

**Czech Technical University in Prague**  
**Faculty of Electrical Engineering**

# **Doctoral Thesis**

*September 2014*

*Ing. Marian Kněžek, ACCA*

Czech Technical University in Prague

Faculty of Electrical Engineering

Department of Economics, Management and Humanities

***RISK MANAGEMENT IN ELECTRICITY  
SALES OF POWER PRODUCER***

**Doctoral Thesis**

***Ing. Marian Kněžek, ACCA***

Prague, September 2014

Ph.D. Programme: Electrical Engineering and Information Technology

Branch of study: Business Management and Economics

**Supervisor: *Prof. Ing. Oldřich Starý, CSc.***

# Annotation

There were on-going developments in the electricity markets in recent years, in particular the establishment of Energy Power Exchanges and OTC markets and the associated transfer from campaign-driven sale to continual trading. It represents a significant change for electricity trading companies and power producers who face these developments on daily basis. These changes force trading companies and power producers to review their business strategy and re-establish their trading processes. The measure of success is based on maximization of profit, calculated by subtracting the realized sales from the realized purchases and in case of power producers incurred production costs.

This work is in some aspects unique. It comes from author's three years' experience as a senior consultant in operational consulting company Grant Thornton and six years' experience as an auditor in Ernst & Young. Most of author's clients are world well-known companies - financial institutions, power engineering companies and IT companies. The aim of this theses is to bring new visions based on deep knowledge and experience of the internal processes of these companies.

By comparing electricity markets with more matured financial markets we get an unique chance to find out directions to reestablish electricity markets and increase their efficiency. It sees power producers as banks and electricity traders as financial market traders. This approach helps to find out new market options of power producers based on similarities identified.

# Acknowledgements

This study would not have been possible without the support and assistance of a number of people. I would like to thank a few of those who in some way contributed to the completion of this doctoral thesis.

First I would like to thank to supervisor of my doctoral thesis Prof. Ing. Ondřich Starý, CSc. for professional leadership, objective comments and useful advices given during the time spent on the thesis. He freely shared his knowledge, his time and guidance. Also I'd like to thank to the head of our department Doc. Ing. Jaroslav Knápek, CSc. who reviewed the thesis and provided me with suggestions and consultations for improvement of the thesis.

Secondly I would like to thank my professors at the Czech Technical University in Prague for the education I gained during my studies. Many thanks belong to my colleagues at work and in the university. Also I am grateful to all the professionals I met and with whom I was able to discuss individual areas of this thesis.

Lastly I would like to express my gratitude to Irina, my wife, with whom I am married for ten years. She always provided me with the patience and encouragement I needed. And also to my parents who sacrificed their lives for me and provided their love and care. Without them I would not have made it this far.

# Content

About author.....	7
Terminology used .....	10
1 Introduction.....	14
1.1 Motivations.....	14
1.2 Goals of the doctoral thesis .....	15
1.3 Hypotheses .....	15
1.4 Methodology used in research.....	16
1.4.1 Delta hedge - Black-Scholes formula and Taylor series.....	16
1.4.2 Gross Margin.....	18
1.4.3 Value at Risk .....	18
2 Current situation.....	20
3 Portfolio valuation through Gross Margin.....	40
3.1 Closed historical position .....	41
3.2 Closed future position .....	41
3.3 Forward price curves.....	42
3.4 Open future position .....	44
3.5 Total Gross Margin and difference from plan fulfilment .....	45
4 Open position risk exposure and its valuation .....	46
4.1 Risk exposure .....	46
4.2 Calculation of VAR .....	47
4.3 Assumptions for calculation of VAR.....	49
4.4 Risk management and limits .....	51
4.5 Breach of Risk Management limits .....	52
5 Financial products used by power producers.....	53
5.1 Greeks .....	54
5.1.1 Delta.....	54
5.1.2 Gamma.....	56
5.1.3 Vega.....	59
5.1.4 Theta .....	60
5.1.5 Rho.....	62
5.1.6 Greeks charts .....	63
5.2 Approximations by using Greeks .....	65
5.2.1 Delta aproximation .....	65
5.2.2 Delta-gamma aproximation.....	68
5.2.3 Delta-gamma-theta approximation .....	70
5.2.4 Delta-gamma-vega-theta-rho approximation .....	71
5.3 Delta hedge.....	73
6 Implementation of the results in practice .....	77
6.1 Physical delivery.....	78

6.2	Dealing on proprietary desk .....	79
6.3	Writing options .....	79
7	Conclusion .....	81
7.1	Summary of the results including the benefits of the thesis.....	82
7.2	Further development of the research .....	85
	Appendix A: Model – Gross Margin portfolio valuation & VAR calculation.....	92
	Appendix B: Greeks formulas .....	93
	Appendix C: Model – Greeks for the option pricing approximation.....	95
	Appendix D: Model – Delta Hedge in the electricity market application.....	97
	References .....	87

# About author

Marian Kněžek is currently living in Denmark. He is chartered certified accountant – ACCA<sup>1</sup>. He speaks fluent English, Russian and Danish. During his university studies he got several academic awards and scholarships – e.g. McKinsey & Company scholarship, General Electric scholarship, Dean's price for the excellent diploma thesis, CEZ scholarship, Preciosa scholarship, etc.

## Ernst & Young

In 2008 he joined Ernst & Young Prague office (Big 4 company). Since January 2012 he works for Ernst & Young Denmark.

As an auditor I got unique opportunity to get an invaluable knowledge of the businesses from different areas. My responsibilities are development and maintenance of productive working relationship with clients, negotiation of audit fees, seeking new opportunities with client, preparation of the budget and follow up on gross margin. Identify internal controls and develop audit approach to the changing client environment. Management of audit teams through effectively delegated tasks and provision of guidance. Selected engagements are:

### Engagement Manager:

- DK: Siemens AS (engineering)
- DK: Intel Copenhagen ApS (IT)
- DK: Intel Denmark ApS (IT)
- DK: Citrix Systems Denmark ApS (IT)
- DK: McAfee ApS (IT)
- DK: Google Aps (IT)
- DK: DigiPos A/S (IT) – Nordics (Denmark, Norway, Sweden, Finland)
- DK: Tellabs Denmark A/S (IT)
- DK: Wonderware Scandinavia (IT) – branch of Wonderware Scandinavia AB (Sweden)

### Supervisor:

- DK: Vattenfall A/S (utilities)
- DK: Hewlet Packard A/S (IT)
- DK: TT Network A/S (Telia and Telenor telecommunication infrastructure company)
- DK: Orange (telecommunications – France Telecom group)

---

<sup>1</sup> The Association of Chartered Certified Accountants (ACCA) is the international body for professional accountants based in London (United Kingdom). This association was founded in 1904 and proffers the Chartered Certified Accountant qualification. The term 'Chartered' in ACCA qualification implies to the Royal Charter granted in 1974 by Her Majesty the Queen in the UK.

- DK: Nomeco A/S (logistics)
- DK: Alfa Laval A/S (production)
- DK: Trevi Foundations A/S (civil engineering)
- DK: Copenhagen Metro I/S (line C - civil engineering)
- DK: Premiere Global Services ApS (IT)
- DK: DigiPos Nordics A/S (IT)

Senior:

- CZ: Komerční banka, a.s. (one of three biggest banks in the Czech Republic – Société Générale group)
- CZ: Česká spořitelna, a.s. (one of three biggest banks in the Czech Republic – ErsteBank group)
- CZ: REICO investiční společnost České spořitelny, a.s. (investment company)
- CZ: Factoring KB, a.s. (factoring company)

Assistant:

- CZ: Aviva životní pojišťovna, a.s., (life insurance)
- CZ: AEGON Pojišťovna, a.s., AEGON Penzijní fond, a.s. (life and pension insurance)
- CZ: ING Životní pojišťovna N.V., ING Penzijní fond, a.s. (life and pension insurance)
- CZ: ČSOB Pojišťovna, a. s. (life and nonlife insurance)
- CZ: ŠKODA PRAHA a.s.. ŠKODA PRAHA Invest s.r.o. (reconstructions of power stations)
- CZ: Boston Scientific Česká republika s.r.o., GUIDANT ČR s.r.o. (medical instruments)

**Grant Thornton**

In 2006 he started to work as a consultant for Grant Thornton advisory services during his university studies at Czech Technical University in Prague. He designed and programmed number of models and applications for the TOP 100 companies in the electricity market in the Czech Republic. His expertise were primarily risk management, processes optimization and business strategy settings. Selected engagements are:

- ČEPS, a.s. (el. transmission)

I designed and programmed a Loss Event Database and a statistical model for operational risk quantification for the whole company. This model works on the Monte-Carlo simulation.

- ČEPS, a.s. (el. transmission)

I set a methodology and programmed a statistical model for managing two main processes of the company Optimization of purchases of Ancillary Services (AnS), Optimization of cross-border capacity supply



- ČEZ, a.s. (electricity producer – the biggest in the Czech Republic)  
I setup a methodology and programmed a statistical model for forward electricity price curves calculation in the markets of Central and Eastern Europe.
- Pražská energetika, a.s. (electricity producer – 2<sup>nd</sup> biggest in the Czech Republic)  
I designed and programmed an application for calculating the FX open position and its valuation by the Value at Risk methodology.
- Pražská teplárenská, a.s. (heat and electricity producer – biggest in the Czech Rep.)  
I prepared a Request for Proposal documentation for creating a software for the optimisation of heat stations utilization in real time.
- Česká spořitelna, a.s. (bank – belongs to three biggest banks in the Czech Republic)  
Business Case preparation for the entry to the newly established Power Exchange in Prague. I analyzed the current electricity market and calculated a Cost-Benefit analysis for 3 scenarios (base case, optimistic, pessimistic).
- UniCredit Bank Czech Republic, a.s. (bank)  
I participated in the project related to Basel II implementation in the bank.
- Kooperativa, a.s. (insurance company – 2<sup>nd</sup> in the Czech market)  
I designed and programmed a Loss Event Database and Risk Catalogue according to Solvency II requirements.
- Ministry of Agriculture  
I have led a project for the optimization of the public procurement.

# Terminology used

## **American options**

An option that can be exercised anytime during its life.

## **Arbitrage**

A transaction which generates profit with zero level of risk.

## **Asian options**

An option that can be both the American and European type. Its payoff is linked to the average value of the underlying asset on a specific set of dates during the life of the option.

## **At the money - ATM**

Instruments whose realisation price is the same as the actual market price of underlying asset.

## **Exchange**

Exchange is an organised market, where standardised contracts are traded. Exchange is a counterparty of each settled contract. In case of electricity market there are PXE, EEX, NordPool, APX, etc.

## **Derivatives**

Derivatives are hedged contracts, whose values are derived from the value of underlying asset. These contracts can be distinguished as linear i.e. forwards, futures, swaps and as nonlinear i.e. options.

## **European option**

An option that can be only be exercised at the end of its life.

## **Forwards**

Linear hedge instruments which represent for buyer an obligation to buy a specific amount of underlying asset for concluded value (realisation price) and date and for seller an obligation to sell that amount in the same conditions. The conditions of the contract such as price, volume and the delivery period are defined in the agreement between the buyer and the seller. Forwards are traded only in OTC markets (over-the-counter), but not in exchanges.

## **Futures**

Linear hedge instruments which represent for buyer an obligation to buy a specific amount of underlying asset for concluded value (realisation price) and date and for seller an obligation to sell that amount in the same conditions. The conditions of the contract such as price, volume and the delivery period are defined in the agreement between the buyer and the seller. In contrast with forwards are futures traded only in organised exchanges, but not in OTC markets. The conditions including standardised underlying asset are set by the exchange.

## **Greeks**

Measures used to evaluation dimension of the risk involved in taking a position in an option. Each measure of the risk is represented by a letter of the Greek alphabet except Vega.

**Δ - Delta**

Delta represents the rate of change between the option's price and the underlying asset's price - in other words, price sensitivity. Delta is used to measure the slope of the option value line at any particular time/price point.

$$\Delta = \frac{\partial \text{option price}}{\partial \text{underlying asset's price}}$$

**Γ - Gamma**

Gamma is the first derivative of Delta. Gamma represents a rate of measures how much Delta changes with the underlying asset value ie how much the delta hedge needs to be adjusted as the underlying asset value changes, for example if the gama is 0,01 this means that for a 1% rise in the underlying asset value the delta should change by a factor of 0,01.

$$\Gamma = \frac{\partial \text{Delta value}}{\partial \text{underlying asset's price}}$$

**Θ - Theta**

Theta represents the rate of change between an option portfolio and time, or time sensitivity. In general the longer to expiry, the more valuable the option is.

$$\Theta = \frac{\partial \text{option value}}{\partial \text{time}}$$

**V - Vega**

Vega represents the rate of change between an option portfolio's value and the underlying asset's volatility, or sensitivity to volatility. In general the more volatile the underlying asset, the more valuable the option is.

$$V = \frac{\partial \text{option value}}{\partial \text{volatility}}$$

**ρ - Rho**

Rho represents the rate of change between an option portfolio's value and the interest rate, or sensitivity to the interest rate. In general the higher the interest rate, the more valuable the option is.

$$\rho = \frac{\partial \text{option value}}{\partial \text{interest rate}}$$

**Hedging**

Hedging is a strategy designed to minimize exposure of market risks. It is based on offsetting of two opposite open positions.

**In the money - ITM**

Instruments whose realisation price is higher than actual market price of underlying asset. The exceptions are call options whose realisation price is lower than market price of underlying asset.

**Linear instruments**

Instruments whose market value is linear dependent of market price of underlying asset. Among typical representatives of these contracts belong forwards, futures and swaps.

**Long position**

- a) Position of derivative's buyer.
- b) Position of portfolio, where the company has excess of volume of underlying asset. This excess is necessary to sell.

**Market position**

Market position represents market net present value of the portfolio. It consists of closed and open position.

**Maturity date / expiration date**

The date accepted within the contract conclusion on which a debt becomes due for payment also called maturity.

**MtM (Mark to Market)**

Net present value of portfolio's open position.

**Nonlinear instruments**

Instruments whose market price is nonlinear dependent on market price of underlying asset. Among typical representatives of these contracts belong options.

**Options**

Nonlinear hedge instruments written by a seller that conveys to the buyer the right but not the obligation to buy (Call options) or to sell (Put options) as of maturity date a prior concluded volume of underlying asset for concluded price (realisation price). In return for granting the option, the seller collects a payment (the premium) from the buyer. The seller has an obligation to buy/sell the underlying asset for the concluded price in case that buyer decides to exercise the option.

**OTC (Over The Counter)**

OTC is the market of non-exchange character, which works on computer based network among participants. In case of electricity market we speak about broker systems as are GFI, TFS, Spectron, Prebon, Icap, aj.

**P&L (Profit and Loss)**

Net present value of profits and losses of portfolio's closed position.

**Portfolio**

Portfolio is an electricity volume of all commitments to produce, to sell or to buy concluded in all contracts in time. The value of portfolio represents market position.

**Proprietary trading**

Trading of financial instruments with the firm's own money to make profit on its own desk.

**Risk**

Is an event which may negatively affect the economic result of the company. This event occurs with certain probability.

**Risk Appetite**

An enterprise's willingness and ability to take on risk which is able to accept in connection with the business activity.

**Risk management**

Risk management is a structured approach of certain precautions which lead to reduction or elimination of risks.

**Out of the money - OTM**

Instruments whose realisation price is lower than actual market price of underlying asset. The exceptions are call options whose realisation value is higher than market price of underlying asset.

**Short position**

- a) Position of derivative's seller.
- b) Position of portfolio, where the company has an shortage of volume of underlying asset.  
This shortage is necessary to buy.

**Spot market**

Market where the underlying asset is traded. This asset will be delivered in range of several hours (Hour ahead market) to one day (Day ahead market).

**Stress testing**

Stress testing is a sensitivity analysis which represents how big negative change has to occur in order to influence the portfolio so much that it has an impact to the economic result of the company.

**Swaps**

Linear purely financial hedge instruments in which two parties agree to exchange one stream of cash flows against another. The swaps are traded in the OTC markets (over-the-counter).

**VAR (Value at Risk)**

VAR is a measure of the risk of a specific portfolio at a specified probability and over a specified time horizon in case of normal market trend. VAR takes into account fixed portfolio which is not changed within the defined period.

# 1 Introduction

Electricity markets have been recently deregulated in Europe and the USA. Electricity became a commodity freely traded. New researches related to modelling and prediction of how markets will react to the new structure appeared. As in all other markets, there is a wholesale market and a retail market and there are the three usual players: the producers, the traders and the consumers. However a more advanced trading pattern quickly developed and new players enter the scene: the exchanges, the brokers. Previously the markets were regulated by the government. There was no competition between the companies as each company had its set of consumers and no choice of which company to use. Prices were set to allow companies to recover their costs. In contrast the only goal of deregulated company is to maximize profit.

Electricity markets became closer to fully developed matured financial markets. Based on similarities, comparisons and analysis of banks position in the financial markets we can observe opportunity gaps for power producers in the electricity markets.

This doctoral thesis is worked out on strong fundamentals described in my diploma thesis [51]. Parts of the thesis have been publically presented on international student conferences on electrical engineering and published –for detail please refer to [52], [53], [54], [55], [56]. There are also articles under review to be published in impacted journal and peer-reviewed journal.

## 1.1 Motivations

Banks have established standard products that are developed to the perfection. The question is whether these products could be transformed into the electricity market given the fact that the underlying asset is completely different, has different features in terms of production and utilization. On the other hand it can be argued that power producers have in some ways better position than banks in the financial market as they are able to generate underlying asset in contrast with banks that are not able to print money. Comparing this advantage with current financial products we might get huge opportunities.

In order to identify market opportunities we need to have a look on the electricity market participants and their roles, product, responsibilities and regulation. In this respect it is necessary to look at the power producer from three perspectives:

- look at the power producer as a standalone entity,
- compare power producer's role in the electricity market with the role of the banks in the financial markets,

- identify products that are used for years in the financial market that can be used in the current electricity markets

The aim of this work will be to identify these electricity market opportunities and find the effective way for the utilization of these opportunities.

## **1.2 Goals of the doctoral thesis**

The aim of this work is to bring new ideas and or directions to the current power producers practice. The main goal would be to evaluate possibility of power producer to provide typically financial market products to its customers that are usually provided by the financial institutions. Especially, whether the power producers are able to effectively write electricity market options and take over this role from the banks.

In order to achieve this goal we need to understand current power producer's situation. It is questionable, what is the right measure for the power producers to evaluate its portfolio. There different contracts agreed with counterparties for wide range of period of time. Some of the contracts have expired while others are still running or have not yet been utilised. Due to this fact the power producers' portfolios get into different positions all the time (closed historical, closed future or open position). The single unitary measure would be much of help to value the portfolio. The measure should be also able to evaluate new products e.g. options the same way. Another goal of this doctoral thesis therefore will be to determine the measure for the power producers' portfolios evaluation.

Power producers may also consider opportunity to trade the electricity on its own desk the same way like electricity market traders do. These speculations on electricity prices movements may get the portfolio into unfavourable position. It would be questionable, what is the maximum risk the electricity producer is willing to face and how to measure the risk. Last goal therefore will be to determine measure the risk of the open position.

This work does not aim to compile findings and results of others university or consulting colleagues researches, thesis, reviews or analysis. Its main aim is to bring new views based on author's knowledge gained from the university and consulting and audit practices.

## **1.3 Hypotheses**

In the light of the main goal above the main hypothesis will be that "Power producers are able to get into arbitrage and generate risk free profit through delta hedge based on self-issued option contracts and possibility to generate underlying electricity."

There are different kinds of measures of individual electricity portfolio positions. To create strong fundamentals for the main hypothesis and in line with necessity to evaluate power

producer's portfolio the sub-hypothesis needs to be determined: "Evaluation of the power producer's portfolio should be based on Gross Margin."

In case of proprietary trading the strengths and weaknesses of power producers need to be analysed. By trading on its own desk power producer gets into risky positions that may cause damage of immense size. The last sub-hypothesis will be: "The most suitable measure to evaluate risk of open position should be based on Value at Risk metrics."

## **1.4 Methodology used in research**

To test theoretical hypothesis established above a combination of different kind of methods will be utilised. The core of the thesis will be based on analogy and experiment as already established well known quantitative techniques used in the financial markets by banks will be analysed and applied to the electricity markets for power producers. It is questionable whether the patterns typical for the electricity markets represent any barriers in order to apply these methods in the electricity market:

- non-storability of electricity, production and consumption has to be perfectly synchronised so the demand and supply are all the time in balance,
- dependence on weather/seasonality (e.g. differences in consumption during the hot summer/cold summer/winter or long and short days),
- high volatility of the market prices,
- limited production capacity and its structure,
- ...

The research was performed in four consecutive steps:

- Analysis of all relevant researches already performed in this area.
- Evaluation and usage of the previous researches in this thesis.
- Evaluation of the goals of the doctoral thesis and hypothesis related,
- Applicability of theory on the practical model.

### **1.4.1 Delta hedge - Black-Scholes formula and Taylor series**

First goal relates to evaluation of possibility of power producer to provide typically financial market products to its customers that are usually provided by the financial institutions – power options.



There are few articles that deal with hedging strategy for power producers, however there is no article with the same or similar approach related to possibility of power producer to write options. Also term of Delta hedging is used for different things in these articles. In most of the articles authors are using word “delta” as a general term for a deviation. For example in [35] Bundalova, DeJong and VanDijken calls delta a difference between sales price of electricity futures and purchase price of fuel. While in this thesis it describes first derivative of Black-Scholes formula according to electricity price.

The only delta hedge with the same meaning is used in the article called Energy Modelling and Management of Uncertainty [43], where Eydeland and Geman says that the delta hedge in the electricity market cannot be implemented due to non-storability of the electricity as electricity has to be bought and kept for some time. The article does not describe this fact in detail and therefore it is hard to understand the reason to provide this statement. However this is strong statement which can be easily disproved. The trading with electricity prices is not about physical settlement and storability but about financial settlement. In this respect it does not really matter whether the electricity market participant actually has the physical power or facility to produce it. Further this thesis deals directly with power producer which can produce agreed amount of electricity at any time in its facility so the electricity is stored in its input, which is fuel (coal, gas, ...).

Delta hedging is a method of ensuring that the value of a portfolio of options will not fluctuate when the price of the underlying asset fluctuates. Delta hedging is based on combination of option pricing formula and Taylor series. The most common option pricing model used is Black-Scholes formula [32]. Many empirical tests have shown that option prices according to the Black-Scholes pricing formula are "fairly close" to the market prices. Using of Black-Scholes pricing formula was also argued in [62], where Pineda and Conejo says that due to non-storability of the electricity Black-Scholes pricing equation “is not generally a good method for pricing electricity derivatives”. The only argument they have is a link to [67], where Wu describes pricing of European options based on the fuzzy pattern of Black-Scholes formula. Wu’s paperwork is based on stock prices and does not relate to electricity market. Wu is trying to modify Black Scholes formula to implement jump diffusion model. Wu admits that even though his modification to the standard Black-Scholes formula gives slightly more accurate results, it boosts (as he says complicates) pricing model greatly. In this respect it is not applicable in practice and leaves this model in the theoretical area and provides reasonable argument that Black-Scholes model is still valid and mostly used option pricing tool in practice. Stochastic modelling of forward electricity prices and using of hedging strategies including analysis of temperature derivatives is published in [3] by brothers Benth and Koekebakker.

Delta hedge is based on application of Taylor series on first partial derivative of Black-Scholes model. It helps us to evaluate the amount of underlying asset needed to maintain neutral portfolio – Delta hedge, get power producer into arbitrage and generate risk free profit. By applying Taylor series on option delta we obtain options value approximation. This

approximation allows effectively reevaluate option contracts in the portfolio. This is convenient due to the high demand on IT solutions bearing in mind the amount of data that need to be processed.

### **1.4.2 Gross Margin**

Gross Margin represents the sales revenues that company retains after incurring the direct costs associated with the production sold by the company. In this respect we would need to identify power producer's portfolio, divide it into positions and assess whether the Gross Margin is the right measure for the evaluation of the individual positions.

There are number of articles that relate to pricing in the wholesale electricity market. For example in [34] Borenstein marginal costing and divides firms into market makers as those who exercises market power when it reduces its output or raises the minimum price at which it is willing to sell output (its offer price) in order to change the market price and price taking firms as those who are taking price given in the market. This is usually typical for the monopoly or oligopoly markets. It might be also the case in the Czech Republic if the biggest producer CEZ is completely privatised. In this case the intervention of power exchange, independent system operator or government regulatory body is necessary to alter the operation of the wholesale electricity market. However as the price-taking firm still sells its output at the market price which is usually above the marginal production cost of all or almost all the production, price-taking firms can still cover their full costs, including their going-forward fixed costs of operation.

The competitiveness and the exercising of power in the electricity market would be above the scope of this thesis. After deep research of the scientific sources there were no articles that are focusing on the portfolio valuation of power producer from the trading perspective. There is a need to categorize market positions and use unitary measure for the business evaluation.

### **1.4.3 Value at Risk**

Power producer is able to generate electricity. Due to this fact its risk of open position might be rather limited or eliminated depending on the strategy and risk appetite of the power producer. To evaluate the risk of open position, JP Morgan [50] developed RiskMetrics methodology called VaR (Value at Risk). VAR is a measure of the risk of a specific portfolio at a specified probability and over a specified time horizon in case of normal market trend. VAR takes into account fixed portfolio which is not changed within the defined period.

According to [58] another approach for VAR calculation might be Extreme Value Theory which can capture extremities better than conventional probability distributions, as it concentrates on the observations that exceed certain limit. The attention is focused on the tail of the price distribution, which is the area where the VAR is estimated. The question is whether these so called fat tails is the main issue why the conventional probability distributions do not describe the pattern of the electricity market prices fluctuations. It is

apparent from Figure 22 that the main issue is given by the shape of the theoretical distributions before the tail starts and not by the tail itself. Also there is no standard interpretation of what the fat tail is. Pearson's chi-squared testing of the probability distribution is attached in the form of XLS file on the attached CD. In the related table for probability distribution testing we can see that the differences between empirical probability distribution and the real market prices are high before the fat tail starts. The question is whether the Log-normal distribution could be used for the Value at Risk calculation at the fat tail area.

Analysis of Extreme Value Theory technique is even in more detail published by Gencay, Selcuk, Ulugulyagci in [45]. This research is based on stock market S&P500 and ISE indexes. As already mentioned in introduction financial markets are more developed and have different pattern than electricity markets. One of the main point will be that stock price can increase several times during a short period and remain on the same level for years, while electricity do not have such a huge jumps and always get back to an average price level in short time period.

Extreme Value Theory became evergreen for past ten years and there are number of authors like Paul Embrechts, Alexander McNeil, Richard Smith, Rüdiger Frey, Francois Longin, Hans Byström and Kimmo Lehikoinen that performed number of articles on this topic. These articles define EVT VAR which is modified Value at Risk technique described above for the extreme electricity market prices fluctuations.

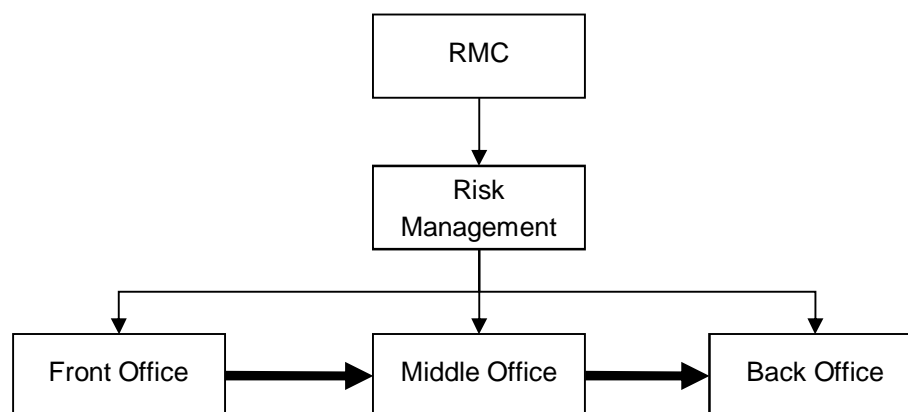
As power producers have possibility to generate missing amount of electricity if they are in a short position or stop producing electricity if they are in the long position, they can always avoid rare situation of high electricity price movements. Due to this fact Extreme Value Theory seems to be redundant for power producers.

## 2 Current situation

Most of the electricity markets in the industrial countries have been deregulated. Electricity turned into an asset freely traded in the market place. Power producers became a part of electricity market and as well as traders tend to sell produced electricity in the most favourable way to increase its profit. Power producers are exposed to electricity market risk through the trading of electricity and electricity related derivatives as well as commodities associated with electricity production. Electricity prices are highly unpredictable and volatile. High volatility reflects lower liquidity of the market products and hence the maturity of the market. Investments in trading activities and risk management expertise are soaring as companies become aware of market risks in their businesses. To ensure the rules of free market the demand side (customers) also has the option to choose electricity supplier.

The risks in general are potential unknown threats which flow from extraordinary situations. In the financial markets there are all kinds of stochastic and deterministic models and their modifications and combinations to address these risks. The field of risk management in the energy sector especially related to power producers is still in its infancy. The comprehensive literature on this topic does not yet exist and most companies, which deal with risk management in the electricity market keeps their best practice in secret.

In order to effectively manage the risks related to the trading operations, the responsibilities, duties and organisation of the trading and risk management group is closely related to basic company organisation whilst maintaining separation between the front, middle and back offices of the trading department.



**Figure 1:** General risk management structure

Front office is responsible for the merchant trading operations with approved instruments and on approved markets within authorized limits.

The Front Office responsibilities are as follows:

- Contract the deals and generate trade ticket.
- Enter new contracts into the trading system.
- Evaluate new opportunities in the electricity market (exchanges or OTCs).
- Evaluate positions against trading strategy.
- Fulfil hedging strategies and requirements agreed at the RMC.
- Operate within individual trading limits approved by RMC.
- Maintain and develop relationships with counterparties to ensure there are sufficient routes to market in order to efficiently hedge commodities.
- Monitor weather forecast.
- Monitor cross border capacities market.
- Provide selected information regarding counter-party.
- Submit requirements for evaluation of counter-party.

Middle office is responsible for processes that support front office, back office and Risk Manager. Middle office analyses markets and evaluates data for other departments. In small enterprises is middle office part of front office.

Middle office responsibilities include:

- Verify concluded contracts.
- Calculate MtM a P&L of portfolio's position and VAR.
- Reports for Risk Manager.
- Forecast electricity and CO<sub>2</sub> prices and future demand vs. supply in the market.
- Gather and prepare information for evaluation of counter-parties.
- Monitor current depletion of limits of involvement.
- Monitor compliance with assessed limits.
- Maintain and distribute a list of approved counterparties.

Back office is responsible for the settlement of all contracts on a daily, weekly and monthly basis. Back office is also responsible for the confirmation of trades and settlement of broker bills.

Back office responsibilities include:

- Ensure trades are checked and confirmations sent to counterparties on the Business Day following execution of the trade.
- Ensure confirmations are signed and returned by counterparties within necessary timescales.
- Weekly and monthly invoicing of the following:
  - CO<sub>2</sub> emissions trades.
  - Electricity trades.
  - Ancillary services trades.

- Derivatives products.
- Agree netting statements with counterparties.
- Checking and settlement of power exchange trades.
- Liaise with Accountants to ensure correct payments.
- Providing information for financial accounting.
- Checking and maintaining of receivables.
- Measure, monitor and report credit exposure by counterparty.
- Reporting of overdue receivables to the Risk Manager.

## **Risk Management**

Risk Management is responsible for establishing and maintaining appropriate risk framework across the trading process. Risk Management has no responsibility for trading activities and must remain independent of trading. Risk Management also takes responsibility for the measurement and management of counterparty credit risk, finance risk, compliance with policies and procedures and ensuring adequate IT systems are available. The overall risk management in deregulated electricity market is described by Rickenlund [63]. Another perspective is provided by Hung-Po in [49] with number of questions for further development. Besides the articles described above a full insight overview has been provided by Leppard in [22], which provide clear understanding of energy risk management issues, illustrates the purpose of energy derivatives, and evaluates their benefits and weaknesses.

Risk Management responsibilities include:

- Ensure that appropriate risk and control frameworks are in place.
- Review and maintenance of risk management policies.
- Co-ordinate and participate in RMC meetings.
- Ensure that trading department operate within the policies, limits and authorities approved by the RMC.
- Measurement, monitoring and reporting of all trading risk positions.
- Ensure maximum benefit from risk exposure.
- Work with Internal Audit to meet the latest governance requirements.
- Ensure appropriate legal documentation is in place.
- Ensure policies, procedures and limits are consistent with the Risk Management Policy.
- Prepare methodology of credit risk management and credit rating.
- Review existing credit limits.
- Propose limits of involvement with counter-party.

## **Risk Management Committee (RMC)**

Risk Management Committee provides advice to the Board and reports on the status of the business risks to the Company through its risk management processes aimed at ensuring risks

are identified, assessed and properly managed. It is the Board's responsibility to ensure that an effective internal control system is in place across the company.

The RMC oversees the financial reporting process, the systems of internal control and risk management, the audit process and the Company's processes for monitoring compliance with laws and regulations. The Committee meets at least four times a year and reports to the full Board following each meeting, including in respect of recommendations of the Committee that require Board approval or action.

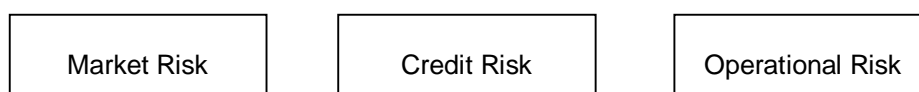
Risk Management Committee responsibilities include:

- Set Risk Management Policy regarding risk management, procedures, and controls.
- Ensure that the Risk Management Policy is aligned with the company's strategic objectives.
- Review with management the system for identifying, managing and monitoring the key risks of the organisation.
- Review the effectiveness of the company's internal controls.
- Create and control the framework and delegation of authority within which risk is managed.
- Define limits on risk that are acceptable to the business and recommend risk limits to the Board.
- Inspect compliance with RMC approved limits.
- Approve trading instruments and commodity types.
- Approves credit risk management methodology
- Approves proposed limits
- Monitor the status of prior defined key risk exposures or incurred incidents.
- Monitor any breaches of policies, procedures or limits and any action taken to remedy the breach.

## **Risks and internal regulations**

Power producer is exposed to different risk positions through its operations. This part outlines the main risk positions as well as the internal regulations used to eliminate or reduce related risks, and ensure power producer's operations.

As well as in financial institutions that must organize their risk management program according to Basel II the risk management for energy trading companies could be divided into these three main risks categories:



**Figure 2:** General overview of main risks categories

Each of these risks must be managed independently. It means that for each of these risks has to be designed risk management guideline which involves processes and procedures and also organisation and responsibilities. The elementary introduction into risk management, with summary chapters on market risk, credit risk and operational risk is worked out by Jorion in Value-at-Risk: The New Benchmark for Managing Financial Risk [20]. There is a brief discussion of risk management in banking, investment management, pension funds, and corporate settings.

## **Market risk**

Market risk is the risk of financial loss resulting from the impact of an adverse electricity price movement. Power producer's portfolio consists of physical and financial contracts with a residual amount of un-hedged output. The impact of adverse price movements is dependent on the availability and efficiency of the power stations.

Market Risk is managed by:

- Continual review of market movements and development
- Quantitative limitations on the scope of authorized activities
  - Maximum cost of one trade.
  - Maximum daily cost of one trader.
  - Maximum daily volume of purchased/soled electricity of one trader.
  - Market price limit of one traded MWh
- Quantitative guidelines and limits for managing risk exposure
  - Maximum daily MtM of open position (continuous evaluating)
  - Maximum monthly MtM of open position (continuous evaluating)
  - Maximum annual MtM of open position (continuous evaluating)
  - Maximum MtM of individual contracts with defined counterparties
- Regular Risk Management Committee meetings
- Regular review of portfolio positions
- Training of the traders and administration
- Open communication with risk manager

## **Credit Risk**

Credit Risk is the risk that a trading partner does not fulfil its obligation in full as at due date or at any time thereafter. This risk is characteristic for bilateral contracts like forwards as futures are guaranteed against default by the power exchange. This risk consists of counterparty concentration risk in the event that there is a lack of diversity of counterparties, and credit grade concentration risk if counterparties with lower credit rating represent a larger part of the portfolio.

Credit risk is managed by:

- Defined methodology for setting credit rating and involvement limit for counterparties.
- Approval of counterparties by RMC



- The adoption of standard agreements for concluding contracts with counterparties
  - EFET (European Federation of Energy)
  - GFA (General Framework Agreement)
- Regular assessment and reporting of credit exposure to each counterparty credit grade concentration risk and counterparty concentration risk.
- Monitoring and evaluation of the value of credit C-VAR as maximum possible loss caused by credit risk at a specified probability and over a specified time period.

It is necessary to set the rules for specifying who we are willing to trade with, setting out the counterparties currently approved for trading as well as revising limits with existing trading parties.

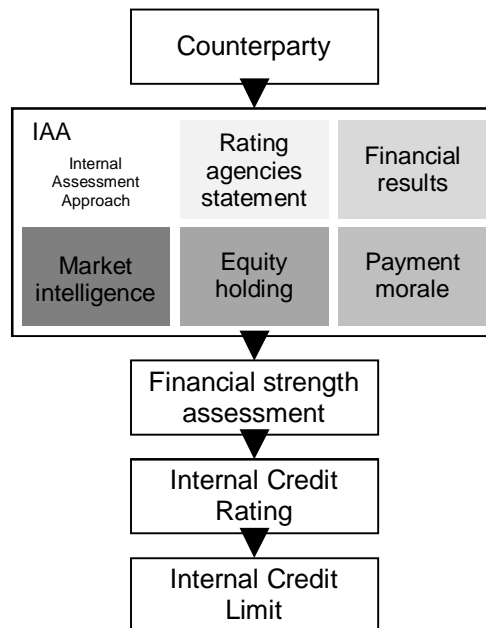
Counterparties that have met the specified requirements and have been approved by the RMC are defined as “Approved Counterparties” and are listed in register, which is regularly updated and approved by the RMC.

The Risks related to the possibility of losses coming from the ability of trading counterparties to keep all contract obligations is limited by credit limits. Credit limit is assigned based on a careful evaluation of the economic situation of each counterparty and defines maximum cumulative payment in form of open positions against each counterparty which includes all valid contracts.

The RMC reviews the credit rating policies. All reviews are to be based upon current counterparty information, the latest financial statements, the current trading relationship and use of credit limits. Counterparty credit limits will be reviewed immediately following an adverse change in credit rating.

If credit exposure to a counterparty increases such that a counterparty credit limit is breached, the Risk Manager will immediately inform the RMC and traders and all trading with such counterparty is blocked until the credit exposure is reduced to less than the credit limit.

The internal assessment approach for counterparty credit risk management evaluation is described on Figure 3.



**Figure 3:** Counterparty credit risk management evaluation process

## Operational Risk

Operational risk is the risk of financial loss resulting from:

- Human error - pricing or data entry
- Breach of limits or authority
- Inadequate supporting systems and tools
- Systems failure or malfunction
- Inadequate or non-compliance of trading operations with internal regulations
- Management failure

Operational Risk is managed by:

- Appropriate training of the operations staff,
- IT application control related to authorization – four eyes check
- Maintain reliable IT systems and other facilities to enable the execution, recording and settlement of all trading transactions and the monitoring, analysis and reporting of risk exposures.
- System backup in case of failure or malfunction.
- Promotions of new internal regulations and monitoring of compliance with internal regulations.
- Maintain adequate management hierarchy, establish clear roles and responsibilities and segregation of duties within trading operations.

## Markets, instruments and counterparties

It is inherent part of the internal policies that power producers have approved markets where the contracts might be concluded, approved instruments describing what kind of contracts might be concluded and approved counterparties with whom these instruments might be concluded.

Effective management of trading operations requires the trading team to engage in various trading channels such as exchanges, OTC etc. Actually approved/valid markets and instruments are listed in the register, which is regularly updated and approved by the RMC.

Market segments and the size of transactions that individual trading authorities can undertake have to be approved by RMC. There are several markets with different types of contracts:

	BILATERAL CONTRACTS	OTC	STANDARDISED
MARKET	Contracts are agreed on individual basis	Contracts are traded through brokers system such as TFS, GFI, ICAP, PREBON, SPECTRON, ...	Contracts are traded through organised exchanges such as PXE, EEX, BelPEX, APX, EXAA, PPX, ...
CONTRACTS DIFFERENCES	<ul style="list-style-type: none"> <li>▪ Contracts are customized according to actual needs of counterparty</li> <li>▪ Contracts are usually concluded for long term period with strategic counterparty, which is usually significant electricity consumption company</li> <li>▪ These contracts form the main part of the portfolio</li> <li>▪ The Credit Risk exposure depends on a counterparty</li> </ul>	<ul style="list-style-type: none"> <li>▪ Contracts are not usually standardised and differ</li> <li>▪ Counterparty could be anonymous according to the preferences</li> <li>▪ Price creation is very non-transparent</li> <li>▪ Costs on trade are usually very low.</li> <li>▪ The Credit Risk exposure depends on a counterparty</li> </ul>	<ul style="list-style-type: none"> <li>▪ Contracts are standardised and always quoted.</li> <li>▪ Trading is anonymous</li> <li>▪ Market makers bid/offer electricity price in every moment</li> <li>▪ Price creation is transparent</li> <li>▪ There is no Credit Risk exposure because the counterparty is always exchange.</li> </ul>

**Figure 4:** Types of markets and instruments

New instruments must be approved by the RMC that the new instrument:

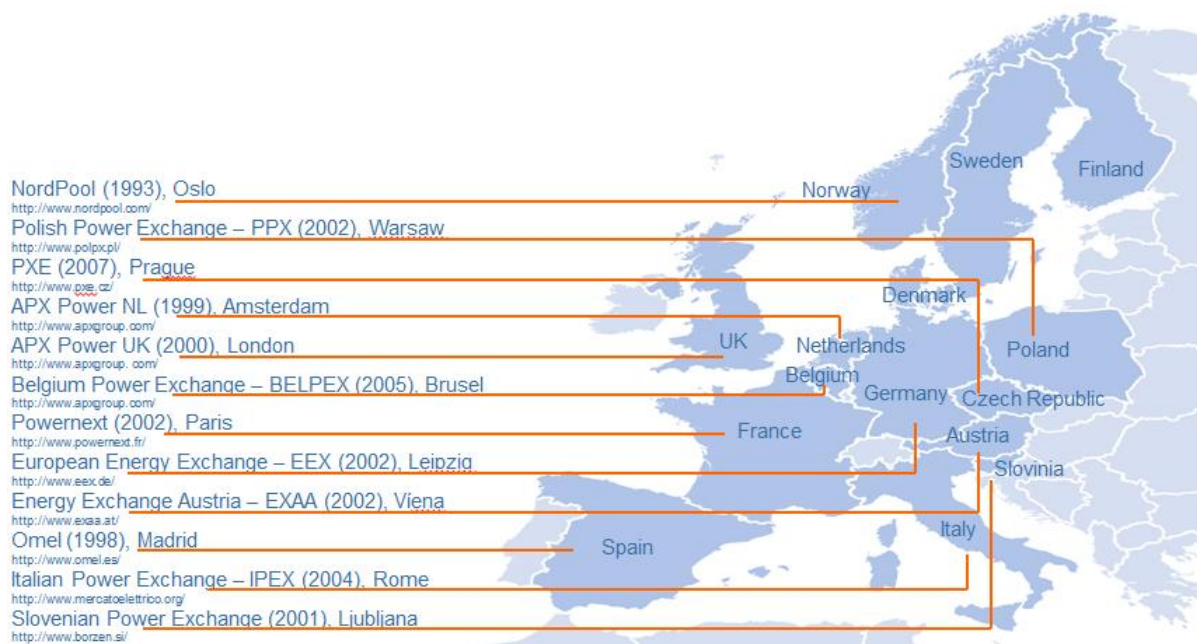
- is available through a recognised trading exchange or established brokers,
- is traded on standardised terms and conditions,
- individual transaction has a duration of no longer than three years,
- if individual transaction has a duration of up to five years may be authorised by the RMC
- is in compliance with companies policy

Before approving the new instrument for use the RMC must be satisfied that:

- the use and need for the trading instrument or market is well understood including its role in existing trading strategies
- all main risks associated with the instrument have been identified and their implications understood

- the appropriate decision support tools for valuing and pricing the instrument are in place
- the instrument can be captured from a front office, risk management and position reporting, and settlements perspective
- any applicable regulatory requirements have been satisfied

The following part has been first introduced on 10<sup>th</sup> International Student Conference on electrical Engineering by Knezek [52] in 2006 and published in diploma thesis [51] in 2007. It has been awarded by 2<sup>nd</sup> place in CEZ competition for the best scientific and technical research.



**Figure 5:** Main power exchanges in Europe

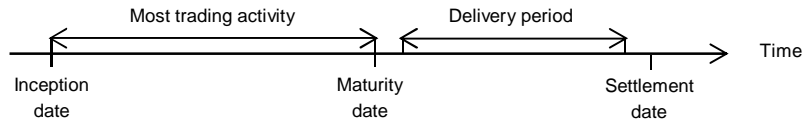
## Financial and non-financial linear instruments

The theory and practice of financial and non-financial linear derivative instruments used in the matured financial markets are described in general in [6], and in more detail in [2], [7], [11]. Application of energy linear derivatives are covered in [13] by Geman. Typical approved instruments used in the electricity market are Forwards and Futures. There might be available other instruments however their usage is quite rare. In this chapter we focus on description of these instruments in terms of timeline and flowchart.

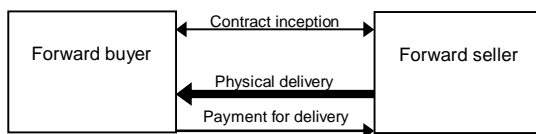
Forwards contracts are linear instruments that represent obligation for buyer to buy certain amount of underlying asset as at defined day for agreed price – strike price and for seller obligation sell that amount of underlying asset under the same conditions. Forwards are traded on bilateral basis or in the OTC market. Unlike from Futures (as per below) Forwards contracts are used for the physical delivery of the underlying asset. There are no variation payments of the difference between forward settlement price and current market price. As

these contracts are agreed on bilateral basis or through OTC there is existing credit risk of the counterparty.

The timeline and the flowchart for futures are presented bellow in Figure 6 and Figure 7.

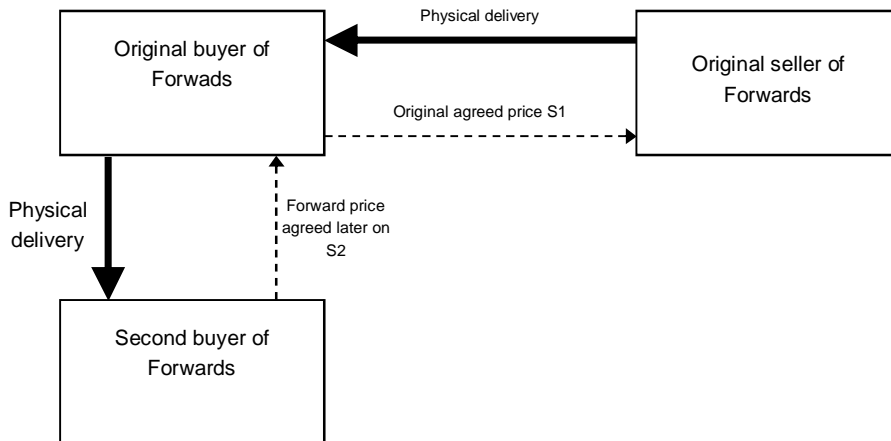


**Figure 6:** Timeline for forward contract



**Figure 7:** Forward flowchart

Forwards is used for the physical delivery of the underlying asset. The market risk of electricity price changes is eliminated as the original electricity price has been already agreed before. Forward might be effectively settled before physical delivery by selling Forwards with the same conditions. In this case of settlement of forwards open position, forwards might be sold to other counterparty than from which these have been initially bought. This is the difference from futures where the counterparty is always power exchange. The flowchart is illustrated in Figure 8:



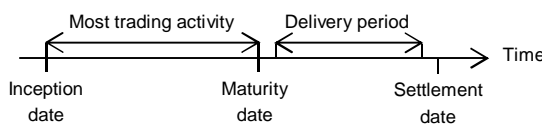
**Figure 8:** Resale of forward flowchart

Profit or loss from this position is equal to price difference  $S2 - S1$ . MtM related to settlement of Forwards contract is same like for Futures contract however the only cash flow is performed as at expiry date.

Futures contracts are linear instruments that represent obligation for buyer to buy certain amount of underlying asset as at defined day for agreed price – strike price and for seller obligation sell that amount of underlying asset under the same conditions. These contracts are traded on organized power exchanges like PXE, EEX or NordPool unlike from Forwards which are traded on bilateral basis or on OTC markets. Conditions related to the contracts are defined by the power exchange where the contracts are traded.

Another difference from Forward is that Futures are not settled by delivery of physical power but financially. In the light of volatility of electricity prices futures are used for the hedging against price deviation. Most of them are closed out before physical delivery. For example clearing of one long future contract in the current month is performed by sale of the same amount of electricity which had been purchased.

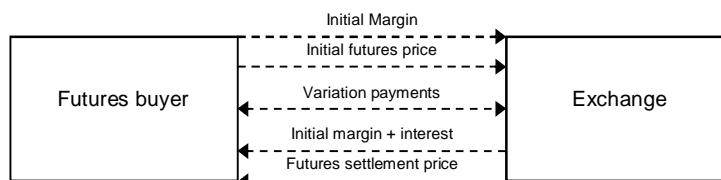
The timeline and the flowchart for futures are presented bellow in Figure 9 and Figure 10.



**Figure 9:** Timeline for futures contract

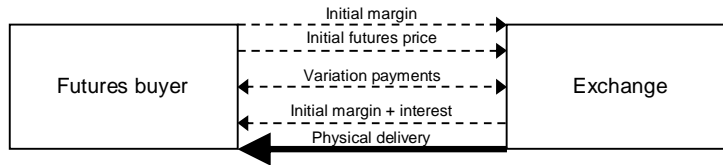
When futures are bought buyer is paying actual price of the future called prevailing future price and initial security margin for the amount of the underlying asset which has been bought. This money is held by the power exchange on its current account. Power exchange performs daily settlement Mark to Market of all future contracts and calculates P&L caused by the market price movement. Based on calculated P&L power exchange performs payments of variation margins to market participants. These payments are deducted from buyers in case the electricity price decreased and are added in case of electricity price increase. Finally when the open position is closed the rest of the initial security margin including interest is returned to the buyer together with final future price.

Future transaction which is closed before the physical delivery is illustrated in Figure 10:



**Figure 10:** Futures flowchart – settlement before physical delivery

In case of physical electricity delivery there will be on-going variation payments. The physical power is then bought on separate daily market. The flowchart is illustrated in Figure 11:



**Figure 11:** Futures flowchart – settlement with physical delivery

### Financial non-linear instruments

The theory and practice of financial non-linear derivative instruments used in the matured financial markets are published in general in [25] and in more detail by Jilek in [17], [18], [19]. Application of different types of energy options including exotic options, spread and basket options, digital option, quanto options and real options are examined in detail by Wengler in [28]. Different perspective is provided by Spinler, Huchzermeier and Kleindorfer in [65], where authors focus on options contracts that enable risk-sharing between the trading partners- buyer and seller.

Valuation of different kind of options through either geometric Brownian motion or mean reverting processes is introduced in [38] by Deng, Johnson and Sogomonian. The valuation results have been used in turn to construct real options. The application of their research in practice has been left for future researches.

Another approach has been used by Eydeland and Geman in [43]. Option prices are examined through production based approach, where they estimate base load price, forward prices of marginal fuel, expected load conditional on the information at time and “power stack”. In the work they list a number of issues they would need to address and therefore this work is more like a discussion paper left for future researches.

Options are non-linear hedging instruments which give buyer (option holder in the long position) rights but not an obligation to buy or sell as at expiry date agreed amount of underlying asset for agree price – strike price. For these rights is paying buyer to seller (option writer in the short position) a payment called option premium. Option premium includes risks related to the option contract which is given by the market.

Option seller has obligation to sell or buy defined amount of underlying asset for agreed price in case the option is exercised by the buyer. Options which give buyer right to buy underlying asset are called call options and options which give buyer right to sell underlying asset are called put options.

There are several types of options:

- European options – Option could be exercised at expiry date.
- American option – Option could be exercised any day during the period of its existence including the expiry date
- Bermuda option – Option could be exercised at predefined dates.
- Asian option – Option could be European or Assian type but its payoff does not depend on underlying asset price at expiry date but on average prices during certain period of time.

Further on we will consider only European type of options. Here we need to define payoff and P&L of European options.

Payoff represents value of option as at expiry date, while P&L represents whether option brings profit or loss as follows:

$$\text{european call payoff} = \max(P - S, 0) \tag{1}$$

$$\text{european put payoff} = \max(S - P, 0) \tag{2}$$

$$\text{european long call P\&L} = \max(P - S, 0) - p \tag{3}$$

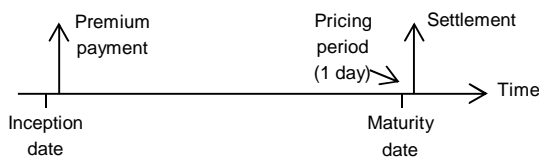
$$\text{european short call P\&L} = p - \max(P - S, 0) \tag{4}$$

$$\text{european long put P\&L} = \max(S - P, 0) - p \tag{5}$$

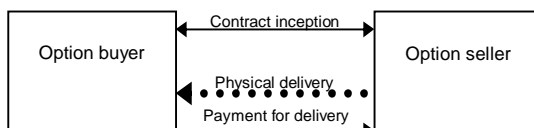
$$\text{european short put P\&L} = p - \max(S - P, 0) \tag{6}$$

Where  $P$  ... electricity market price at expiry date  
 $S$  ... strike price of option  
 $p$  ... premium paid for option

The timeline and the flowchart for options are presented bellow in Figure 12 and Figure 13:



**Figure 12:** Timeline for option contract



**Figure 13:** Option flowchart



Correct use of these instruments allows electricity participants to mitigate their positions in a controlled way. These instruments take place both on organized exchanges and on OTC markets.

Based on price of underlying asset option contracts could be divided into three types:

- In the money (ITM) – strike price is more favourable than market price
- Out of the money (OTM) – strike price is less favourable than market price.
- At the money (ATM) – strike price is the same like market price.

Even it is not possible to exercise European option before expiry date, this option could be sold to other market participant.

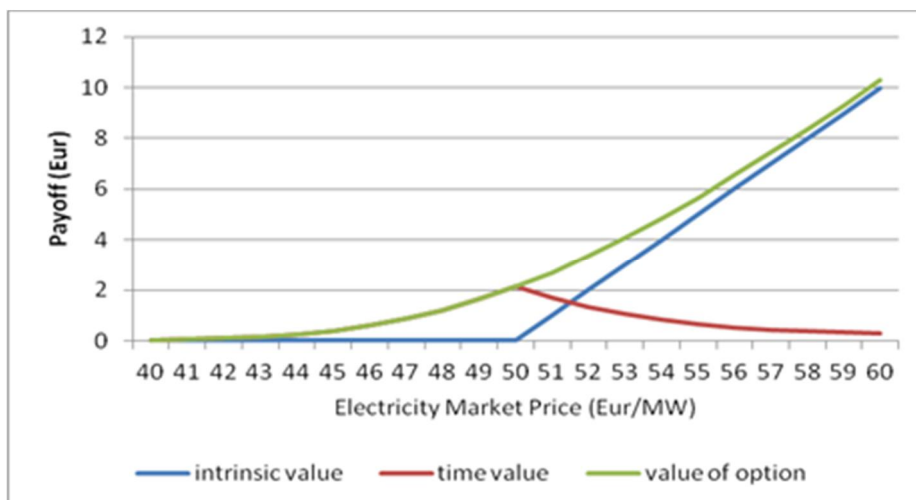
It is important to take into account time remaining to the maturity date and the volatility of market prices of the underlying assets. For this reason, the option value cannot be zero. This fact allows us to divide the total value of the option into two components:

- Intrinsic (internal) option value measures how much the option is ITM, OTM, ATM. (Intrinsic value will be positive if the option is ITM and zero if OTM).
- Time value of the option is given by the possible movement of the market price of the underlying asset to maturity date in a more positive direction. (The time value is always greater than zero, provided that there remains some time to maturity and volatility of the underlying asset exists.)

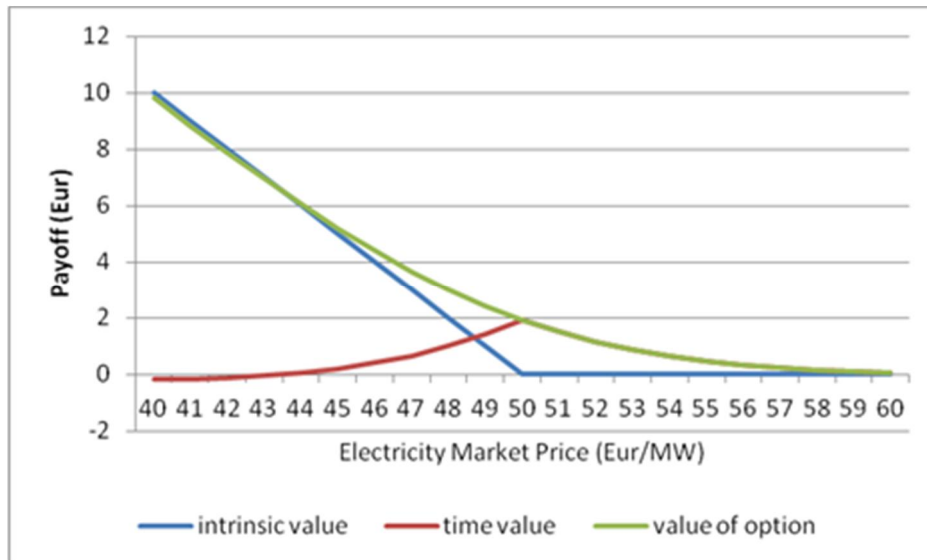
The total value of the option can never be less than the intrinsic value of the option, due to the potential market price developments to maturity date. Intrinsic value of the option is the present value of the payoff at maturity provided using current market prices.

$$\text{intrinsic value of a call option} = e^{-rt} \max(S - P, 0) \tag{7}$$

$$\text{intrinsic value of a put option} = e^{-rt} \max(P - S, 0) \tag{8}$$



**Figure 14:** Call option – Intrinsic, time and total option value



**Figure 15:** Put option – Intrinsic, time and total option value

If the owner decides to exercise the option, owner will receive compensation equal to the difference between the current futures price and the option exercise price. The value of such an option depends on the following five main variables:

1. electricity market price,
2. option exercise price,
3. time to maturity of the option,
4. discount rate used,
5. market price volatility.

These factors affect the value of the option in the following way:

- Market price and exercise price determines intrinsic value of the option and option return.
- Time to maturity of the option and the market price volatility determines the set of possible changes in the option price. The greater the time to maturity, the higher the volatility and the greater the probability of option becoming ITM from OTM is and vice versa.
- Discount rate determines the present value of the option. (The higher the discount rate, the lower the present value of the option is).

The key factor is volatility. Without volatility would not exist the time value of the option. Volatility is a measure of how much and how fast market prices move. Volatility is usually determined on an annual basis. It can be divided into:

- Historical volatility - says how much market prices changed in the past. (Historical volatility is a measure of market price variability)
- Expected volatility - says how high price variability is expected between now and the maturity date. (Expected volatility is a measure of uncertainty in the future market price development.)

The most common option pricing model used is Black-Scholes formula [32]. Many empirical tests have shown that the Black-Scholes price is "fairly close" to the market prices. Option value is based on the above mentioned factors and the general theoretical assumptions of market behaviour.

Call option value:

$$P_{call} = P \cdot N(d_1) - S \cdot N(d_2) \cdot e^{-rt} \quad (9)$$

Put option value:

$$P_{put} = S \cdot N(-d_2) \cdot e^{-rt} - P \cdot N(-d_1) \quad (10)$$

where:

$$d_1 = \frac{\ln\left(\frac{P}{S}\right) + (r + 0,5 \cdot \sigma^2) \cdot t}{\sigma \sqrt{t}} \quad (11)$$

$$d_2 = d_1 - \sigma \sqrt{t} \quad (12)$$

$P$	...	electricity market price
$S$	...	option exercise price
$t$	...	time to maturity of the option
$r$	...	discount rate used
$\sigma$	...	market price volatility

and where  $N(x)$  represents value of the distribution function of the normal distribution at  $x$ :

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{x^2}{2}} dx = 1 - \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{x^2}{2}} dx \quad (13)$$

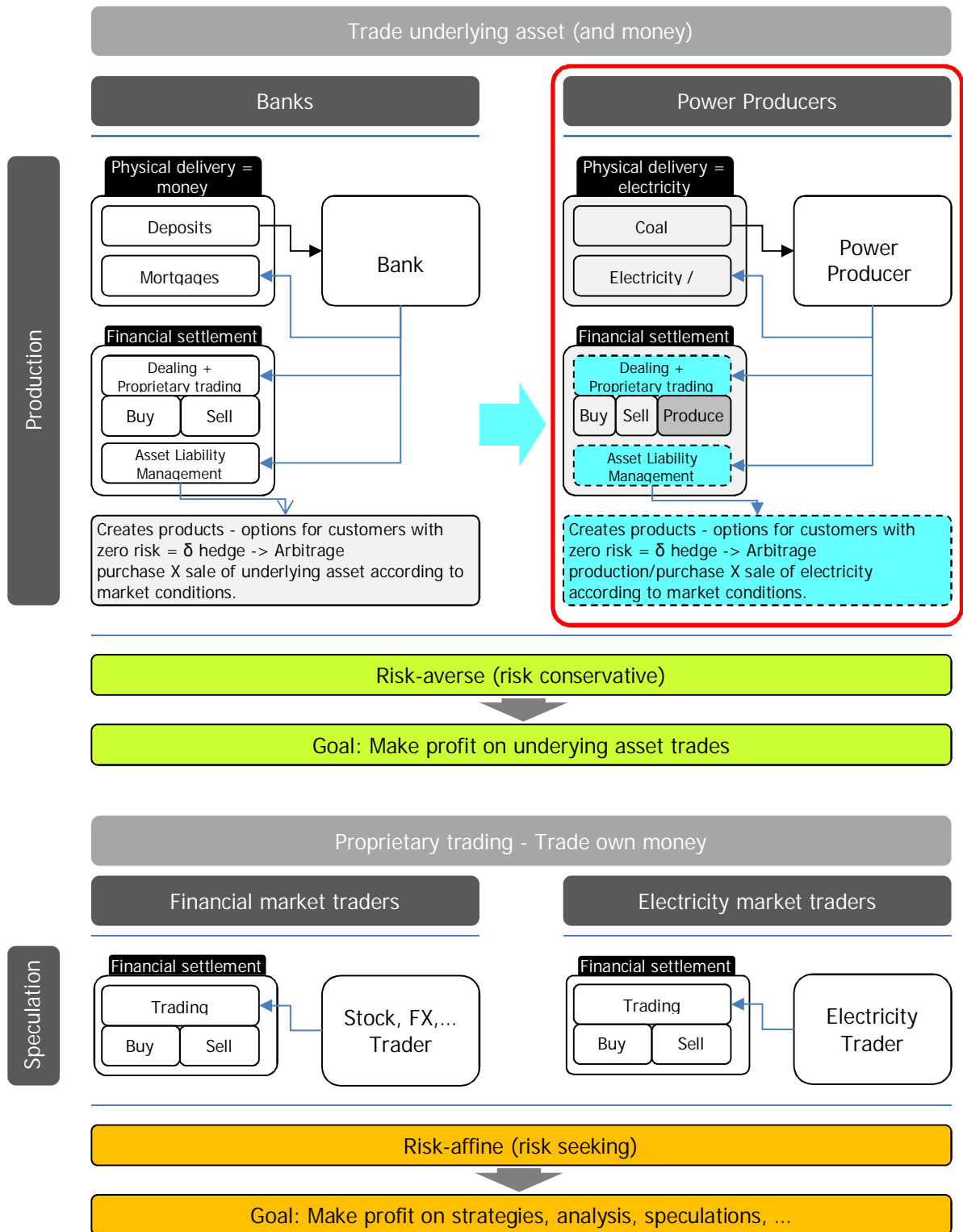
In order to use Black-Scholes pricing formula it is important to establish theoretical assumptions of market behaviour:

- The market is efficient, with all information about the historical prices and without possibility of arbitrage.
- The market is infinitely liquid, it is possible to buy resp. sell any amount of underlying assets without any impact on the market
- There are no transaction fees and other fees, so it is possible to buy or. sell for the same price at any time in the future.
- Trading is continuous, without closing time, 24 hours a day.
- Discount rate is constant.
- The prices of the underlying assets change in a way that they have the character of a normal probability distribution.

### **Opportunity gaps**

Power producers have tremendous competitive advantage opposed to electricity traders to produce electricity. In this respect power producers have in their hands possibilities to generate higher revenues with lower risks. However this potential opportunity has not been fully utilised. Power producers have high knowledge of running the business in terms of generating electricity and selling it to the market either through bilateral contracts or through organised power exchanges however the higher profitable part of the business – sales to portfolios of individual customers has been partially overtaken by electricity traders whose business is based on cheaper purchase and higher price sale.

In order to achieve this marginal resale traders have sophisticated tools to assess the electricity consumption of its customers' portfolio for certain period of time in advance. In the light of the above the number of electricity traders increased several times in last couple of years as electricity trading became highly profitable business. However this business bears risks as traders might not be able to purchase cheaper electricity on the market. Some of their contracts binding them to sale the electricity to their customers for specified price and these contracts might become loss making. In contrast power producers would have possibility to produce the electricity themselves and reduce or eliminate these losses.



**Figure 16:** Comparison of financial and electricity markets.

## **Banks vs. Power producers**

Banks are offering various structured products like options without having necessary knowledge of the electricity market. This lack of knowledge makes the option contracts expensive and unattractive. Power Producers may easily takeover their role and create and offer structured products themselves with extensive profits without any risk exposure. This will result in higher trading activity in the organized power exchanges and OTC markets and in cheaper electricity for the end-users.

The key to success is to establish Asset Liability Management department which is one of the most important department within the bank – sometimes called “the bank in the bank”. This department will have the only responsibility to create structured option products based on Delta hedging strategy

## **Power producers vs. Electricity market traders**

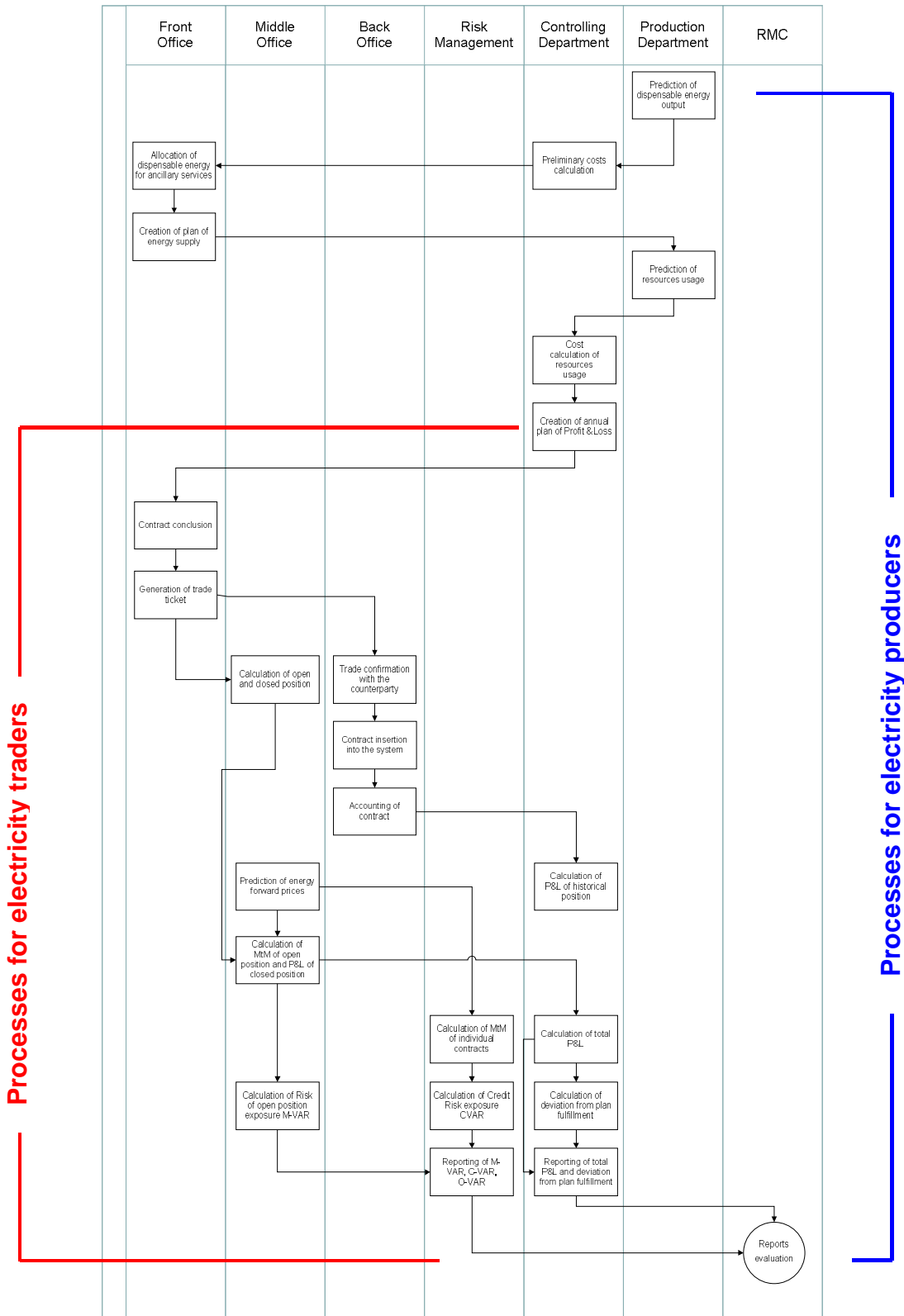
The main goal of power producer in current market is to generate profit based on provided ancillary services to national transmission system operator and deliver of electricity supply to their own customers based on agreed consumption diagram.

- Ancillary services – power producers keep reserve in production capacity for ancillary services. This reserve is compensated by a fixed payment plus additional payment in case of production / provision of the ancillary services to the electricity transmission operator.
- Electricity sale – power producers produce electricity in order to sell it through bilateral contracts, organized exchanges or through OTC (Over-The-Counter) markets.

Electricity traders are generating profit based on the opportunity gap in the electricity market which is given by the power producers self-regulations. This gap might be easily replaced by the power producers.

Proprietary trading – Power producers have competitive advantage in form of possibility to generate electricity in case when it is too expensive in the market and sell it and purchase electricity in case when it is cheaper than the production. They may use remaining production capacity and invaluable knowledge of the electricity market for speculative purposes. This will increase trading activity in the market, diminish inefficiencies and finally result in cheaper electricity for the end-users.

# Comparison of operations

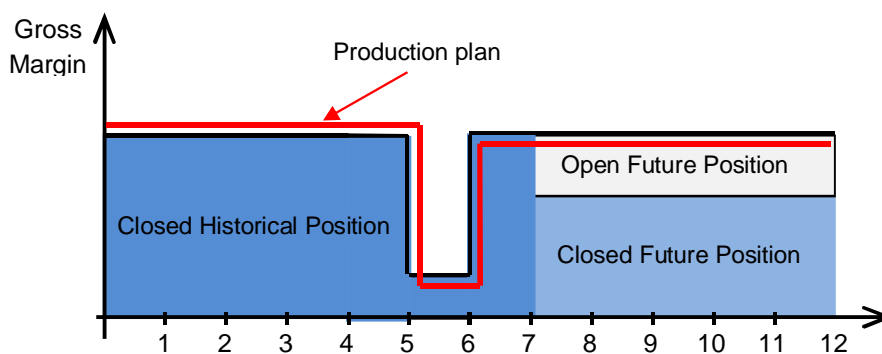


**Figure 17:** Processes in the electricity trading companies – traders/producers

### 3 Portfolio valuation through Gross Margin

Gross margin is a difference between selling price and cost. Gross margin represents a key factor behind many of the most fundamental business considerations, including budgets and forecasts. All managers should be and usually they are aware of the margins they generate in the industry they operate in. Electricity markets have been recently liberalised. There are contracts representing revenues that have been concluded many years ago on bilateral basis, there are costs relating to the production and administration and there are capacities that need to be sold. Some of these revenues have been already generated and costs incurred and some of them should be generated and incurred. Revenues and costs need to be matched and show whether the company generates profit or is loss making not only in the past but also in the future, so the management has company under control and prepare reliable budget and track the forecast. However there is still production that is not yet sold. There are questions related to valuation of this future production.

The following part has been first introduced on 12<sup>th</sup> International Student Conference on Electrical Engineering by Knezek [54] in 2008 and also published in peer-reviewed journal Acta Polytechnica [55]. The situation described above represents power producer portfolio positions. As discussed there are two types of market positions – open position and closed position.



**Figure 18:** Portfolio positions of power producer

Closed position is a position which has been already settled. Closed position which brought company a loss automatically decreases the Actual Limit of an open position of the day and naturally that position, which was settled successfully, brings the company profit and increases the Actual limit of open position. Closed positions are not sensitive to the price movements and that's why we do not need to evaluate its riskiness.

Closed position profitability is calculated using the GM (Gross Margin) which is equal to the difference between sales revenues and costs of electricity purchase and/or generation.



On the other hand those contracts which are going to be settled create open positions. Open positions are usually divided in two parts: the long-one and the short-one.

- The long position is a position in which the company is a net holder of contract, i.e. still has capacities to sell electricity.
- The short position is a position in which the company owes the quoted asset in the contract, i.e. the company has agreed to deliver more electricity than currently has.

The open position profitability indicates what “profit might be expected from an open position, if it was settled today”. In other words, it is the current market valuation of contracts representing the open position. This valuation is called Mark to Market.

### 3.1 Closed historical position

A closed historical position is a position for the period from the beginning of the current year to the present date, and is calculated as the sum of the concluded contract volumes per hour and day of the relevant period, while the following applies:

$$\sum_{td} qS_{td} = \sum_{td} qG_{td} + \sum_{td} qP_{td} \quad (14)$$

where  $qS_{td}$  is the volume of sold electricity for hour  $t$  of day  $d$  [MWh],  $qG_{td}$  is the generated volume of electricity for hour  $t$  of day  $d$  [MWh] and  $qP_{td}$  is the volume of electricity purchased for hour  $t$  of day  $d$  [MWh]

The gross margin is used for determining the profitability of a closed historical position. GM from the closed historical position is to be calculated as the difference between the sales revenues (multiplied by the sold volume) and the production costs (the weighted value of the production prices multiplied by the produced volume) and the purchase costs (the weighted value of the purchase price multiplied by the purchase volume).

$$GM_{-CHP} = \sum_{td} GM_{-CHP_{td}} = \sum_{td} qS_{td} \cdot \Phi pS_{td} - \sum_{td} qG_{td} \cdot \Phi pG_{td} - \sum_{td} qP_{td} \cdot \Phi pP_{td} \quad (15)$$

where  $\Phi pS_{td}$  is the weighted value of the sales prices [Kč],  $\Phi pG_{td}$  is the weighted value of the production prices [Kč], and  $\Phi pP_{td}$  is the weighted value of the purchased prices [Kč].

### 3.2 Closed future position

A closed future position is a position for the period from the next day within the selected future time window, and is calculated as the sum of concluded contract volumes per hour and day of the relevant time window, while the following applies:

$$\sum_{td} qS_{td} = \sum_{td} qPG_{td} + \sum_{td} qP_{td} \quad (16)$$

where  $qS_{td}$  is the volume of sold electricity for hour  $t$  of day  $d$  [MWh],  $qPG_{td}$  is the planned electricity generation for hour  $t$  of day  $d$  [MWh],  $qP_{td}$  is the volume of electricity purchased for hour  $t$  of day  $d$  [MWh]

The gross margin is used for determining the profitability of a closed future position. GM from the closed future position is to be calculated as the difference between the sales revenues (multiplied by the sold volume) and the planned production costs (the planned value of the production prices multiplied by the produced volume) and the purchase costs (the weighted value of the purchase price multiplied by the purchase volume).

$$GM\_CFP = \sum_{td} GM\_CFP_{td} = \sum_{td} qS_{td} \cdot \Phi pS_{td} - \sum_{td} qPG_{td} \cdot pPG_{td} - \sum_{td} qP_{td} \cdot \Phi pP_{td} \quad (17)$$

where  $\Phi pS_{td}$  is the weighted value of the sales prices [Kč],  $pPG_{td}$  is the planned value of the production prices [Kč] and  $\Phi pP_{td}$  is the weighted value of the purchase prices [Kč]

### 3.3 Forward price curves

To determine the market value of future contractual obligations in the form of individual portfolio positions electricity forward price curve need to be determined. There were many different models to determine / model future electricity prices. Probably the easiest way and the most accurate is to find electricity prices in power exchange or OTC markets as used in [51]. These exchanges quote electricity prices from hour ahead to six years ahead. The fundamentals of forward price curve constructions has been described in [44] by Fleten and Lemming.

In [35] Bundalova, Dejong and Vandijken determined electricity prices based on Magrabe's formula which is based on an option to exchange one risky asset for another risky asset at maturity. Also the same work used another approach called full simulation based on 200 scenarios of Monte Carlo simulation. In both cases there was used a number of estimates and limitations, which leaves these methods in theoretical area rather than usage in practice. Deng and Oren describes in [39] two main approaches of modelling electricity prices – fundamental approach and technical approach. Fundamental approach simulates production facilities and operations to arrive at electricity market price while technical approach is based on modelling stochastic behaviour of market prices from historical data and statistical analysis. Simulation of electricity prices with utilization of Monte Carlo technique as described by Dudorkin in [9],[10], Glasserman in [14] and Bertocchi, Consigli, Dempster in [4] are developed by Baughman and Lee in [31].

An article with fairly new approach of forward price modelling and quite a number of citations has been published by Chen, Deng, Huo in [36]. The approach stands on novel non-parametric approach for the analysis and prediction of electricity price curves by applying the manifold learning methodology. The accuracy has been demonstrated on historical prices from Eastern U.S. electricity power markets. The proposed price curve model performs well

in forecasting both short-term price such as the day-ahead prices and longer term price such as the week-ahead prices.

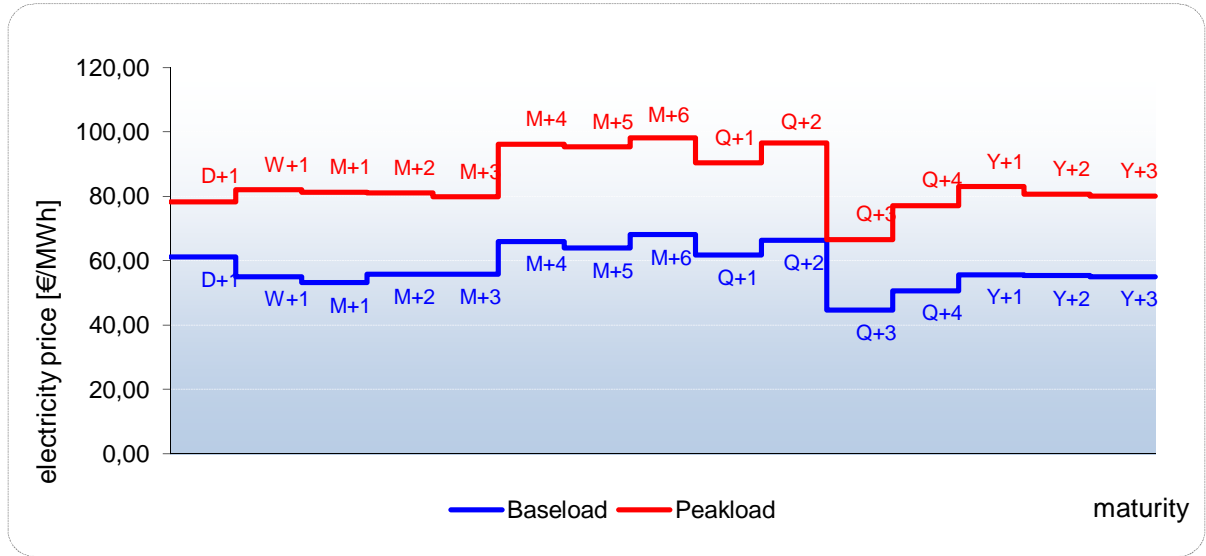
Calculation of hourly price forward curve by using mathematical regression methods, like polynomial, radial basic function neural networks and furrier series is examined in detail in article from Abdolkhalig in [30]. The criteria for choosing the model were dependent on minimization of the Root Mean Squared Error (RMSE), using the correlation analysis approach. The results provided showed comparisons of the methods used, however the applicability of the theory used in practice is questionable.

Another approach introduced by Borak and Weron in [33] is based on a dynamic semiparametric factor model for electricity forward curves. According to authors model's biggest advantage is that it leads not only to smooth, seasonal forward curves extracted from exchange traded futures and forward electricity contracts, but also to a parsimonious factor representation of the curve. This is very theoretical approach and the applicability of the model in practice is doubtful bearing in mind robust IT equipment needed for the data processing.

For our purposes the most accurate representation of the forward price curves are used data from German power exchange [www.EEX.de](http://www.EEX.de). The forward price curve is used to evaluate future part of power producer's portfolio. An example of the forward price curve determined from futures contracts traded on the power exchange [www.EEX.de](http://www.EEX.de):

<b>Futures type</b>	<b>Delivery date</b>	<b>Base load</b>	<b>Peak load</b>
M	Jul-2006	43,35	66,41
M	Aug-2006	43,60	66,43
M	Sep-2006	48,93	69,70
M	Oct-2006	49,80	72,50
M	Nov-2006	61,78	93,75
M	Dec-2006	60,32	92,00
Q	Jul-2006	45,25	67,48
Q	Oct-2006	57,25	85,99
Q	Jan-2007	64,10	93,18
Q	Apr-2007	42,48	64,58
Q	Jul-2007	47,07	72,63
Q	Oct-2007	59,42	90,83
Q	Jan-2008	63,70	93,23
Y	Jan-2007	53,24	80,35
Y	Jan-2008	56,15	81,50
Y	Jan-2009	57,15	82,49
Y	Jan-2010	58,33	83,25
Y	Jan-2011	58,45	83,30
Y	Jan-2012	58,49	83,35

The graphical representation of the forward electricity prices is as follows:



**Figure 19:** Graphical representation of the forward price curve.

### 3.4 Open future position

An open future position is a position for the period from the next day within the selected future time window and is calculated as the difference between the planned volume of sold electricity, the planned volume of generated electricity and the planned volume of purchased electricity per hour and day of the relevant time window, while the following applies:

$$\sum_{td} qPS_{td} = \sum_{td} qPG_{td} + \sum_{td} qPP_{td} \quad (18)$$

where  $qPS_{td}$  is the planned volume of sold electricity for hour  $t$  of day  $d$  [MWh],  $qPG_{td}$  is the planned volume of generated electricity for hour  $t$  of day  $d$  [MWh] and  $qPP_{td}$  is the planned volume of electricity purchased for hour  $t$  of day  $d$  [MWh]

The gross margin is used for determining the profitability of an open future position. GM from the open position is to be calculated as the difference between the planned sales revenues (the planned sales volume multiplied by the planned sales price – forward price curve) and the planned production costs (the planned generation volume multiplied by the planned production cost) and the purchase costs (the planned purchased volume multiplied by the relevant planned purchase price).

$$GM_{-OFP_{td}} = \sum_{td} GM_{-OFP_{td}} = \sum_{td} qPS_{td} \cdot pPS_{td} - \sum_{td} qPG_{td} \cdot pPG_{td} - \sum_{td} qPP_{td} \cdot pPP_{td} \quad (19)$$

where  $pPS_{td}$  is the planned value of the sales prices [Kč],  $pPG_{td}$  is the planned value of the production prices [Kč] and  $pPP_{td}$  is the planned value of the purchased prices [Kč].

For the purposes of an open position Mark-to-Market valuation, the open position per hour has to be observed in order to be valued by the hourly forward price curve, which differentiates the PEAK and OFF-PEAK prices for sales and purchase.

### 3.5 Total Gross Margin and difference from plan fulfilment

The total GM value is calculated as the sum of the gross margins of the closed historical and future position and open future position, as follows:

$$GM = GM\_CHP + GM\_CFP + GM\_OFP \quad (20)$$

The total difference from plan fulfilment will be given as the sum of the actual historical variance and the expected future variance in a one-year time window:

$$\Delta = \sum_{td} \Delta Act_{td} + \sum_{td} \Delta Fut_{td} \quad (21)$$

The actual difference from plan fulfilment represents the profit/loss realized from plan fulfilment for the period from the beginning of the current year to the present date. It is to be calculated as the difference between the planned value of the GM of the closed historical position and the actual value of the GM of the closed historical position to the present date:

$$\Delta Act_{td} = GM\_CHPpl - GM\_CHPact \quad (22)$$

The expected difference from plan fulfilment represents the projected unrealized profit/loss from plan fulfilment for the period from the next day within the selected future time window. It will be calculated as the difference between the planned value of the GM of the future position and the sum of the actual value of the GM of the closed future position and the GM of the open future position to the present date:

$$\Delta Fut_{td} = GM\_FPpl - GM\_CFPact - GM\_OFPact \quad (23)$$

## 4 Open position risk exposure and its valuation

### 4.1 Risk exposure

Power producers, unlike electricity traders (involved in pure electricity purchase/sale and managing their position based on trading only), provide their sold electricity primarily by controlled generation, with purchases being of a complementary nature only. To avoid the risk exposure of an open position, their internal regulations in general states that:

- producers are only allowed to purchase electricity for a price that is lower than the cost price by a certain coefficient,
- producers are allowed to sell electricity for a price that is higher than the cost price by a certain coefficient,
- producers are allowed to sell electricity up to the volume that they are able to generate.

For the above reasons, power producers are usually not exposed to risks implied by the fluctuating market price of electricity from an open position and, therefore, the calculation of Value at Risk for conservative power producer would have no rationale.

However as discussed above in chapter 2, power producers may easily exploit their competitive advantage in form of possibility to generate electricity in case when it is too expensive in the market and sell it and purchase electricity in case when it is cheaper than the production. They may use remaining production capacity and invaluable knowledge of the electricity market for speculative purposes. This leads to exposure to the volatility of market prices and open portfolio positions.

One of the main market risk related to trading the electricity market price is the volatility. Volatility may expose trader with the electricity to extreme losses. There are several ways how to estimate volatility of the future prices.

There were several articles that relate to this topic. Probably one of the most in depth research in this area has been performed by Hadsell, Marathe, Shawky in [47]. The model is based on data from May 1996 till September 2001 (5 years, 3 months) across five markets in the USA. They encountered in all five markets strong evidence of unstable expected volatility, which is explained by “good and bad news for electricity market”.

The simulation of spot prices and volatility calculation with investigation of its impact on future electricity market of Spanish and Portuguese forthcoming integration has been performed by Sousa, Lagarto and Pestana in [57]. The interesting part is a variety of techniques they used and comprehensive approach they applied.

For the purpose of this thesis I have recalculated volatility to evaluate Value at Risk of open position based on data of spot prices from 1.1.2002 till 31.12.2009 (8 years). The volatility calculated is 35,12%. Also the pattern of market prices has been tested against

empirical probability distribution through Pearson's chi-squared test – for more detail please refer to chapter 4.3.

One of the hedging techniques that mitigate this market risk to determined level is Value at Risk. Value at Risk ( VAR ) - is a measure that is telling us at certain level of probability the amount of loss incurred in the portfolio due to the adverse market prices movements in case of normal market developments during the specified period. VAR takes into account a fixed portfolio that does not change during the specified period.

There are several books related to Value at Risk. The fundamentals are explained in detail by Holton in [15] and by Jorion in [20]. These books are not market specific and deals with Value at Risk in general. An application of parametric models and Monte Carlo simulation in Value at Risk metrics has been introduced by Cheung and Powell in [37]. A positive part of this publication is its application in practice. The data used are from financial markets. Another application of Monte Carlo simulation on Value at Risk is described by Glasserman, Heidelberger and Shahabuddin in [46]. One of the most important results from their empirical study found is that market returns exhibit greater kurtosis and heavier tails than can be captured with a normal distribution.

In order to manage risk exposure appropriate risk management framework has to be established. This framework covers organization and responsibilities, risk assessment and internal regulations and approved markets, instruments and counterparties – described in chapter 2.

## **4.2 Calculation of VAR**

Value at risk is generally based on probabilistic scenarios under normal market conditions, while stress testing deals with the extreme scenarios for which it is difficult to determine the probability.

The company' risk manager is responsible for calculating and monitoring VAR and stress testing, as well as for setting the limits. VAR technique is not very accurate due to the fact that the entire portfolio which has a number of risk factors is assessed only by a single number. The main advantage of VAR is its simplicity. It briefly provides management with the view of the risk position to which the portfolio is exposed to.

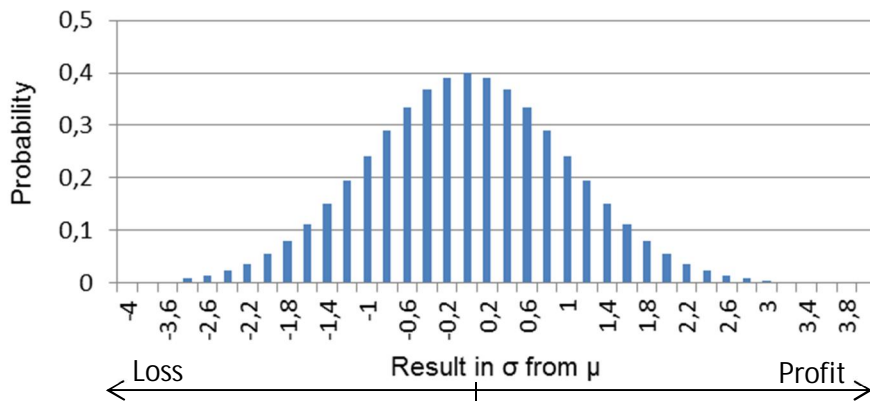
Techniques VAR and stress testing express a range of P&L indicators. P&L is calculated as the difference of potential future discounted MtM and current MtM . So we can get the P&L for many different probability scenarios. By using VAR management is focusing on examination of those scenarios that have the least favourable impact on the company and these are further reported to senior management.

Suppose VAR of our portfolio, which worth € 10 million is € 657,944 with a level of confidence of 95 % within a specified period of 10 days.

VAR tells us with 95% level of confidence that in the next 10 days there will be no loss in our portfolio due to changes in the electricity market price higher than € 657,944. Accordingly there is probability of 5 % that the loss in our portfolio exceeds €657,944.

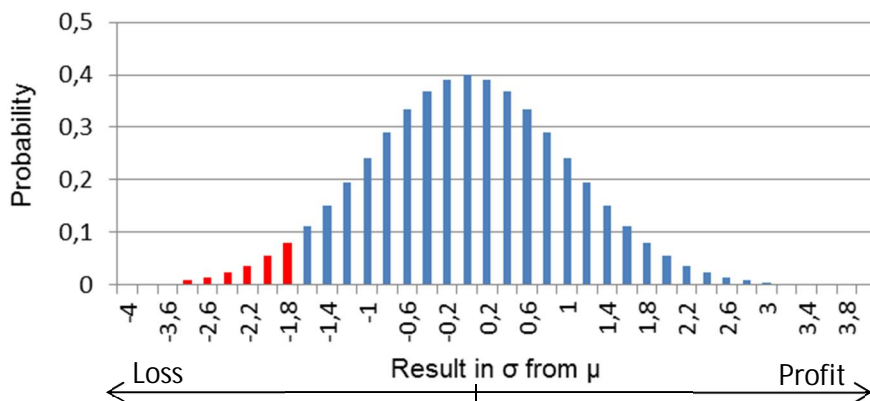
The time used for calculation of VAR is the time in which the development of the market price movements is considered. The longer the period is, the higher the range of market prices that may occur and impact our portfolio is.

The confidence level for the VAR is a confidence level of one tail of the probability distribution, since we only consider the adverse price movements. In the Figure 20 is illustrated normal probability distribution with mean of one and standard deviation of null.



**Figure 20:** The normal probability distribution  $N(0,1)$

The result that can be achieved with a probability of 95% is illustrated in the Figure 21, which generally corresponds to the distance of 1,645 times the standard deviation from the mean.



**Figure 21:** Left tale of the probability distribution - VAR = 95%



The loss calculated through VAR which could be achieved with 95% probability is marked in red and is called the left tale of the probability distribution. VAR therefore indicates the 95% quantile of the normal distribution. Sometimes where the higher accuracy is needed the VAR is calculated at 99% level of confidence

95 % quantile of the normal distribution corresponds to a distance 1,645 times standard deviation from the mean 0 then we can write:

$$P(x > 1,645) = 0,95$$

In generally, the volatility of the electricity prices increases linearly with time and therefore the standard deviation increases with the square root of time. Since we assume a annual volatility and we need to calculate VAR for the next ten days we have to recalculate volatility for ten days :

$$\sigma_{10} = \sigma_{250} \cdot \sqrt{\frac{10}{250}} = 4 \%$$

Now we can calculate VAR of the portfolio with a 95% level of confidence for the next 10 days according to the formula:

$$VAR_{95\%,10} = 10.000.000 \cdot \sigma_{10} \cdot (-1,645) = -657.944$$

In case of 99 % level of confidence, the value of VAR for the next 10 days would be €930,545.

With the increasing level of confidence, the absolute value of VAR increases. In other words, it increases potential loss at lower probability level.

### 4.3 Assumptions for calculation of VAR

In the chapter above we have defined VAR and its calculation based on normal probability distribution. In praxis electricity market prices might not be fully described by the normal probability distribution. In order to define which probability distribution describes the electricity market price the best we need to take a sample of the electricity market prices and perform Pearson's chi-squared test as described in [23], [26], [27] and in more detail Kaňok in [21] and Rektorys in [24].

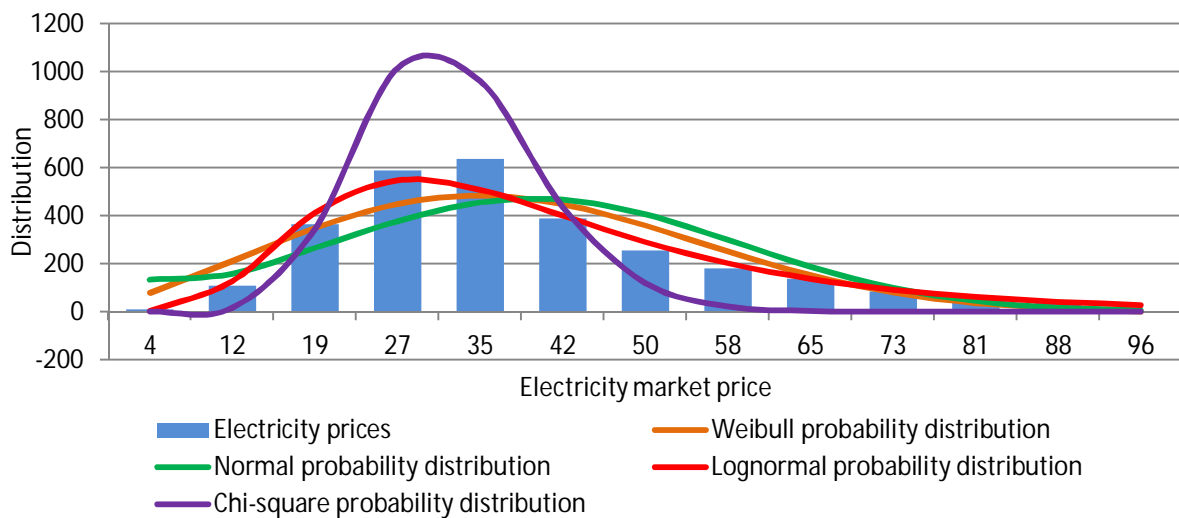
The sample used for the chi-squared test covered electricity market prices of baseload of EEX ([www.eex.de](http://www.eex.de)) from 1.1.2002 till 31.12.2009. There were tested several types of the empirical continuous probability distributions through Pearson's chi-squared test.

- Normal probability distribution
- Lognormal probability distribution

- Weibull probability distribution
- Chi-squared probability distribution

The closest results of the real electricity market prices to the empirical probability distribution was given by the log-normal distribution, however none of the probability distribution gave us null hypothesis true. In this respect we would need to assess how much the use of the lognormal distribution misleads the results from the real historical data.

Another approach would be to use different probability distribution however further work has not been performed as it would be over the scope of this thesis. The interpretation of the real historical data and empirical probability distribution is on Figure 22.



**Figure 22:** Left tale of the probability distribution - VAR = 95%

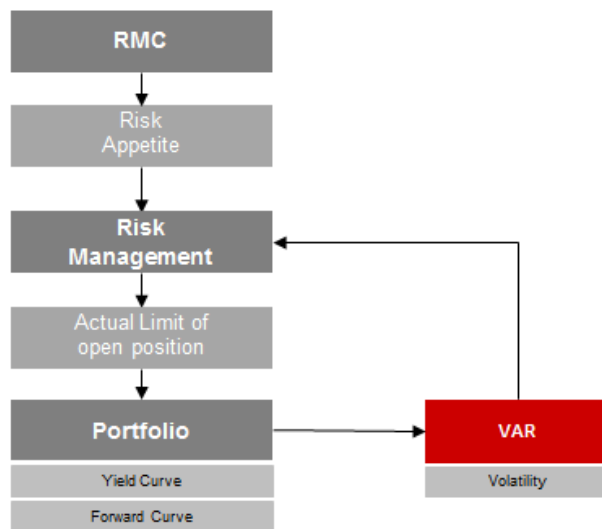
According to [58] another approach for VAR calculation might be Extreme Value Theory which can capture extremities better than conventional probability distributions, as it concentrates on the observations that exceed certain limit. The attention is focused on the tail of the price distribution, which is the area where the VAR is estimated. The question is whether these so called fat tails is the main issue why the conventional probability distributions do not describe the pattern of the electricity market prices fluctuations. It is apparent from Figure 22 that the main issue is given by the shape of the theoretical distributions before the tail starts and not by the tail itself. Also there is no standard interpretation of what the fat tail is. Pearson's chi-squared testing of the probability distribution is attached in the form of XLS file on the attached CD. In the related table for probability distribution testing we can see that the differences between empirical probability distribution and the real market prices are high before the fat tail starts. The question is whether the Log-normal distribution could be used for the Value at Risk calculation at the fat tail area.

Extreme Value Theory became evergreen for past ten years and there are number of authors like Paul Embrechts, Alexander McNeil, Richard Smith, Rüdiger Frey, Francois Longin, Hans Byström and Kimmo Lehikoinen that performed number of articles on this topic. These articles define EVT VAR which is modified Value at Risk technique described above for the extreme electricity market prices fluctuations. This is eventually useful for the electricity market traders as these do not have possibility to generate electricity to avoid these rare situations. Due to this possibility this metrics is redundant for power producers.

Testing of probability distribution through Pearson’s chi-squared test is a by-product of this thesis, however the results are very valuable and present a strong fundament for another researches to be performed in this area.

#### 4.4 Risk management and limits

First of all the board of directors must set the Annual Risk Appetite for the risk of open position. Risk Appetite is an amount of capital that a company is willing to lose in order to generate a potential profit. This information is necessary for the Risk Management due to setting of Actual Limit of open positions in portfolio. The Actual Limit is very variable and depends on the market positions of the contracts which were traded. The following part has been first introduced on 11<sup>th</sup> International Student Conference on Electrical Engineering by Knezek [53].



**Figure 23:** Risk Management of open position

Open positions are very sensitive to the volatile prices of electricity and therefore it is necessary to evaluate its risk. To evaluate the risk of open position JP Morgan [50] developed RiskMetrics methodology called VaR (Value at Risk). VaR is a measure of the maximum potential loss in the value of open position with a given probability over a specified time period.

Risk Management continuously evaluates VaR for their actual open position and according to the Actual Limit decreases or increases the size of open position.

#### **4.5 Breach of Risk Management limits**

In case of any open position limit is threatened the RMC has right to take appropriate measurement which eliminates the threatening risk and stop any action, which could cause limit breach. In the event that any limit is breached, the RMC is obliged to evaluate the seriousness of the potential breach as follows:

##### **The foreseeable limit breach**

In case of a foreseeable limit breach, where breach is against producer trading strategy and approved limits, front office is obliged to announce such situation to RMC. Then front office has to take appropriate measure, to restrict the impact of the limit breach.

In case of a foreseeable limit breach where trading activity is evidently profitable, the RMC has the right to increase level of limit related to that trading activity.

##### **The limit breach has already occurred**

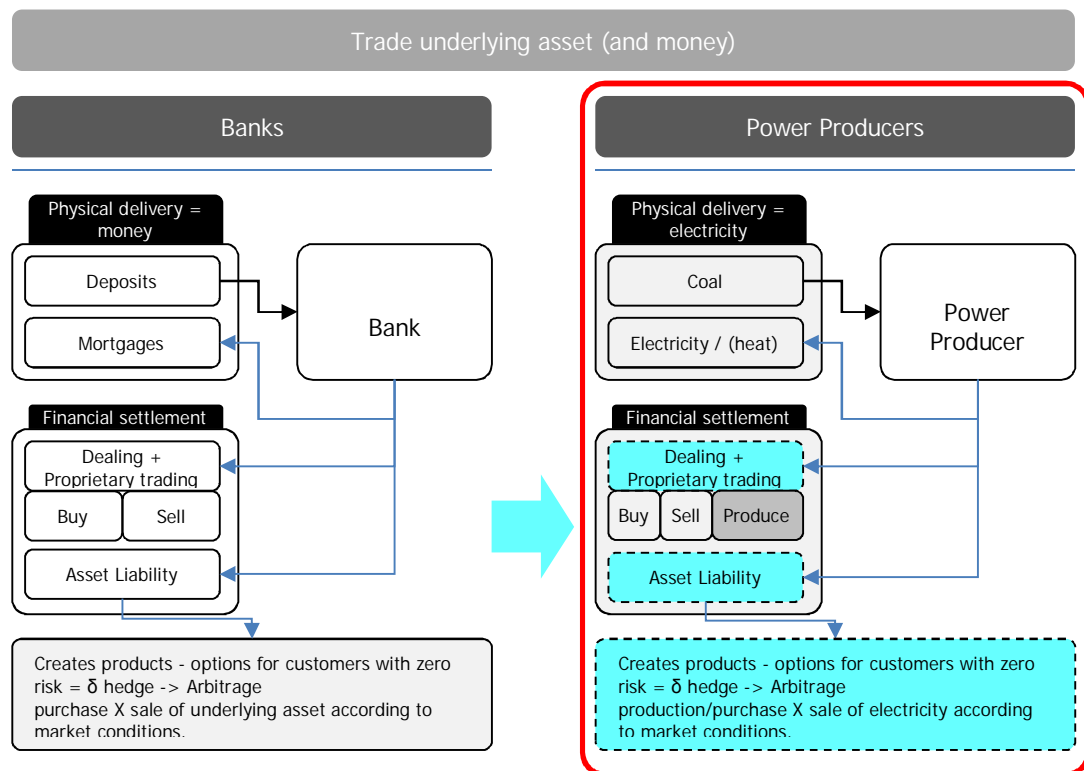
In case that already concluded trading activity caused breach of existing limit which is against producer trading strategy and approved limits and has no economic justification the RMC has to inform Board of Directors of such breach. This will include a short description of the breach and any immediate action proposed or taken.

In case that already concluded trading activity caused breach of existing limit and breach of limit reduced/eliminated economic losses RMC will evaluate the related trading activity, its rightfulness and scope of limit breach. If all findings are positive RMC can approve such limit breach.

In any case of a foreseeable or actual limit breach mentioned above, it is reported in the monthly RM report.

## 5 Financial products used by power producers

The following part has been first presented on 17<sup>th</sup> International Student Conference on Electrical Engineering by Knezek [56] in 2013.



**Figure 24:** Power Asset Liability Management

The general overview of hedging in electricity market is described by Morbee and Willems in [60]. Their discussion paper deals with different kind of derivatives in the portfolio, incompleteness of electricity market, welfare effects and CRRA (Constant Relative Risk Aversion) utility function. The results of their paper are mostly theoretical. Another perspective based on cross-hedging is provided by Hoang, Horowitz and Woo in [48]. Deng and Xia describe in pricing and electricity supply agreements by using tolling agreements in [40]. All three publications, have number of citations and give an invaluable insight into electricity market hedging from the power producer perspective, however their approach is different from the approach used in this thesis. Despite the fact that there are number of articles related to hedging in the electricity market, none of these deals with delta hedge. General concept of delta hedge with application in the financial market has been introduced in [59] by Lyuu.

This chapter deals with another opportunity gap for the power producer which is generally handled by the banks. Power producers have better conditions to write call and put options for its customers. As they may decide if the option is getting out of the money whether they will

produce or purchase missing electricity in case of exercised call options or reduce production or sell electricity in case of exercised put options.

The key idea behind the model is to hedge the sold options (either call or put) by buying and selling the underlying asset to "eliminate risk" and get "risk free profit" in form of premium. This hedge is called delta hedging and is the basis of more complicated hedging strategies such as those engaged in by investment banks and hedge funds. The hedge implies that there is a unique price for the option which is given by the Black–Scholes formula.

The creation of options based on delta hedge is performed by Asset Liability Management department in the banks, which is one of the most important department within the bank – sometimes called "the bank in the bank". This department is responsible for the creation of structured products with risk free profit. For example natural hedge inform of using short term savings/deposits and providing long-term mortgages/loans with higher interest rate, creation of fixed/variable or variable/fixed swaps based on portfolio of bonds with variable/fixed coupons or other swaps, creation of option contracts based on delta hedge, etc.

Power Producers may easily takeover their role and create and offer structured products themselves with extensive profits without any risk exposure. The key to success is either to establish similar department or increase the role of the risk management department. This department/sub-department will have the only responsibility to create structured option products based on delta hedge strategy.

## **5.1 Greeks**

Greeks and delta hedging in the financial market has been described in [41] by Elder. Application of Greeks and hedging is further described by Pelletier in [61]. A little bit more detail of Greeks could be found in [1]. However all these publications relate to hedging of stock prices and provide only brief introduction for the future researches in this area. In this respect this topic is much further developed in this thesis and could be perceived as an invaluable source of information for the future researches in this area.

In order to understand delta hedge strategy we need to define Greeks. Greeks collectively tell us about the sensitivity of the option price to its underlying input. In other words they help us to evaluate dimension of the risk involved in taking a position in an option contracts. Each measure of the risk is represented by a letter of the Greek alphabet.

### **5.1.1 Delta**

Delta represents the rate of change between the option's price and the underlying asset's price - in other words, price sensitivity. Delta is used to measure the slope of the option value line at any particular time/price point.

Option which is deeply ITM will almost certainly be exercised. In case of price change of underlying asset the value of the option changes as well. In contrary option which is deeply OTM is almost insensitive to the change in price of the underlying asset, since it is very unlikely to be realized. This is referred as an indicator delta. Delta is the expected change in price of the option if the underlying asset price changes by one unit. For better Below are some borderline cases:

- Options deeply OTM have delta equal to zero.
- Call options deeply ITM have delta close to +1 as the change in the price of electricity for every price unit will cause a change of the option price also almost a price unit.
- Put options deeply ITM have delta close to -1, as the change in the price of electricity for every price unit will cause a change of the option price also almost a price unit.

Delta is calculated as the first partial derivative of the option price according to price of the underlying asset, where the option price is calculated using the Black-Scholes model.

General formula:

$$\Delta = \frac{\partial \text{option price}}{\partial \text{underlying asset's price}} \quad (24)$$

We can rewrite the formula as follows:

Delta - call option:

$$\Delta_{call} = \frac{\partial P_{call}}{\partial P} = \frac{\partial (P \cdot N(d_1) - S \cdot N(d_2) \cdot e^{-rt})}{\partial P} \quad (25)$$

Delta - put option:

$$\Delta_{put} = \frac{\partial P_{put}}{\partial P} = \frac{\partial (S \cdot N(-d_2) \cdot e^{-rt} - P \cdot N(-d_1))}{\partial P} \quad (26)$$

And after calculation of partial derivative:

Delta - call option:

$$\Delta_{call} = N(d_1) \quad (27)$$

Delta - put option:

$$\Delta_{put} = N(d_2) - 1 \quad (28)$$

### 5.1.2 Gamma

Gamma is the first derivative of Delta. Gamma represents a rate of measures how much Delta changes with the underlying asset value i.e. how much the delta hedge needs to be adjusted as the underlying asset value changes, for example if the gamma is 0,01 this means that for a 1% rise in the underlying asset value the delta should change by a factor of 0,01. Gamma is calculated as second partial derivative of option price according to price of the underlying asset, where the option price is calculated using Black-Scholes model.

General formula:

$$\Gamma = \frac{\partial \text{Delta value}}{\partial \text{underlying asset's price}} \quad (29)$$

We can rewrite the formula in form of second partial derivative. In this case the formula for Gamma is the same for call and put options:

$$\Gamma = \frac{\partial^2 P_{call}}{\partial P^2} = \frac{\partial^2 (P \cdot N(d_1) - S \cdot N(d_2) \cdot e^{-rt})}{\partial P^2} = \frac{\partial^2 P_{put}}{\partial P^2} = \frac{\partial^2 (S \cdot N(-d_2) \cdot e^{-rt} - P \cdot N(-d_1))}{\partial P^2} \quad (30)$$

And after calculation of partial derivatives:

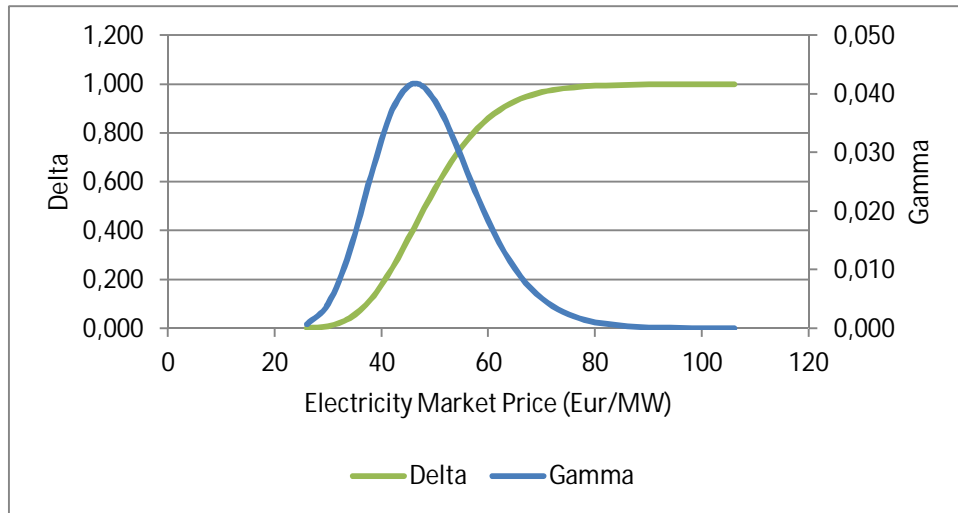
Gamma:

$$\Gamma = \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-d_1^2}{2}}}{P \cdot \sigma \cdot \sqrt{t}} \quad (31)$$

If the value of delta represents the change in value of the option when the underlying asset price changes by one unit, then the value of gamma says how sensitive is the change in value of the option when underlying asset price changes. The delta for options that are deep ITM or OTM almost does not change with the change of the underlying asset price and therefore corresponding gamma value is close to zero, while for options whose market value is around the strike price are measures delta and gamma very sensitive.

The dependence of parameters delta and gamma on the price of electricity and time to expiry for the call option is shown in Figure 25 and Figure 26 below:

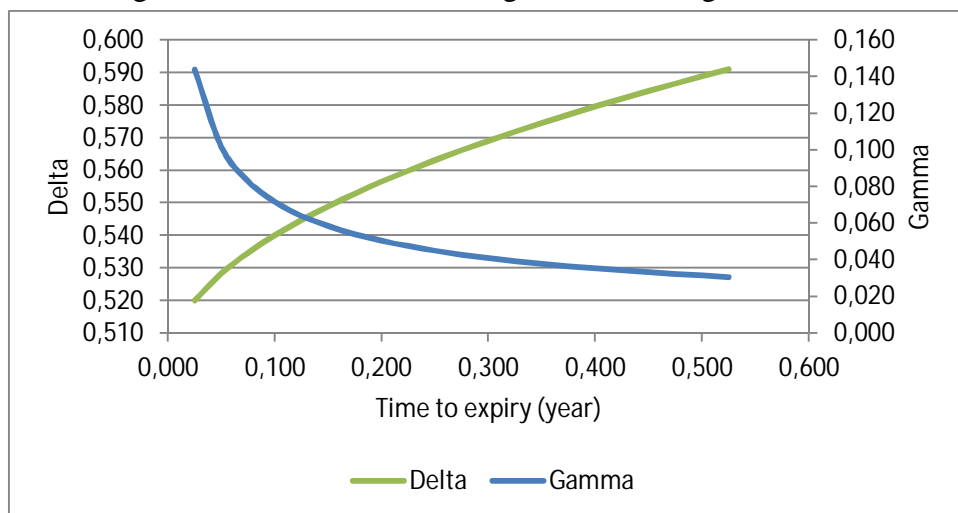




**Figure 25:** Dependence of delta and gamma on electricity price for call option

From the Figure 25 above we can see that:

- Delta is bounded by 1 and 0.
- As the electricity market price increases the option becomes progressively more in the money such that the delta converges and becomes close to one.
- As the electricity market price decreases the option is becoming progressively out of the money and the delta converges to zero.
- Gamma tends to peak when the option is at the money and that's where the gamma is changing the most rapidly. As it has the steepest slope right about at the money.
- With increase of electricity market price the delta is converging to 1 but becoming more stable so gamma is tending towards zero.
- With decrease of electricity market price the delta is converging to zero and also becoming more stable its rate of change is slower so gamma is also close to zero.

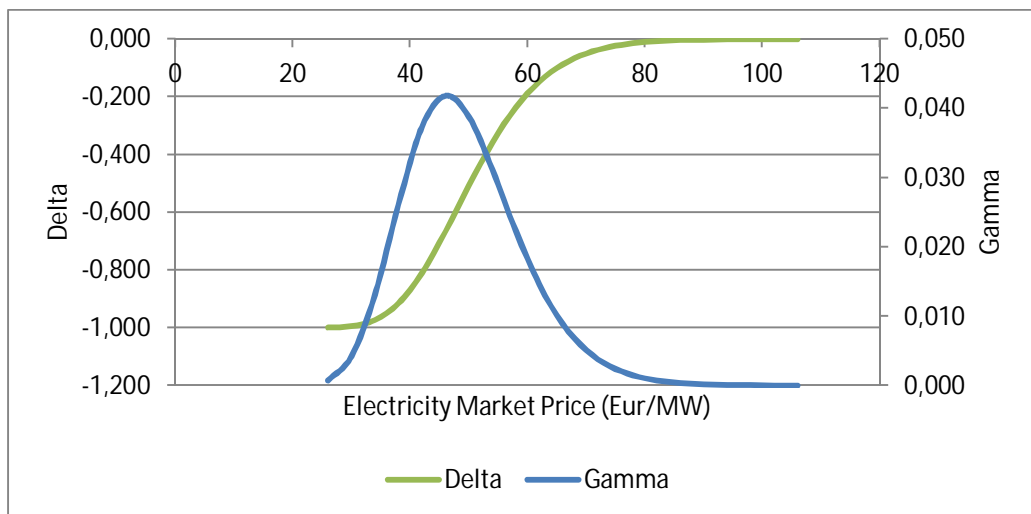


**Figure 26:** Dependence of delta and gamma on time to expiry for call option

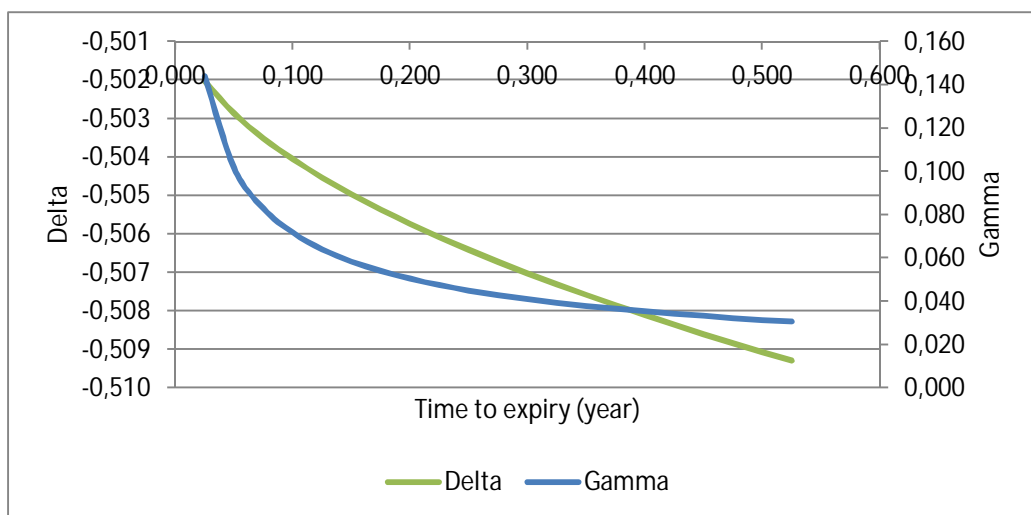
From the Figure 26 above we can see that:

- The longer the term of the option the greater the delta the greater its sensitivity to the electricity market price.
- And for gamma the longer the term the slower the rate of change and the lower the gamma.

In case of a put option will be dependence of parameters delta and gamma on the price of electricity symmetrically reversed. Indicator gamma is independent of the type of option and remains the same as shown in the Figure 27 and Figure 28 below:



**Figure 27:** Dependence of delta and gamma on electricity price for put option



**Figure 28:** Dependence of delta and gamma on time to expiry for put option

### 5.1.3 Vega

Vega represents the rate of change between an option portfolio's value and the underlying asset's volatility, or sensitivity to volatility. In general the more volatile the underlying asset, the more valuable the option is.

General formula:

$$V = \frac{\partial \text{option value}}{\partial \text{volatility}} \quad (32)$$

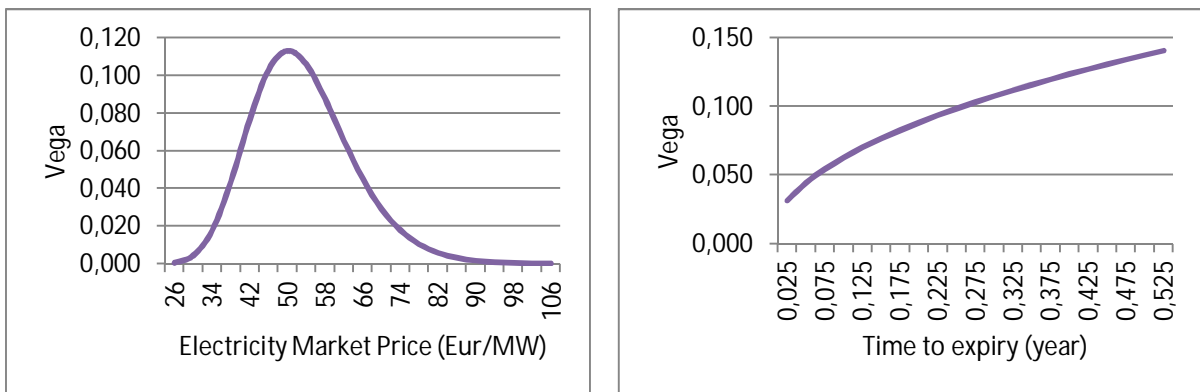
In this case the formula for Vega is the same for call and put options. We can rewrite the formula as follows:

$$V = \frac{\partial P_{call}}{\partial t} = \frac{\partial (P \cdot N(d_1) - S \cdot N(d_2) \cdot e^{-rt})}{\partial t} = \frac{\partial P_{put}}{\partial t} = \frac{\partial (S \cdot N(-d_2) \cdot e^{-rt} - P \cdot N(-d_1))}{\partial t} \quad (33)$$

And after calculation of partial derivative:

Vega – same for call and put options:

$$V = P \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{d_1^2}{2}} \cdot \sqrt{t} \quad (34)$$



**Figure 29:** Dependence of Vega on electricity price and on time to expiry for call option

From the chart above we can see that:

- Like gamma vega tends to peak at the money, so at the money option has higher sensitivity to changes of volatility.
- Vega is increasing function of the option life. The longer the term more sensitive the option price is to changes of volatility.

### 5.1.4 Theta

Theta is also sometimes called time decay and represents the rate of change between an option portfolio and time, or time sensitivity. In general the longer to expiry, the more valuable the option is.

General formula:

$$\Theta = \frac{\partial \text{option value}}{\partial \text{time}} \quad (35)$$

We can rewrite the formula as follows:

Theta - call option:

$$\Theta_{call} = \frac{\partial P_{call}}{\partial t} = \frac{\partial (P \cdot N(d_1) - S \cdot N(d_2) \cdot e^{-rt})}{\partial t} \quad (36)$$

Theta - put option:

$$\Theta_{put} = \frac{\partial P_{put}}{\partial t} = \frac{\partial (S \cdot N(-d_2) \cdot e^{-rt} - P \cdot N(-d_1))}{\partial t} \quad (37)$$

And after calculation of partial derivative:

Theta - call option:

$$\Theta_{call} = -\frac{P \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{d_1^2}{2}} \cdot \sigma}{2\sqrt{t}} - r \cdot S \cdot e^{-rt} \cdot N(d_2) \quad (38)$$

Theta - put option:

$$\Theta_{put} = -\frac{P \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{d_1^2}{2}} \cdot \sigma}{2\sqrt{t}} + r \cdot S \cdot e^{-rt} \cdot N(-d_2) \quad (39)$$

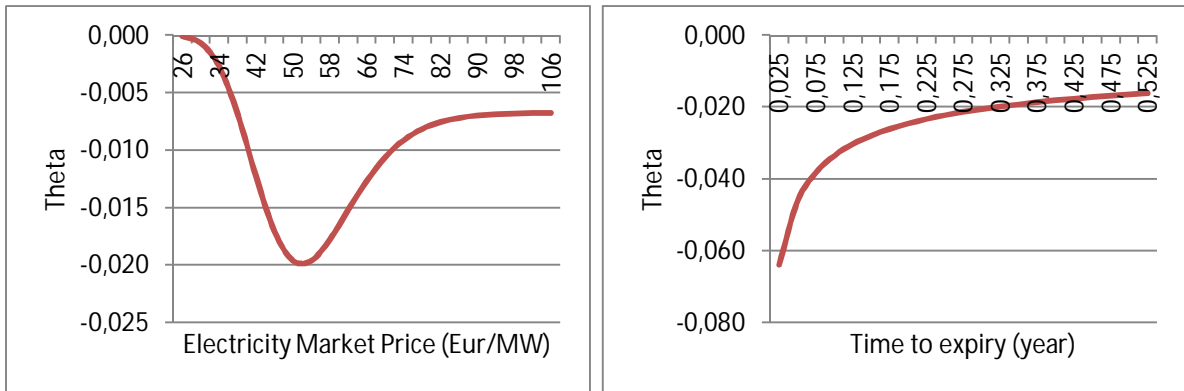
Theta is usually quoted per calendar day (T=365) or per trading day (T=252):

Theta - call option:

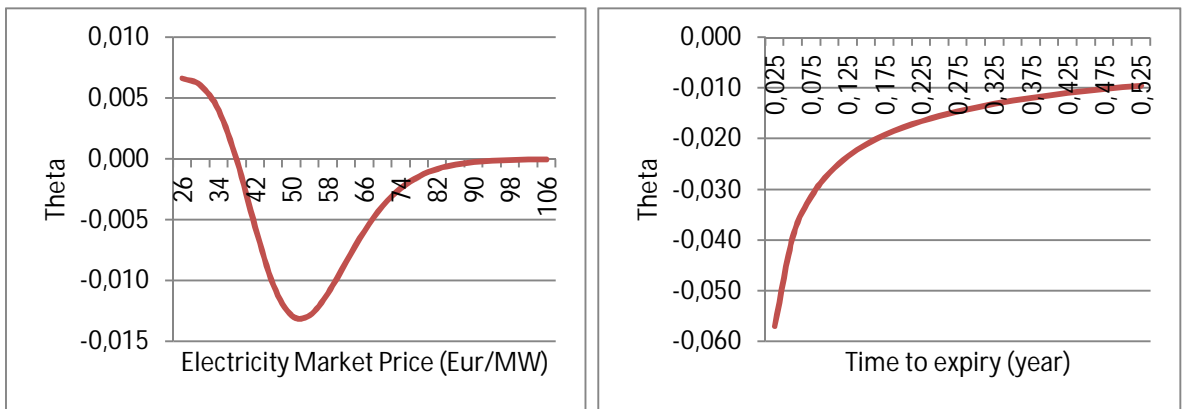
$$\Theta_{call} = \frac{-\frac{P \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{d_1^2}{2}} \cdot \sigma}{2\sqrt{t}} - r \cdot S \cdot e^{-rt} \cdot N(d_2)}{T} \quad (40)$$

Theta - put option:

$$\Theta_{put} = \frac{P \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{d_1^2}{2}} \cdot \sigma}{2\sqrt{t}} + r \cdot S \cdot e^{-rt} \cdot N(-d_2)$$
(41)



**Figure 30:** Dependence of Theta on electricity price and on time to expiry for call option



**Figure 31:** Dependence of Theta on electricity price and on time to expiry for put option

From the Figure 30 and Figure 31 above we can see that:

- Theta is generally less than zero
- As time passes and the option is getting closer to expiration its becoming less valuable. So time is eroding the value of the option. And as it turns out the closer the option gets to the expiration the more rapidly that theta is acting on the option. It is sort of exelcating function as we get closer to expiration.

### 5.1.5 Rho

Rho represents the rate of change between an option portfolio's value and the interest rate, or sensitivity to the interest rate. In general the higher the interest rate, the more valuable the option is.

General formula:

$$\rho = \frac{\partial \text{option value}}{\partial \text{interest rate}} \quad (42)$$

We can rewrite the formula as follows:

Rho - call option:

$$\rho_{call} = \frac{\partial P_{call}}{\partial r} = \frac{\partial (P \cdot N(d_1) - S \cdot N(d_2) \cdot e^{-rt})}{\partial r} \quad (43)$$

Rho - put option:

$$\rho_{put} = \frac{\partial P_{put}}{\partial r} = \frac{\partial (S \cdot N(-d_2) \cdot e^{-rt} - P \cdot N(-d_1))}{\partial r} \quad (44)$$

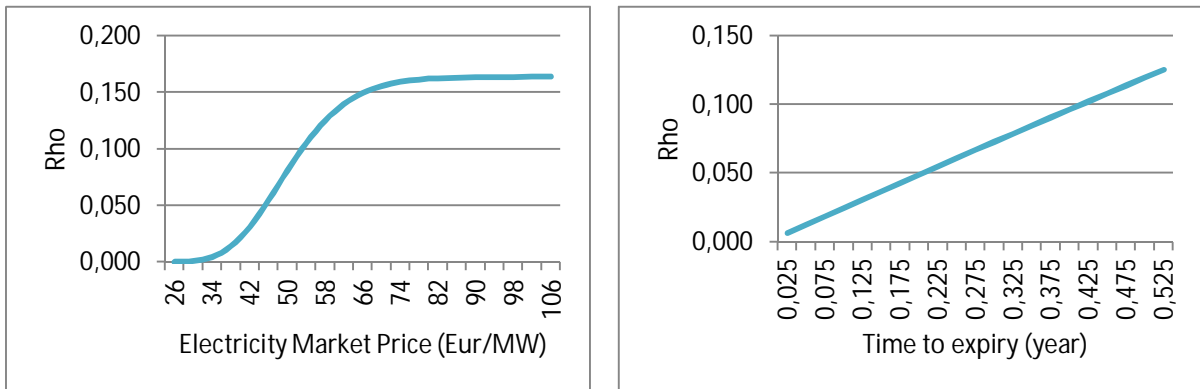
And after calculation of partial derivative:

Rho - call option:

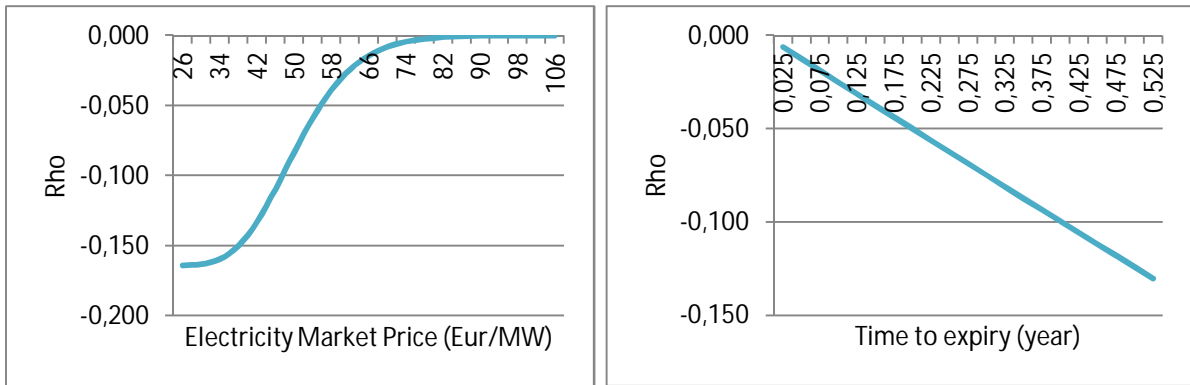
$$\rho_{call} = S \cdot t \cdot e^{-rt} \cdot N(d_2) \quad (45)$$

Rho - put option:

$$\rho_{put} = -S \cdot t \cdot e^{-rt} \cdot N(-d_2) \quad (46)$$



**Figure 32:** Dependence of Rho on electricity price and on time to expiry for call option



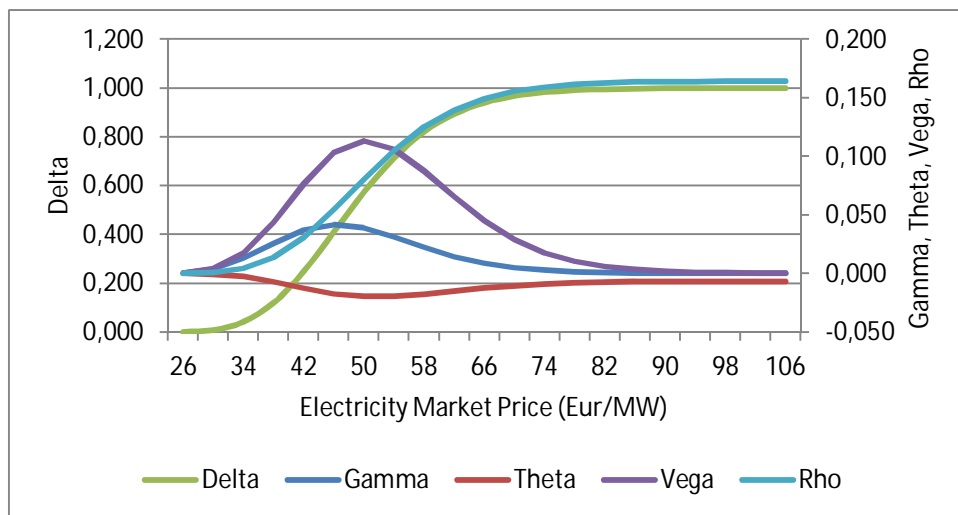
**Figure 33:** Dependence of Rho on electricity price and on time to expiry for put option

From the Figure 32 and Figure 33 above we can see that:

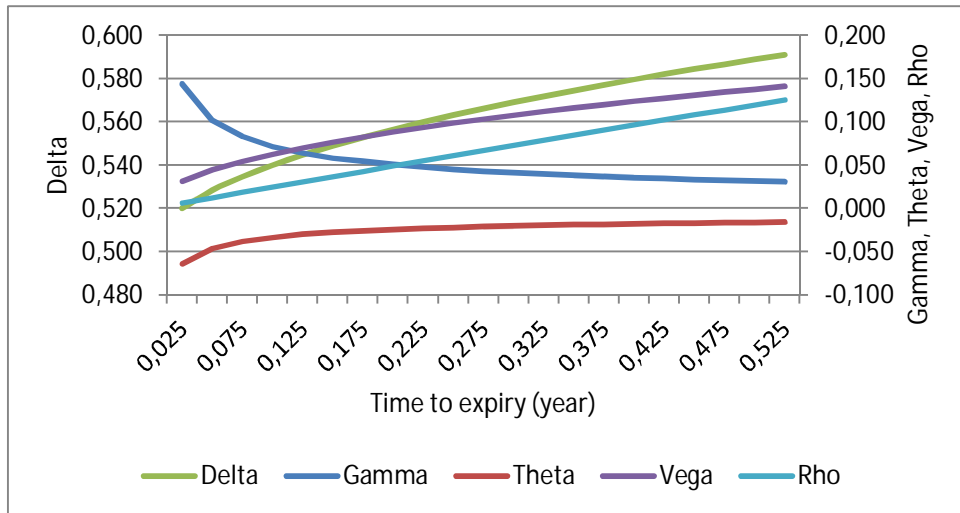
- Relative to the other inputs the sensitivity of the option to the interest rate is a little modest.
- Rho is increasing function of the remaining life of the option. The longer the term more sensitive the option price is to changes of interest rate.

### 5.1.6 Greeks charts

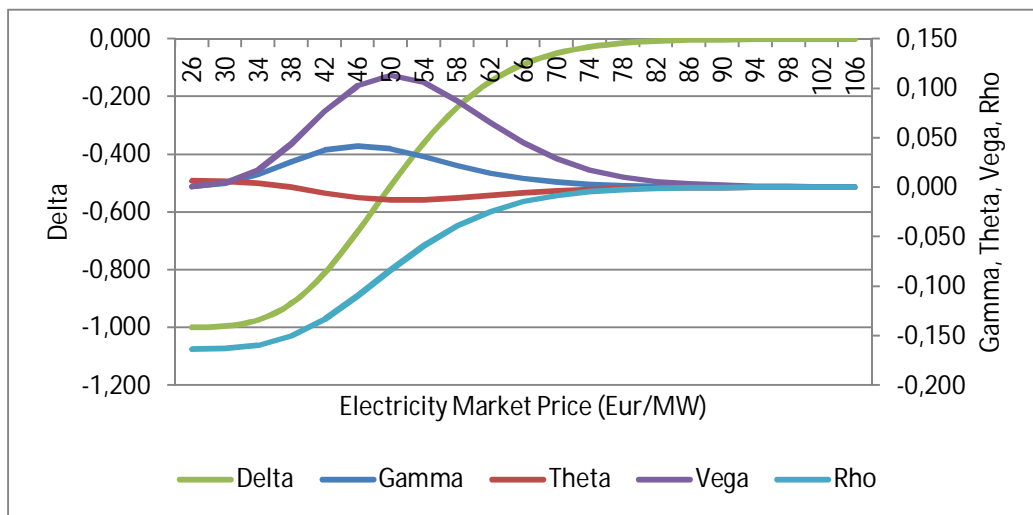
Dependence of all Greeks on electricity market price in the above example is demonstrated in Figure 34 and Figure 35 for call option and in Figure 36 and Figure 37 for put option:



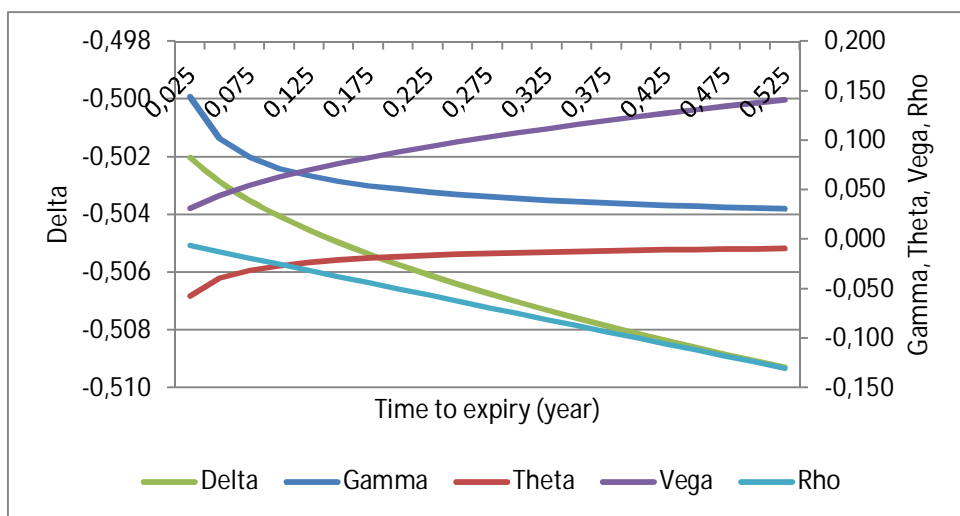
**Figure 34:** Dependence of Greeks on electricity price for call option



**Figure 35:** Dependence of Greeks on time to expiry for call option



**Figure 36:** Dependence of Greeks on electricity price for put option



**Figure 37:** Dependence of Greeks on time to expiry for put option



## 5.2 Approximations by using Greeks

Greeks could be used to get approximate option price more easily without recalculation option price through Black-Scholes formula as the valuation of options for large portfolios presents a trade-off between speed and accuracy, with the fastest methods relying on rough approximations and the most realistic approach – Black-Scholes – often too slow to be practical. In this respect Greeks approximations represent invaluable possibility for delta hedge neutral options portfolio revaluation with great accuracy within required time period. This can greatly reduce the time that would be otherwise needed. The only limitation is that the approximation could be used for small changes of the option input factors as described in Chapter 2 (electricity market price, option exercise price, time to maturity of the option, interest rate, market price volatility).

Approximations are based on Taylor series:

$$f(x) = f(x_0) + f'(x_0) \cdot \frac{(x - x_0)}{1!} + f''(x_0) \cdot \frac{(x - x_0)^2}{2!} + f'''(x_0) \cdot \frac{(x - x_0)^3}{3!} \dots \quad (47)$$

Where

$$f(x) = P_{call}(P) \quad \text{or} \quad f(x) = P_{put}(P) \quad (48)$$

By applying Taylor series on Black-Scholes option pricing formula we obtain sum of individual functions to infinity.

Call option:

$$P_{call}(P_1) = P_{call}(P_0) + \frac{\partial P_{call}(P_0)}{\partial P_0} \cdot \frac{(P_1 - P_0)}{1!} + \frac{\partial^2 P_{call}(P_0)}{\partial P_0^2} \cdot \frac{(P_1 - P_0)^2}{2!} + \dots \quad (49)$$

Put option:

$$P_{put}(P_1) = P_{put}(P_0) + \frac{\partial P_{put}(P_0)}{\partial P_0} \cdot \frac{(P_1 - P_0)}{1!} + \frac{\partial^2 P_{put}(P_0)}{\partial P_0^2} \cdot \frac{(P_1 - P_0)^2}{2!} + \dots \quad (50)$$

The individual functions are becoming after first couple of steps immaterial and we can truncate them so we get truncated Taylor series.

### 5.2.1 Delta approximation

Delta approximation is truncated equation above from the second degree further, so we get the following equation:

Call option – delta approximation:

$$P_{call}(P_1) \approx P_{call}(P_0) + \Delta_{call} \cdot \Delta P \quad (51)$$

Put option – delta approximation:

$$P_{put}(P_1) \approx P_{put}(P_0) + \Delta_{put} \cdot \Delta P \quad (52)$$

Suppose we know the value of a call option on the market price of electricity 50 €MW, we know the delta and we would like to find out how much the value of the options at the market price of electricity 51 €MW. The time to maturity of the option is 4 months, the discount rate is 5% and the volatility of electricity prices is 35%. Option value at electricity market price of 50 €MW, according to Black-Scholes model will be:

$$P_{call}(50) = 4,417 \text{ €MW}$$

Delta value will be:

$$\Delta_{call} = 0,573$$

Option value at electricity market price of 51 €MW using the approximation will be:

$$P_{call}(51) \approx P_{call}(50) + (\Delta \times 1) = 4,990 \text{ €MW.}$$

Real option value calculated through Black-Scholes model will be: 5,009 €MW.

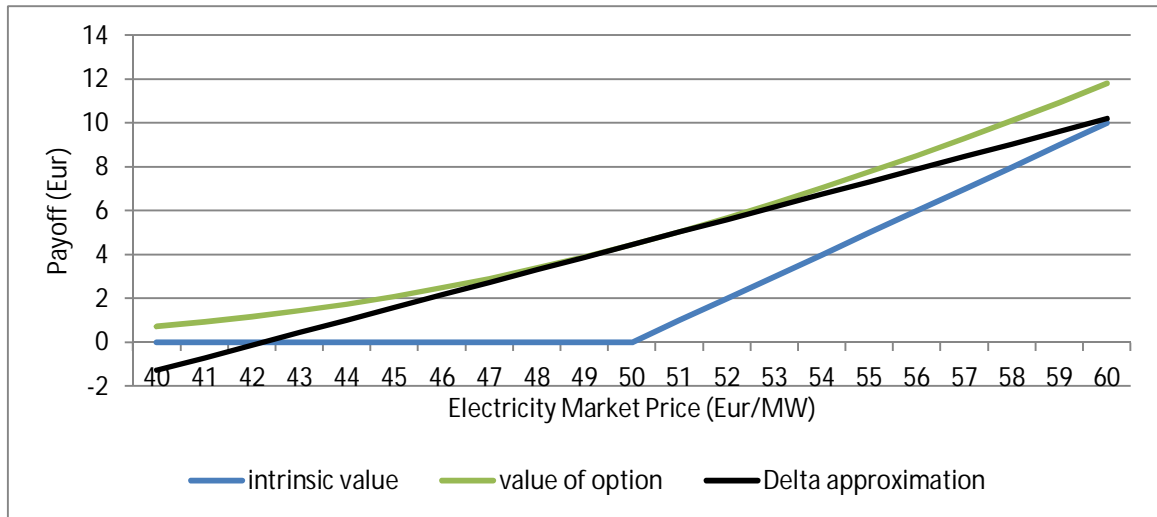
Absolute difference is 0,019 €MW

In case the electricity price is 49,5 €MW option value will be:

$$P_{call}(49,5) \approx P_{call}(50) - (\Delta \times 0,5) = 4,131 \text{ €MW}$$

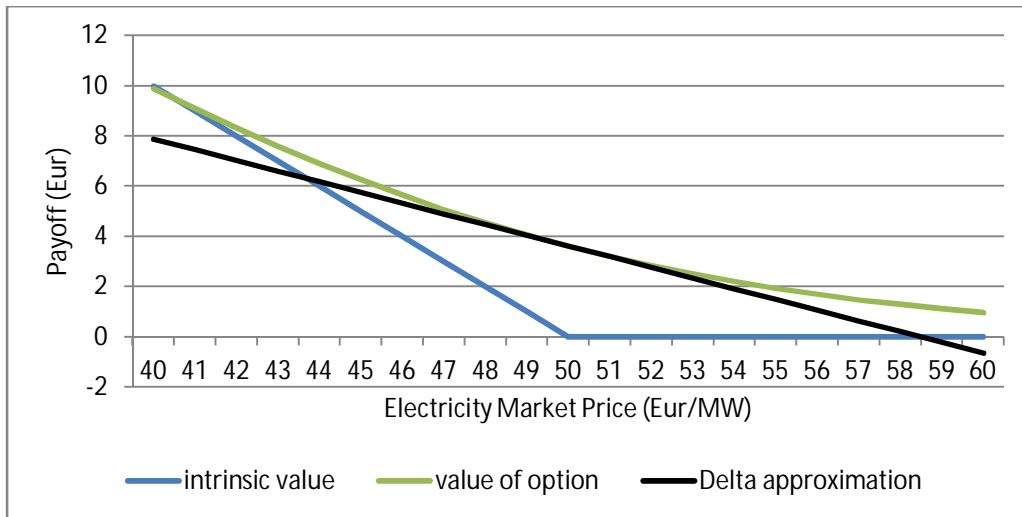
Real option value calculated through Black-Scholes model will be: 4,136 €MW and absolute difference is 0,005 €MW.

Dependence of option value based on delta approximation on electricity price is shown in Figure 38.



**Figure 38:** Delta approximation on electricity market price for call option

In case of put option with the same parameters as in the example above the delta approximation would be as follows - Figure 39:



**Figure 39:** Delta approximation on electricity market price for put option

From the graph above it is apparent that with the difference in electricity prices from the original value for which delta approximation was calculated becomes approximation less accurate. It is obvious that the approximation using the delta indicators can be used only for electricity, which does not differ too much from the original value. Delta is therefore used to quickly recalculate the approximate value of the options portfolio for small changes in the electricity prices.

### 5.2.2 Delta-gamma approximation

In above example we have described how we can use the delta indicator for a quick revaluation of options in a portfolio for change in the market price of electricity. By using second degree of the equation we get formula for the Delta-gamma approximation. It gives us more precise results as we have truncated the equation from the third degree further:

Call option – Delta-gamma approximation:

$$P_{call}(P_1) \approx P_{call}(P_0) + \Delta_{call} \cdot \Delta P + \Gamma_{call} \cdot \frac{\Delta P^2}{2} \quad (53)$$

Put option – Delta-gamma approximation:

$$P_{put}(P_1) \approx P_{put}(P_0) + \Delta_{put} \cdot \Delta P + \Gamma_{put} \cdot \frac{\Delta P^2}{2} \quad (54)$$

So let's see how we can using both indicators to better approximate the value of the option in our portfolio with electricity market price change.

Suppose we know the value of a call option on the market price of electricity 50 €/MW, we know the delta and we would like to find out how much the value of the options at the market price of electricity 51 €/MW. The time to maturity of the option is 4 months, the discount rate is 5% and the volatility of electricity prices is 35%. Option value at electricity market price of 50 €/MW, according to Black-Scholes model will be:

$$P_{call}(50) = 4,417 \text{ €/MW}$$

Delta and gamma values will be:

$$\Delta = 0,573$$

$$\Gamma = 0,039$$

Option value at at electricity market price of €51 / MW using the approximation will be:

$$P_{call}(51) \approx P_{call}(50) + (\Delta \times 1) + \frac{1}{2} \times \Gamma \times (1)^2 = 5,010 \text{ €/MW.}$$

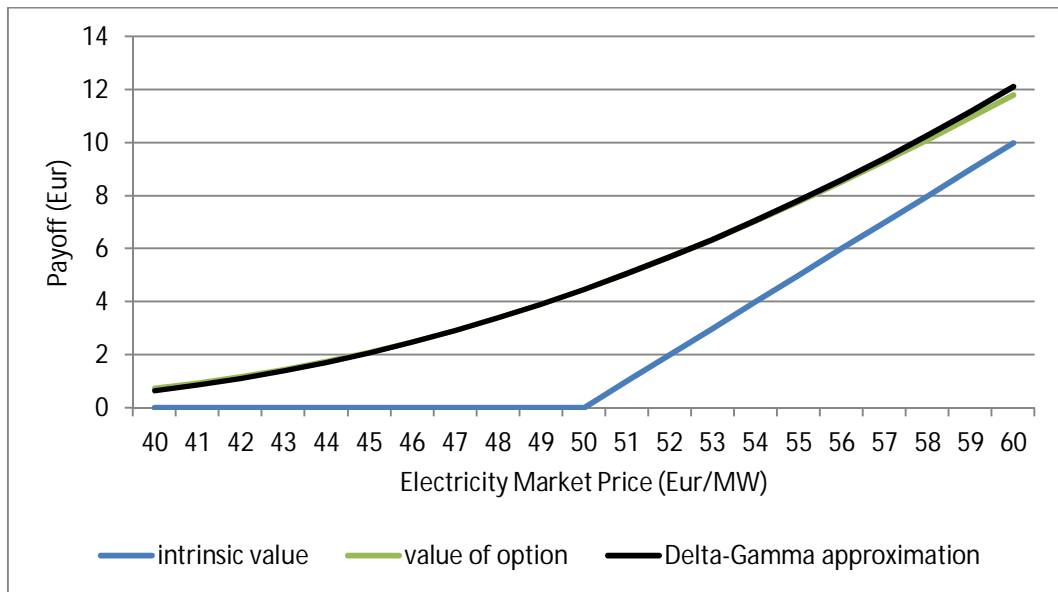
Real option value calculated through Black-Scholes model will be: 5,009 €/MW. Absolute difference is 0,001 €/MW.

In case the electricity price is 49,5 €/MW option value will be:

$$P_{call}(49,5) \approx P_{call}(50) - (\Delta \times 0,5) + \frac{1}{2} \times \Gamma \times (-0,5)^2 = 4,136 \text{ €/MW}$$

and the real value calculated through Black-Scholes model will be: 4,136 €/MW. Absolute difference is 0,000 €/MW

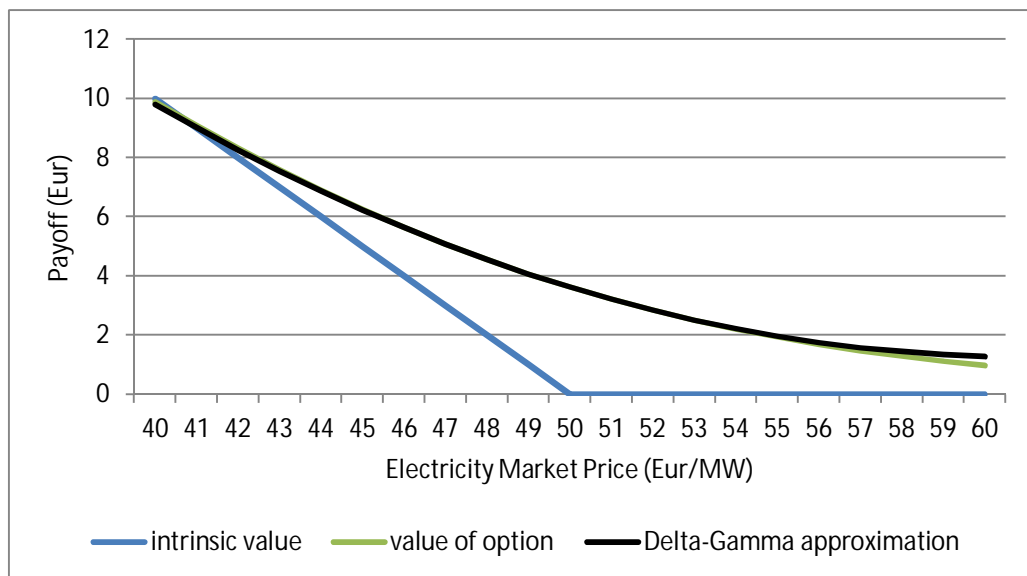
Dependence of option value based on delta-gamma approximation on electricity price is shown on graph below:



**Figure 40:** Delta-gamma approximation on electricity market price for call option

From the Figure 40 it is apparent that the delta-gamma approximation follows the value of the option in a wider area than the delta approximation. With the difference in the electricity price of electricity from the option exercise price becomes approximation less accurate.

In case of put option with the same parameters as in the example above the delta-gamma approximation would be as follows - Figure 41:



**Figure 41:** Delta-gamma approximation on electricity market price for put option

### 5.2.3 Delta-gamma-theta approximation

Above we have discussed Taylor approximation in one variable – electricity market price. Assuming that the volatility of electricity market price is constant and also interest rates are constant within short period of time and option value depends only on electricity market price and time to expiry, we can use delta-gamma-theta approximation.

General formula – we can modify formula (47) into the form of more variables:

$$f(x, y) = f(x_0, y_0) + f'(x_0) \cdot \frac{(x - x_0)}{1!} + f'(y_0) \cdot \frac{(y - y_0)}{1!} + f''(x_0) \cdot \frac{(x - x_0)^2}{2!} + f''(y_0) \cdot \frac{(y - y_0)^2}{2!} + \dots$$

Where

$$f(x, y) = P_{call}(P, t) \text{ or } f(x, y) = P_{put}(P, t) \quad (55)$$

By applying Taylor series on Black-Scholes option pricing formula we obtain sum of individual functions to infinity:

Call option:

$$P_{call}(P_1, t_1) = P_{call}(P_0, t_0) + \frac{\partial P_{call}(P_1, t_1)}{\partial P_1} \cdot \Delta P + \frac{\partial P_{call}(P_1, t_1)}{\partial t_1} \cdot \Delta t + \frac{\partial^2 P_{call}(P_1, t_1)}{\partial P_1^2} \cdot \frac{\Delta P^2}{2} + \frac{\partial^2 P_{call}(P_1, t_1)}{\partial t_1^2} \cdot \frac{\Delta t^2}{2} + \dots \quad (56)$$

As all the individual functions with variable – electricity price would be immaterial from the third degree further and time to expiry will be immaterial from the second degree we can truncate them. The delta-gamma-theta approximation will be as follows:

$$P_{call}(P_1, t_1) \approx P_{call}(P_0, t_0) + \frac{\partial P_{call}(P_1, t_1)}{\partial P_1} \cdot \Delta P + \frac{\partial^2 P_{call}(P_1, t_1)}{\partial P_1^2} \cdot \frac{\Delta P^2}{2} + \frac{\partial P_{call}(P_1, t_1)}{\partial t_1} \cdot \Delta t \quad (57)$$

The shorten format of the above equation by using Greeks will be as follows:

$$P_{call}(P_1, t_1) \approx P_{call}(P_0, t_0) + \Delta \Delta P + \Gamma \times \frac{\Delta P^2}{2} + \Theta \Delta t \quad (58)$$

From the formula above (60) we can express a formula for the change of call option price:

$$\Delta P_{call} = \Delta \Delta P + \frac{1}{2} \times \Gamma \times \Delta P^2 + \Theta \Delta t \quad (59)$$

The same formula is applicable for put option price:

$$\Delta P_{put} = \Delta \Delta P + \frac{1}{2} \times \Gamma \times \Delta P^2 + \Theta \Delta t \quad (60)$$

Suppose we have the same call option as in previous example with current market price of electricity 50 €/MW, strike price 50 €/MW, maturity period 4 months, the discount rate 5% and the volatility of electricity prices 35%.

$$P_{call}(50, 4M) = 4,3846 \text{ €/MW}$$

Greeks values will be:

$$\Delta = 0,5728$$

$$\Gamma = 0,0388$$

$$\Theta = -0,0196$$

We would need to find out the value of the call option if the market parameters change as follows: market price of electricity 51 €/MW and time to expiry decreases by 1 day.

Option value using the Delta-gamma-theta approximation will be:

$$P_{call}(51, 4M - 1D) = 4,3846 + 0,5728 \times 2 + 0,0388 \times \frac{2^2}{2} - 0,0196 \times \frac{1}{365} = 4,9768 \text{ €/MW}$$

Real option value calculated through Black-Scholes model will be:

$$P_{call}(51; 4M - 1D) = 4,9562 \text{ €/MW}$$

Absolute difference is -0,0206 €/MW

As there are two variables in the formula (59) it makes the delta-gamma-theta approximation more precise than delta-gamma approximation. With higher difference from original value the approximation is becoming less and less accurate.

#### 5.2.4 Delta-gamma-vega-theta-rho approximation

Delta-gamma-vega-theta-rho approximation is most complex approximation. As we have used formula (55) for two variables we can easily use that for more variables. This formula can be used for all the option inputs – price, volatility, time and interest rate. Then we get Delta-gamma-vega-theta-rho approximation:

$$\begin{aligned}
P_{call}(P_1, \sigma_1, t_1, r_1) = & P_{call}(P_0, \sigma_0, t_0, r_0) + \frac{\partial P_{call}(P_1, \sigma_1, t_1, r_1)}{\partial P_1} \cdot \Delta P + \frac{\partial P_{call}(P_1, \sigma_1, t_1, r_1)}{\partial \sigma_1} \cdot \Delta \sigma + \\
& + \frac{\partial P_{call}(P_1, \sigma_1, t_1, r_1)}{\partial t_1} \cdot \Delta t + \frac{\partial P_{call}(P_1, \sigma_1, t_1, r_1)}{\partial r_1} \cdot \Delta r + \frac{\partial^2 P_{call}(P_1, \sigma_1, t_1, r_1)}{\partial P_1^2} \cdot \frac{\Delta P^2}{2} + \\
& + \frac{\partial^2 P_{call}(P_1, \sigma_1, t_1, r_1)}{\partial \sigma_1^2} \cdot \frac{\Delta \sigma^2}{2} + \frac{\partial^2 P_{call}(P_1, \sigma_1, t_1, r_1)}{\partial t_1^2} \cdot \frac{\Delta t^2}{2} + \frac{\partial^2 P_{call}(P_1, \sigma_1, t_1, r_1)}{\partial r_1^2} \cdot \frac{\Delta r^2}{2} + \dots
\end{aligned} \tag{61}$$

As all the individual functions with variable – electricity price would be immaterial from the third degree further and all the individual functions with variables – volatility, time and interest rate will be immaterial from the second degree we can truncate them. The Delta-gamma-vega-theta-rho approximation will be as follows:

$$\begin{aligned}
P_{call}(P_1, \sigma_1, t_1, r_1) \approx & P_{call}(P_0, \sigma_0, t_0, r_0) + \frac{\partial P_{call}(P_1, \sigma_1, t_1, r_1)}{\partial P_1} \cdot \Delta P + \frac{\partial^2 P_{call}(P_1, \sigma_1, t_1, r_1)}{\partial P_1^2} \cdot \frac{\Delta P^2}{2} + \\
& + \frac{\partial P_{call}(P_1, \sigma_1, t_1, r_1)}{\partial \sigma_1} \cdot \Delta \sigma + \frac{\partial P_{call}(P_1, \sigma_1, t_1, r_1)}{\partial t_1} \cdot \Delta t + \frac{\partial P_{call}(P_1, \sigma_1, t_1, r_1)}{\partial r_1} \cdot \Delta r
\end{aligned} \tag{62}$$

The shorten format of the above equation by using Greeks will be as follows:

$$P_{call}(P_1, \sigma_1, t_1, r_1) \approx P_{call}(P_0, \sigma_0, t_0, r_0) + \Delta \Delta P + \Gamma \times \frac{\Delta P^2}{2} + V \Delta \sigma + \Theta \Delta t + \rho \Delta r \tag{63}$$

From the formula above (58) we can express a formula for the change of call option price:

$$\Delta P_{call} = \Delta \Delta P + \frac{1}{2} \times \Gamma \times \Delta P^2 + V \Delta \sigma + \Theta \Delta t + \rho \Delta r \tag{64}$$

The same formula is applicable for put option price:

$$\Delta P_{put} = \Delta \Delta P + \frac{1}{2} \times \Gamma \times \Delta P^2 + V \Delta \sigma + \Theta \Delta t + \rho \Delta r \tag{65}$$

Suppose we have the same call option as in previous example with current market price of electricity 50 €/MW, strike price 50 €/MW, maturity period 4 months, the discount rate 5% and the volatility of electricity prices 35%.

$$P_{call}(50, 35\%, 5\%) = 4,3846 \text{ €/MW}$$

Greeks values will be:

$$\Delta = 0,5728$$



$$\Gamma = 0,0388$$

$$V = 0,1132$$

$$\Theta = -0,0196$$

$$\rho = 0,0807$$

We would need to find out the value of the call option if the market parameters change as follows: market price of electricity 51 €/MW, time to expiry decreases by 1 day, the discount rate is 5,5% and the volatility of electricity prices is 34,5%.

Option value using the Delta-gamma-vega-theta-rho approximation will be:

$$P_{call}(51;34,5\%;4M - 1d;5,5\%) = 4,3846 + 0,5728 \times 2 + 0,0388 \times \frac{2^2}{2} + 0,1132 \times (-0,5\%) +$$

$$-0,0196 \times \frac{1}{365} + 0,0807 \times 0,5\% = 4,9767 \text{ €/MW}$$

Real option value calculated through Black-Scholes model will be:

$$P_{call}(51;34,5\%;4M - 1d;5,5\%) = 4,9433 \text{ €/MW}$$

Absolute difference is -0,0334 €/MW

As there are more variables in the formula (65) the delta-gamma-vega-theta-rho approximation gives us the best possible approximation that could be used in the real life. In order to present usefulness of Greeks approximation in the electricity market practice I have prepared a comprehensive model. For detail please refer to appendix C to this thesis and for electronic file with formulas please refer to CD attached to this thesis. Please note that with higher difference in option input factors (as described in chapter 2) from original values the approximation is becoming less and less accurate. In order to avoid this lack of accuracy value of option could be revalued through Black-Scholes option pricing formula if one or more variables change by certain percentage. Another option would be a development of dynamic correction model. However this out of scope of this thesis and will be included as a further development of this research.

### 5.3 Delta hedge

Delta hedge is a trading strategy that dynamically maintains a delta neutral portfolio based on Taylor series approximation applied across portfolio level by using offsetting underlying asset to reduce outright direction of risk associated. Delta neutral portfolio is hedged in a way that it is immunized against small changes in the electricity market price. It needs to be rebalanced continuously in order to maintain neutrality.

In perfect hedge the gain/loss on option sold should be mitigated by the loss/gain on underlying asset purchased, so the option writer always gets the premium in full. However the delta hedge does not replicate the call option perfectly due to its nature as described in Delta approximation chapter above. Delta hedging strategy is the dynamic process of keeping delta of the portfolio as close to zero as possible. In practice maintaining a zero delta might be very complex because there are risks associated with actively hedging large movements in the underlying asset price. The forcefulness of the underlying asset moves will increase with the gamma of the option. With increased frequency of rehedging of the option portfolio will get higher probability of getting the higher portion of the option premium. Note that by using delta hedge strategy we never get gain higher than the option premium. Net gain from the option written is as follows:

- Loss on exercised / 0 not exercised option
  - ± Gain/Loss on underlying asset purchased/sold
  - + Gain on option premium received
- Net gain on option written

Suppose we want to write X call options on Y electricity units which will be hedged by using delta hedging. Then the delta neutral portfolio should include N=Δ units of electricity which could be either purchased or produced. The equation of the portfolio value will be as follows:

$$\Pi = N \times P - P_{call} \times X \times Y \quad (66)$$

The delta neutral portfolio would be given by the first derivative of the portfolio according to electricity market price equal to zero:

$$\frac{\partial \Pi}{\partial P} = \frac{\partial(N \times P)}{\partial P} - \frac{\partial(P_{call} \times X \times Y)}{\partial P} = 0 \quad (67)$$

Which will be:

$$N - \Delta \times X \times Y = 0 \quad (68)$$

$$N = \Delta \times X \times Y \quad (69)$$

Suppose we want to write X=100 call options on Y=10 MW. The strike price is 50 €/MW, volatility 30%, interest rate 6%, and time to delivery 4/52 to 0/52 weeks:

S	$\sigma$	r	T	P	d1	d2	Nd1	Nd2	Option value	Delta	Change in Delta	No. of units needed	Cumulative No. of units in hedged portfolio
50	30%	6,00%	0,08	50	0,097	0,014	0,5387	0,506	1,773	0,539		539	539
			0,06	51	0,359	0,287	0,6402	0,613	2,110	0,640	0,1015	101	640
			0,04	49	0,275	0,334	0,3918	0,369	0,77	0,392	-0,2484	-248	392
			0,02	50	0,049	0,007	0,5194	0,503	0,86	0,519	0,1276	128	519
			0,00	51	#####	#####	1,0000	1,000	1,00	1,000	0,4806	481	1.000

In each week we perform rebalancing the portfolio based on delta to get delta-neutral portfolio.

In case the option ends up in the money we end up with 1000 units either purchased or produced. In the above example the net gain of the option writer will be:

-	1.000 €	...	Loss on exercised option
+	227 €	...	Gain on underlying asset purchased and sold
+	<u>1.773 €</u>	...	<u>Gain on option premium received</u>
+	1.000 €	...	Net gain on option written

In case the option ends up out of the money we end up with 0 units as we have everything sold. The above example will look as follows:

S	$\sigma$	r	t	P	d1	d2	Nd1	Nd2	Option value	Delta	Change in Delta	No. of units needed	Cumulative No. of units in hedged portfolio
50	30%	6,00%	0,08	50	0,097	0,014	0,5387	0,506	1,773	0,539		539	539
			0,06	51	0,359	0,287	0,6402	0,613	2,110	0,640	0,1015	101	640
			0,04	49	0,275	0,334	0,3918	0,369	0,77	0,392	-0,2484	-248	392
			0,02	50	0,049	0,007	0,5194	0,503	0,86	0,519	0,1276	128	519
			0,00	49	#####	#####	0,0000	0,000	0,00	0,000	-0,5194	-519	0

And the net gain of the option writer will be:

	0 €	...	Option is not exercised
-	773 €	...	Loss on underlying asset purchased and sold
+	<u>1.773 €</u>	...	<u>Gain on option premium received</u>
+	1.000 €	...	Net gain on option written

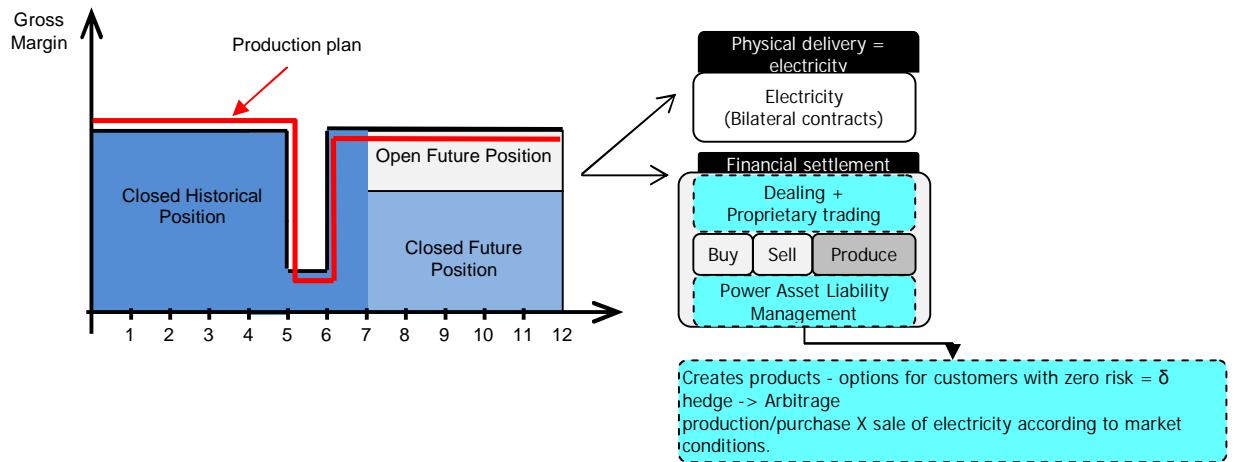
By performing rebalancing continually the net gain of the option will be close to the option premium.

The valuation of options for large portfolios presents a trade-off between speed and accuracy, with the fastest methods relying on rough approximations and the most realistic approach – Black-Scholes – often too slow to be practical. In this respect Greeks

approximations represent invaluable possibility for delta hedge neutral options portfolio revaluation with great accuracy within required time period. This can greatly reduce the time that would be otherwise needed. The application of all Greeks and their presentation on practical example gives a huge benefit of this thesis.

In order to further present usefulness of delta hedge in the electricity market practice I have prepared a comprehensive model. For detail please refer to appendix D to this thesis and for electronic file with formulas please refer to CD attached to this thesis.

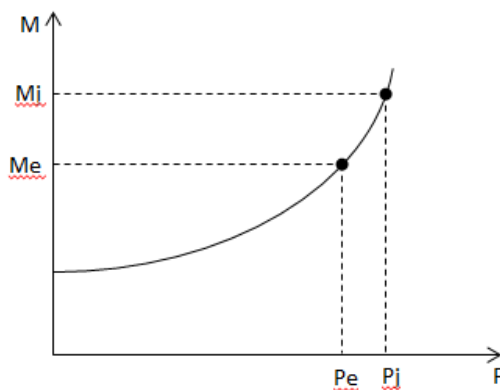
## 6 Implementation of the results in practice



**Figure 42:** Allocation of the future open position

In general there are several ways where power producers are able to generate profit. In the Figure 42 we can see that power producer may sell produced electricity through bilateral contracts or power exchanges and/or trade some of the electricity on its own desk by using its deep knowledge of the electricity market and/or create options contracts with risk free profit.

In any case the power producer needs to be aware of the capacity of electricity which is able to produce. So the short position in sold electricity through bilateral contracts, trading position and written options is within the possibilities of the production facility in case of delivery or closing the positions. In this respect the power producer has to take into consideration the efficiency of the power station i.e. the consumption of the steam and the electricity power. The dependence is non-linear. Up to certain point – called economic point ( $P_e, M_e$ ) of the consumption curve the efficiency is increasing however behind the economic point to the nominal point ( $P_j, M_j$ ) the efficiency is decreasing. The consumption characteristics can be interpreted as follows - Figure 43:



**Figure 43:** Consumption characteristic of the power station on power produced

In general the economic power of the power station represents 75%-90% of the nominal power. In this respect we can still use the difference between nominal and economic amount if the gain received from the market (either by using written options, trading or other) covers increased costs incurred.

The strategy of the power producer will be to generate maximum profit with taken reasonable risk. The open future position might be therefore allocated into the different categories based on current market needs. For example if the power producer has an opportunity to sell risk free call options hedged by delta hedging strategy then it might be, bearing in mind all the circumstances e.g. risk position, production costs, etc., more desirable way than selling the energy through bilateral contracts or trading the energy on proprietary desk. Each of the market opportunity has its own 'pros and cons that are highly dependent on current market development. Therefore it is not possible to say which of them will be more sufficient for the power producer. The final allocation of the production is up to power producer's defined strategy and risk appetite. The summary of comparison of the power producer's market opportunities is in table below.

Market opportunity	Risk involved	Profit generated	Requirements
Physical delivery	No risk	Standard margin, no loss	Standard requirements
Dealing on proprietary desk	Risk limited, based on VAR and could be covered by produced electricity	Higher profit with increased risk	Establishment of Trading desk and Risk Management function. Setting up the limits and Risk appetite and Risk capacity.
Writing options	No risk	Standard margin + option premium	Establishment of Power Asset Liability Management function.

## 6.1 Physical delivery

Physical delivery is based on the bilateral contracts with electricity distribution companies or customers. It also covers provision of ancillary services that are provided national power operator in order to maintain reliable electricity network.

There is no risk involved as the electricity is sold either through bilateral contracts or through organised power exchange to the customer. The profit generated represents standard margin that covers production and administration costs. This is the core business of the power producers. The valuation of the power producer's portfolio is described in chapter 3.

The final part of this goal has been demonstrated on a comprehensive model to present usefulness in the electricity market practice. For detail please refer to appendix A to this thesis. For electronic file with formulas please refer to CD attached to this thesis.

## **6.2 Dealing on proprietary desk**

Dealing on proprietary desk covers speculation techniques on market prices movements. This market is maintained by the electricity traders. Power producers have competitive advantage in form of possibility to generate electricity in case when it is too expensive in the market and sell it and purchase electricity in case when it is cheaper than the production. They may use remaining production capacity and invaluable knowledge of the electricity market for speculative purposes. This will increase trading activity in the market, diminish inefficiencies and finally result in cheaper electricity for the end-users.

The risk involved in proprietary trading is increasing with the risk appetite of the power producer. It might be significant and it might be zero depending on the strategy chosen by the power producer. Power producer might be conservative and trade electricity for speculative purposes up to the maximum capacity that its production facility is able to produce. Also power producer might bear some risk up to a certain level – risk appetite. Then the establishment of the risk management department and continual evaluation of the open position through Value at Risk technique will be necessary. The valuation of the open position through Value at Risk is described in chapter 4.

The final part of this goal has been demonstrated on a comprehensive model to present usefulness in the electricity market practice. For detail please refer to appendix A to this thesis. For electronic file with formulas please refer to CD attached to this thesis.

## **6.3 Writing options**

Power Asset Liability Management is based on creation of financial products like options for its customers. This market is maintained by the banks.

Call option - Power producers may write call options where the strike price will be on the same level as the standard production cost plus margin. Power producers have competitive advantage in form of possibility to generate electricity in case the option is in the money and is exercised by its holder. In this respect power producer realizes risk free profit in form of margin for the delivery of electricity plus premium for the written call option. If the option is out of the money the option holder will not exercise the option and the power producer realizes profit in form of premium and is able to sell the electricity on day to day market or just decrease the production and safe production costs.

Put option - Power producer may also write put options where the strike price will be on the same level as its production costs. In this respect power producer pays the same amount for the electricity taken without any necessity to produce it in case the option is in the money. If the option is out of the money – not exercised power producer realizes profit in form of premium.

In both cases power producer does not incur any losses.

Large option portfolios could be revaluated through Greeks approximations as described in chapter 5. This technique represents a trade-off between speed and accuracy, with the fastest methods relying on rough approximations – Greeks and the most realistic approach – Black-Scholes – often too slow to be practical. In this respect Greeks approximations represent invaluable possibility for delta hedge neutral options portfolio revaluation with great accuracy within required time period. This can greatly reduce the time that would be otherwise needed. The only limitation is that the approximation could be used for small changes of the option input variables as described in Chapter 2 (electricity market price, option exercise price, time to maturity of the option, interest rate, market price volatility). This part represents a huge benefit of this thesis as all Greeks formulas have been derived from the Black-Scholes model by application of Taylor series and partial derivatives. These Greeks formulas have been presented in Appendix B, and the comprehensive model for the Greeks option pricing approximation has been presented in Appendix C to this thesis. For electronic file with formulas please refer to CD attached to this thesis.

The options could be hedged through delta hedging strategy. Delta hedge gives power producer unique opportunity to maintain minimum electricity capacity necessary to be produced or purchased depending on current market conditions and likelihood of options being exercised. Delta hedge gives also power producer unique opportunity to exploit his competitive advantage to banks in form generate and not-generate electricity depending if the option is in the money or out of the money. By using the right dynamic continuous hedging strategy the power producer may incur profit equal of option premium without any risk. This is called arbitrage. The delta hedging strategy is described in chapter 5.

The final part of this goal has been demonstrated on a comprehensive model to present usefulness in the electricity market practice. For detail please refer to appendix D to this thesis. For electronic file with formulas please refer to CD attached to this thesis.



## 7 Conclusion

In the last few years companies witnessed steady growth in needs to manage risks they are facing to. Risk Management has become an important topic, especially since the business is related to the conditions of uncertainty. Recent developments in the financial industry highlighted the importance of effective risk management procedures. These needs have been in turn passed on other industries, especially energy sector. Energy sector represents strategic area for each country. This thesis focused on risk management in electricity sales of power. It is based on analysis of differences and similarities between financial and energy markets and also on analysis of strengths and weaknesses between power producers and electricity market traders. This approach gave a unique opportunity to find out opportunity gaps in these areas which have been examined in this thesis.

In chapter 1 I have defined motivations that lead to write the doctoral thesis, goals that I would like to achieve and related hypothesis that I would like to verify. Further there I have described methodology that has been used throughout the thesis.

In chapter 2 I have described current situation on the electricity market including best practice and knowledge used in the electricity markets - designed general risk management structure, identified related risks involved and specified internal regulations that addressed these risks. I have explained the authorization process of available markets, instruments and counterparties, the description of mostly traded instruments in terms of timelines and flowcharts. More focus is related to options which are more complex and represent market opportunity for the power producers.

Further I have introduced opportunity gaps and presented a comprehensive roadmap (Figure 16) describing opportunities gaps between banks and power producers and between power producers and electricity traders. This roadmap is a crucial basis for the preparation of the whole thesis. It describes market opportunities of power producers as opposed to financial institution and electricity market traders based on their competitive advantage to produce electricity.

In chapter 3 I have described challenges related to power producer portfolio valuation. Further I have defined current portfolio positions of power producers and described techniques to evaluate the portfolio by using the gross margin. This part has been first introduced on 11<sup>th</sup> International Student Conference on Electrical Engineering by me [54]. It won several awards including 3<sup>th</sup> place awarded by Czech Technical University committee and award of main sponsor – General Electric. Also it has been published in peer-reviewed journal *Acta Polytechnica* [55].

In chapter 4 I have described risk exposure in which the open future position can be and proposed deterministic model as a technique to measure this exposure through VAR. Power producers may easily exploit their competitive advantage in form of possibility to generate

electricity in case when it is too expensive in the market and sell it and purchase electricity in case when it is cheaper than the production. They may use remaining production capacity and invaluable knowledge of the electricity market. In this respect they can replace current role of the electricity traders and trade on their own desk. This is called in the financial markets proprietary trading. Further I have defined assumptions for the VAR calculation. In this part of the work I have tested different probability distributions on electricity market prices data from eex.com from 2002 till 2009 to find out which probability distribution describes the nature of the electricity market prices the best. These tests have been based on Pearson's chi-squared test. None of the probability distribution gave us null hypothesis true. The closest results of the real electricity market prices to the empirical probability distribution were given by the log-normal distribution. In addition I have defined risk management function and risk management limits that are used to manage the market risk.

In chapter 5 I have established Power Asset Liability Management function and worked out opportunity gap for the power producers, which is generally handled by the banks. Power producers have better conditions to write call and put options for its customers as they may decide if the option is getting out of the money whether they will produce or purchase missing electricity in case of exercised call options or reduce production or sell electricity in case of exercised put options. By using appropriate hedging strategy power producers may generate risk free profit. I have described Greeks as measures of sensitivity of the option price to electricity market price and illustrated usage of these Greeks in practice on different electricity market price approximations. These Greeks are widely used in the financial markets however need to be slightly modified for the usage in the electricity market. The introduction of these Greeks in the electricity market is a huge benefit of this thesis. By buying and selling the underlying asset based on the options in the portfolio I have presented delta hedge strategy as another tool for the power producer that helps to eliminate the risk associated with the option portfolio and get "risk free profit" in form of premium of the written options. In the financial markets is delta hedge engaged by banks and hedge funds. The usage of the delta hedge in the electricity market is another benefit of this thesis.

In chapter 6 I have prepared an overview of the benefits of the thesis in relation to the implementation of the individual opportunities related to defined goals in practice. It describes allocation of the production facility possibilities to individual market opportunities and represents basis of the business strategy framework for power producers.

## **7.1 Summary of the results including the benefits of the thesis**

This thesis is based on analysis of current electricity market situation and examination of opportunity gaps found during the comparison of electricity markets with more matured financial markets and also comparison of strengths and weaknesses of power producers as opposed to the electricity market traders. This thesis provides unique directions to re-establish electricity markets and increase their efficiency.

In the introductory chapter I have defined three goals to be achieved in this research and come to the following results:

- Determine the proper measure for the power producers portfolio evaluation

In connection with this goal I have hypothesized that: “Evaluation of the power producer’s portfolio should be based on Gross Margin.” In order to evaluate the portfolio I have divided power producer’s portfolio into several positions. The Gross Margin as a measure used for the valuation of power producer’s portfolio has been assessed as the most appropriate one. This part is worked out in chapter 3.

The goal has been achieved and related hypothesis has been validated.

- Determine the possibility of power producer to trade the electricity on its own desk the same way like electricity market traders

In connection with this goal I have hypothesized that: “The most suitable measure to evaluate risk of open position should be based on Value at Risk metrics.” The competitive advantage of power producers has been described in the chapter 2. The approach of trading electricity and the risk management strategy related including the evaluation of the open position based on Value at Risk was worked out in chapter 4 with references to risk management framework as described in chapter 2.

The goal has been achieved and related hypothesis has been validated.

- Evaluate the possibility of power producer to provide typically financial market products like options to its customers that are usually provided by the financial institutions.

In line with this goal I have hypothesized that: “Power producers are able to get into arbitrage and generate risk free profit through delta hedge based on self-issued option contracts and possibility to generate underlying electricity.” By using partial derivative of options value according to different variables I got Greeks modified for the electricity market. These Greeks helped me to evaluate the amount of underlying asset needed to maintain neutral portfolio – Delta hedge and verify the hypothesis that power producer is able to get into arbitrage and generate risk free profit. By-product of this thesis is that I have applied Taylor series [66] on these Greeks and obtained options value approximations. The valuation of options for large portfolios presents a trade-off between speed and accuracy, with the fastest methods relying on rough approximations and the most realistic approach – Black-Scholes – often too slow to be practical. In this respect Greeks approximations represent invaluable possibility for delta hedge neutral options portfolio revaluation with great accuracy within required time period. This can greatly reduce the time that would be otherwise needed. The application of all Greeks and their presentation on practical example gives a huge benefit of this thesis. This part is worked out in chapter 5.

The goal has been achieved and related hypothesis has been validated.

For a better understanding of the topic described individual chapters are supplemented by model calculations illustrating how to use the techniques, metrics and/or instruments in practice. In order to provide a complete overview of presented results in context I have created a comprehensive model, where individual parts of the thesis are presented – reference is made to appendix A, B, C and D to this thesis. For electronic file with formulas please refer to CD attached to this thesis.

One of the main focus of the thesis was the applicability and implementation of suggested solutions in practice. Individual goals are not standalone goals that deal with selected part of the business but rather are connected and interdependent on the production facility. In this respect I have prepared an overview in relation to the implementation of the opportunities related to defined goals in practice – reference is made to chapter 6. Proposed solutions described in the thesis solve the goals stated and are implementable in the practice in the form as described.

As stated in the introduction paragraph the aim of this thesis wasn't to compile findings and results of others university or consulting colleagues researches, thesis, reviews or analysis. But based on their findings, mathematical techniques and based on my experience and knowledge, of the electricity market and financial market I have been able to produce the thesis that will bring new ideas and directions in the current power producers market situation.

This work is in some aspects unique. It presents usage of proprietary trading, Greeks and delta hedge both hedging of the portfolio position and optimization of the portfolio position in the electricity market. It is based on application of theoretical mathematical models and techniques in practice. It also provides an overview of other important risk management issues in the electricity market and can be used as a basis for further work on this topic. This work might be helpful for the top management of the power producing companies as it helps to setup business strategy framework and establish Risk Management and Power Asset Liability Management functions governing the management of the risks and management of all positions across all of the trading activities. It gives a unique chance to find out directions to re-establish electricity markets and increase their efficiency.

Power producing companies can now optimize the sale of their production capacities with the objective of maximizing profit from wholesale electricity, financial instruments and supporting services.

Individual areas of this thesis have been presented on international student conferences on electrical engineering and published in peer-reviewed journal. Some of the theses won several awards:

- 2nd place in CEZ competition for the best scientific and technical research

- 3th place awarded by Czech Technical University committee on 11<sup>th</sup> International Student Conference on Electrical Engineering
- Award of main sponsor – General Electric on 11<sup>th</sup> International Student Conference on Electrical Engineering

Last but not least this doctoral thesis is based on diploma thesis which has been awarded by dean's price.

As a by-product of the thesis has been performed a testing of probability distribution through Pearson's chi-squared test. Even though none of the tested probability distribution gave us null hypothesis true, the closest results of the real electricity market prices to the empirical probability distribution were given by the log-normal distribution. These results are very valuable and present a strong fundament for researches to be performed in this area.

## **7.2 Further development of the research**

As described above there might be several ways for further development of this thesis. These proposals for developments come from findings or the limitations of the thesis presented.

### **Empirical probability distribution describing electricity market prices**

One of them might be assessment of the probability distribution describing market prices. As described above in chapter 4.3 I have tested several empirical probability distributions that could be applied for description of the electricity market prices. Probability distributions used were:

- Normal probability distribution
- Log-normal probability distribution
- Weibull probability distribution
- Chi-squared probability distribution

Tested data were taken from EEX and covered base load (24h), peak load (from 8:00 – 20:00) and individual prices per hour from 1.1.2002 till 31.12.2009. Null hypothesis for evaluation of the similarity between empirical probability distribution and real market data was based on Pearson's chi-squared test. None of the probability distribution gave us null hypothesis true. The closest results of the real electricity market prices to the empirical probability distribution were given by the log-normal distribution. Another approach would be to use different probability distribution however further work has not been performed as it would be over the scope of this thesis. The interpretation of my results of the real historical data and empirical probability distribution is on Figure 22.

In this respect there might several options to solve this problem by using:

- Other already known continuous probability distribution,
- Combination of probability distributions – one for the market prices under normal market conditions and one applied for the fat tail,
- K-transformation of normal probability distribution as described by Kanok in [21] or
- Develop brand new probability distribution.

Any results solving this problem should be proved by null hypothesis of Pearson's chi-square test, Kolmogorov–Smirnov test and/or back testing of randomly generated figures based on potential empirical distribution through Monte-Carlo simulation to real market prices [8].

### **Unpredictable electricity price movements**

Another place for development is the uncertainty of occurrence of unpredictable price movements and its evaluation. The evaluation of the option value is based on Black-Scholes pricing model which covers several assumptions. However the only assumption which might bring drawbacks to our delta hedge calculation is that markets are efficient and work continuously with smooth electricity spot price development. This is not always true and there are unpredictable movements that might cause adverse movement in the delta hedge of our portfolio. Drawbacks of all the other assumptions are eliminated by performing continual rebalancing of the options and physical electricity portfolios.

The applicable approach for evaluation of these unpredictable changes with low frequency and high volume impact to power producer portfolio might be Extreme Value Theory [58]. The result solving this problem should be back tested to the real market prices – historical simulation or through randomly generated figures through Monte-Carlo simulation in case we find out the appropriate probability distribution.

### **Correction model of Greeks approximations**

As described in Chapter 5, the higher differences in option input variables as described in Chapter 2 (electricity market price, option exercise price, time to maturity of the option, interest rate, market price volatility) from original values the approximation is becoming less and less accurate. In order to avoid this lack of accuracy, value of option could be revalued through Black-Scholes option pricing formula if one or more variables change by certain percentage. Another option would be a development of dynamic correction model.

# References

## Books:

- [1] ACCA P4 Advanced Financial Management, BPP Learning Media, ISBN 9780751776003
- [2] BANKS E., SIEGEL P.: The options applications handbook: hedging and speculating techniques for professional investors. McGraw-Hill Professional 2006, ISBN 0071453156
- [3] BENTH F.E., BENTH J. S., KOEKEBAKKER S.: Stochastic Modeling of Electricity and Related Markets, World Scientific, 2008, ISBN 9789812812308
- [4] BERTOCCHI M., CONSIGLI G., DEMSTER M.A.H.: Stochastic Optimization Methods in Finance and Energy, Springer, ISBN 1441995854
- [5] BLÁHA, Z. S. – JINDŘICHOVSKÁ, I.: Opce, swapy, futures, Management Press, 1997. ISBN 8085943298
- [6] BREALEY R.A., MYERS S.C.: Teorie a praxe firemních finance, Computer Press 2000, ISBN 8072261894
- [7] BRIYS, E., MAI, H., BELLALAH, M., VARENNE, F.: Options, Futures and Exotic Derivatives, Wiley 1998. ISBN 0417969095
- [8] DE WEERT F.: An Introduction to Options Trading, John Wiley & Son Inc., 2006, ISBN 0470029706
- [9] DUDORKIN J.: Operační výzkum, skriptum FEL ČVUT 1991
- [10] DUDORKIN J.: Systémové inženýrství a rozhodování, skriptum FEL ČVUT, 1990
- [11] DVOŘÁK, P.: Finanční deriváty, University of Economics, Prague, 1995. ISBN 8070799056
- [12] EPPS T.W.: Pricing Derivative Securities, World Scientific Pub Co Inc, ISBN 9789812700339
- [13] GEMAN H.: Commodities and Commodity Derivatives – Modelling and Pricing for Agricultures, Metals and Energy, Wiley 2005, ISBN 978-0-470-01218-5
- [14] GLASSERMAN P.: Monte Carlo Methods in Financial Engineering, Springer 2004. ISBN 9780387004518
- [15] HOLTON, G. A.: Value at Risk, Elsevier Academic Press 2003. ISBN 0123540100
- [16] HUNG-PO CH., HILLARD G.: Designing Competitive Electricity Markets, Hardcover 1998. ISBN 079238282
- [17] JÍLEK, J.: Termínové a opční obchody, Grada, 1995. ISBN 8071691836
- [18] JÍLEK, J.: Finanční a komoditní deriváty, Grada, 2004. ISBN 8024703424

- [19] JÍLEK, J.: Finanční a komoditní deriváty v praxi, Grada, 2005. ISBN 8024710994
- [20] JORION P., Value at Risk: The New Benchmark for Managing Financial Risk, McGraw-Hill 2006. ISBN 978-0071464956
- [21] KAŇOK M.: Statistické metody v management, skriptum FEL ČVUT Praha, 2005
- [22] LEPPARD, S.: Energy Risk Management, Risk Books 2005. ISBN 1904339743
- [23] McCLAVE, J. T., BENSON, P. G., SINCICH, T.: Statistics for Business and Economics (7th Edition), Prentice Hall 1998. ISBN 0139505458
- [24] REKTORYS, K.: Přehled užití matematiky 3. vydání, SNTL Praha, 1973
- [25] SHARPE F., Alexander G.J.: Investice, Victoria Publishing 1994, ISBN 8085605473
- [26] ŠOR, J.B.: Statistické metody analýzy a kontroly jakosti a spolehlivosti, SNTL Praha 1965
- [27] TERRELL D.: Business Statistics, Houghton Mifflin company, Boston 1989
- [28] WENGLER, J.: Managing energy risk, Pennwell Books 2001. ISBN 0878147942
- [29] WONNACOT, T. H., WONNACOT R.J.: Statistika pro obchod a hospodářství, Victoria Publishing, ISBN 8085605090

### **Articles & Researches:**

- [30] ABDOLKHALIG A.: Optimized Calculation of Hourly Price Forward Curve, World Academy of Science, Engineering and Technology, International Journal of Electrical, Robotics, Electronics and Communications Engineering Vol:2 No:9, 2008
- [31] BAUGHMAN M.L., LEE W.W.: A Monte Carlo model for calculating spot market prices of electricity, Power Systems, IEEE Transactions, 1992
- [32] BLACK F., SCHOLES M.: The Pricing of options and Corporate Liabilities, The Journal of Political Economy, 1973, University of Chicago
- [33] BORAK S., WERON R.: A semi parametric factor model for electricity forward curve dynamics, The Journal of Energy Markets, 2008
- [34] BORENSTEIN S.: Understanding Competitive Pricing and Market Power in Wholesale Electricity Markets, The Electricity Journal, Elsevier, 2000
- [35] BUNDALOVA A., DEJONG C., VANDIJKEN H.: Power Plant Hedging Strategies, Energy Risk, 2012
- [36] CHEN J., DENG S., HUO X.: Electricity Price Curve Modeling and Forecasting by Manifold Learning, IEEE Transactions on Power Systems, 2008, work has been partially supported by National Science Foundation grants 0604736 and 0134210



- [37] CHEUNG Y.S., POWELL R.J.: Anybody can do Value at Risk: A Teaching Study using Parametric Computation and Monte Carlo Simulation, Australian Accounting, Business and Finance Journal, 2013
- [38] DENG S.J., JOHNSON B., SOGOMONIAN A.: Exotic Electricity Options and the Valuation of Electricity Generation and Transmission Assets, Elsevier, 2001
- [39] DENG S.J., OREN S.S.: Electricity Derivatives and Risk Management, Elsevier, 2006
- [40] DENG S.J., XIA Z.: Pricing and Hedging Electricity Supply Contracts: a Case with Tolling Agreements, Discussion Paper, Institute of Technology, Atlanta, 2005
- [41] ELDER J.: “Greeks” - Hedging with Options, 2009, Colorado State University
- [42] EYDELAND A., WOLYNIEC K.: Energy and Power Risk Management – New developments in Modelling, Pricing and Hedging, Wiley, 2002
- [43] EYDELAND A., GEMAN H.: Fundamentals of Electricity Derivatives, Risk Books, 1999
- [44] FLETEN S., LEMMING J.: Constructing forward price curves in electricity markets, Energy Economics, Elsevier, 2003
- [45] GENÇAY R., SELÇUK F., ULUGULYAGCI A.: High volatility, thick tails and extreme value theory in value-at-risk estimation, Elsevier, Insurance: Mathematics & Economics 33: 337-356.
- [46] GLASSERMAN P., HEIDELBERGER P., SHAHABUDDIN P.: Efficient Monte Carlo Methods for Value-at-Risk, 2000, Columbia University, New York
- [47] HADSELL L., MARATHE A., SHAWKY A.: Estimating the Volatility of Wholesale Electricity Spot Prices in the US, The Energy Journal, 2004
- [48] HOANG K., HOROWITZ I., WOO C.K.: Cross Hedging and Forward-Contract Pricing of Electricity, Energy Economics, 23, 1-15. (Lead article and one of the 10 most cited recent papers published in Energy Economics), 2001
- [49] HUNG-PO Ch.: Risk Management in Electricity Market Design, Conference: System Sciences, 2006. HICSS '06. Proceedings of the 39th Annual Hawaii International Conference on, Volume: 10, 2002
- [50] J.P.MORGAN/REUTERS: Risk Metrics - Technical Document, Fourth Edition 1996
- [51] KNĚŽEK M.: Řízení rizik na trhu s elektřinou, Diploma Thesis 2007, Czech Technical University in Prague
- [52] KNĚŽEK M.: Řízení rizik na trhu s elektřinou, International Student Conference on Electrical Engineering - Poster 2006, Czech Technical University in Prague
- [53] KNĚŽEK M.: Electricity Market Risk Management, International Student Conference on Electrical Engineering - Poster 2007, Czech Technical University in Prague

- [54] KNĚŽEK M.: Power Producer Production Valuation, International Student Conference on Electrical Engineering - Poster 2008, Czech Technical University in Prague
- [55] KNĚŽEK M.: Power Producer Production Valuation, Acta Polytechnica 2008, Czech Technical University in Prague
- [56] KNĚŽEK M.: Power Producer Risk Management, International Student Conference on Electrical Engineering - Poster 2013, Czech Technical University in Prague
- [57] LAGARTO J., SOUSA J., PESTANA R.: Spot Prices Simulation and Volatility Analysis in the Future Iberian Electricity Market, Power Systems Computation Conference in Liege (Belgium), 2005
- [58] LEHIKOINEN K.: Extreme Value Theory in Risk Management for Electricity Market, 2007, Helsinki University of Technology, Finland.
- [59] LYUU Y.: Delta Hedge, 2007, National Taiwan University
- [60] MORBEE J., WILLEMS B.: Risk Management in Electricity Markets: Hedging and Market Incompleteness, Discussion Paper 08.23, Center for Economic Studies, K.U.Leuven, Belgium, 2008
- [61] PELLETIER D.: Elements of Asset Pricing, 2006, North Carolina State University
- [62] PINEDA S., CONEJO A.J.: Managing the Financial Risks of Electricity Producers of Using Options, Elsevier, 2012
- [63] RICKENLUND I.: Risk Management on a Deregulated Electricity Market, 1997, Stockholm, Sweden
- [64] ROZARIO R.: Estimating Value at Risk for the Electricity Market Using a Technique from Extreme Value Theory, 2002, University of New South Wales, Australia.
- [65] SPINLER, S., HUCHZERMEIER A., KLEINDORFER P.: Risk hedging via options contracts for physical delivery, Springer Ling, 2003
- [66] TAYLOR B.: Methodus incrementorum directa et inversa and Linear Perspective, 1715
- [67] WU H.: Pricing European options based on the fuzzy pattern of Black–Scholes formula, Elsevier, 2004

### **Internet pages:**

- Risk related:

<http://www.derivativesmodels.com>

<http://www.gloriamundi.org>

<http://www.investopedia.com>

<http://www.jpmorgan.com/RiskManagement/RiskMetrics/RiskMetrics.html>

<http://www.risk.net>

<http://www.riskglossary.com>

<http://www.riskmetrics.com>

▪ Power exchanges:

<http://www.eex.de>

<http://www.exaa.at>

<http://www.ote-cr.cz>

<http://www.polpx.pl>

<http://www.pxe.cz>

<http://www.hupx.hu>

<http://www.apxgroup.com>

<http://www.nordpoolspot.com>

<http://www.belpex.be>

<http://www.powernext.com>

<http://www.omie.es>

<http://www.mercatoelettrico.org>

<http://www.desmie.gr>

<http://www.bsp-southpool.com>

<http://www.opcom.ro>

▪ OTC brokers:

<http://tfsenergy.com>

<http://www.marexspectron.com>

<http://www.icapenergy.com>

<http://www.gfigroup.com>

<http://www.tullettprebon.com>

# Appendix A: Model – Gross Margin portfolio valuation & VAR calculation

## Portfolio valuation through Gross Margin + VAR 31.12.2009

**Purpose:** To present portfolio positions and its valuation through Gross Margin and calculate Risk of open position through Value at Risk.  
 For illustration lets imagine we are in 31.12.2009. Our portfolio is closed so far.  
**January** - Power producer is in long position (purchases exceeded sales) in January month. This position will need to be covered. Power producer has several options.  
 In case that power producer is not able to sell the rest of the open position through monthly contract, then it might be sold on daily market.  
 Also power producer, unlike from electricity trader, can decide to decrease its own production.  
**February** - Same scenario applies for February month.  
**March** - In March power producer is in short position. There are also several options to cover this open position.  
 In case that power producer is not able to purchase the rest of the open position through monthly contract, then it might be purchased on daily market.  
 Also power producer, unlike from electricity trader, can decide to increase its own production, depending on the available capacity.

Sum of Power[MW]		
	Purchase	Sales
2.009	15.000	15.000
8	3.100	3.100
9	2.700	2.700
10	3.100	3.100
11	3.000	3.000
12	3.100	3.100
2.010	41.860	11.480
1	5.580	3.100
2	4.480	2.800
3	3.100	5.580
4	4.200	
5	3.100	
6	3.000	
7	3.100	
8	3.100	
9	3.000	
10	3.100	
11	3.000	
12	3.100	



Portfolio		Market Positions			
Gross Margin	4.707.652 €	Open position	1.365.907 €	Closed position	3.341.745 €
		- Long	871.432 €	- Future	2.860.281 €
		- Short	494.475 €	- Historical	481.464 €

VAR		
VAR <sub>10,00%</sub>		-122.945 €
VAR <sub>10,05%</sub>		-157.798 €
VAR <sub>10,08%</sub>		-223.177 €

# Appendix B: Greeks formulas

## Greeks – formulas

**Purpose:** To present formulas of individual Greeks for put and call options and illustrate its values on example.  
 These formulas have been derived from Black-Scholes option pricing formula by using partial derivatives and Taylor series.

P	Underlying asset value	50
S	Exercise price	50
r	Risk free rate	5,00%
σ	Volatility	35%
t	Time to expiration	124/365

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{x^2}{2}} dx = 1 - \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{x^2}{2}} dx$$

$$d_1 = \frac{\ln\left(\frac{P}{S}\right) + (r + 0,5 \cdot \sigma^2)t}{\sigma \sqrt{t}}$$

d1= 0,185

$$d_2 = d_1 - \sigma \sqrt{t}$$

d2= -0,019

**Delta**

**CALL Option**

$$P_{call} = P \cdot N(d_1) - S \cdot N(d_2) \cdot e^{-rt}$$

Pcall = 4,463

**PUT Option**

$$P_{put} = S \cdot N(-d_2) \cdot e^{-rt} - P \cdot N(-d_1)$$

Pput= 3,621

$$\Delta_{call} = N(d_1)$$

Δcall = 0,573

$$\Delta_{put} = N(d_1) - 1$$

Δput = -0,427

<b>Gamma</b>	$\Gamma = \frac{\frac{1}{2\pi} \cdot e^{-\frac{d_1^2}{2}}}{P \cdot \sigma \cdot \sqrt{t}}$ $\Gamma =$ <span style="background-color: yellow;">0,038</span>	Same for both types of option
<b>Theta</b>	$\Theta_{call} = -\frac{P \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{d_1^2}{2}} \cdot \sigma}{2\sqrt{t}} - r \cdot S \cdot e^{-rt} \cdot N(d_2)$ $\Theta_{call} =$ <span style="background-color: yellow;">-7,098</span>	$\Theta_{put} = -\frac{P \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{d_1^2}{2}} \cdot \sigma}{2\sqrt{t}} + r \cdot S \cdot e^{-rt} \cdot N(-d_2)$ $\Theta_{put} =$ <span style="background-color: yellow;">-0,013</span>
<b>Vega</b>	$V = P \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{d_1^2}{2}} \cdot \sqrt{t}$ $V =$ <span style="background-color: yellow;">11,429</span>	Same for both types of option
<b>Rho</b>	$\rho_{call} = S \cdot t \cdot e^{-rt} \cdot N(d_2)$ $\rho_{call} =$ <span style="background-color: yellow;">8,225</span>	$\rho_{put} = -S \cdot t \cdot e^{-rt} \cdot N(-d_2)$ $\rho_{put} =$ <span style="background-color: yellow;">-8,475</span>

# Appendix C: Model – Greeks for the option pricing approximation

## Greeks approximation of options market price - Delta, Gamma, Theta, Vega, Rho

Example of using Greeks for the option pricing approximation.

Exercice price of option to buy electricity	S	50	EUR /MWh
Electricity price	P1	50	EUR /MWh
Standard deviation	r1	5,00%	
Risk free rate	$\sigma$ 1	35%	
in 4 months	t1	124/365	

Value of option	<u>4,463</u>	EUR /option
d1	0,185	
d2	-0,019	
N(d1)	0,573	
N(d2)	0,493	
intrinsic value =	0,000	EUR /option
time value =	<u>4,463</u>	EUR /option
value of option =	<u>4,463</u>	EUR /option

Greeks:	Delta =	0,573
	Gamma =	0,038
	Theta =	-7,098
	Vega =	11,428552
	Rho =	8,2252901

Change in price	P2	52
Change in interest rate	r2	4,00%
Change in volatility	$\sigma$ 2	33%
Change in time	t2	123/365

Value of option after change	<u>5,346</u>	EUR /option
d1	0,371	
d2	0,179	
N(d1)	0,645	
N(d2)	0,571	
intrinsic value =	2,000	EUR /option
time value =	<u>3,346</u>	EUR /option
value of option =	<u>5,346</u>	EUR /option

<b>Delta approximation</b>		
Option price - approximation value	5,610	EUR
Option price - real value	5,346	EUR
Difference:	<u>0,264</u>	EUR

<b>Delta-Gamma approximation</b>		
Option price - approximation value	5,687	EUR
Option price - real value	5,346	EUR
Difference:	<u>0,341</u>	EUR

<b>Delta-Gamma-Theta approximation</b>		
Option price - approximation value	5,667	EUR
Option price - real value	5,346	EUR
Difference:	0,322	EUR
<b>Delta-Gamma-Theta-Vega approximation</b>		
Option price - approximation value	5,439	EUR
Option price - real value	5,346	EUR
Difference:	0,093	EUR
<b>Delta-Gamma-Theta-Vega-Rho approximation</b>		
Option price - approximation value	5,357	EUR
Option price - real value	5,346	EUR
Difference:	0,011	EUR

Based on changes in market price, interest rate, volatility and time we can see that the best estimate of the option market price gives Delta-Gamma-Theta-Vega-Rho approximation, compared to real option value.



# Appendix D: Model – Delta Hedge in the electricity market application

## Delta Hedge

Example of using Delta Hedge in the electricity market.

The key question is - How many MWs should an option writer be prepared to guarantee to generate to be able to maintain a delta hedge neutral portfolio on x written call options.

No of written call options (1 option gives opportunity to buy 1MW) =

100 pieces of call options

### Day 1

Electricity price

P 50 EUR /MW

Exercise price of option to buy electricity

S 50 EUR /MW

Standard deviation

$\sigma$  35%

Risk free rate

r 5,00%

in 4 months

t 244/365

Value of option	<u>6,455</u>	EUR /option
d1	0,260	c
d2	-0,026	c
N(d1)	0,603	c
N(d2)	0,490	c
intrinsic value =	0,000	EUR /option
time value =	<u>6,455</u>	EUR /option
value of option =	<u>6,455</u>	EUR /option

To maintain Delta neutral portfolio power producer needs to purchase/produce electricity amount:

60 MWs

(In case the option is available to be exercised now)

Premium received for sold options:

100 options  
6,455 EUR /option  
645 Gain in EUR

Purchased amount of electricity - Day 1:

60 MWh  
50 EUR /MW  
-3.013 Loss in EUR

If option position canceled today then following amount of electricity has to be sold &:

60 MWh  
50 EUR /MW  
3.013 Gain in EUR

& option with following premium has to be bought:

100 options  
6,455 EUR /option  
-645 Loss in EUR

**Net gain** **0** Portfolio is Delta Neutral

**Day 2:**

Electricity price	P	49,0	EUR /MWh
Exercise price of option to buy electricity	S	50	EUR /MWh
Standard deviation	$\sigma$	35%	
Risk free rate	r	5,00%	
in 4 months	t	243/365	

Value of option	<u>5,852</u>	EUR /option
d1	0,189	c
d2	-0,097	c
N(d1)	0,575	c
N(d2)	0,461	c
intrinsic value =	-1,000	EUR /option
time value =	<u>6,852</u>	EUR /option
value of option =	<u>5,852</u>	EUR /option

To maintain Delta neutral portfolio power producer needs to purchase/produce electricity amount: **57,5** MWh

Premium received for sold options: 100 options  
6,455 EUR /option  
645 Gain in EUR

Purchased amount of electricity - Day 1: 60 MWh  
50 EUR /MW  
-3.013 Loss in EUR

Need to be sold - Day 2: -3 MWh  
49 EUR /MW  
136 Gain in EUR

If option position cancelled today then following amount of electricity has to be sold &: 57 MWh  
49 EUR /MW  
2.817 Gain in EUR

& option with following premium has to be bought: 100 options  
6 EUR /option  
-585 Loss in EUR

**Net gain** **0** Portfolio is Delta Neutral

**Day 3**

Electricity price	P	50	EUR /MWh
Exercise price of option to buy electricity	S	50	EUR /MWh
Standard deviation	$\sigma$	35%	
Risk free rate	r	5,00%	
in 4 months	t	242/365	

To maintain Delta neutral portfolio power producer needs to purchase/produce electricity amount:

60 MWh

Premium received for sold options:

100 options  
6,455 EUR /option  
645,491 Gain in EUR

Purchased amount - Day 1:

60 MW  
50 EUR/MW

Sold amount - Day 2:

-3 MW  
49 EUR/MW  
-2.877 Loss in EUR

Need to be purchased - Day 3:

3 MWh  
50 EUR /MW  
-137 Loss in EUR

If option position cancelled today then following amount of electricity has to be sold &:

60 MWh  
50 EUR /MW  
3.011 Gain in EUR

& option with following premium has to be bought:

100 options  
6,426 EUR /option  
-643 Loss in EUR

Value of option	6,426	EUR /option
d1	0,259	c
d2	-0,026	c
N(d1)	0,602	c
N(d2)	0,490	c
intrinsic value =	0,000	EUR /option
time value =	6,426	EUR /option
value of option =	6,426	EUR /option

(In case the option is available to be exercised now)

**Net gain** 0 Portfolio is Delta Neutral

**Day 4**

Electricity price	P	51	EUR /MWh
Exercise price of option to buy electricity	S	50	EUR /MWh
Standard deviation	$\sigma$	35%	
Risk free rate	r	5,00%	
in 4 months	t	241/365	

To maintain Delta neutral portfolio power producer needs to purchase/produce electricity amount:

63 MWs

Premium received for sold options:

100 options  
6,455 EUR /option  
645,491 Gain in EUR

Purchased amount - Day 1:

60 MW  
50 EUR/MW

Sold amount - Day 2:

-3 MW  
49 EUR/MW

Purchased amount - Day 3:

3 MW  
50 EUR/MW

Need to be purchased - Day 4:

-3.013 Loss in EUR  
3  
51  
-135 Loss in EUR

If option position cancelled today then following amount of electricity has to be sold &:

63 MWh  
51 EUR /MW  
3.205 Gain in EUR

& option with following premium has to be bought:

100 options  
7,026 EUR /option  
-703 Loss in EUR

**Net gain**

0 Portfolio is Delta Neutral

Value of option	<u>7,026</u>	EUR /option
d1	0,328	c
d2	0,044	c
N(d1)	0,629	c
N(d2)	0,517	c
intrinsic value =	1,000	EUR /option
time value =	<u>6,026</u>	EUR /option
value of option =	<u>7,026</u>	EUR /option

(In case the option is available to be exercised now)

We have maintained Delta neutral portfolio by purchasing/producing/selling electricity in four consecutive days with changing electricity market price.

