INVITED PAPER

Bulk Metamaterials Made of Resonant Rings

The realization of 3-D magnetic metamaterials at microwave frequencies is discussed in this paper and an exciting application of such materials is described.

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ABSTRACT | In this brief review, we present the fundamentals of bulk resonant ring metamaterial (RRM) theory. Metamaterials made of resonant rings are discussed, and some basic design rules are provided. Homogenization (including spatial dispersion) of 3-D resonant ring latices is reviewed, with emphasis in isotropic designs. Edge effects in finite size metamaterial samples are discussed. Finally, possible applications and future trends are briefly reviewed.

KEYWORDS | Artificial media; homogenization theory; magnetic resonance imaging; magnetic resonators; magneto-inductive waves; metamaterials; spatial dispersion

I. INTRODUCTION

Diamagnetic properties of closed inductive loops were well known in the past (see, for instance, [1]). It was also known that this effect can be enhanced by adding a chip capacitor [2] to the ring. However, it was not until recent years that these effects were systematically studied in order to develop artificial media (or metamaterials) with negative magnetic permeability [3], which may be eventually combined with conducting plates or wires [4] in order to provide a medium with simultaneously negative magnetic permeability and permittivity [5], i.e., the negative refractive index (NRI) medium predicted many years ago [6], [7]. After these seminal works, metamaterial theory became a "hot" scientific topic, with thousands of published scientific papers (for RRM specifically see, for instance, [8], [9], and references therein). Although other alternatives besides resonant rings have been proposed for negative μ metamaterial design, resonant ring technology can be still considered as the "standard" approach to this goal, at least up to optical frequencies, where the combined effects of the kinetic inductance of electrons [10] and high frequency dissipation [11] introduce severe limitations to this approach. In this paper, we will shortly review the fundamentals of resonant ring metamaterial (RRM) theory, with emphasis in isotropic 3-D effective media design. The paper ends with a short discussion on possible applications and future trends for bulk metamaterial technologies.

II. RESONANT RINGS FOR METAMATERIAL DESIGN

A. Resonant Ring Basic Concepts

A closed conducting ring of inductance L and resistance R provides a magnetic moment $\mathbf{m} = -(j\omega\pi^2 r^4/Z)\mathbf{B}_\perp$, where r is the ring radius, $\mathbf{B}_\perp$ is the external magnetic field component perpendicular to the ring, and $Z = j\omega L + R$ is the ring impedance. Since $L \sim \mu_0 r$, this magnetic moment, though opposite to the magnetic field, is not sufficient for providing a negative effective permeability [8]. This effect can be enhanced if the ring impedance $Z = j\omega L + R$ is

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modified by the presence of a series connected capacitance C [2], so that $Z = j\omega L + R + 1/(j\omega C)$ and

$$\mathbf{m} = \alpha(\omega)\mathbf{B}_\perp = \frac{\pi^2 r^4}{L}\frac{\omega^2}{\omega_0^2 - \omega^2 + j\omega R/L}\mathbf{B}_\perp \quad (1)$$

where $\omega_0 = \sqrt{1/(LC)}$ is the frequency of resonance. Lumped capacitors are available at radio and microwave frequencies. However, attaching a lumped capacitor at microwave frequencies may not be a very practical approach because of some unavoidable parasitic inductance, which cannot be neglected. For this reason, lumped capacitors are rarely used except at radio frequencies. At higher frequencies, it may be advantageous to use conventional printed circuit techniques and substitute the lumped capacitor by a distributed capacitance. This leads to the split ring resonator (SRR), already known and used for some specific applications [12], but first proposed as metamaterial element in [3].

In order to simplify the analysis, avoiding magneto-electric couplings (see below), the geometry proposed in [12] is used in the following. This structure (see Fig. 1) is a broadside-coupled SRR (BC-SRR). As has been shown in [13], near the resonance, the total current (i.e., the sum of the current flowing on both rings) is almost uniform, forming a closed current loop. This current flows from the upper to the lower ring and *vice versa* through the gap between them, as an electric displacement current. Therefore, the total capacitance of the resonator is the series connection of the capacitances through both resonator halves. The resonator inductance can be approximated by the inductance of a single ring (assumed closed) and the magnetic polarizability is still given by (1).

A main advantage of the BC-SRR over the design proposed in [3] is that broadside coupling provides a much higher capacitance than edge coupling, thus allowing for a much smaller electrical size at resonance [13]. The price to pay for this advantage is a more complicated fabrication process, which includes two levels of metalization. Another important advantage (for most applications) of the BC-SRR is the aforementioned absence of magneto-electric coupling. Magneto-electric coupling appears in many SRR designs due to the simultaneous excitation of an electric and a magnetic dipole at resonance. According to reciprocity, the excitation of an electric dipolar moment by an external magnetic field implies the excitation of a magnetic dipolar moment by an external electric field. Therefore, magneto-electric coupling makes the metamaterial bi-anisotropic [14]. Magneto-electric coupling is not present if the SRR is invariant by spatial inversion, as it happens in Fig. 1. In addition to the reported resonant magnetic (or magneto-electric) polarizability, SRRs also show a nonresonant electric polarizability, which can be approximated as the polarizability of a metallic disk of the same radius [15]. Besides the aforementioned edge-coupled and broadside-coupled SRRs, many other SRR designs have been proposed, aimed to specific applications. The interested reader is referred to [8] for a description of some of these proposals. Finally, it may be worth to mention that high permittivity dielectric rings also show a resonant magnetic polarizability similar to (1) [16], due the combined effects of the electric field confinement inside the ring and the internal capacitance, both associated to the high value of the dielectric constant.

Closely related to resonant rings, there are other resonant structures useful for metamaterial design, which deserve some comments. One of them is the "swiss roll," also proposed in [3], which provides a very strong magnetic response at very low frequencies, thus being useful for applications in the megahertz range (see Section IV). Another interesting structure is the complementary SRR (CSRR), proposed in [17]. This resonator is the complementary screen of any planar SRR, whose properties can be related to those of the SRR using Babinet theorem: magnetic and electric polarizabilities are interchanged. However, it must be emphasized that the polarizabilities of the CSRR are only "effective," having opposite sign at both sides of the resonator [17]. This fact makes this element useless for 3-D metamaterial design (although certainly very useful for 2-D metamaterial design).

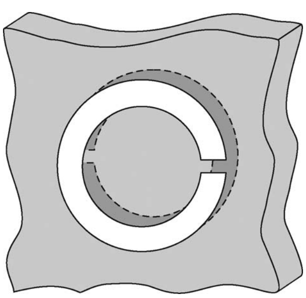

Fig. 1. ***Broadside-coupled SRR (BC-SRR). The supporting dielectric board is in light gray, whereas metallic parts are in white (upper ring) and dark gray (lower ring).***

B. SRRs at Optical Frequencies

Upon the success of SRRs at radio and microwave frequencies, such metamaterial elements were studied for operation at terahertz [18] and infrared frequencies [19]. At few terahertz, metals still behave as good conductors. However, at frequencies in the infrared and the visible range, metals are better described as solid plasmas with a complex permittivity approximated by

$$\varepsilon = \varepsilon_0\left[\varepsilon_i - \frac{\omega_{\mathrm{p}}^2}{\omega(\omega - jf_{\mathrm{c}})}\right] \quad (2)$$

where ε_i is the permittivity of the ionic background, ω_p is the angular plasma frequency, and f_c is the frequency of collision of electrons. For metamaterial design, the electrical size of the SRR should be substantially smaller than the wavelength, and the SRR details (for instance, the wire section) even smaller. Therefore, the characteristic lengths of the SRR become of the same order as the mean free path of electrons—which for good conductors is of several tenths of nanometers—and collisions with the SRR boundaries become important [20]. This effect dramatically increases f_c and, therefore, losses, as it has been analyzed in [11] and [21]. Other effects, such as interband electron transitions and surface roughness, also contribute to an increase of metal losses at optical frequencies [11].

Even more important than losses are the effects of the kinetic inductance of electrons L_k [10]. This effect can be also understood as the effect of the negative permittivity of the metal [8], which provides a negative internal capacitance associated with the displacement current inside the metallic ring. This negative internal capacitance is, in fact, equivalent to a positive extra inductance

$$L_k = -\frac{1}{\omega^2 C_{\text{int}}} = -\frac{1}{\omega^2}\frac{2\pi r}{\Re(\varepsilon)S} \approx \frac{2\pi r}{\varepsilon_0 S \omega_p^2} \qquad (3)$$

where r is the ring radius and S is the wire section. This additional kinetic inductance must be added to the magnetic inductance L in the expressions for the polarizability of the SRR (1). When the SRR is scaled down in order to achieve resonance at optical frequencies, the kinetic inductance (3) scales as $L_k \sim 1/r$, whereas the magnetic inductance scales as $L \sim r$. Therefore, the kinetic inductance becomes dominant and the frequency of resonance saturates to the constant value $\omega_s = \sqrt{1/L_k C}$ (the capacitance C scales down as $C \sim r$). In addition, the amplitude of the magnetic susceptibility scales as $\chi_m \sim N\alpha$, where $N \sim 1/r^3$ and α is given by (1). Therefore, $\chi_m \sim r/L_k \sim r^2$. That is, the amplitude of the susceptibility decreases dramatically when the SRR is scaled down. Since the magnetic inductance varies as $L \approx \mu_0 r$ with the ring radius r, from (3) follows that the kinetic inductance becomes dominant when the section of the SRR wire becomes of the same order as the plasma wavelength of the metal [8]. For good conductors this effect appears when the wire section approaches to several tens of nanometers. Therefore, both effects analyzed in this section appear simultaneously, making the SRRs useless below these dimensions.

III. THREE-DIMENSIONAL RESONANT RING METAMATERIALS

There is a very high variety of phenomena that may appear in 3-D arrays of resonant rings, even if the analysis is restricted to homogenizable mixtures. In order not to make the analysis endless, we will restrict ourselves to the important case of isotropic RRM, which has been addressed in [22], [23], and [24] among others. Most of the concepts developed for isotropic designs can be extended to more complex structures, and the reader interested in such composites is referred to the available bibliography [8], [9].

A. Lorentz Homogenization Theory for Cubic Lattices of Resonant Rings

Let us consider an isotropic cubic lattice of electrically small resonant rings, as is sketched in Fig. 2(a). We will consider rings with only magnetic polarizability (1), so that magneto-electric coupling is not considered. We will also neglect the details of the ring design, assuming that the rings can be described by a closed current loop with some internal inductance, capacitance, and resistance (the conditions for the validity of such assumptions have been discussed in [24]). Lorentz homogenization theory of cubic crystals is based on the well-known relation [25] $\mathbf{H}_l = \mathbf{H} + \mathbf{M}/3$ between the local magnetic field $\mathbf{H}_l$, the macroscopic magnetic field $\mathbf{H}$, and the macroscopic magnetization $\mathbf{M}$. This relation is valid for cubic lattices of point magnetic dipoles. However, in cubic lattices of resonant rings of radius r comparable to the lattice constant a, the interaction between the closest rings cannot be approximated as the interaction between two magnetic dipoles. In this case, the local field component normal to the ring $\mathbf{H}_{l,\perp}$ is better approximated by

$$\mathbf{H}_{l,\perp} = \mathbf{H}_\perp + \left\{\frac{a^3}{\mu_0(\pi r^2)^2}(4M_c + 2M_a) + \frac{1}{3}\right\}\mathbf{M}_\perp \qquad (4)$$

where M_c and M_a are the mutual inductances between parallel coplanar and co-axial nearest rings, respectively (couplings between rings placed over perpendicular planes cancel each other). Taking into account the exact mutual

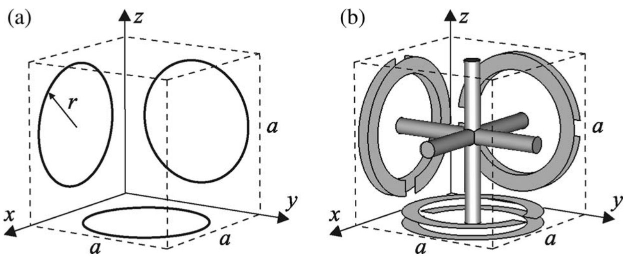

Fig. 2. ***(a) The unit cell of an ideal cubic lattice of resonant rings of self-inductance L, self-capacitance C, and resistance R. (b) Unit cell of a NRI medium made by a combination of BC-SRRs and wires.***

inductance between more distant neighbors [26] does not significantly change the result for a cubic lattice. From (4), the magnetic susceptibility of the lattice can be readily found as a function of the ring polarizability α. For α given by (1), this expression is

$$\chi(\omega) = \frac{\omega^2 \mu_0 \alpha_0/(a^3)}{\omega_0^2 - \omega^2[1 + 2M_a/L + 4M_c/L + \mu_0\alpha_0/(3a^3)] + j\omega R/L} \tag{5}$$

where $\alpha_0 = \pi^2 r^4/L$ is the magnitude of the polarizability of an ideal lossless ($R = 0$) and nonresonant ($\omega_0 = 0$) ring.

B. Magneto-Inductive Waves in Resonant Ring Metamaterials

In addition to electromagnetic TEM waves with phase constant $k = \omega\sqrt{\mu\epsilon}$, lattices of resonant rings also support magneto-inductive waves [27]. These waves appear at frequencies near the ring resonance and come from the inductive coupling between rings. For a linear chain of rings, with periodicity a, the general dispersion equation for these waves is [27]

$$\frac{\omega_0^2}{\omega^2} = 1 + 2\sum_{n=1}^{\infty} \frac{M_n}{L}\cos(nka) - j\frac{R}{\omega L} \tag{6}$$

where M_n is the mutual inductance between two rings separated by a distance na. In the nearest neighbors approximation, (6) reduces to

$$\frac{\omega_0^2}{\omega^2} = 1 + \frac{M_1}{L}\cos(ka) - j\frac{R}{\omega L}. \tag{7}$$

For an accurate quantitative analysis, (6) must be used, and the summation taken up to the appropriate convergence. However, in order to capture the most salient features of these kind of waves, (7) is sufficient [27]. Therefore, we will use this approximation in our analysis. For a linear chain of co-axial rings, $M_1 \equiv M_a > 0$, and the dispersion relation (7) shows that the magneto-inductive wave is a forward wave. However, for a linear chain of coplanar rings, $M_1 \equiv M_c < 0$, and (7) shows that the magneto-inductive wave is a backward wave. Equation (7) also shows that magneto-inductive waves have a narrow bandwidth $\Delta\omega/\omega_0 \sim M_1/L$ around the frequency of resonance ω_0.

It is also possible to induce magneto-inductive waves in unbounded lattices of resonant rings, the general formula for these waves being a straightforward generalization of (6) [27]. It is however interesting to explicitly write these expressions for the cubic lattice shown in Fig. 2(a). If we restrict our analysis to waves propagating along a main axis, namely the z-axis of Fig. 2(a), we can distinguish two branches, corresponding to "longitudinal" waves with axial coupling between the rings, and to "transverse" waves with coplanar coupling between rings. For longitudinal waves, we have

$$\frac{\omega_0^2}{\omega^2} = 1 + 2\frac{M_a}{L}\cos(ka) + 4\frac{M_c}{L} - j\frac{R}{\omega L} \tag{8}$$

and for transverse waves

$$\frac{\omega_0^2}{\omega^2} = 1 + 2\frac{M_c}{L}\cos(ka) + 2\frac{M_c}{L} + 2\frac{M_a}{L} - j\frac{R}{\omega L}. \tag{9}$$

From the signs of M_a and M_c it comes out that longitudinal waves are forward and transverse waves are backward.

C. Connection Between Magneto-Inductive Waves and Electromagnetic Waves. Spatial Dispersion

Magneto-inductive and TEM electromagnetic waves are both present in 3-D lattices of resonant rings. Moreover, the interesting region of negative permeability is very close to the frequency of resonance of the rings, where magneto-inductive waves also appear. Therefore, it is crucial to elucidate which are the connections between magneto-inductive and TEM electromagnetic waves. This connection was first investigated in [28] using a transmission line model. Subsequently, it was investigated for a cubic lattice of rings in [29]. In this section, we will mainly follow the analysis reported in [29], which has been adapted to our present purpose. Other approaches to spatial dispersion in metamaterial structures, which provide similar results in many cases, can be found in the literature [30], [31].

Let us consider, for simplicity, a z-polarized magnetic wave propagating along one of the main axis of the structure shown in Fig. 2(a), with propagation constant $\mathbf{k}$, macroscopic magnetic field $H_z = H_{0,z}\exp(-j\mathbf{k}\cdot\mathbf{r})$, and magnetization $M_z = M_{0,z}\exp(-j\mathbf{k}\cdot\mathbf{r})$, associated to a distribution of currents on the rings oriented perpendicular to the z-axis $I_{n_x,n_y,n_z} = I_0\exp(-ja(k_x n_x + k_y n_y + k_z n_z))$. In the nearest neighbors approximation, and for propagation along one of the main axes, couplings between rings located on orthogonal planes cancel. Thus, a straightforward generalization of (4) leads to the following equation for the current I_0 on the ring at $n_x = n_y = n_z = 0$:

$$\left(\frac{\omega_0^2}{\omega^2} - 1 + j\frac{R}{\omega L}\right) I_0^z = \frac{\Phi_{\text{ext}}}{L} = \frac{\pi r^2 \mu_0}{L}\left\{H_{0,z} + \frac{a^3}{\mu_0 (\pi r^2)^2} \times \left[2M_c \cos(k_x x) + 2M_c \cos(k_y y) + 2M_a \cos(k_z z)\right] M_{0,z} + \frac{1}{3} M_{0,z}\right\}. \quad (10)$$

The macroscopic wave equation for the magnetization is

$$(\omega^2 \mu_0 \varepsilon - k^2)\mathbf{H}_0 + (\omega^2 \mu_0 \varepsilon - \mathbf{k}\mathbf{k}\cdot)\mathbf{M}_0 = 0 \quad (11)$$

where ε is the macroscopic effective dielectric constant of the lattice. By combining (10), (11), and $M_{0,z} = \pi r^2 I_0 / a^3$, the wave equation for the currents on the rings is obtained. For waves propagating along the z-axis, we obtain "longitudinal" waves with the dispersion equation

$$\frac{\omega_0^2}{\omega^2} = 1 + 2\frac{M_a}{L}\cos(k_z a) + 4\frac{M_c}{L} - \frac{2}{3}\frac{\mu_0 \alpha_0}{a^3} - j\frac{R}{\omega L} \quad (12)$$

and for waves propagating along the x-axis (or y-axis), we obtain "transverse" waves with the dispersion equation, shown at the bottom of the page, where $k_m = \omega\sqrt{\varepsilon \mu_0}$. Of course similar results can be obtained for the currents on rings oriented perpendicular to the x- and y-axis of Fig. 2(a). Therefore, both branches of longitudinal and transverse waves coexist along any main axis of the structure.

Equation (12) corresponds to the longitudinal magneto-inductive wave (8) slightly modified by the presence of the volume magnetization. In the long wavelength limit, where $\cos(k_x a) \approx 1$, substitution of (12) into (5) leads to $\chi(\omega) = -1$, i.e., $\mu(\omega) = 0$. Therefore, longitudinal magneto-inductive waves in cubic lattices of resonant rings are the short wavelength continuation of the longitudinal magneto-plasmons that appear in continuous media when $\mu = 0$.

Regarding the long wavelength limit of (13), it corresponds to the transverse electromagnetic waves propagating in a medium of $\mu = \mu_0(1 + \chi_m(\omega))$, with $\chi_m(\omega)$ given by (5). In the short wavelength limit, when $k_x^2 \gg k_m^2$, (13) reduces to

$$\frac{\omega_0^2}{\omega^2} = 1 + \frac{2M_c}{L}\cos(k_x a) + \frac{2(M_a + M_c)}{L} + \frac{\mu_0 \alpha_0}{3a^3} - j\frac{R}{\omega L}. \quad (14)$$

Equation (14) corresponds to the transverse magneto-inductive waves (9) slightly modified by the effect of the volume magnetization. Therefore, transverse magneto-inductive waves in cubic lattices of resonant rings are the short wavelength continuation of transverse electromagnetic waves. Equation (13) also provides the spatially dispersive magnetic susceptibility $\chi_m(k_x, \omega)$ for transverse plane waves propagating along the x-axis of the lattice (and actually along any other main axis). The general expression for $\overline{\overline{\chi}}_m(\mathbf{k}, \omega)$ for any value of $\mathbf{k}$ becomes much more involved, because the mutual inductances between rings lying on orthogonal planes do not cancel. These expressions can be found in [29], where the reported theory was also checked by careful numerical computations. Finally, it may be worth to mention that, when spatial dispersion is present, the magnetic susceptibility becomes a tensor [29], even if the conditions for an isotropic behavior in the long wavelength limit [24] are fulfilled, thus destroying the isotropic behavior of the metamaterial.

In order to illustrate the typical behavior of TEM electromagnetic and magneto-inductive waves in RRM, the dispersion diagrams for longitudinal (12) and transverse (13) waves along a main axis of Fig. 2(a) are shown in Fig. 3 (solid lines) for some realistic values of the structure parameters. For positive values of the macoscopic permittivity ε, a negative μ forbidden band gap can be clearly appreciated near the resonance $\omega = \omega_0$. As is expected from its magneto-inductive nature, the longitudinal branch presents a flat dispersion curve. The transverse branch behaves as a nondispersive TEM electromagnetic wave at low frequencies, and as a slightly backward magneto-inductive wave when the frequency approaches the forbidden band gap (the reason for which this backward behavior is not clearly seen in the figure is because (14) corresponds to the limit $k_x \to \infty$ and, in practice, k_x reaches the end of the first Brillouin zone long before this limit).

$$\frac{k_x^2}{k_m^2} - 1 = \chi_m(k_x, \omega) = \frac{\omega^2 \mu_0 \alpha_0 / a^3}{\omega_0^2 - \omega^2[1 + (2M_c/L)\cos(k_x a) + 2(M_a + M_c)/L + \mu_0 \alpha_0/(3a^3)] + j\omega R/L} \quad (13)$$

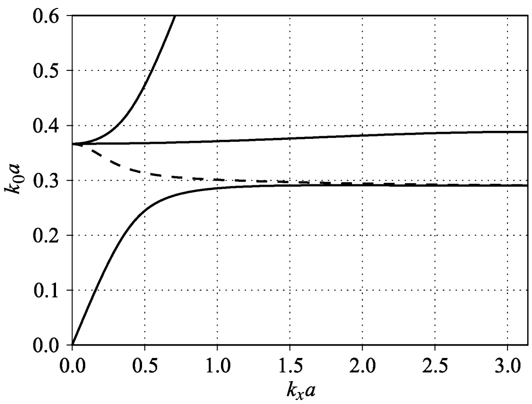

Fig. 3. ***Dispersion diagrams for longitudinal (12) and transverse (13) waves in the lattice of rings sketched in Fig. 2(a), for two different values of the macoscopic permittivity of the metamaterial. Solid lines: $\varepsilon = 2.5\varepsilon_0$. Dashed lines: $\varepsilon = -\varepsilon_0$ [longitudinal waves (12) are not affected by the macroscopic permittivity]. Structural parameters are $a = \lambda_0/20$, where λ_0 is the wavelength at the frequency of resonance ω_0, $r = 0.45a$, $M_c/L = 0.02$, $M_a/L = -0.015$, $L = 1.1\mu_0 a$, and $R/\omega L \rightarrow 0$. The macroscopic permittivity $\varepsilon = 2.5\varepsilon_0$ approximately corresponds to the background static permittivity of the lattice of rings.***

D. Negative Refractive Index Metamaterials

Equation (11) is the wave equation in a medium with $\mathbf{E} = \varepsilon\mathbf{D}$ and magnetization $\mathbf{M}$, regardless of the relation between $\mathbf{H}$ and $\mathbf{M}$. Therefore, ε in (11) is the macroscopic permittivity of the metamaterial, which may be different from the permittivity of the host medium. If this permittivity is negative, then $k_m = \omega\sqrt{\varepsilon\mu_0}$ in (13) becomes imaginary, and a backward passband appears in the regions of negative magnetic permeability. This effect is illustrated in Fig. 3 where the dashed line shows the dispersion equation for transverse waves when $\varepsilon = -\varepsilon_0$. The question arising now is how to make negative the macroscopic permittivity without affecting the permeability of the metamaterial.

The simplest possibility, at least conceptually, is to place the resonant rings in a continuous host medium of negative dielectric permittivity (assuming that such medium is available). This strategy works for rings loaded by a chip capacitor or for dielectric resonant rings [16]. However, for SRRs, the presence of a host medium of negative dielectric permittivity drastically affects the SRR capacitance, making it negative, and therefore equivalent to an inductance. Then, the resonance disappears as well as the negative polarizability. In order to avoid this effect (and also because negative ε media are not easily available at radio frequencies) arrays of metallic wires [4], [5] are commonly used to complement SRR lattices. However, it must be taken into account that not any combination of SRRs and wires provides a left-handed behavior [31], [32]. For this purpose, both sublattices must be combined in such a way that the quasi-static properties (like inductance and/or capacitance) of the elements of each sublattice are not substantially affected by the presence of the complementary ones. An example of such configurations is shown in Fig. 2(b) [33]. Finally, it may be worth to mention that NRI without wires can be achieved in some specific bi-isotropic SRR cubic lattices [34].

E. Quasi-Static Resonances in Finite Size Resonant Ring Metamaterials

Until now we have focused our analysis on unbounded lattices of resonant rings. However, practical metamaterials must have a finite size, and scattering by the edges may lead to the appearance of eigenmodes that would not be excited in unbounded metamaterials. Actually, even homogeneous finite size negative ε or μ samples present quasi-satic resonances that are excited at the edges and corners of the structure (see [35] and references therein). Moreover, realistic metamaterials are mesoscopic systems, made of a finite number of elements much smaller than the number of atoms in any macroscopic sample. Therefore, it is not clear if the resonances that may appear in finite samples of metamaterials will even correspond to the resonances that may appear in the corresponding homogenized samples of a hypothetical continuous medium with the corresponding effective parameters.

In order to take into account these effects, we have recently developed a code able to solve large (but finite) samples of RRM under arbitrary excitations [36]. This code is based on the computation of the whole impedance matrix for all rings, which is then solved for the specific external excitation [27], [36]. Our preliminary results [37] reveal that indeed extra resonances appear in finite samples of RRM, when they are illuminated by a TEM plane wave. These resonances emerge even when realistic losses are taken into account, and are observed in structures up to at least 10 000 elements. The frequency dispersion of the polarizability of finite samples is qualitatively different from that of samples of continuous media under similar excitation. Therefore, although the homogenization theory of unbounded RRM is now quite well established (at least for the simplest lattices), the behavior of finite samples in the region of negative effective permeability (which is usually the region of interest) is far from being fully understood. Further research is necessary on this topic, of key importance for practical applications [38].

IV. APPLICATIONS AND FUTURE TRENDS

Metamaterial is a relatively new concept, which has provided in the recent years new ideas for the design of old devices such as antennas, frequency selective surfaces, and microwave circuits and filters. Some of these applications of RRM can be seen in [8] and references therein. However, 3-D metamaterial technology itself, that is, the technology for the development of bulk effective artificial media providing new electromagnetic effects, such as super-resolution or cloaking, is still in a very initial stage.

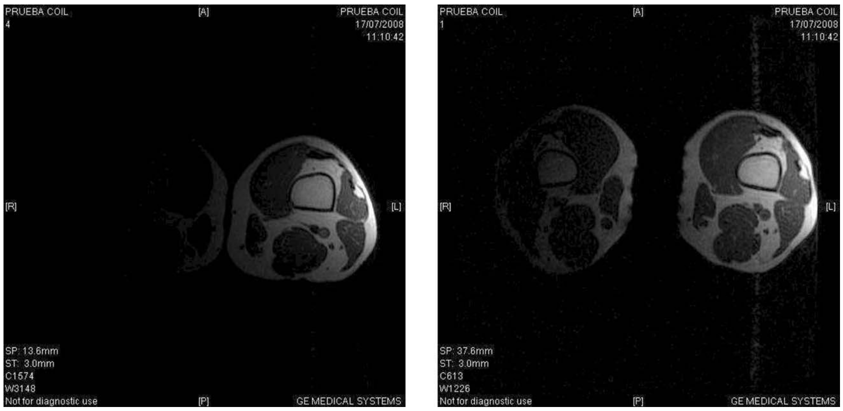

Fig. 4. ***MRI images of two human knees obtained without (left) and with (right) the help of a $\mu = -1$ metamaterial lens. The lens is located between both knees (black region the image). Reprinted with permission from [40]. Copyright 2008, American Institute of Physics.***

Therefore, it is not strange if practical applications of bulk metamaterials are still far from being developed. At the present stage of the technology, low-frequency narrow-band applications—for instance, in magnetic resonance medical imaging (MRI) using "swiss rolls" [39], and resonant rings [40]–[43] 3-D metamaterials—seem to be the most promising ones. An example of this kind of applications is shown in Fig. 4, where the enhancement of the image of two human knees obtained using a $\mu = -1$ RRM lens can be seen. Other examples can be seen in [43], where $\mu \rightarrow 0$ and $\mu \rightarrow \infty$ RRM slabs are used in order to enhance the sensitivity of MRI surface coils.

Applications of bulk RRM in microwave and millimeter wave technology can also be envisaged, with the technological obstacles related to the fabrication process, which may imply the assembling of thousands of micro- or nano-structured elements. Applications at optical frequencies are more challenging, due to the effects analyzed in Section II-B.

From a theoretical point of view, the properties of bulk RRM still present important challenges, mainly with regard to the analysis and characterization of realistic finite samples, which should probably be analyzed using the theoretical tools of the physics of mesoscopic systems. ■

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