Analytical Calculation Methods

- Classical Laminate Theory
  \[ c_{xy} = S_{xy}^{-1} = \begin{bmatrix} \frac{1}{E_x} & -\frac{v_{xy}}{E_x} & 0 \\ \frac{v_{xy}}{E_y} & \frac{1}{E_y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \]
  \[ U = U_{M_0} + U_I \]
  \[ \sigma = \sum_{k=1}^{n} E_k \cdot I_y \]

- ABD Matrices
  \[ [N] = [A] \cdot [e] \quad [M] = [B] \cdot [D] \]
  \[ E_{eq} = \left( A_{11} - [A]_{12} \cdot A_{13} \cdot [A]_{33}^{-1} \cdot [A]_{31} \right) \frac{1}{t} \]

- Average values from Stiffness and Compliance matrix
  \[ E_{eq} = \frac{E_J + E_I}{2} \]

- A new calculation method
  \[ (E_J)_{\text{equivalent}} = E_J \text{tensile} + E_J \text{bending} = (E_J)_{\alpha^2} + (E_J)_{\beta^2} \]

FE methods:
- Conventional shell
- Continuum shell (Solid shell)
- Volume model

Tensile Modulus Eq from Analytical and FEM Models

- Deflection from the moment:
  \[ v_M = \frac{M \cdot l^2}{2E_J} \]
  \[ E_{eq}(M) = \frac{M \cdot l^2}{2v_M} \]

- Bernoulli's and Timoshenko's theorem is used
  \[ U = U_{M_0} + U_I = \int_0^l \frac{M_0^2(x)}{2E_J} dx + \int_0^l \frac{\beta \cdot t^3(x)}{2E_J} dx \]

Results

- Percentage deviations from experimental data

Conclusion

In most cases, the results of the new stiffness matrix and compliance matrix average value approaches and the geometry-based method showed good agreement with the experimental results. With a deviation of less than 20% from the experimental results for thick-walled pipes and a deviation between 20% and 25% from the experimental results for thin-walled tubes from the base series of samples. For these thin-walled tubes, greater inaccuracies involving local deformations in the load members during the experiment are assumed. For an additional series of thick-walled specimens, the values are even closer to the experimental data with a maximum deviation of 17% from the experimental value. The results of both methods are much closer to experimental data than other analytical methods and FEM models. Since the new approach with a geometric approach predicts a lower stiffness than the experiment, the results of this method are appropriately conservative and can be evaluated as safe. Both methods can be recommended for preliminary calculations in new designs of mechanical systems with tubular composite beams.