# Load impedances of complex slotline terminations

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## Abstract

Single slotline has gained new interest due to application in uniplanar circuits, antennas or active radiators, all these including slotline short circuits or even more complex terminations. This paper gives a new approach to calculation of reflection coefficient and load impedances of such terminations using the spectral domain method. Furthermore, slotline resonators are analysed in the similar way, and their complex resonant frequency is calculated. That is used in an alternative determination of the slotline load impedance by application of the transmission line theory. Computed results compare well with experiment. Closed-form formulae for CAD purposes are proposed.

## Introduction

Slotline plays an important role for antennas [1], radiating resonators [2], slotline filters and couplers [3 - 6], or uniplanar integrated circuits [7]. In many slotline components as well as in transitions to other planar waveguides like microstrip or coplanar line, e.g. [8], [9], short- and open-circuited slotlines are required. To improve bandwidth, different slotline terminations as shown in Fig. 1 are used. While the simple slotline short circuit has been analysed by different authors [10], [11], more complex terminations have been evaluated largely experimentally [12] and theoretically by the method of finite differences [13]. This paper presents a spectral domain method (SDM) for the analysis of general slotline terminations. In contrast to [13], SDM is much more suitable to evaluate the contributions of leaky wave excitation and radiation, and it can include easily dispersive effect of these terminations. Open resonant structures have been analysed in [14]. Now the technique used in [14] and [15] has been applied to the analysis of slotline resonators.

## Analytical Approaches

First a reflection coefficient of the slotline termination will be derived. Generally, the SDM is based on the procedure given in [10] or [11]. The slotlines in the structures as shown in Fig. 1 are divided using a suitable rectangular mesh. The tangential electric field along the slot is then described by a summ of piecewise sinusoidal functions (PWSFs). One of the field elements is impressed by a given amplitude, in dependence on which the other amplitudes are computed. The reflection coefficient of the termination $\Gamma$ is then calculated by means of the standing wave pattern in the feeding slot which is fitted to the function

$$E(z) = A[\exp(-jk_s z) + \Gamma \exp(jk_s z)], \qquad (1)$$

by the least squares method. A is an amplitude, $k_s$ is the propagation constant of the wave guided in the z direction by the slotline [16]. $\Gamma$ and A can be found analytically minimizing the least squares discrepancy between (1) and the calculated standing wave pattern. Finally the reflection coefficient is

$$\Gamma = \frac{-\sum_{i=1}^{P} \exp(2jk_s y_i) \sum_{i=1}^{P} E_i \exp(-jk_s y_i) + P \sum_{i=1}^{P} E_i \exp(jk_s y_i)}{P \sum_{i=1}^{P} E_i \exp(-jk_s y_i) - \sum_{i=1}^{P} \exp(-2jk_s y_i) \sum_{i=1}^{P} E_i \exp(jk_s y_i)} \qquad (2)$$

where $E_i$ are calculated values of the tangential field along the slotline sampled at points $Y_i$ number of which is P. Load impedance is found by a simple recalculation of known $\Gamma$.

A modified approach can be used for calculation of the slotline load impedance. The procedure is based on the analysis of the slotline resonator terminated at both ends by the structure under consideration. The concept of the complex resonant frequency [14], [15] is used. In the sourceless case a set of equations solved for the unknown PWSF's amplitudes is homogeneous. It has non-trivial solution when the determinant of the system matrix equals zero. Accomplishment of this constraint provides the complex resonant frequency. The load impedance is then determined by means of the complex resonant frequency using the transmission line theory combined with the lumped element model of the termination. The slotline resonator is treated as a section of the slotline terminated at the distance L by impedances normalized values of which are Z = R+ jX Terminating impedances are normalized to the characteristics impedance of the slotline. After setting the known complex resonant frequency $f = f_r + jf_i$ and the length of the resonator L into the resonant condition Z can be find from

$$\mathrm{Im}\left[ Z + \frac{Z + j\mathrm{tg}(k_s L)}{1 + jZ\mathrm{tg}(k_s L)} \right] = 0 \qquad (3)$$

$$Q = f_r/(2f_i) = k_s L\left(1 - \left|\frac{Z-1}{Z+1}\right|^2\right)^{-1} \qquad (4)$$

where Q is the resonator quality factor. This approach is valid as far the equivalent circuit of the resonator holds, i.e. the width of the slotline is negligible in comparison with the resonator's length.

## Results Evaluation

A number of different slotline terminations on the substrate 1.27 mm thick with dielectric constant of 11 were investigated both theoretically and experimentally. To measure the circuit, excitation of the slot mode with low losses at respective transitions is important. Therefore, the wave was transmitted from the waveguide to the finline and further from the finline to the open slotline. Nevertheless, radiation and standing waves occured at these transitions and resulted in the strong ripple of the measured characteristics. Behaviour of the slotline resonators was tested using the structure shown in Fig. 2.

Figs. 3, 4, 5 show calculated and measured return losses of three slotline terminations given in Fig. 1a, b, c. Although in all cases the measured pattern show strong ripple, the general behaviour of the curves fits very well.

Calculated normalized terminal resistance and reactance of the short-circuited slotline in the span of width from 0.1 to 3.25 mm for $H/\lambda_o$ in the range from 0.00425 to 0.0845 are given in Fig. 6 where permittivity of the substrate is 11 and H denotes its thickness.

Fig. 7 shows calculated normalized terminating impedance of the short-circuited slotline obtained by the present method with that published in [10] and with measured values given in [17]. A good agreement is evident. Similar comparison is made in Fig. 8. The agreement of normalized terminal reactance with data obtained in [18] by the integral equation technique is quite good. Resistances can be compared at low frequencies only with the discrepancy growing for the frequency raise.

As to the values shown in [13]: leaky waves are excited by the slotline ($\varepsilon_r = 9.8$, H= 1.5 mm, W = 0.75 mm) from app. 28 GHz upward as follows from calculation according to [16]. Above this frequency the slotline does not transmit bound wave and consequently definition of the terminal impedance is sensless.

The main contribution to the losses at the slotline terminations is caused by excitation of surface waves and by radiation. Losses in dielectric play a minor role only as it has been proved by accounting its complex permittivity.

To measure the resonant frequency $f_r$ and the quality factor Q of the short-circuited slotline resonator the circuit shown in Fig. 2 was manufactured on the substrate with permittivity 11 and 1.27 mm thick. The resonator is loosely coupled to microstrip feeding lines on the opposite side of the substrate. Measured data and corresponding values resulting from the resonator analysis mentioned above agree well. For illustration Tab. 1 gives calculated $f = f_r + jf_i$ and measured $f_m = f_{rm} + jf_{im}$ complex resonant frequencies along with relevant Q and $Q_m$ respectively for short-circuited resonators of unequal lengths. Relatively higher errors Δfi follow from measurement accuracy of $Q_m$.

Comparison of normalized short-circuited slotline terminating impedance calculated directly by SDM with values provided by the transmission line theory utilizing calculated complex resonant frequency of the slotline resonator is shown in Fig. 9.

Slotline resonators with terminations shaped in accordance with Fig. 1b, c have been manufactured and their resonat frequencies were measured. The rectangular patch had 2x2 mm size and the fan patch with 90° angle had 2 mm radius. Both resonators were 14 mm long with 0.15mm wide slot. They were fed according to Fig. 2. Their characteristics are nearly identical.

## Closed-Form Formulae of Short-Circuited Slotline

An exact model of the short-circuited slotline based on SDM can not be implanted into CAD packages. For that a set of impedances Z = R+ jX normalized to the characteristic impedance of the slotline calculated by the present technique quoted above has been fitted by the least squares method. The resultant closed-form formulae are

$$R(x,y) = A_1.\exp(A_2 y) - A_3.\exp\left(-A_4|y - A_5|^{A_6}\right) \tag{5}$$

$$X(x,y) = \left(B_1 + B_2|y - B_3|^{B_4}\right)^{-1} + B_5.\exp\left(B_6|y - B_7|^{1.93}\right) \tag{6}$$

where $x = W/H$, $y = H/\lambda_o$, W is width of the slotline and H is thickness of the substrate. Further

$$A_1 = 0.3199.\exp(0.26107x) + 0.003911.\exp[3.70742(x - 1.46771)] - 0.310381 - 0.05084865x \tag{7}$$

$$A_2 = 737.87342.\exp(-0.28926x) - 697.59731 + 198.70971x - 22.28357x^2 \tag{8}$$

$$A_3 = 1.00879.\exp(0.176596x) + 0.0022.\exp[3.42346(x - 1.11394)] - 0.94566 - 0.00104x \tag{9}$$

$$A_4 = 200.40584.\exp(-0.827945x) - 49.27144 + 64.64021x - 16.76561x^2 \tag{10}$$

$$A_5 = 0.06253 + 0.05357x - 0.05139(x + 0.02198)^{1.04007} \tag{11}$$

$$A_6 = \left(0.876 + 0.01796x^{1.08986}\right)^{-2.83102} \tag{12}$$

$$B_1 = 9(x + 1.3)^{-7.5} + 0.35\exp\left(-0.8|x - 0.3|^2\right) + 0.5(x + 3.5)^{-5} + 1.025 + \exp[-2(x + 0.5)] \tag{13}$$

$$B_2 = 900.\exp\left(-0.5|x - 0.179|^{7.5}\right) + 258 + 50(x + 0.88)^{-20} - 20.7x - 150.\exp\left(-8|x - 0.88|^2\right) + $$
$$+12000\left(0.001 + 1000|x - 1.182|^{0.41}\right)^{-1} \tag{14}$$

$$B_3 = 0.0903.\exp(-0.076x) - 0.000085\left(0.05 + 150|x - 0.81|^{12}\right)^{-1} - 0.000095\left(0.05 + 500|x - 1.55|^5\right)^{-1} + $$
$$+ 0.0002x^{1.3} \tag{15}$$

$$B_4 = 0.335.\exp\left(-3.1|x - 0.55|^{7.5}\right) + 1.5 \tag{16}$$

$$B_5 = 0.053 + 0.025.\exp\left(-0.95|x - 2.19|^{4.5}\right) - 0.011.\exp\left(-14|x - 1.53|^2\right) - 0.0024.\exp\left(-5|x - 2.28|^{1.9}\right) - $$
$$-20(x + 1.7)^{-13} - 10(x + 2.5)^{-9.5} - 0.00032\left(0.05 + 50|x - 0.15|^5\right)^{-1} + $$
$$+0.0005\left(0.16 + 100|x - 1.8|^{2.1}\right)^{-1} + 0.025[\exp(-100|x - 1.278|) - 1].(x - 1.278).|x - 1.278|^{-1} \tag{17}$$

$$ {}_6 = \left[850 - 900\left(1 + 50|x - 1.278|^{0.9}\right)^{-1}\right](x - 1.278).|x - 1.278|^{-1} + 1150 + 8x^{0.5} + $$

$$ +121\left(x + 150|x - 1.13|^{3}\right)^{-1} + 30.\exp\left(-15|x - 0.3|^{2}\right) - 0.0016(x - 0.07)^{2} \qquad (18) $$

$$ {}_7 = 0.0179 + 0.005x^{0.48} - 2.9(x + 1.24)^{-20} - 0.0014.\exp\left(-40|x - 0.98|^{2}\right) + 0.0016.\exp\left(-3|x - 1.9|^{2.5}\right) \qquad (19) $$

Eqns. (5) and (6) are valid in the frequency range from1/2 to 20 GHz for a slot line with the slot width from 0.1 to 3.0/1.5 mm on the substrates 1.27/0.635mm thick having permittivity $\varepsilon_r$=11. In the other words they hold for $\varepsilon_r$=11 and y $\in$(0.00425,0.0846), x $\in$ (0.0787,2.362) when H =1.27mm and x $\in$ (0.157,2.362) when H=0.635mm.

## Conclusions

Two different techniques based on the spectral domain method for calculating the load impedance of the complex slotline terminations have been presented and verified by experiments with a good agreement. Influence of the surface wave leakage and radiation is clearly seen at higher frequencies when the reflection coefficient is going down. Dielectric losses mostly play minor role. For CAD purposes, terminating impedance of the short-circuited slotline is approximated by the closed-form formulae with sufficient accuracy.


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## References


[1] Yoshimura Y.: A microstripline slot antenna, IEEE Trans. Microwave Theory Techn., vol. MTT-20, 1972, No. 11, pp. 760-762.

[2] Luy J.F., Buechler J., Thieme M., Biebl E.M.: Matching an active mm-wave slotline antennas, Electron Lett., vol. 29, Sept.1993, pp. 1772-1774.

[3] Mariani E.A., Agrios J.P.: Slot-line filters and couplers, IEEE Trans. Microwave Theory Techn., vol. MTT-18, 1970, No. 12, pp. 1089-1095.

[4] Schieck B., Kohler J.: Improving the isolation of 3-dB couplers in microstrip-slotline technique, IEEE Trans. Microwave Theory Techn., vol. MTT-26, 1978, No. 1, pp. 5-7.

[5] Ho C.H., Fan L., Chang K.: A broad-band uniplanar slotline hybrid ring coupler with over one octave bandwidth, IEEE MTT-S Digest, 1993, pp. 585-588.

[6] Ho C.H., Fan L., Chang K. :Broad-band uniplanar hybrid-ring and branch-line couplers, IEEE Trans. Microwave Theory Techn., vol. MTT-41, 1993, No. 12, pp. 2116-2125.

[7] Ogava H., Minagawa A.: Uniplanar MIC balanced multiplier - a proposed new structure for MIC's, IEEE Trans. Microwave Theory Techn., vol. MTT-35, 1987, No. 12, pp. 1363-1368.

[8] Schuppert B.: Microstrip/slotline transitions: modeling and experimental investigation, IEEE Trans. Microwave Theory Techn., vol. MTT-36, 1988, No. 8, pp. 1272-1282.

[9] Ho C.H., Fan L., Chang K.: Experimental investigations of CPW-slotline transitions of uniplanar microwave integrated circuits, IEEE MTT-S Digest, 1993, pp. 877-880.

[10] Yang H.Y., Alexopoulos N.G.: A dynamic model for microstrip-slotline transition and related structures, IEEE Trans. Microwave Theory Techn., vol. MTT-36, 1988, No. 2, pp. 286-293.

[11] Rozzi T., Morini A., Pallota A., Moglie F.: A modified dynamic model for planar microwave circuits, IEEE Trans. Microwave Theory Techn., vol. MTT-39, 1991, No. 12, pp. 2148-2153.

[12] Chramiec J.: Reactances of slotline short and open circuits on alumina substrate, IEEE Trans. Microwave Theory Techn.,vol. MTT-37, 1989, No. 10, pp. 1639-1641.

[13] Fusco V.F., Chen Q: Slotline short and open circuit analysis by the finite-difference time-domain method, Proc. of 24thEuMC, Cannes, France, Sept. 1994, vol. 2, pp.1720-1726.

[14] Itoh T., Menzel W.: A full-wave analysis method for open microstrip structures, IEEE Trans. Antennas Propag., vol. AP-29,1981, No.I, pp. 63-68.

[15] Bhat B., Koul S.K.: Analysis, design and applications of fin lines, Artech House, Norwood, 1987.

[16] Machac J., Zehentner J.: Leaky waves on a slot line, Proc. of 1995 International Symposium on Electromagnetic Theory, URSI, St. Petersburg, Russia, May 1995, pp. 761-763.

[17] Knorr J.B., Saenz J.: End effect in a shorted slot, IEEE Trans. Microwave Theory Techn., vol. MTT-21, 1973, No. 9, pp. 579-580.

[18] Drissi M., Hanna V.F., Citerne J.: Analysis of coplanar waveguide radiating end effects using the integral equation technique, EEE Trans. Microwave Theory Techn., vol. MTT-39,1991, No. 1, pp. 112-116.


Tab. 1 Calculated f, Q and measured $f_m$, $Q_m$ complex resonant frequencies and quality factors of the short-circuited slotline resonators (w=0.15 mm, H=1.27 mm, $\varepsilon_r$ = 11)

| L (mm) | $f = f_r + jf_i$ (GHz) | Q | $f_m = f_{rm} + jf_{im}$ (GHz) | $Q_m$ | $\Delta f_r$ (%) | $\Delta f_i$ (%) | $\Delta Q$ (%) |
|---|---|---|---|---|---|---|---|
| 14.95 | 4.308+j0.0352 | 42.11 | 4.353+j0.0324 | 44.82 | 1.04 | 7.95 | 6.46 |
| 3.75 | 14.146+j0.441 | 15.35 | 14.340+j0.4813 | 13.81 | 1.37 | 9.14 | 10.03 |

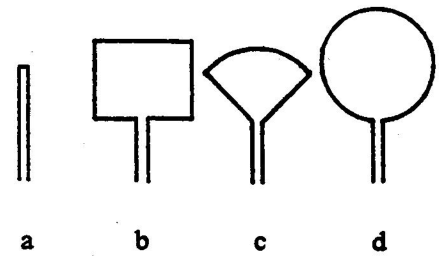


Fig. 1 Short- and open-circuited slotline

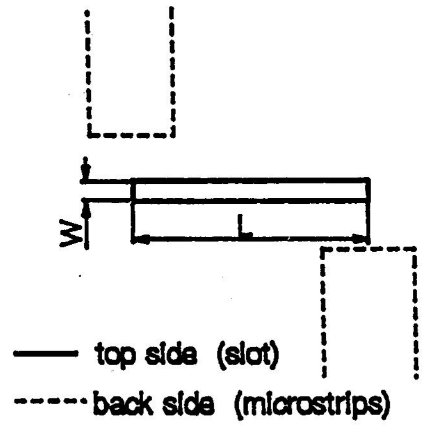


Fig. 2 Slotline resonator fed by microstrip lines

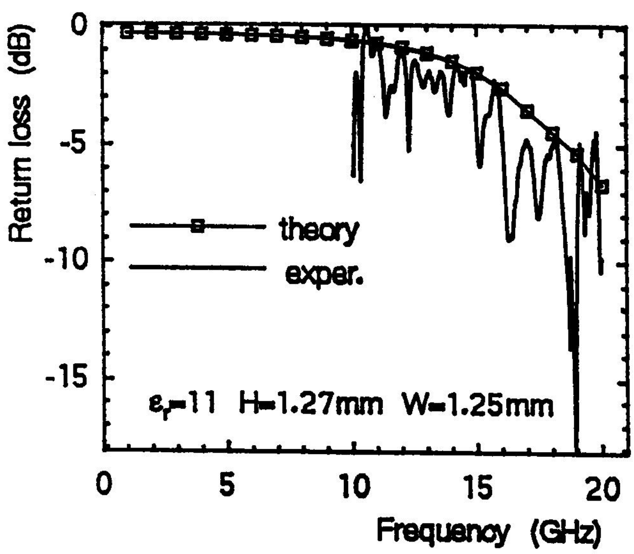


Fig 3. Return loss of the short-circuited slotline

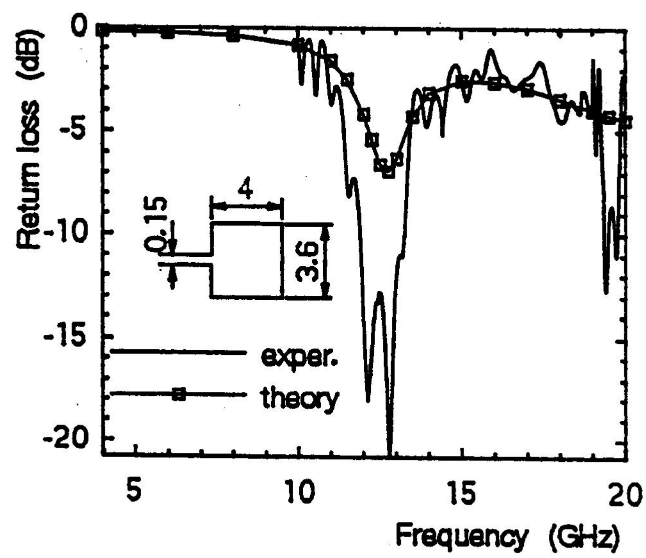


Fig. 4 Return loss of the open-circuited slotline

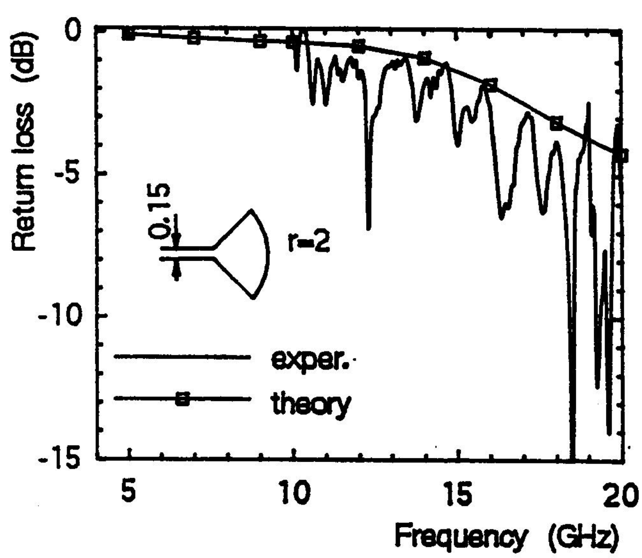


Fig. 5 Return loss of the open-circuited slotline

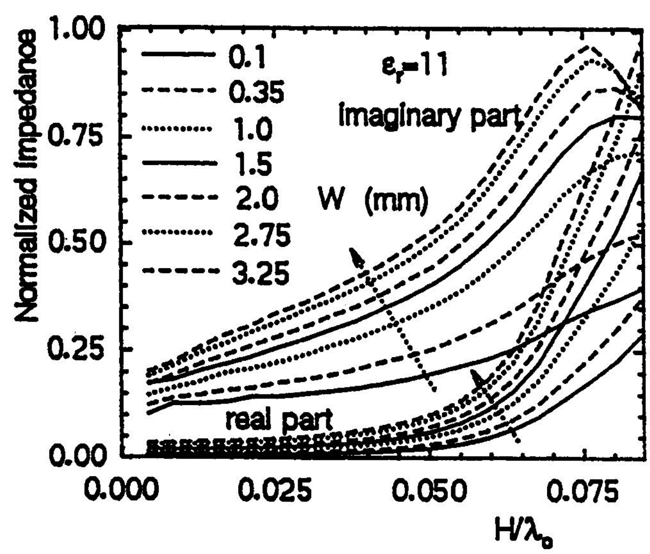


Fig. 6 Normalized resistance and reactance of the short-circuited slotline

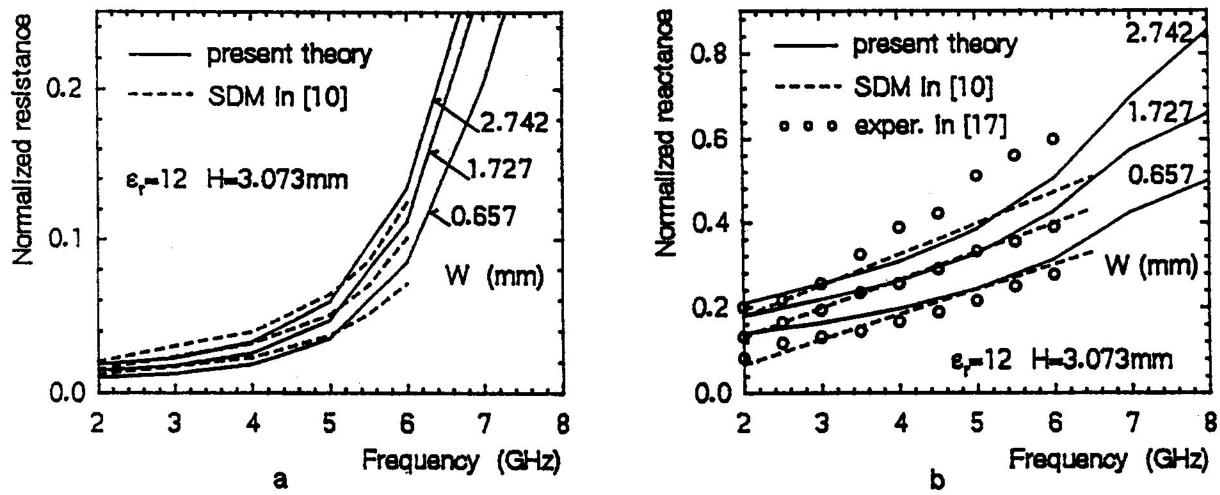


Fig. 7 Comparison of normalized terminating impedance of the short-circuited slotline calculated by the present method and that given in [10] and [17]

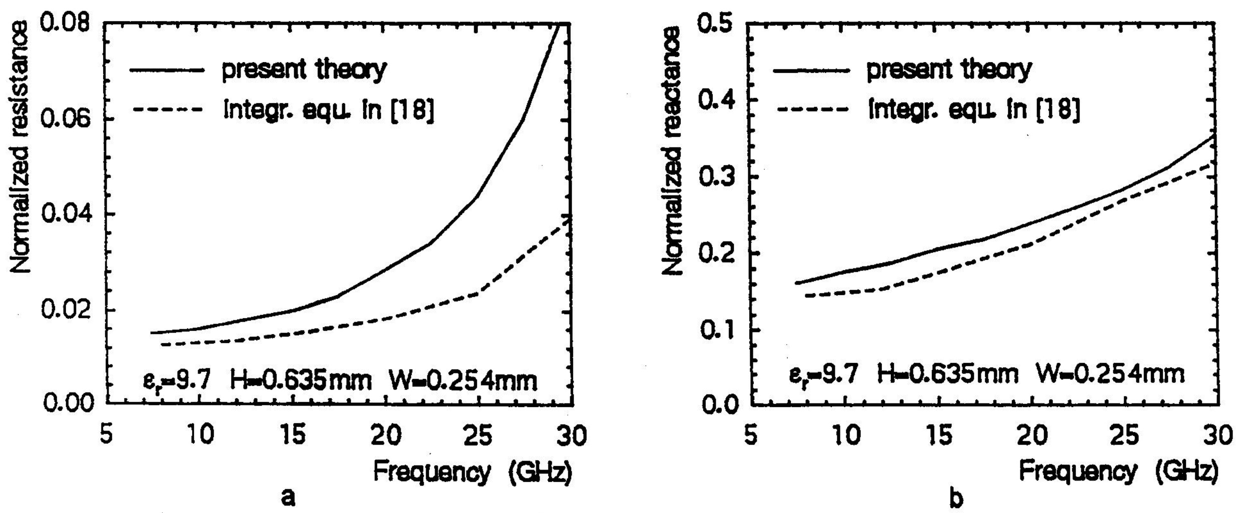


Fig. 8 Comparison of normalized terminating impedance of the short-circuited slotline calculated by the present method and that given in [18]

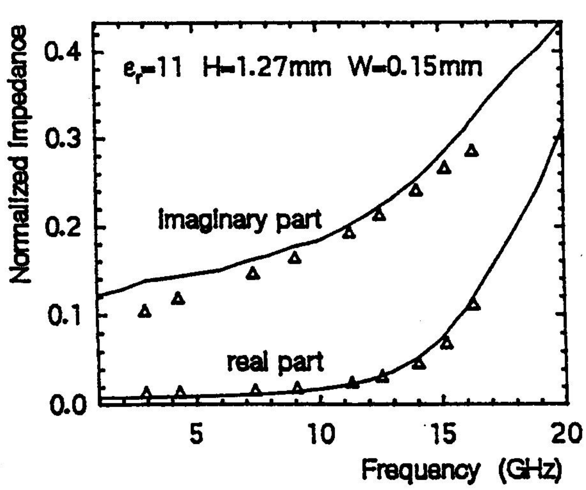


Fig. 9 Load impedance of the short-circuited slotline calculated: —— directly by SDM, Δ Δ Δ from the complex resonant frequency of the slotline resonator