger windings current density in dry transformers is lower. And that means possibility of higher short-term overload.
    - This ability is supported by high thermal capacity of epoxy resin.

Because of their price and lower losses oil transformers are used much more often. Actually they are practically everywhere where they can be. Exceptions are cases where oil transformer cannot be installed. There may be more reasons. Probably the most common one is fire safety, for which oil transformers cannot be placed in buildings, underground, mines and some other places of industrial operation. Next one can be the impossibility to construct safety accessories for oil transformers like fire walls or catch pit nearby. [15] [18]

5.4 Cooling

We must again split this topic into two categories, because cooling systems of transformers depend on their isolation system. So I am going to talk about dry and oil transformers separately. What is common for both is that they need to somehow transfer and release waste heat from inside into air.

5.4.1 Dry transformers

I am going to start with dry transformers because their case is simpler. They transfer heat into environment directly because of the fact that their windings encapsulated in epoxy resin are cooled directly by air. Dry transformers mostly rely only on passive cooling. That means they are cooled only by free flowing of heated air. Because of it windings of these transformers are often bigger than they need to be in order to provide bigger surface and simplify heat transfers. Naturally it is possible to add fans for forced blowing. This greatly improves heat dissipation and it allows increased performance of transformer. This type of cooling is used in cases when we need to improve transferred power by long time overloading. Increase can be up to 40 %. However this distinctly increases losses.

Primary and secondary windings are separated and they a have gap in between which increase surface and allow air flow. With the same exact principle axial holes or gaps directly in windings during the process of potting by epoxy resin can be created. It is more technologically difficult but it very helps with cooling.

5.4.2 Oil transformers

Oil serves as medium for heat transfer. By natural thermal flowing it transfers heat from core and windings to walls of container where the heat is dispersed into surroundings. Hot oil expands gains lower density and rises around walls then it cools down and flows down. However containers have cuboid shape with small surface which has not sufficient dispersive abilities. For smaller transformers it can be enough but with rising power there must still be better cooling systems.

This is done by radiators placed on sides of transformers and connected with them via oil pipes. This system may be further enhanced by fans and oil pumps. Fans same like in dry transformers create forced blowing. Pumps support natural flowing and increase oil circulation in transformer by pushing oil into radiators. Oil flows through radiators also without pump, but for big transformers pumps are necessary.

For improvements of inner heat transfer are used systems of channels and vents which are placed lengthwise and radially across whole windings. Gaps are also between individual windings and core. These allow oil to flow through whole machine and carry heat also from the innermost zones.

6 Transformer diagnostics

In this part I describe several most common methods for analysis of transformers. Regular checks of their condition are necessary for ensuring of long lifespan. These methods are focused on various aspects of transformers. Insulation, winding, magnetic core, oil and other aspects are monitored. Some methods examine just one aspect some are even more complex.

All distribution and power transformers are checked periodically usually once or twice a year. Also after every potentially damaging situation, for example lightning hit. Shutdown of transformers and especially big ones can be quite an expensive affair and every minute counts. So methods which need just a short shutdown or do not need it at all are the most valuable.

6.1 Sweep frequency response analysis and low voltage impulse test

Due to a lot of inductances and parasitic capacitances transformers are complex devices with complicated frequency responses. Especially those parasitic capacitances are presented in between of phases, grounding and every turns of wire. And this is creating many resonances. Every transformer is little different – windings are not identical, random deviations on magnetic core, various air gaps and so on. Because of it also the frequency response is unique for every transformer. But it is enough to little move just a small piece of device and the frequency response will be defferent because some parameters will be changed and they make new different resonances. And this effect is great for detection of mechanical damage typically by short-circuit fault, caused by lightning or transportation.

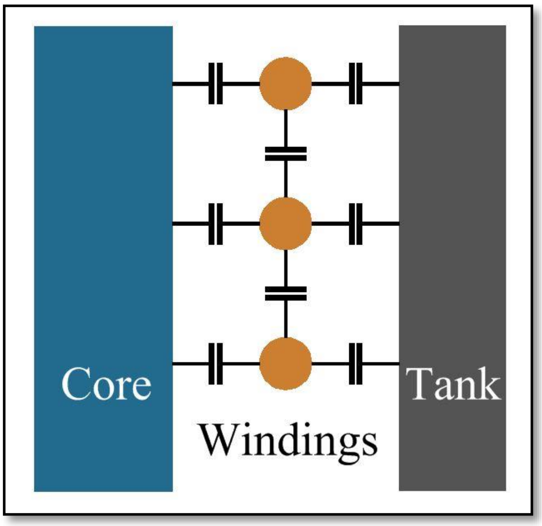

Figure 14. Parasitic capacitances in transformer

Unfortunately, we must have reference measurement undertaken before potential damage for comparison. But if we have it this method has practically 100% efficiency to confirm or exclude any change in connection or position of parts. It is possible to find out axial or vertical displacement of winding, its deformation, damaged magnetic core or its bad grounding etc. If the FRA test cannot be performed in the factory, they should be conducted as an acceptance test before energizing to establish a baseline. It is good to use the same test equipment or the results may not be comparable.

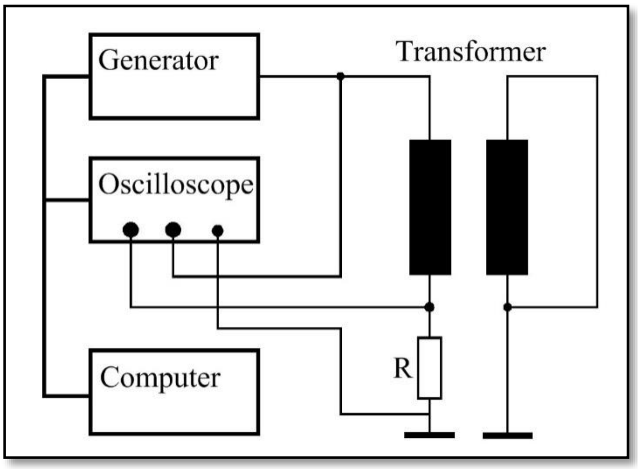

Figure 15. Basic measurement circuit FRA

There are two ways of injecting the wide range of necessary frequencies. We can use low voltage impulse test or make frequency sweep by using sinusoidal signal. Sweep frequency response method is better in accuracy and precision and it is more often used but it has its disadvantage in longer measurement time (but still less than 15 minutes).

The transformer must be disconnected and set to base value on tap changer. In the Figure 15 the basic measurement circuit is shown. This is only for recording of amplitude characteristics. It is also possible to measure a phase changing but this method is quite rare nowadays.

It is necessary to measure all phases one by one. Used frequency range is at a rough guess 10 Hz to 10 MHz Connection to transformer is formed by coaxial cables. Same resistance of those is required for all measurements. For the best results it is good to take into account also the effect of temperature, magnetizing of core, aging of oil and interference from extraneous electromagnetic flux.

Output from measure is a graph where dependence of signal response (amplitude) on frequency is shown. It is a draw with logarithmic axis by standard. Example of it may be seen in Figure 16 below. Low frequencies (<1 kHz) are specific for resonance of magnetizing inductance and shunt capacitance of the winding. At medium frequencies there are influences of air-cored capacitances of the winding. In the highest frequencies is added effect of serial capacitances. In this part is impact of used cables more significant.

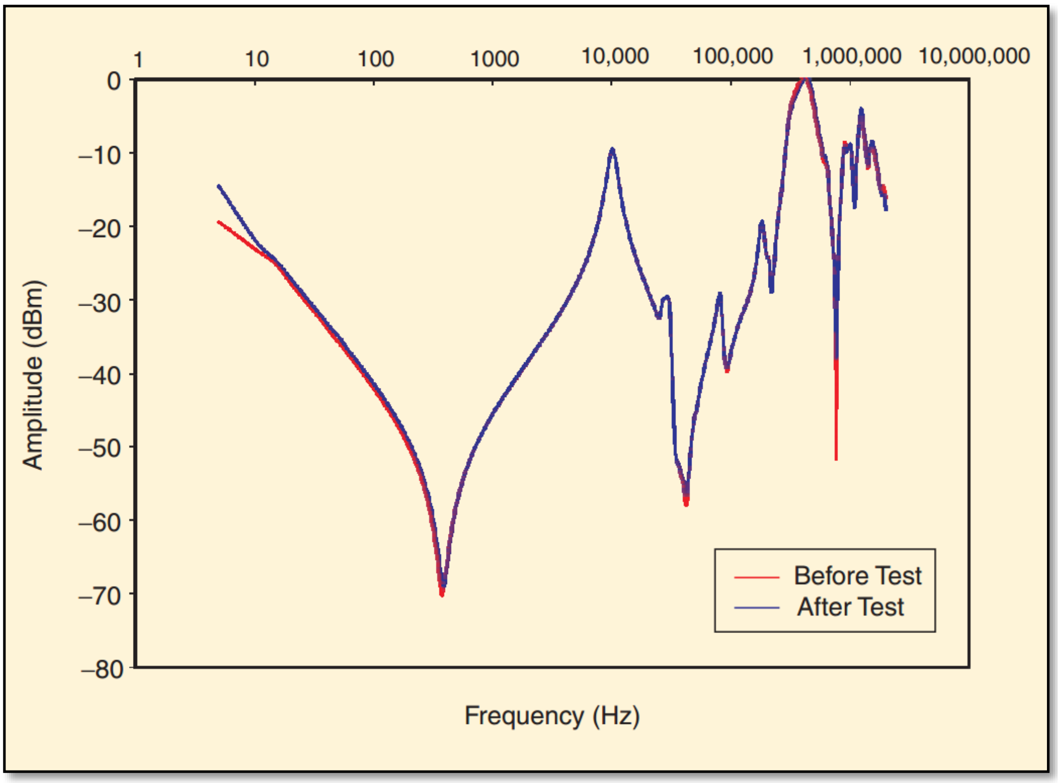

Figure 16. Illustration of typical result FRA [RYDER, Simon A. Diagnosing transformer faults using frequency response analysis. Electrical Insulation Magazine, IEEE, 2003, 19.2: 18.]

Measuring with low voltage pulses is similar. Outgoing signal after pulse applying is recorded. Voltage of impulse is 300 V. Fast Fourier transformation (FFT) is used on records and output is again graph of waveforms. [1] [2] [3]

6.2 Measuring of isolation resistance and dielectric absorption test

In this test is nondestructively measured electrical resistance of isolation system by direct current. It is also known as Megger. The higher resistivity means the better condition of isolation. Ideally it should be infinite but because of leakage current it is not. Recommended measurement voltages are 1, 2.5 and 5 kV for high voltage equipment but it should be always less than nominal voltage of the specific transformer. Suitable ohmmeter is connected on, united primary winding present one electrode, secondary windings is second electrode and grounded transformer housing is the third one. Due to dependence of resistivity on voltage we cannot increase the voltage arbitrarily. With rising voltage the resistivity is decreasing. And if we would continue we could possibly break through the isolation. These phenomena can be used for approximation of breakdown voltage in some cases. Another important variable is time which has strong influence because of transitions by inner capacitance of the transformer. From that reason is good to measure for longer time. After five minutes the inaccuracy is usually negligible and practically only wanted leakage current is flowing through. Also because of resistivity dependence on temperature same temperature at all times is required.

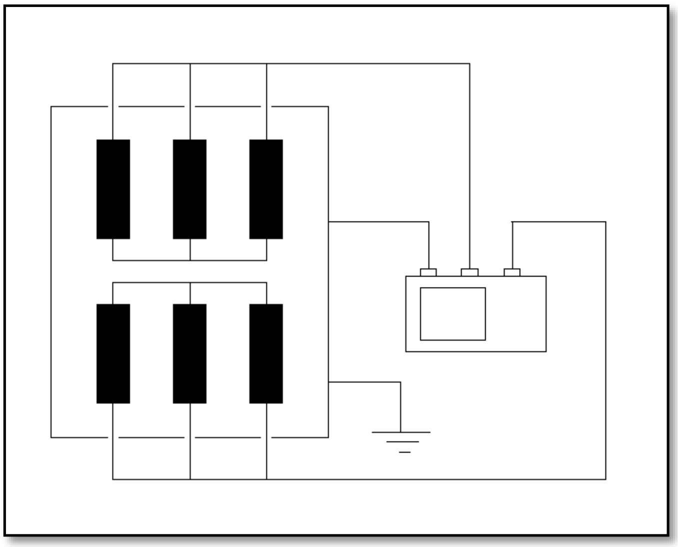

Figure 17.Connection for isolation measurement

We can directly measure sectional resistivity between our three electrodes or there is another possibility – indirect measurement. In the first case there are some disadvantages:

- We need an ohmmeter with shielding terminal.
- In case of good conditions of isolation our values usually have big measurement errors.
- There is higher chance on measurement error due to residual charge.

For reducing or removing these, it is better to use indirect measure. We have three electrodes in both cases but in direct method we measure resistivity between each other separately while with the second method we measure resistivity of every one against the other two united together. From these three values we calculate resistivity with higher precision by simple relations.

This method is used like simple and cheap way for monitoring of isolation condition. It means irreversible changes consists of conductive ways (damaged or carbonized segments of isolation, breakdowns, metal particles) and overall increase of conductivity (wetting, chemical changes, aging and especially for oil also sludge and debris). Unfortunately this test is sometimes distorting. For example this measurement may not detect disruption on isolation paper in oil because voltage is not high enough for creating new breakdown in oil. Or settled sludge of oil may not be threatening the transformer. So isolation resistance decrease indicates its deterioration but it must not be always critical and on the other hand good results are not guarantee of quality. Critical values for this test are prescribed by standards.

Except of resistivity with this measuring we can also measure dielectric absorption ratio (DAR) and polarization index (PI) which are other status indicators of isolation quality, mainly oil. Their dimensionless values are based on change of resistivity during measuring by formulas:

$$DAR = \frac{R_{60}}{R_{30}} \tag{5}$$

$$PI = \frac{R_{600}}{R_{60}} \tag{6}$$

R_{30}, R_{60} and R_{600} are values of resistivity after 30, 60 respectively 600 seconds after beginning of measure. Chosen time does not need to be the same everywhere. Dielectric absorption current is rising with growing amount of ion pollution in isolation system which is related with its aging. Characteristic time curve of this current is flatter and flatter with aging and also DAR and PI decrease. Typical values are slightly different for different machines but in general DAR value should not be lower than 1.2. Values higher than 1.6 are typical for isolation systems in good condition. Rules for interpreting PI results are following: lower than 2 are risky, good are around 3 and more. [4] [5]

6.3 Dissipation factor tan δ and capacitance

It is one from the oldest nondestructive diagnostic methods but voltage dependence of capacitance and dissipation factor is still useful mainly for long time aging monitoring of electrical machines isolation systems. It is difficult conclusively evaluate results from this method and usually is necessary take into account also another measures. It’s also known as dielectric loss angle (DLA) test.

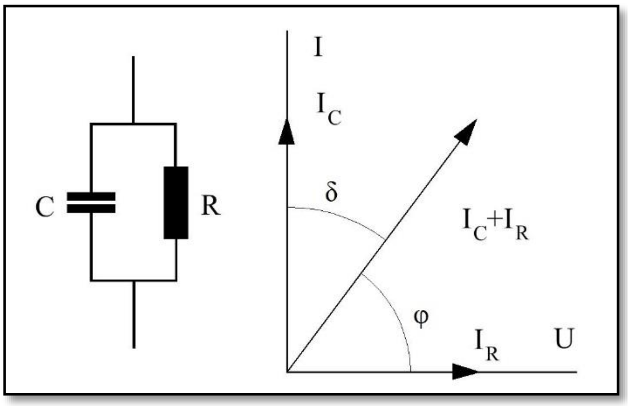

Figure 18. Relationship between capacitance and loss factor

Aging of isolation means losing of its resistivity and with it growing loss angle δ and dissipation factor tan δ. For measuring we use Schering’s bridge. Most often we work with industrial frequency of 50 or 60 Hz. Voltage should be between 25% and 60% of nominal voltage. Maximal test voltage of 10 kV is allowed in case of nominal power higher than 20 kV. For elimination of interference from environment, it is recommended to conduct measurement twice with different polarity. [6] [7]

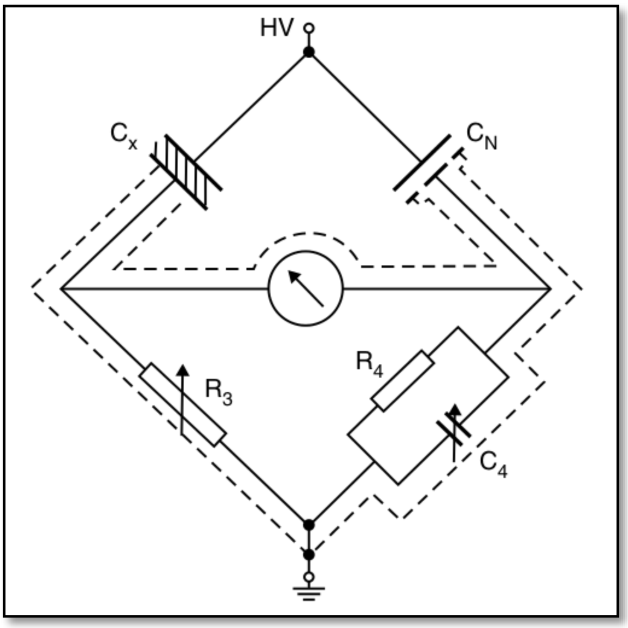

Figure 19. Schering bridge

“As mentioned earlier, the dissipation factor can be measured directly using a Schering capacitance bridge circuit (named after Harald Schering). The basic circuit is shown in the figure right where Cx is the unknown capacitance under test. Like other bridge circuits (such as the Wheatstone bridge), the circuit is tuned until the current through the middle of the bridge is zero and the circuit is balanced. For a capacitance Cn with low losses the unknown capacitance and dissipation factor can be calculated as follows:” [7]

$$C_x = C_N \frac{R_4}{R_3} \quad (7)$$

$$\tan\delta = \omega C_4 R_4 \quad (8)$$

6.3.1 Capacitance ratio C_2/C_{50}

Capacity of isolation containing water is dependent on frequency while in case of dry one the capacity is nearly constant. These phenomena are used for this diagnostic of testing the presence of water. Measurement of isolation capacity is same like in previous test but now it is necessary measure capacity on frequencies 2 and 50 Hz (or similar) with low working voltage 100 V. Critical value of ratio has dependence on temperature of winding, so standards contain values for different temperatures. [4]

6.4 Partial discharge test

Used when we have some local anomaly in isolation with lowered isolation ability. Typical problem for transformer are air cavities. This weak spot may cause partial breakdown if the voltage between conductors is high enough. In the Figure 20 you can see this graphically interpreted. Isolation is disturbed by air cavity. Isolation around cavity is still strong enough for safe work but on sides of cavity the charge is accumulating and when this potential is higher than breakdown voltage of this cavity the partial discharge is created inside. It works like spark gap. This small electric arc is not dangerous at beginning but every time it arises the arc damages isolation around a little. Duration of discharge is very short in order of nanoseconds and whole process is going to be repeated many times during every amplitude. It may be in hundreds or thousands cycles every second. Because of high frequency of discharge it may be interfering

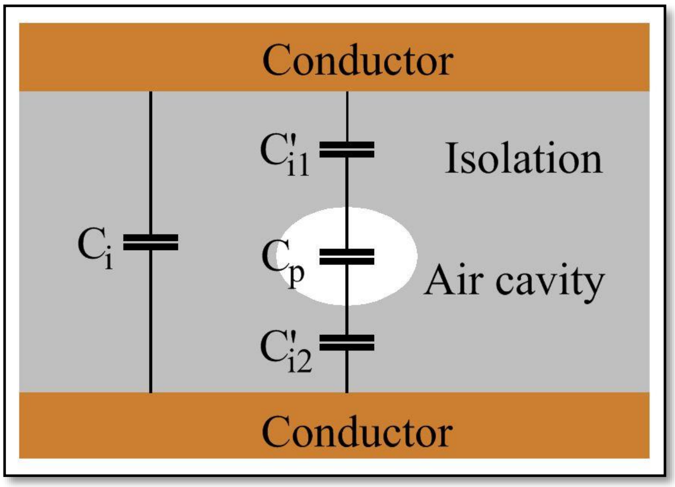

Figure 20. Illustration of partial discharge

telecommunication. With more damaged isolator the process is accelerated and it may end with total breakdown.

This test can be done by more ways but these most popular are based on observation of current impulses in power system. These impulses are generated right by partial discharges. [8]

6.5 Transformer oil testing

This is little specific part of diagnostic because it concerns only oil transformers. But as they constitute the majority so this is also very important. Characteristic attribute of these diagnostic methods is in their impracticability on the spot. They need laboratory equipment and that means that results will be ready after some time. For analyzing purposes we must take samples of oil. Oil is taken from bottom of the machine through sampling valve.

6.6 Dissolved gas analysis (DGA)

DGA is direct testing of transformer oil samples. Every technical problem in transformer including arcing, overheating and partial discharge causes local or total overheat of insulation. It has response in form of gases dissolved in oil. Electrical and thermal stresses cause decomposition of isolation.

Whatever the cause, these stresses will result in the chemical breakdown of some of the oil or cellulose molecules constituting the dielectric insulation. The main degradation products are gases, which entirely or partially dissolve in the oil where they are easily detected at the ppm level by DGA analysis. [9]

It is possible to install special sensors inside of transformer for continuous monitoring of dissolved gasses but for precious analysis we always need a sample of oil. In laboratory the sample is analyzed in gas chromatograph. Typical gases from faults are H_2, CH_4, C_2H_2, C_2H_6, CO and CO_2. Their relative concentrations show us which type of fault did happen because each specific gas needs specific temperature in order to arise. Interpretation of results may be slightly different in dependence on applied standards for evaluation. [9]

6.7 Furanic compounds in oil

Cellulose paper used in isolation system is composed from natural polymers. During time fibbers of cellulose are degrading and the polymer chains are breaking down. This is irreversible process and can be accelerated by improper handling of transformer. Direct testing of paper durability is very complicated expensive and requires de-energization of the equipment. But products of degradation are released into oil and by its examination it is possible to determine condition of cellulose paper what is important for prediction of isolation durability. Most common product is 2-furaldehyde but several similar derivates based on furan are also tested.

It is important to know what kind of cellulose paper is used because it may contain cellulose from different sources which has different evolution. Commonly used are cotton-based and wood-based. For expressing of polymer length is used degree of polymerization (DP). It is average number of glucose monomers in polymer chain. DP of new cellulose isolation paper before installation is normally 1200 to 1400. For new installation it is 800 to 1200. Degree of polymerization of life ending isolation is around 200. Below this number the quality of isolation is so poor it has not enough capability for normal working in a transformer. All mechanical and electrical stresses destroy rests of it in short time.

From the laboratory test results of oil sample we are able to recognize causes of prospective degradation. Typical causes are described in Table 1. [10] [11]

Table 1. Causes of specific furanic compounds

Compound	Observed cause
2FAL 2-furaldehyde	General overheating or ageing
5M2F 5-methyl-2-furaldehyde	High temperatures
5H2F 5-hydroxymethyl-2-furaldehyde	Oxidation
2ACF 2-acetyl furan	Rare, causes not fully defined
2FOL 2-furfuryl alcohol	High moisture

7 Economical operation of transformer

This chapter is about economy of transformer. Like in every economical study we want reduce total costs on possible minimum. Transformers are usually necessary or forced investment so I do not count with alternatives. Simply we need some transformers and our target is to select the best one for our desired purposes.

The main operation costs of transformers are energy loses. I describe them and I also explain their dependence on transferred power and how to count them. These are not the only costs; there are also another costs involved for example costs on maintenance. But these are negligible because they are relatively small and also similar for every considered variant. From this is evident that we want reduce losses on minimum. Obvious way is to take transformer with lowest losses. But these are usually also the most expensive ones and total costs are composed of operation and acquisition costs. We want find the fittest compromise.

Another possible way is to make project with two or more transformers working in parallel connection. In some specific cases it can distinctly reduce losses, but on the other hand they have higher investment costs.

I also pursue valuation of these losses. Lost energy can be calculated quite simply, but now we have problem with its price. And for economical comparison it is needed. Unfortunately for us real price of electrical energy is not stable and it changes steadily so there I am forced to use some simplification.

7.1 Investment costs

Transformers are not cheap machines and their purchase price is important. As it happens superior ones with lower losses are more expensive. Except the price of transformer itself there are also costs on transportation, eventual construction work and support devices as protections or disconnectors. These accompanying costs can be substantial especially in case of more transformers working in parallel connection.

7.2 Operating costs

Operating cost has generally two parts – fixed and variable. As the names imply fixed cost are still constant and variable costs are changing in dependency on usage. Variable costs are more important here. They are based on lost electrical energy in transformer, respectively on price and amount of these losses.

$$C_o = C_{of} + C_{ov} \tag{9}$$

C_o	*[CZK]*	Operating costs per year
C_{of}	*[CZK]*	Fixed operating costs per year
C_{ov}	*[CZK]*	Variable operating costs per year

7.2.1 Energy losses

As I explained in one of previous parts, there are several different kinds of losses in transformer. But usually there are usher only two numbers which describe whole losses. Load losses and no-load losses. Eddy currents in magnetic core are the biggest portion of no-load losses. These losses are practically constant and they are generated all the time when transformer is connected into electric network. Main portion of load power dissipation is energy loss in windings caused by electrical resistance. Since the electrical power depends on square of current by formula (10) also the load loss depends on square of transferred power. Value of load losses is measured at nominal power of transformer.

$$P = R \cdot I^2 \tag{10}$$

Then I can describe total power dissipation like sum of these partial losses and I get this formula:

$$P_d = P_0 + P_{ln} \cdot \left(\frac{S}{S_n}\right)^2 \tag{11}$$

P_d	*[kW]*	Total power dissipation
P_0	*[kW]*	Power dissipation by no-load losses
P_{ln}	*[kW]*	Power dissipation by load losses at nominal power
S	*[kVA]*	Current apparent power of transformer
S_n	*[kVA]*	Nominal apparent power of transformer

You can see graphical interpretation of this dependence in Figure 21.

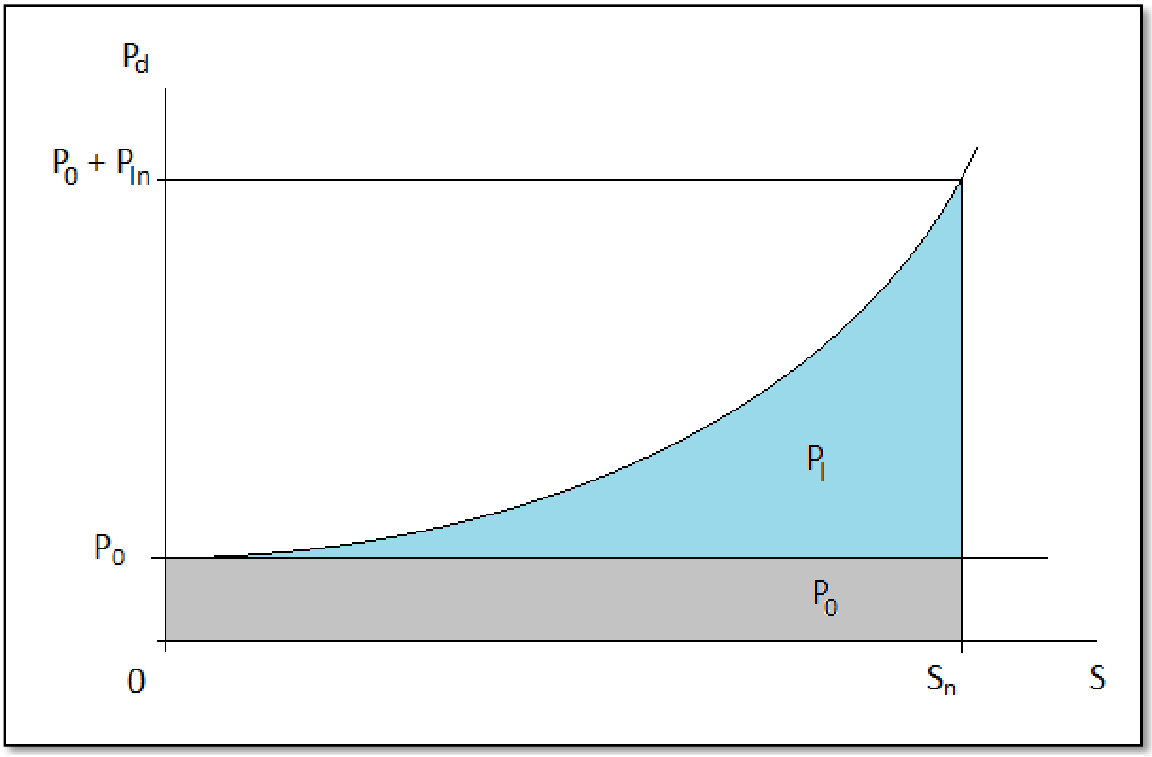

Figure 21. Dependence of losses on load

7.2.2 Load diagram and loss equivalent hours

Actual load of transformer is usually unstable with lot of changes. Progression of load is normally typified in load diagram which is time record or assumption of load during some time. That time is usually one day because most of our activities are happening repeatedly every day and also the load diagram is nearly same every day.

Formula (11) for total dissipation power is non-linear and with changing load it is not handy for calculation of lost energy. Because of it we use loss equivalent hours. This parameter transfers whole diagram into one number I desire. It is defined by formula (12).

"Loss equivalent hours are the number of hours of peak load which will produce the same total losses as is produced by the actual loads over a specified period of time." [20]

$$T_{leh} = \frac{1}{S_{max}^2} \cdot \int_0^T S(t)^2 dt \qquad (12)$$

T_{leh}	*[h]*	Loss equivalent hours
S_{max}	*[kVA]*	Maximal load
S	*[kVA]*	Actual load

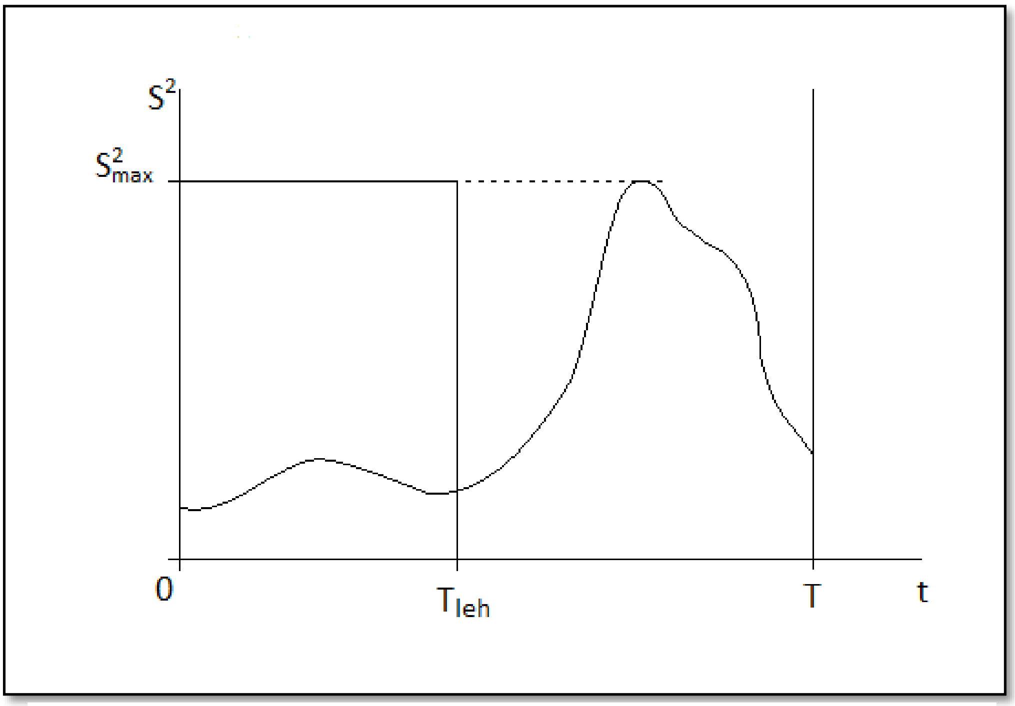

Figure 22. Graphical interpretation of LEH

7.2.3 Total energy loss

We already know how to get LEH so now I will demonstrate you how to use it in calculation of total energy loss. I will multiply LEH with nominal short circuit losses and square ratio of maximal load to nominal one. It represents all short circuit energy losses. To this number I must also add the open circuit losses multiplied by time of connection to power grid which mean open circuit losses. Mathematical interpretation:

$$W_d = P_0 \cdot T_c + P_{ln} \cdot \left(\frac{S_{max}}{S_n}\right)^2 \cdot T_{leh} \tag{13}$$

W_d	*[kWh]*	Total dissipated energy
T_c	*[h]*	Time of connection

7.2.4 Parallel operation

Possible way for decreasing lost energy may be the use of parallel connection. Two or more similar transformers connected together have several advantages. First is economical – lower losses transcript into saving money. The second one is practical. When we have more transformers usually it is not problem to turn off one of them for maintenance or it can serve as a backup.

Parallel connection need to meet several conditions. Here they are:

- Same voltage ratio of transformers
- Same phase displacement
- Same phase sequence
- Very similar percentage impedance
  - Maximal recommended deviation is 10%
- Similar nominal load
  - Ratio between them should be lower than 3.2

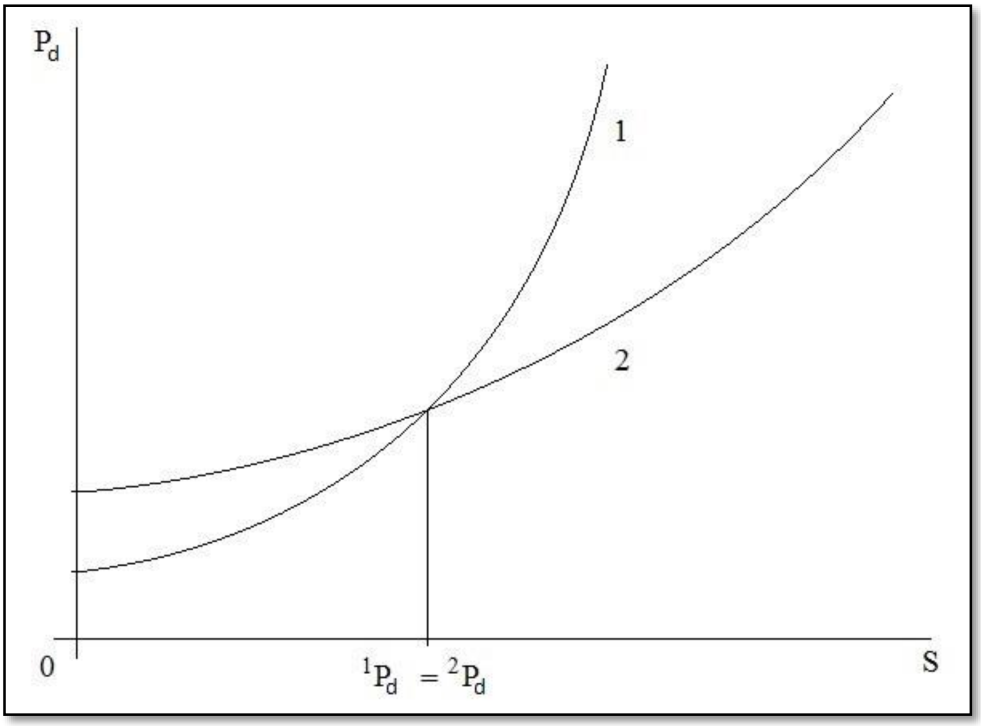

Figure 23. Dependence of losses on load for more transformers

For better understanding how more transformers can save energy I created Figure 23 where you can see comparison between loss curves. Curve number one refers to loss dependence of single transformer. Number two is for pair of same transformers like in case one. Note that initial open circuit losses are twice so big for pair of transformers. On the other hand maximal or nominal load is also twice so high. Written in equations, formula for dissipated power will be changed to following:

$$^{n}P_d = n \cdot P_0 + \frac{1}{n} P_{ln} \cdot \left(\frac{S}{S_n}\right)^2 \quad (13)$$

n	*[-]*	Number of transformers

With low load is obviously better to use only one transformer because every transformer has open circle losses when it is connected to power supply. So when I connect more of them losses will increase. With raising load the losses increase also but much more in case of one transformer. It is because transferred power is divided between more transformers (equally for case with same transformers) and losses rise with square of load.

The point where the lost power is same for both curves (${}^{1}P_l$ = ${}^{2}P_l$) is so called transient power or transient point. When the transferred power is under this point it is better to turn off one of devices and conversely turn on both of them when transferred power is above the value of transient power. However usage of high voltage switch is not free of charge respectively there always are some marks of usage after switching. Because of it there is usually some interval around transient power for triggering of switch to avoid unnecessary damage. It may be caused by often switching when transferred power is fluctuating around transient point. I neglected this fact in my calculations.

When I want to know exact value of transition power I start from the equality of losses for *n* and *n-1* transformers.

$$(n-1)\cdot P_0 + \frac{1}{n-1}P_{ln}\cdot\left(\frac{S_{tr}}{S_n}\right)^2 = n\cdot P_0 + \frac{1}{n}P_{ln}\cdot\left(\frac{S_{tr}}{S_n}\right)^2 \qquad (14)$$

S_{tr} *[kVA]* Transient power

And after solution we got:

$$S_{tr} = S_n\sqrt{(n-1)n\frac{P_0}{P_{ln}}} \qquad (15)$$

With this formula, we are able to determine how many transformers should be connected at a certain load. For finding of total dissipated energy in some time we must split the time on intervals depending on how many machines are currently running. That means calculated separately losses during time for $1, 2, \dots n$ transformers because they all have different formulas. For each number of machines we get some dissipated energy and total lost is obviously the sum of them. Final form is in formula (16). Different times of connection T_c and loss equivalent hours T_{leh} are identified by index.

$$W_d = \left[P_0 \cdot T_{c1} + P_{ln} \cdot \left(\frac{S_{max1}}{S_n}\right)^2 \cdot T_{leh1}\right] + \left[2 \cdot P_0 \cdot T_{c2} + \frac{P_{ln}}{2} \cdot \left(\frac{S_{max2}}{S_n}\right)^2 \cdot T_{leh2}\right] + \cdots + \left[n \cdot P_0 \cdot T_{c,n} + \frac{P_{ln}}{n} \cdot \left(\frac{S_{maxn}}{S_n}\right)^2 \cdot T_{leh,n}\right] \quad (16)$$

7.2.5 Value of lost energy

Price of electricity is unstable and its actual value depends on actual matching supply with demand and closed contracts. The same applies to the price of lost electricity. To avoid problems with unpredictable price of losses I use long time average price of lost electricity for what it is buying by distribution companies. There will be some deviations in results but it is still the only legitimate way how to get some results.

This price is applied on losses in entrance point on the 110 kV voltage level because that is the point where distribution companies buy it. However losses on distributors' transformers are on the second side of the power line or on lower voltage level. And during transportation of energy for transformers' losses are generated another losses on power line or higher level transformer. Because of this it is necessary to increase price of lost electricity multiplying it by coefficient of voltage level.

7.2.6 Fixed operating costs

In case of transformer fixed costs are usually only costs on maintenance, diagnostics, wages or repairs. Biggest part of these costs is intended to breakage of machine. Breakage of transformer represents big problem. In that case is necessary to repair or replace it, to pay transportation, workers, fines for non-supply and so on. It can be reduced by readiness of another transformer in reserve nearby, but most of expenses remain.

Appropriate diagnostic method can reveal oncoming end of service life but is unpredictable in the long term. I decided to use method of average fixed operating costs. It is suitable especially for distribution companies which operate many transformers. We do not know which machine will break down but when we have many of them we know the year average. Based on it we can determine average year cost on one machine which will be expressed in percent of purchase price. It is usually around 2 % and I will call it quotient of fixed costs and I will refer to this as letter β.

7.3 Net present value

The effective value of money is changing during time. For comparison of costs during lifetime of transformer we must adjust the costs for each year by cumulated discount rate for that year and transform that price onto presence level. For that discount is ordinarily used WACC of investor. This index can be different every year but difference is normally not that high. I use it as constant because the prognosis is impossible to articulate for me.

Net present value (NPV) is index used for comparison of investment projects or for assessment of their worth. It is applied on cash flows in most cases, because investment project should bring some profit. In my work I do not calculate with any incomes from operation of transformer and I take it only just as forced investment because determination of these incomes is not part of this thesis so I apply it only on costs. NPV transfer nominal cash flows each year (or just costs as in my case) into their effective value and sum them so it gives adjusted value for whole lifespan and add initial investment costs.

$$NPV_C = C_i + NPV_{C_o} = C_i + \sum_{t=1}^{T_{lt}} \frac{C_{o,t}}{(1+i)^t} \tag{17}$$

NPV_C	*[CZK]*	Net present value of costs
C_i	*[CZK]*	Investment costs
NPV_{Co}	*[CZK]*	Net present value of operational costs
T_{lt}	*[years]*	Lifetime
$C_{o,t}$	*[CZK]*	Operational cost at year *t*
i	*[-]*	Discount rate

7.4 Examples of calculations

In this part I show usage of formulas from previous pages on examples based on real problems. I created two of them. First one is simple and shows you the difference in operation of one and two transformers. Second one is complex and its task is to select the best possible choice for purchase of new transformers.

7.4.1 Advantage of parallel operation

Distributor is forced to ensure the reliability of electricity supply otherwise it threatens penalties. To increase the reliability on transformation level in substation is often usage of reserve transformer. It can be connected to parallel, stayed unconnected nearby or be in storage and shared for more substations. Decision depends on importance of costumer's policy of company and economic criteria like costs and size of the fine in case of power failure.

So it often creates situation where we have two same or similar transformers in substation but they were not been intended for parallel operation. And in this example I want to look closer on possibilities of savings in this case. I will try to calculate savings of energy and money during one year created by effective usage of parallel operation of machines compared to initial state when only one transformer is running. I do not calculate with investment costs, changes of prices and so on.

In the beginning I need information and parameters about transformers. I work with two same transformers because it simplifies calculations. Anyway these transformers should by very similar because one is reserve for the second one and they probably were purchased together. So let's presume they are same type.

I decided to select distribution transformers with reduced losses and nominal power 1600 kVA. Details are in Table 2. Nominal power, load and no-load losses are real value from catalogue sheet.

Table 2. Basic information about selected type of transformer

Type	S_n	P_0	P_{ln}	S_{tr}
	kVA	*W*	*W*	*kVA*
CoBk 1600	1600	1700	14000	788,5

Transient power S_{tr} was calculated by formula (15):

$$S_{tr} = S_n \sqrt{(n-1)n\frac{P_0}{P_{ln}}} = 1600 \cdot \sqrt{(2-1) \cdot 2 \cdot \frac{1700}{14000}} = 788{,}5\ kVA \tag{18}$$

Next, I need to know load. In my examples I use year average daily load diagram. Technically it is not absolutely correct and I should look at load development, difference between work and weekend days, difference between winter and summer. But it cannot be ideal anyways it is only about the level of our pedantry. In addition, it makes no sense to worry about it because accurate prediction of consumption will again be based only on conjecture. I think the year average is good enough. I am using hourly values.

Table 3. Daily load diagram

Hour	S [kVA]	Hour	S [kVA]
1.	320	13.	940
2.	290	14.	1100
3.	280	15.	1250
4.	300	16.	1220
5.	425	17.	1175
6.	470	18.	930
7.	600	19.	865
8.	855	20.	745
9.	900	21.	835
10.	870	22.	755
11.	1090	23.	540
12.	950	24.	380

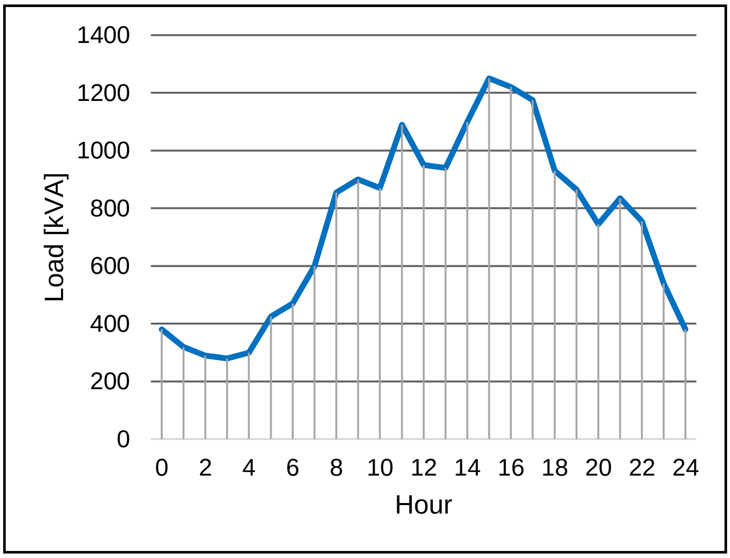

Figure 24. Daily load diagram

7.4.1.1 Operation of one transformer

I want to find costs on operation of one transformer firstly. It is easier part. I set up the time of connection T_c on 364 days (8736 hours) in a year as I suppose there are some small power cuts or maintenance disconnections so the transformer will not work the whole year. With the time of connection and load diagram I am able to calculate loss equivalent hours T_{leh}. I calculate, as if every day was the same diagram, so I can determine T_{leh} just for one day and then multiply it by number of connection days. I use modification of formula (12). I have only hourly measures so technically I have only discrete values and the integral turns into sum here. In the Table 3 I will find out maximal load $S_{max} = 1250\ kVA$.

$$T_{leh} = \frac{T_c[d]}{S_{max}^2} \cdot \int_0^T S(t)^2 dt = \frac{364}{1250^2} \cdot \sum_{i=1}^{24} S_i^2 = 3704\ h \qquad (19)$$

$T_c[d]$ *[days]* Time of connection in days

Now it is simple to find total dissipated energy for whole year W_d. All I need is formula (13).

$$W_d = P_0 \cdot T_c + P_{ln} \cdot \left(\frac{S_{max}}{S_n}\right)^2 \cdot T_{leh} = \\ = 1{,}7 \cdot 8736 + 14 \cdot \left(\frac{1250}{1600}\right)^2 \cdot 3704 = 46\ 505\ kWh \qquad (20)$$

To find the annual cost of losses or variable operational costs I just calculate these losses. Price for electricity losses I used is 1,46 CZK/kWh. We must not forget the coefficient of voltage level which will be 1,017. It is not so much but it can make difference in some cases. Here is the formula:

$$C_{ov} = W_d \cdot P_l \cdot \alpha_v \qquad (21)$$

p_l *[CZK/kWh]* Price of electricity loss

α_v *[-]* Coefficient of voltage level

And the calculation is following:

$$C_{ov} = W_d \cdot P_l \cdot \alpha_v = 46\ 505 \cdot 1{,}46 \cdot 1{,}017 = 69\ 052\ CZK \qquad (22)$$

As you can see, when only one machine is running costs on lost electricity are roughly 69 thousand CZK per one year.

7.4.1.2 Parallel operation of two transformers

Calculation for parallel operation is very similar to previous part. The main difference is in splitting of load diagram into parts or ranges by the number of machines currently working. I already know transient power S_{tr}. I look on daily load diagram and see that there are three ranges with values of power under threshold defined by our transient power which is 788,5 kVA. These are hours 1 to 7; 20 and 22 to 24. During these hours only one machine should run. Two of them should run rest of time. Note that the hours 0 and 24 are identical and they must be counted only once. Their values are same and whole diagram is continuing.

After summation of time when only one machine operates we get 11 hours. It represents time of connection per day. Multiplied it by days of connection it is 4004 hours in year. To get loss equivalent hours we must again use formula (12) but only for selected hours this time. Watch out for that small index *1* signifying values apply to one running transformer, especially $S_{max,1}$ because that means there should be maximal value of power but only from selected ranges. It is highest value under the threshold.

$$T_{leh1} = \frac{T_c[d]}{S_{max1}^2} \cdot \left\{ \sum_{i=1}^{7} S_i^2 + S_{20}^2 + \sum_{i=22}^{24} S_i^2 \right\} = \frac{364}{755^2} \cdot \{\ldots\} = 1710\ h \qquad (23)$$

And I do same procedure for two transformers which means for hours 8 to 19 and 21. That is 13 hours per day and it is 4732 hours of connection in year.

$$T_{leh2} = \frac{T_c[d]}{S_{max2}^2} \cdot \left\{ \sum_{i=8}^{19} S_i^2 + S_{21}^2 \right\} = \frac{364}{1250^2} \cdot \{\ldots\} = 3081\ h \qquad (23)$$

Now we know all necessary for calculation of total energy loss. I must just apply formula (16).

$$W_d = \left[P_0 \cdot T_{c1} + P_{ln} \cdot \left(\frac{S_{max1}}{S_n}\right)^2 \cdot T_{leh1}\right] +$$

$$+ \left[2 \cdot P_0 \cdot T_{c2} + \frac{P_{ln}}{2} \cdot \left(\frac{S_{max2}}{S_n}\right)^2 \cdot T_{leh2}\right] =$$

$$= \left[1{,}7 \cdot 4004 + 14 \cdot \left(\frac{755}{1600}\right)^2 \cdot 1710\right] + \tag{24}$$

$$+ \left[2 \cdot 1{,}7 \cdot 4732 + \frac{14}{2} \cdot \left(\frac{1250}{1600}\right)^2 \cdot 3081\right] =$$

$$= 41387\ kWh$$

We can already spot the difference amount of dissolved electricity between one transformer and two of them. Parallel operation saved more than 5 MWh in this case. To finish my example I just even count the costs.

$$C_{ov} = W_d \cdot p_l \cdot \alpha_v = 41\,387 \cdot 1{,}46 \cdot 1{,}017 = 61\,453\ CZK \tag{25}$$

7.4.1.3 Results

By applying of parallel operation we possibly can save up to 5 MWh every year. The difference in annual variable operational costs is roughly 7.5 thousand CZK in present values. It represents 11 % of original value and that can be significant saving especially when it costs nothing. It is important to understand that transformers in this example were not planned for parallel operation and they have not optimal nominal power for it. In another cases the parallel operation may save much more.

7.4.2 Looking for optimal variant

In second example I will compare variants of new transformers. I prepared selection of various transformers with different powers, qualities and prices. And I want to find the economically best choice for whole lifespan. I will do it by calculation of NPV for all costs.

7.4.2.1 Load

As in first example I will work again with year average daily load diagram. We know it or we are able to predict it with sufficient accuracy for present but for the more distant future it's starting to be more and more vague. Usually the load is rising with time as new consumers are connected to grid. I decided to simplify the model by creation of daily load diagram for three different time sections – one applies to first five years, second one to next five year (years 6 to 10) and the last one applies to rest of transformer lifespan. I have chosen lifespan of thirty years. This solution is still quite accurate the model is simplified and I do not need to calculate costs for all the years but only for three of them. I think the allocation of years according to diagrams is correct because the consumption of energy is ordinarily growing faster in the beginning and after some time it began to be relatively constant. Moreover due to discount the farther years have much less impact on total costs so inaccuracy there is not so important.

Table 4. Diagram for years 1 to 5

Hour	*S* *[kVA]*	Hour	*S* *[kVA]*
1.	195	13.	375
2.	180	14.	465
3.	160	15.	495
4.	155	16.	485
5.	170	17.	470
6.	185	18.	380
7.	245	19.	345
8.	330	20.	280
9.	370	21.	335
10.	350	22.	290
11.	410	23.	240
12.	380	24.	200

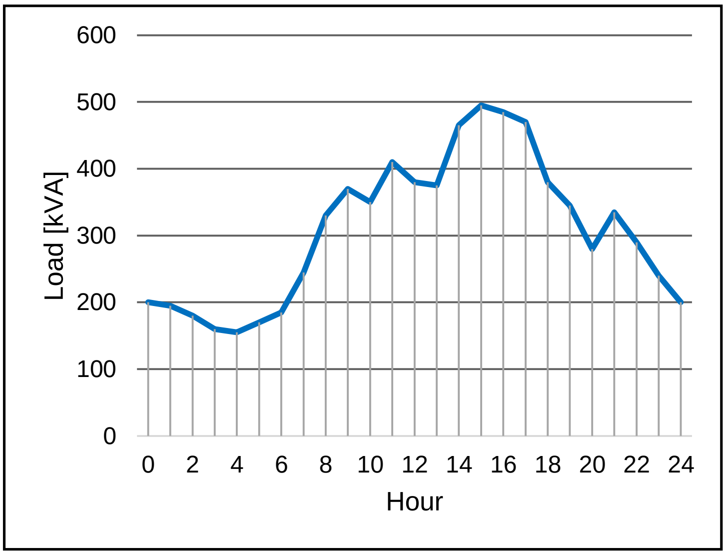

Figure 25. Daily load diagram for years 1 to 5

Table 5. Diagram for years 6 to 10

Hour	*S* [kVA]	Hour	*S* [kVA]
1.	200	13.	420
2.	200	14.	400
3.	190	15.	480
4.	180	16.	520
5.	195	17.	500
6.	200	18.	510
7.	240	19.	450
8.	350	20.	390
9.	410	21.	300
10.	415	22.	350
11.	435	23.	310
12.	950	24.	240

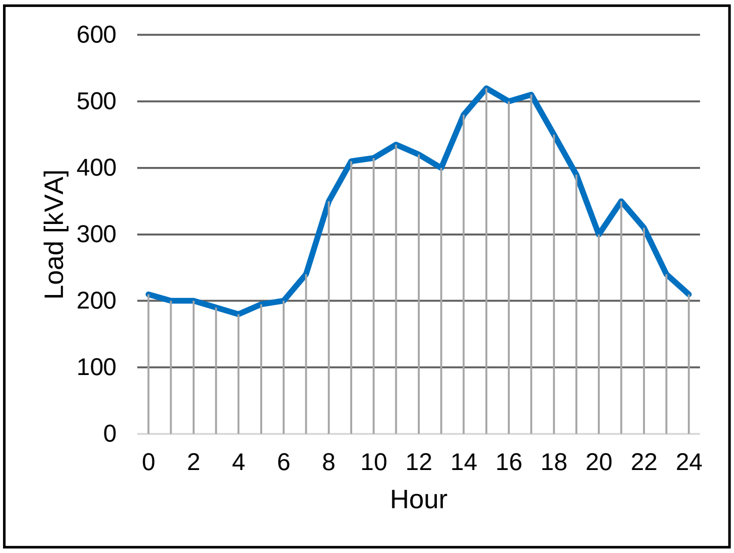

Figure 27. Daily load diagram for years 6 to 10

Table 6. Diagram for year 11 to 30

Hour	*S* [kVA]	Hour	*S* [kVA]
1.	210	13.	430
2.	205	14.	500
3.	200	15.	530
4.	195	16.	520
5.	205	17.	525
6.	250	18.	450
7.	300	19.	400
8.	405	20.	305
9.	420	21.	340
10.	435	22.	330
11.	460	23.	250
12.	450	24.	215

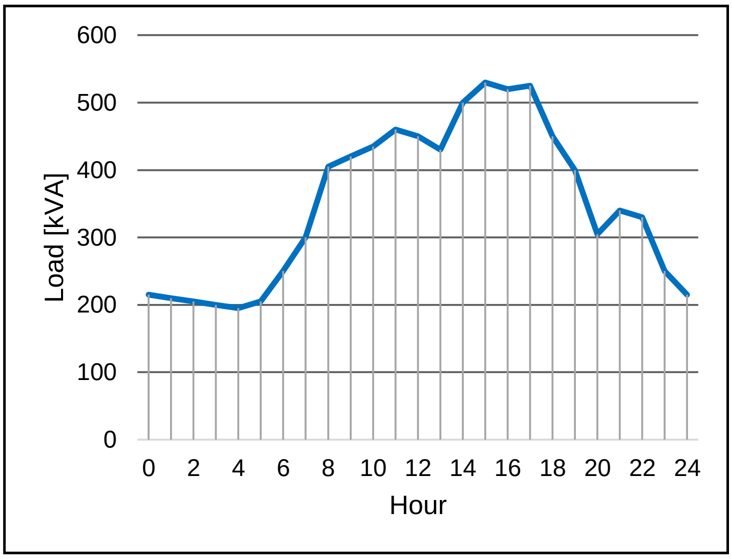

Figure 28. Daily load diagram for years 11 to 30

7.4.2.2 Transformers

In the following Table 7 you can see the selection of machines for which I will count the costs. There is given type, price, nominal power, no-load and load losses. Codes in column type correspond with labeling of transformers by their quality. It represent offering cross-section of machines on market suitable for my task. There are machines with too small power for solo-working but that is correct because I plan to calculate also results for parallel operation of two and even three of them.

Table 7. Transformers overview

Type	***Price***	***S_n***	***P_0***	***P_{ln}***
	thsd CZK	*kVA*	*W*	*W*
EoDk 250	131	250	650	4200
CoCk 250	133	250	425	3250
CoBk 250	143	250	425	2750
AoBk 250	160	250	300	2750
EoDk 400	156	400	930	6000
CoCk 400	156	400	610	4600
CoBk 400	174	400	610	3850
AoBk 400	190	400	430	3850
EoDk 500	173	500	1100	7200
CoBk 500	196	500	720	4600
AoBk 500	213	500	510	4600
EoDk 630	180	630	1300	8400
CoCk 630	209	630	860	6500
CoBk 630	219	630	860	5400
AoBk 630	239	630	600	5400
EoDk 800	234	800	1400	10500
CoBk 800	277	800	930	7000
AoBk 800	299	800	650	7000

7.4.2.3 Losses

Unlike the previous example I will not show all the steps of losses calculations here because they are analogical. I must find out annual dissipated energy for all three daily diagrams and all three connection options – one machine, pair and triplet. In following Table 8 I wrote down all results. Blank cells are invalid variants meaning variants where nominal power of transformers is not high enough. You can see that in my example the lowest losses have transformers with highest power. All partial results as loss equivalent hours or other you can find in attachment.

Table 8. Dissipated energy for all variants

	1 Transformer			2 Transformers			3 Transformers		
Type	Years 1-5	Years 6-10	Years 11-30	Years 1-5	Years 6-10	Years 11-30	Years 1-5	Years 6-10	Years 11-30
EoDk 250							37631	41334	43984
CoCk 250							27276	30142	32162
CoBk 250							24626	27050	28784
AoBk 250							21669	24086	25753
EoDk 400				33269	36531	38825	32492	35179	37045
CoCk 400				23969	26462	28162	23086	25012	26348
CoBk 400				21545	23639	25115	21071	22824	24031
AoBk 400				18830	20565	22197	17778	18984	20312
EoDk 500				31123	33616	35569	31085	33452	35289
CoBk 500				20129	21722	22976	20117	21637	22822
AoBk 500				17125	18726	19871	16841	18254	19222
EoDk 630	31523	34841	35752	29419	31451	32974	29419	31451	32974
CoCk 630	23118	25685	26391	20885	22373	23560	20885	22373	23560
CoBk 630	20477	22610	23196	19204	20528	21516	19204	20528	21516
AoBk 630	18206	20338	20925	15809	17021	17973	15809	17017	17946
EoDk 800	27863	30435	31142	27162	29283	30650	27162	29283	30650
CoBk 800	18546	20261	20732	18074	19483	20392	18074	19483	20392
AoBk 800	16100	17815	18286	14934	15961	16755	14934	15961	16755

7.4.2.4 Prediction of price developments

I already know dissipated power for every year but unfortunately the price of electrical losses will develop somehow and also the value of money is changing every year. As I said I will correct the value of money by discount of some distribution company what will be its WACC. For Czech Republic the value of discount is typically around six percent and in my example I will also use this exact number. I expect that the discount will be constant for whole machine lifespan.

Prediction of electricity price development is much more complicated. Nobody can exactly know what the price of electricity will be in five years much less in thirty. Mainstream prognosis of development says that the price of electricity will few years continue in actual downward trend and then it may be rising again. Therefore I prepared three forecasts based on it but with different ratios of growth or decline. Start price is actual value and that is 1,46 CZK/kWh. On Figure 29 you can see all three variants in process during the lifetime. Exact values for all years are in attachment.

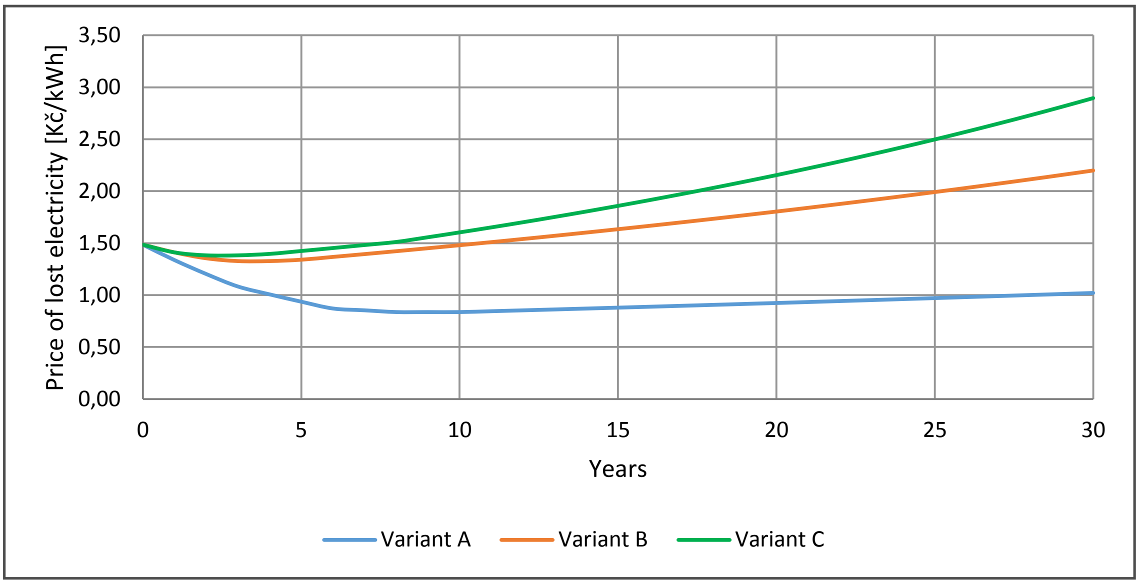

Figure 29. Graphical interpretation of dissipated electricity price development

7.4.2.5 NPV

In next step I will explain how I calculated total net present value of all costs. I split that into sum of three partial calculations as in equation 26.

$$NPV_C = C_i + NPV_{C_{of}} + NPV_{C_{ov}} \tag{26}$$

NPV_{cof}	*[CZK]*	Net present value of fixed operational costs
NPV_{cov}	*[CZK]*	Net present value of variable operational costs

Investment costs C_i are one-time charge in the beginning and thus it is not corrected by the discount. This item consists only of purchase price of transformer multiplied by number of them.

$$C_i = n \cdot p_T \tag{27}$$

p_T	*[CZK]*	Purchase price of machine

NPV of fixed costs looks complicated at first sight but in my case it can be substantially simplified. Fixed operational costs C_{of} consist of quotient of fixed costs and investment costs. Investment costs depend on chosen machine and the rest is constant. Inference and calculation is in formula 28.

$$NPV_{C_{of}} = \sum_{t=1}^{T_{lt}} \frac{C_{of}}{(1+r)^t} = \sum_{t=1}^{T_{lt}} \frac{C_i \cdot \beta}{(1+r)^t} = C_i \cdot \beta \cdot \sum_{t=1}^{T_{lt}} (1+r)^{-t} =$$
$$= C_i \cdot 0{,}02 \cdot \sum_{t=1}^{30} (1+0{,}06)^{-t} = C_i \cdot 0{,}02 \cdot 13{,}765 = C_i \cdot 0{,}275 \tag{28}$$

β	*[-]*	quotient of fixed costs

The last part is most difficult but because I have the same value of dissipated energy for several years it can be also little simplified.

$$NPV_{C_{ov}} = \sum_{t=1}^{T_{lt}} \frac{C_{ovt}}{(1+r)^t} = \sum_{t=1}^{T_{lt}} \frac{W_{dt} \cdot p_{lt} \cdot \alpha_v}{(1+r)^t} \tag{29}$$

Voltage level coefficient α_v is constant so it can be reproached before sum and it is again 1,7. Moreover because I have only three values of dissipated energy I can split the sum into three smaller and simpler parts where is already only one variable. W_{d1} is lost electricity in first year but the value is same for all first five years similarly W_{d6} applies to years 6 to 10 and W_{d11} for the rest of lifetime meaning years 11 to 30.

$$NPV_{C_{ov}} = \alpha_v \cdot \left[W_{d1} \cdot \sum_{t=1}^{5} \frac{p_{lt}}{(1+r)^t} + W_{d6} \cdot \sum_{t=6}^{10} \frac{p_{lt}}{(1+r)^t} + W_{d11} \cdot \sum_{t=11}^{30} \frac{p_{lt}}{(1+r)^t} \right] \tag{30}$$

7.4.2.6 Results

After calculation according to formulas in previous part I got results for all combinations of variants. I put them together and I created overview of results which you can see in following Table 9. I recall that variants A, B and C are related to possible development of loss electricity prices.

Table 9. Overview of NPV_C results. Green is the best (the lowest costs), red is the worst (the highest costs).

	Variant A			Variant B			Variant C		
Type	1 Tr.	2 Tr.	3 Tr.	1 Tr.	2 Tr.	3 Tr.	1 Tr.	2 Tr.	3 Tr.
EoDk 250			939			1290			1403
CoCk 250			797			1053			1135
CoBk 250			786			1016			1090
AoBk 250			797			1002			1068
EoDk 400		880	932		1190	1229		1290	1325
CoCk 400		747	798		971	1009		1044	1077
CoBk 400		756	823		956	1016		1021	1078
AoBk 400		758	824		935	986		992	1038
EoDk 500		886	962		1171	1245		1263	1336
CoBk 500		787	874		972	1057		1031	1116
AoBk 500		790	880		949	1034		1000	1083
EoDk 630	681	875	956	969	1140	1221	1062	1225	1306
CoCk 630	599	829	923	812	1019	1113	880	1079	1173
CoBk 630	572	830	928	759	1003	1102	819	1059	1157
AoBk 630	568	835	942	736	979	1086	790	1025	1132
EoDk 800	694	983	1088	945	1229	1334	1026	1308	1413
CoBk 800	616	963	1088	784	1127	1252	838	1180	1304
AoBk 800	612	974	1108	760	1109	1243	807	1152	1286

Results from my example are not general and depend on task, but you can see some trends there. Obviously is always better to not stint and pay some extra money for higher quality machines with lower losses. Even in case with low electricity price the investment returns is higher than input. As it turned out that the parallel connection gives worse results than standard standalone transformer.

But let us look at the specific numbers. Conclusively best solution for my task is one transformer AoBk 630 kVA. This is expensive model but decreased losses proved their worth over the machine’s lifespan. As I already wrote above two or more transformers are sometimes required in substations from several reasons. If this was the case it would be optimal to use a pair of machines AoBk 400 kVA. It is not best in A category where it is actually on third place but the difference is not high and it gives the best results in comparison to other variants. It also shows the impact of different electricity price predictions on solution. There is big difference between variants and NPV varies by tens of percent but order remains nearly same.

8 Conclusions

I put myself through a task of trying to acquaint readers with issue of transformer operation and I believe the result might be useful. In the first two chapters I explained what the transformer is, how it works and how it is constructed. I started with theoretical principles and then I dealt with the real machines. I described materials and technologies used for every part of machine in details so even a layman will be able to understand. Then I showed some advantages of possible alternatives and I even provided a comparison of the advantages.

In the next part I wrote about diagnostic possibilities. These methods can discover hidden problems caused by aging, manufacturing defects, short-circuit or something else. Revealing of defect allows you to take immediate action in order to correct and thus prevent greater damage and the associated expenditures. It is great tool for machine’s lifespan extension.

The last part was about the economy of transformer and decision criteria for buying of new equipment. I showed how to calculate costs based on parameters of transformers and expected load. There was also explanation of parallel work and inference of formulas for it included. For clarity of economical calculations I prepared two examples where I used all of those formulas. In the first one I showed advantages of parallel operation. In the second one there was complete calculations of costs for whole lifespan and comparison of results for several different transformer types. I did it mainly as an illustration but it is based on real numbers and shows us some general trends especially improved profitability of more expensive hi-quality devices.

It is difficult to imagine the world and our modern life without electricity which transformers mediate to us. It is necessary part of our lifestyle without realizing of it because transformers work ceaselessly and reliably tucked somewhere in substations.

9 List of abbreviations and symbols

a	[-]	Turn ration of transformer
c	[J/kgK]	Specific heat capacity
C_i	[CZK]	Investment costs
C_o	[CZK]	Operational cost per year
C_{of}	[CZK]	Fixed operating costs per year
C_{ov}	[CZK]	Variable operating costs per year
D	[kg/m^3]	Density
I	[A]	Electric current
i	[-]	Discount rate
n	[-]	Number of transformers
NPV_C	[CZK]	Net present value of costs
NPV_{Co}	[CZK]	Net present value of operational costs
NPV_{Cof}	[CZK]	Net present value of fixed operational costs
NPV_{Cov}	[CZK]	Net present value of variable operational costs
P	[W]	Power
P_0	[kW]	Power dissipation by no-load losses
P_d	[kW]	Total power dissipation
P_l	[CZK/kWh]	Price of electricity loss

P_{ln}	[kW]	Power dissipation by load losses at nominal power
P_T	[CZK]	Purchase price of machine
R	[Ω]	Resistivity
S	[kVA]	Actual load
S_{max}	[kVA]	Maximal load
S_n	[kVA]	Nominal apparent power of transformer
S_{tr}	[kVA]	Transient power
T_c	[h]	Time of connection
T_{leh}	[h]	Loss equivalent hours
T_{lt}	[years]	Lifetime
U	[V]	Voltage
W_d	[kWh]	Total dissipated energy
α_v	[-]	Coefficient of voltage level
β	[-]	quotient of fixed costs
ρ	[Ωm]	Specific resistivity
Φ	[Wb]	Main magnetic flux
Φ_σ	[Wb]	Stray magnetic flux

10 List of references

[1] DICK, E. P.; ERVEN, C. C. Transformer diagnostic testing by frequency response analysis. *Power Apparatus and Systems, IEEE Transactions on*, 1978, 6: 2144-2153.

[2] RYDER, Simon A. Diagnosing transformer faults using frequency response analysis. *Electrical Insulation Magazine, IEEE*, 2003, 19.2: 16-22.

[3] POWERTECH LABS. Low voltage impulse test and analysis.

[4] KEMPA, Miloslav. Elektrické zkoušky v diagnostice transformátorů. 2013.

[5] Insulation resistance test. In: *OpenElectrical* [online]. 2012 [of 2014-03-28]. Available from: www.openelectrical.org/wiki/index.php?title=Insulation_Resistance_Test

[6] PETR, J.; ANTFEIST, F.; MAREK, V. Diagnostika izolačních systémů elektrických strojů pomocí ztrátového činitele a kapacity. 2007.

[7] DLA Testing. In: *OpenElectrical* [online]. 2013 [of 2014-03-28]. Available from: www.openelectrical.org/wiki/index.php?title=DLA_Testing

[8] PRSKAVEC, Ladislav. Měření částečných výbojů. 1999

[9] DUVAL, M. Dissolved gas analysis: It can save your transformer. *Electrical Insulation Magazine, IEEE*, 1989, 5.6: 22-27.

[10] EMSLEY, A. M., et al. Degradation of cellulosic insulation in power transformers. Part 2: Formation of furan products in insulating oil. In: *Science, Measurement and Technology, IEE Proceedings*-. IET, 2000. p. 110-114.

[11] STEBBINS, R. D.; MYERS, D. S.; SHKOLNIK, A. B. Furanic compounds in dielectric liquid samples: review and update of diagnostic interpretation and estimation of insulation ageing. In: *Properties and Applications of Dielectric Materials, 2003. Proceedings of the 7th International Conference on*. IEEE, 2003. p. 921-926.

[12] HARLOW, James H. Electric power transformer engineering. 2nd ed. Boca Raton: CRC Press, c2007, 1 v. (various paging). ISBN 08-493-9186-5.

[13] SALMIA, T. Amorphous metals as core materials. 2009.

[14] OLIVARES-GALVAN, J. C., et al. Selection of copper against aluminium windings for distribution transformers. *Electric Power Applications, IET*, 2010, 4.6: 474-485.

[15] KRAUS, Miroslav. Není transformátor jako transformátor. In: Elektro: odborný časopis pro elektrotechniku. Praha: FCC PUBLIC s. r. o., roč. 2005, č. 08. ISSN 1210-0889. Available from: http://www.odbornecasopisy.cz/index.php?id_document=26443

[16] AGARWAL, R. K. Principles of electrical machine design. Fourth edition. Delhi: S.K. Kataria, 2002, 602 p.

[17] VAŠMUCIUS, Adam. Oleje aplikovatelné ve výkonových transformátorech. 2012.

[18] SGB STARKSTROM. Suché transformátory. Catalog, 2009.

[19] GÖNEN, Turan. Electric power distribution system engineering. 2nd ed. Boca Raton: CRC Press, c2008, 834 p. ISBN 14-200-6200-X.

[20] BAYLISS, C.; HARDY, B. Transmission and distribution electrical engineering. 4th ed. Boston: Elsevier/ Newnes, 2012, 1145 p. ISBN 00-809-6912-7.

[21] PAVLOVSKÝ, B. Ztráty v přenosu a rozvodu elektrické energie. 1st ed. Prague: SNTL, 1959, 280 p. DT 621.316.1.017

[22] TŘÍSKA, J. Zlepšení účiníku v energetice. 2nd ed. Prague: Práce, 1956, 278p. D-562593

[23] V-I-P-X. Oceňování ztrát v distribuční síti PREdi pro výpočty ekonomické efektivnosti. Prague, 2012, 18 p.

[24] SCHNEIDER ELECTRIC, Power and Distribution Transformers. Catalogue, 2015

[25] PILOUS, M. Parametry transformátorů vn/nn a jejich technické a ekonomické porovnání. Prague, 2012, 42 p.

11 Attachments

11.1 Partial results from second example

One transformer

Type	Years 1-5			Yars 6-10			Yars 11-30		
	Tc	*Tleh*	*Wd*	*Tc*	*Tleh*	*Wd*	*Tc*	*Tleh*	*Wd*
	hrs.	*hrs.*	*kWh*	*hrs.*	*hrs.*	*kWh*	*hrs.*	*hrs.*	*kWh*
EoDk 250	8736	3889	69711	8736	4104	80244	8736	4104	83140
CoCk 250	8736	3889	53262	8736	4104	61412	8736	4104	63653
CoBk 250	8736	3889	45639	8736	4104	52536	8736	4104	54431
AoBk 250	8736	3889	44547	8736	4104	51444	8736	4104	53339
EoDk 400	8736	3889	43857	8736	4104	49735	8736	4104	51351
CoCk 400	8736	3889	32724	8736	4104	37230	8736	4104	38469
CoBk 400	8736	3889	28257	8736	4104	32029	8736	4104	33066
AoBk 400	8736	3889	26685	8736	4104	30456	8736	4104	31493
EoDk 500	8736	3889	37052	8736	4104	41566	8736	4104	42807
CoBk 500	8736	3889	23823	8736	4104	26707	8736	4104	27500
AoBk 500	8736	3889	21988	8736	4104	24872	8736	4104	25665
EoDk 630	8736	3889	31523	8736	4104	34841	8736	4104	35752
CoCk 630	8736	3889	23118	8736	4104	25685	8736	4104	26391
CoBk 630	8736	3889	20477	8736	4104	22610	8736	4104	23196
AoBk 630	8736	3889	18206	8736	4104	20338	8736	4104	20925
EoDk 800	8736	3889	27863	8736	4104	30435	8736	4104	31142
CoBk 800	8736	3889	18546	8736	4104	20261	8736	4104	20732
AoBk 800	8736	3889	16100	8736	4104	17815	8736	4104	18286

Two transformers

		Years 1-5								Years 6-10							
Type	S_{tr1-2}	T_{c1}	S_{max1}	T_{leh1}	W_{d1}	T_{c2}	T_{leh2}	W_{d2}	W_d	T_{c1}	S_{max1}	T_{leh1}	W_{d1}	T_{c2}	T_{leh2}	W_{d2}	W_d
	kVA	*hrs.*	*kVA*	*hrs.*	*kWh*	*hrs.*	*hrs.*	*kWh*	*kWh*	*hrs.*	*kVA*	*hrs.*	*kWh*	*hrs.*	*hrs.*	*kWh*	*kWh*
EoDk 250	139	0	0	0	0	8736	3889	43373	43373	0	0	0	0	8736	4104	48640	48640
CoCk 250	128	0	0	0	0	8736	3889	32200	32200	0	0	0	0	8736	4104	36275	36275
CoBk 250	139	0	0	0	0	8736	3889	28389	28389	0	0	0	0	8736	4104	31837	31837
AoBk 250	117	0	0	0	0	8736	3889	26205	26205	0	0	0	0	8736	4104	29653	29653
EoDk 400	223	2548	200	2031	5416	6188	3557	27853	33269	2548	210	2234	6064	6188	3739	30468	36531
CoCk 400	206	2548	200	2031	3890	6188	3557	20079	23969	2184	200	2061	3703	6552	3799	22759	26462
CoBk 400	225	2548	200	2031	3509	6188	3557	18036	21545	2548	210	2234	3925	6188	3739	19714	23639
AoBk 400	189	1820	185	1544	2054	6916	3673	16776	18830	364	0	0	157	8372	4060	20408	20565
EoDk 500	276	3276	245	2067	7176	5460	3383	23947	31123	3276	240	2438	7648	5460	3584	25968	33616
CoBk 500	280	3276	245	2067	4641	5460	3383	15487	20129	3276	240	2438	4943	5460	3584	16779	21722
AoBk 500	235	2548	200	2031	2794	6188	3557	14331	17125	2548	210	2234	3112	6188	3739	15614	18726
EoDk 630	350	5460	350	2870	14539	3276	2454	14880	29419	4732	350	2427	12445	4004	3004	19006	31451
CoCk 630	324	4004	290	2178	6444	4732	3141	14441	20885	4004	310	2166	6853	4732	3334	15520	22373
CoBk 630	356	5460	350	2870	9479	3276	2454	9725	19204	4732	350	2427	8115	4004	3004	12412	20528
AoBk 630	297	4004	290	2178	4895	4732	3141	10914	15809	3276	240	2438	3876	5460	3584	13145	17021
EoDk 800	413	7280	410	3682	20346	1456	1363	6816	27162	5824	410	2809	15900	2912	2357	13383	29283
CoBk 800	412	7280	410	3682	13540	1456	1363	4534	18074	5824	410	2809	10580	2912	2357	8902	19483
AoBk 800	345	4732	335	2350	5960	4004	2813	8974	14934	4004	310	2166	4880	4732	3334	11081	15961

		Years 11-30							
Type	**Str1-2**	***Tc1***	***Smax1***	***Tleh1***	***Wd1***	***Tc2***	***Tleh2***	***Wd2***	***Wd***
	kVA	*hrs.*	*kVA*	*hrs.*	*kWh*	*hrs.*	*hrs.*	*kWh*	*kWh*
EoDk 250	139	0	0	0	0	8736	4345	52365	52365
CoCk 250	128	0	0	0	0	8736	4345	39158	39158
CoBk 250	139	0	0	0	0	8736	4345	34276	34276
AoBk 250	117	0	0	0	0	8736	4345	32092	32092
EoDk 400	223	2184	215	1988	5476	6552	4018	33348	38825
CoCk 400	206	1456	205	1404	2584	7280	4135	25578	28162
CoBk 400	225	2184	215	1988	3543	6552	4018	21572	25115
AoBk 400	189	0	0	0	0	8736	4345	22197	22197
EoDk 500	276	2912	250	2198	7160	5824	3856	28410	35569
CoBk 500	280	2912	250	2198	4624	5824	3856	18351	22976
AoBk 500	235	2184	215	1988	2804	6552	4018	17066	19871
EoDk 630	350	4368	340	2472	11725	4368	3328	21249	32974
CoCk 630	324	3640	305	2193	6471	5096	3619	17089	23560
CoBk 630	356	4368	340	2472	7644	4368	3328	13872	21516
AoBk 630	297	2912	250	2198	3616	5824	3856	14357	17973
EoDk 800	413	5096	405	2461	13757	3640	2908	16893	30650
CoBk 800	412	5096	405	2461	9154	3640	2908	11237	20392
AoBk 800	345	4368	340	2472	5964	4368	3328	10790	16755

Three transformers

			Years 1-5											
Type	S_{tr1-2}	S_{tr2-3}	T_{c1}	S_{max1}	T_{leh1}	W_{d1}	T_{c2}	S_{max2}	T_{leh2}	W_{d2}	T_{c3}	T_{leh3}	W_{d3}	W_d
	kVA	kVA	hrs.	kVA	hrs.	kWh	hrs.	kVA	hrs.	kWh	hrs.	hrs.	kWh	kWh
EoDk 250	139	241	0	0	0	0	2912	240	1774	7220	5824	3472	30412	37631
CoCk 250	128	221	0	0	0	0	2548	200	2031	4278	6188	3557	22998	27276
CoBk 250	139	241	0	0	0	0	2912	240	1774	4724	5824	3472	19902	24626
AoBk 250	117	202	0	0	0	0	2548	200	2031	3316	6188	3557	18353	21669
EoDk 400	223	386	2548	200	2031	5416	4368	380	3300	17059	1820	1613	10017	32492
CoCk 400	206	357	2548	200	2031	3890	2912	350	2207	7439	3276	2454	11757	23086
CoBk 400	225	390	2548	200	2031	3509	4368	380	3300	11062	1820	1613	6500	21071
AoBk 400	189	327	1820	185	1544	2054	2184	290	1550	3447	4732	3141	12278	17778
EoDk 500	276	479	3276	245	2067	7176	4732	470	2961	19828	728	713	4081	31085
CoBk 500	280	485	3276	245	2067	4641	4732	470	2961	12831	728	713	2645	20117
AoBk 500	235	408	2548	200	2031	2794	4368	380	3300	8839	1820	1613	5208	16841
EoDk 630	350	607	5460	350	2870	14539	3276	495	2454	14880	0	0	0	29419
CoCk 630	324	561	4004	290	2178	6444	4732	495	3141	14441	0	0	0	20885
CoBk 630	356	616	5460	350	2870	9479	3276	495	2454	9725	0	0	0	19204
AoBk 630	297	514	4004	290	2178	4895	4732	495	3141	10914	0	0	0	15809
EoDk 800	413	716	7280	410	3682	20346	1456	495	1363	6816	0	0	0	27162
CoBk 800	412	714	7280	410	3682	13540	1456	495	1363	4534	0	0	0	18074
AoBk 800	345	597	4732	335	2350	5960	4004	495	2813	8974	0	0	0	14934

	Years 6-10											
Type	***Tc1***	***Smax1***	***Tleh1***	***Wd1***	***Tc2***	***Smax2***	***Tleh2***	***Wd2***	***Tc3***	***Tleh3***	***Wd3***	***Wd***
	hrs.	*kVA*	*hrs.*	*kWh*	*hrs.*	*kVA*	*hrs.*	*kWh*	*hrs.*	*hrs.*	*kWh*	*kWh*
EoDk 250	0	0	0	0	3276	240	2438	8978	5460	3584	32356	41334
CoCk 250	0	0	0	0	2548	210	2234	4727	6188	3739	25415	30142
CoBk 250	0	0	0	0	3276	240	2438	5874	5460	3584	21176	27050
AoBk 250	0	0	0	0	2184	200	2061	3124	6552	3799	20962	24086
EoDk 400	2548	210	2234	6064	2184	350	1623	7791	4004	3004	21324	35179
CoCk 400	2184	200	2061	3703	2548	350	1754	6198	4004	3004	15111	25012
CoBk 400	2548	210	2234	3925	2548	390	1671	6167	3640	2799	12732	22824
AoBk 400	364	0	0	157	3640	310	2044	5493	4732	3334	13334	18984
EoDk 500	3276	240	2438	7648	4004	450	2969	17466	1456	1361	8337	33452
CoBk 500	3276	240	2438	4943	4368	480	2973	12593	1092	1051	4101	21637
AoBk 500	2548	210	2234	3112	2912	400	1953	5845	3276	2584	9297	18254
EoDk 630	4732	350	2427	12445	4004	520	3004	19006	0	0	0	31451
CoCk 630	4004	310	2166	6853	4732	520	3334	15520	0	0	0	22373
CoBk 630	4732	350	2427	8115	4004	520	3004	12412	0	0	0	20528
AoBk 630	3276	240	2438	3876	5096	510	3348	12039	364	364	1102	17017
EoDk 800	5824	410	2809	15900	2912	520	2357	13383	0	0	0	29283
CoBk 800	5824	410	2809	10580	2912	520	2357	8902	0	0	0	19483
AoBk 800	4004	310	2166	4880	4732	520	3334	11081	0	0	0	15961

	Years 11-30											
Type	***Tc1***	***Smax1***	***Tleh1***	***Wd1***	***Tc2***	***Smax2***	***Tleh2***	***Wd2***	***Tc3***	***Tleh3***	***Wd3***	***Wd***
	hrs.	*kVA*	*hrs.*	*kWh*	*hrs.*	*kVA*	*hrs.*	*kWh*	*hrs.*	*hrs.*	*kWh*	*kWh*
EoDk 250	0	0	0	0	2184	215	1988	5926	6552	4018	38057	43984
CoCk 250	0	0	0	0	2184	215	1988	4245	6552	4018	27917	32162
CoBk 250	0	0	0	0	2184	215	1988	3878	6552	4018	24907	28784
AoBk 250	0	0	0	0	728	200	710	1062	8008	4244	24691	25753
EoDk 400	2184	215	1988	5476	2184	340	1677	7697	4368	3328	23871	37045
CoCk 400	1456	205	1404	2584	2912	340	1961	6812	4368	3328	16952	26348
CoBk 400	2184	215	1988	3543	2184	340	1677	4997	4368	3328	15491	24031
AoBk 400	0	0	0	0	3640	305	2193	5585	5096	3619	14727	20312
EoDk 500	2912	250	2198	7160	4368	460	3266	19562	1456	1396	8568	35289
CoBk 500	2912	250	2198	4624	4368	460	3266	12648	1456	1396	5549	22822
AoBk 500	2184	215	1988	2804	2912	405	1901	5839	3640	2908	10579	19222
EoDk 630	4368	340	2472	11725	4368	530	3328	21249	0	0	0	32974
CoCk 630	3640	305	2193	6471	5096	530	3619	17089	0	0	0	23560
CoBk 630	4368	340	2472	7644	4368	530	3328	13872	0	0	0	21516
AoBk 630	2912	250	2198	3616	4732	500	3128	10999	1092	1072	3331	17946
EoDk 800	5096	405	2461	13757	3640	530	2908	16893	0	0	0	30650
CoBk 800	5096	405	2461	9154	3640	530	2908	11237	0	0	0	20392
AoBk 800	4368	340	2472	5964	4368	530	3328	10790	0	0	0	16755

11.2 Development of lost electricity prices

Year	Variant A		Variant B		Variant C	
0		1,48		1,48		1,48
1	-10,0%	1,34	-5,0%	1,41	-5,0%	1,41
2	-10,0%	1,20	-4,0%	1,35	-2,0%	1,38
3	-10,0%	1,08	-2,0%	1,33	0,0%	1,38
4	-7,0%	1,01	0,0%	1,33	1,0%	1,40
5	-7,0%	0,94	1,0%	1,34	2,0%	1,42
6	-7,0%	0,87	2,0%	1,37	2,0%	1,45
7	-2,0%	0,85	2,0%	1,39	2,0%	1,48
8	-2,0%	0,84	2,0%	1,42	2,0%	1,51
9	0,0%	0,84	2,0%	1,45	3,0%	1,56
10	0,0%	0,84	2,0%	1,48	3,0%	1,60
11	1,0%	0,84	2,0%	1,51	3,0%	1,65
12	1,0%	0,85	2,0%	1,54	3,0%	1,70
13	1,0%	0,86	2,0%	1,57	3,0%	1,75
14	1,0%	0,87	2,0%	1,60	3,0%	1,80
15	1,0%	0,88	2,0%	1,63	3,0%	1,86
16	1,0%	0,89	2,0%	1,67	3,0%	1,91
17	1,0%	0,90	2,0%	1,70	3,0%	1,97
18	1,0%	0,91	2,0%	1,73	3,0%	2,03
19	1,0%	0,91	2,0%	1,77	3,0%	2,09
20	1,0%	0,92	2,0%	1,80	3,0%	2,15
21	1,0%	0,93	2,0%	1,84	3,0%	2,22
22	1,0%	0,94	2,0%	1,88	3,0%	2,29
23	1,0%	0,95	2,0%	1,91	3,0%	2,35
24	1,0%	0,96	2,0%	1,95	3,0%	2,43
25	1,0%	0,97	2,0%	1,99	3,0%	2,50
26	1,0%	0,98	2,0%	2,03	3,0%	2,57
27	1,0%	0,99	2,0%	2,07	3,0%	2,65
28	1,0%	1,00	2,0%	2,11	3,0%	2,73
29	1,0%	1,01	2,0%	2,16	3,0%	2,81
30	1,0%	1,02	2,0%	2,20	3,0%	2,90