Triple wire medium for use in isotropic metamaterials

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Abstract— **In the paper a triple wire medium is described as an isotropic negative permittivity medium. A numerical experiment was performed to validate the propagation of electromagnetic waves inside the wire mesh below the plasma frequency, where the effective medium permittivity is negative. For a practical realization, a form of the triple wire medium using a planar technology is proposed.**

I. Introduction

The utilization of artificial materials in microwave engineering is not a new concept. The composite medium consisting of conventional dielectrics and metals has been widely used as an artificial dielectrics especially in the antenna technique. The pioneering work was done mainly by Cohn [1], Brown [2] and Rotman [3] and the properties of artificial dielectrics itself with references can be found in [4]. Various homogenization methods have been developed to describe these systems when the wavelength of the incident radiation is much longer than the intrinsic geometric dimensions of the system.

In this paper, we will focus on the use of wire media (WM)[1] as a negative dielectric permittivity material. Pendry *et al.* [5], [6] and Sievenpiper [7] have independently demonstrated that metallic wire-mesh structures have a low frequency stop band from zero frequency to the cutoff frequency which they attributed to the motion of electrons in the metal wires. This low-frequency stop band can be attributed to effective negative dielectric permittivity and the wire medium can be modeled as a nonmagnetized cold plasma and the permittivity tensor can be described with a Drude dispersion model. When the wire lattice dimensions are chosen properly, negative permittivity can be obtained even at microwave frequencies [5] in a wide frequency band. This is a big advantage in comparison to other negative permittivity structures, since they are mainly resonant and the negative permittivity can be observed only in a narrow frequency band. Various

[1]A lattice of parallel wires in one direction is referred to as a (*single*) WM, a three dimensional lattice formed of two mutually perpendicular single WM arrays is referred to as a *double* WM, a three dimensional lattice formed by three mutually perpendicular single WM arrays is referred to as a *triple* WM.

nonisotropic forms of the ε-negative material have been known, e.g., single WM [6] or electric-field-coupled resonators [8], [9].

The unit cell of the WM is shown in Fig. 1. The wire mesh has a period a, and the wires have radius r_w. It has been shown that a general wire medium (single, double or triple) suffers from a spatial dispersion [10], [11], nevertheless it can be considered as an isotropic negative permittivity medium near the plasma frequency [12]. A spatial dispersion can be defined as a nonlocal dispersive behavior of the material, i.e., the constitutive permittivity and permeability tensors depend not only on the frequency, but also on the spatial derivatives of the electric and magnetic field vectors or, for plane electromagnetic waves, on the wave-vector components determining the direction of propagation.

Our goal is to use the connected triple wire medium for a construction of an *isotropic* negative permittivity material and possibly to combine it with an isotropic negative permeability material to get a double-negative material. Such a material would have been characterized by many interesting properties and its fabrication still remains challenging, especially at optical frequencies.

II. Triple wire medium

A. Numerical model

When we assume that the connected triple WM lattice period a (Fig. 1) is much smaller than the wavelength of the the incident electromagnetic wave, the medium can be homogenized. Such a homogenization was performed in [11] and the relative effective permittivity dyadic describing the medium is of the form

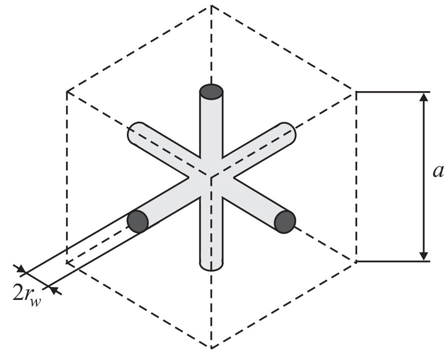

Fig. 1. Triple wire mesh structure formed by a lattice of infinitely long connected wires.

$$\overline{\overline{\varepsilon}} = \overline{\overline{\mathbf{I}}} - \frac{k_p^2}{k_0^2}\left(\overline{\overline{\mathbf{I}}} - \frac{\mathbf{kk}}{k^2 - l_0 k_0^2}\right), \tag{1}$$

where k_p is the plasma wavenumber, k_0 is the free-space wavenumber, and

$$l_0 = \frac{3}{1 + 2k_p^2/\beta_1^2}, \tag{2}$$

$$\frac{1}{\beta_1^2} = \left(\frac{a}{2\pi}\right)^2 \sum_{l\neq 0} \frac{\left[J_0\left(\frac{2\pi l r_w}{a}\right)\right]}{l^2}, \tag{3}$$

where a is the lattice period, r_w is the wire radius and J_0 stands for the Bessel function of the first kind and zero order. In [13] there are two formulas for the plasma wavenumber of such a wire array

$$(k_p a)^2 \approx \frac{2\pi}{\ln\left(\frac{a}{2\pi r_w}\right) + 0.5275} \qquad (r_w < a/100)\,, \tag{4}$$

$$(k_p a)^2 \approx \frac{2\pi}{\ln\left(\frac{a^2}{4r_w(a - r_w)}\right)} \qquad (a/20 < r_w < a/5)\,. \tag{5}$$

It was shown in [12], however, that the calculated propagation constant near the plasma frequency suffers from a weak spatial dispersion, moreover it is the same for Γ–X and Γ–M directions and only differs in the the Γ–R direction in the first Brillouin zone[2]. Thus we can assume that the medium is practically isotropic and we can adopt the permittivity tensor in the simplified form

$$\bar{\bar{\varepsilon}} = \left(1 - \frac{k_p^2}{k_0^2}\right)\bar{\bar{\mathbf{I}}}. \tag{6}$$

Consequently, the wave propagates through the medium in all the important lattice directions with the same phase constant. Several numerical simulations were performed in the CST Microwave Studio (MWS) [14] full-wave simulator to verify the theoretical results. The simulated wire mesh is depicted in Fig. 2. It is formed by a lattice of $8\times8\times8$ wires with a rectangular cross-section[3] 1×1 mm with the lattice period a = 20 mm.

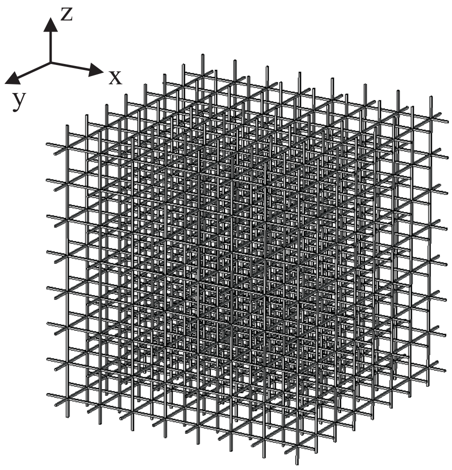

Fig. 2. Cubic wire mesh formed by a lattice of $8 \times 8 \times 8$ connected wires.

[2]In crystallography, it is common to denote the wave propagation along one of the unit cell axes as the Γ–X direction, the propagation along the unit cell face diagonal as the Γ–M direction and the propagation along the unit cell diagonal as the Γ–R direction.

[3]The theoretical formulas (4) and (5) hold for wires with circular cross-section, nevertheless the rectangular cross-section is much more suitable for the numerical experiment and the error of the determination of the plasma frequency is acceptably small within the effective medium approximation.

The mesh is illuminated by a plane wave in all the important directions, i.e., the Γ–X, Γ–M and Γ–R directions. The plasma frequency of the mesh calculated using (5) is approx. f_p = 3.9 GHz. The distribution of the electric field intensity in the plane crossing the WM along all the important directions at the frequency 2.5 GHz is depicted in Figs. 3, 4 and 5.

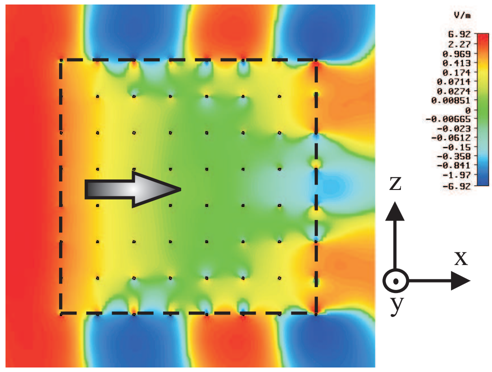

Fig. 3. The distribution of the electric field intensity component E_y in the x-z plane crossing the wire mesh from Fig. 2 at the the half height between two wire layers. The plane wave illuminating the WM propagates in the Γ–X direction. The mesh boundary is marked with the dashed line.

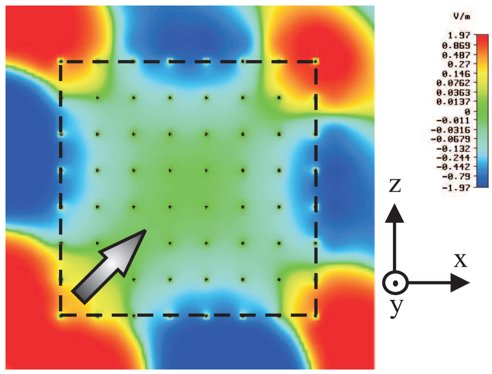

Fig. 4. The distribution of the electric field intensity component E_y in the x-z plane crossing the wire mesh from Fig. 2 at the the half height between two wire layers. The plane wave illuminating the WM propagates in the Γ–M direction. The mesh boundary is marked with the dashed line.

It is evident from Figs. 3, 4 and 5 that the wave

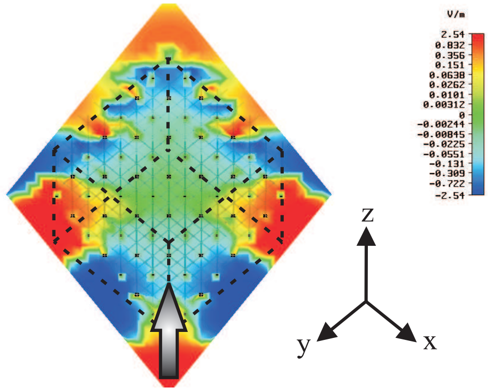

Fig. 5. The distribution of the transversal component of the electric field of the plane wave illuminating the structure in the Γ–R direction. The mesh boundary is marked with the dashed line.

is an evanescent wave within the WM with a purely imaginary propagation constant

$$k = \omega\sqrt{\varepsilon_0 \varepsilon_{\text{eff}} \mu} \equiv j\alpha. \quad (7)$$

The attenuation constant can be determined from the behavior of the plane wave. The intensity of the electric field is [4]

$$E(r) = E_0 e^{-\alpha r}, \quad (8)$$

where r is the coordinate in the propagation direction and E_0 is the amplitude of the electric field intensity at the plane wave source. As an example, the electric field intensity inside the wire mesh for the direction of the propagation Γ–X is depicted in Fig. 6 for several frequencies below the lattice plasma frequency. From the calculated distribution of the electric field it is easy to find the attenuation constant α and consequently the effective permittivity ε_{eff} of the lattice by fitting the electric field distribution inside the structure to the eq. (8). The calculated effective permittivity is depicted in Fig. 7. The effective permittivity is according to (6) equal in all the important propagation directions. The differences observed in Fig. 7 are caused only by the error of the determination of the permittivity from the electric field distribution.

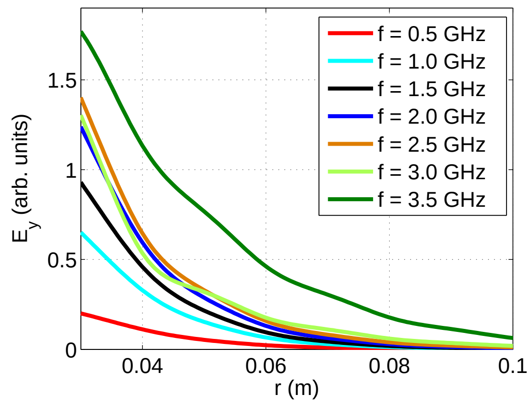

Fig. 6. Amplitude of the E_y component in the wire mesh from Fig. 2 (x-z plane, the lattice is crossed in the half of its height).

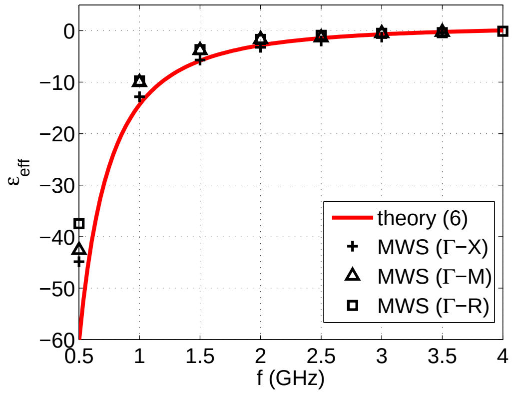

Fig. 7. The calculated effective permittivity of the wire mesh from Fig. 2.

B. *Proposal of the practical realization*

The idea of a triple WM as an isotropic ε-negative material is quite simple, nevertheless the practical fabrication becomes cumbersome. By the authors' knowledge, nobody has experimentally verified the theoretical results so far. We propose an idea how to fabricate an analogy to the wire mesh depicted in Fig. 2 using the planar technology, Fig. 8. It consists of a set of parallel dielectric sheets, the 2D planar mesh is etched in the x-y plane on each substrate layer, the third orthogonal set of planar wires is etched in the y-z planes and the set of substrate layers without metallization in the x-z planes is inserted due to the preservation of the 3D structure symmetry, Fig. 8. The galvanic connection of the planar wires in the y-z plane is performed by via holes and miniature wires soldered in the x-y plane substrate layers. The lattice period 20 mm is the same as that of the structure from Fig. 2 and the strip width is twice the wire diameter [15]. Since the wire mesh is not a resonant structure (in the long wavelength limit), the presence of the supporting dielectric substrates does not affect the plasma frequency of the lattice significantly.

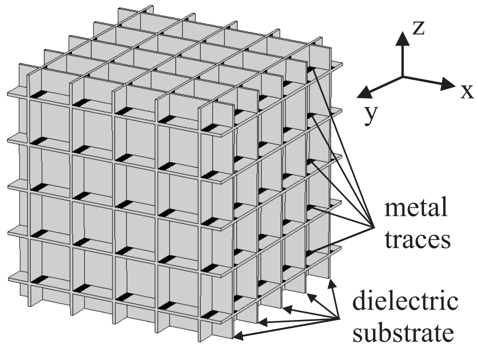

Fig. 8. Proposal of the wire mesh from Fig. 2 manufactured with a planar technology.

The realization of the mesh by a planar technology results in a not fully isotropic structure, nevertheless the propagation of the electromagnetic wave in all the important directions does not differ dramatically in comparison with the ideal mesh, Fig. 2. The effective permittivity determined from the electric field distribution inside the lattice fabricated by the planar technology is shown in Fig. 9. It can be seen that it only differs slightly rom the values determined in Fig. 7. The proposed form of the 3D structure fabricated by the planar technology is suitable for an eventual combination with an isotropic negative permeability material, e.g. [16]. Such a combination, nevertheless, would require much more detailed analysis and some issues are still open.

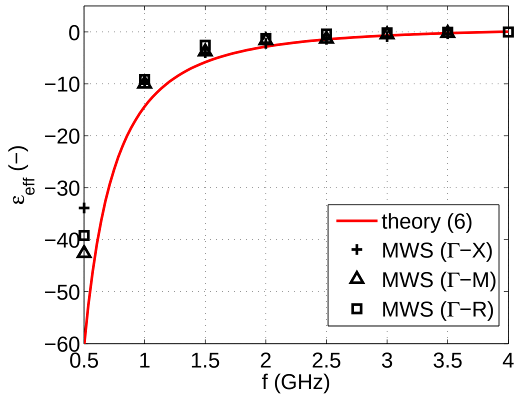

Fig. 9. The calculated effective permittivity of the wire mesh from Fig. 8.

III. Conclusion

In the paper the triple wire medium was denoted as a suitable candidate for an isotropic negative permittivity medium. Set of numerical experiments was performed to validate the theoretical propagation of electromagnetic waves inside the wire mesh below the plasma frequency. In the framework of the effective medium theory the wire mesh effective permittivity is negative below the plasma frequency. The main advantage in comparison with the other ε-negative structures is a wide band, where the negative effective permittivity can be achieved. For a practical realization, the planar form of the triple wire medium is proposed. It consists of three sets of mutually orthogonal dielectric substrate layers, which support cross connected etched planar wires. It is shown that the properties of the proposed planar lattice are very similar to those of the ideal wire lattice. A promising future combination with an isotropic negative permeability material is discussed.

The proposed sample is ready for fabrication and measurement of its properties.

ACKNOWLEDGMENT

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