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STRUCTURAL FLEXURAL STRENGTHENING THROUGH MATERIAL BONDED TO THE CONCRETE SUBSTRATE

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Abstract. This article proposes an alternative method for the structural design of reinforced concrete elements strengthened in bending by metallic plates or fiber-reinforced polymer (FRP) bonded to the concrete substrate. It is proposed a new calculation procedure for the strengthening using thin adhered material bonded to the element surface that dispenses the iterative process generally used in the design. The proposed routine is validated by comparison with other methods. A practical example is also presented, applying the procedure to an element of a building where a load change was foreseen. As result, it was verified that the proposed procedure provides values similar to the trial-and-error method used in the FRP strengthening design. Results are also coherent with other methods available in the literature for metallic plates. Therefore, since this routine obtains similar values without using an iterative method, its applicability in the design becomes advantageous.

Keywords: Attached materials, calculation guide, structural strengthening.

1. Introduction

When constructions do not fit the required capacity for correct use, rehabilitation is necessary. The reasons for rehabilitation may be pathological manifestations, accidents, natural disasters or changes in the building use. To restore the necessary resistance of the building, actions such as recovery or structural strengthening may be necessary [1–3].

The primary function of a structural retrieval is to improve the resistant capacity of any structure. The strengthening can be performed by associating new materials to the original structure, forming a composite system that supports the applied loads. Examples of these processes are the addition of steel bars to the original section, reinforced concrete jacketing, external prestressing tendons and elements adhered to the substrate, such as steel sheets and composite materials [4].

This study proposes a new calculation methodology for the flexural design of reinforced concrete elements associated with small thickness bonded materials to the surface, as external steel sheets or fiber-reinforced polymer (FRP). The main goal herein is to establish a calculation guide for structural strengthening design, which fits the requirements of the Brazilian standard NBR 6118: 2014 [5]. It should be noted that only the flexural capacity is concerned in this paper. Considerations related to anchor capacity or the possibility of debonding of the strengthening are not made.

2. Literature review and numerical simulation

This section presents some methods commonly used for the flexural strengthening design of reinforced concrete beams and slabs present in the literature. These methods consider the insertion of steel sheets and carbon fiber reinforced polymer sheets bonded to the surface. The calculation guide proposed in this paper follows the literature review.

2.1. Calculation models for metal sheet strengthening

There are several calculation methods in the literature for the flexural strengthening using bonded metal sheets and steel profiles. Some of the most relevant are described below.

2.1.1. J. Bresson Model

Metal sheet dimensioning with Bresson's method is carried out by balancing the bending moments in relation to the most compressed fiber. Materials are

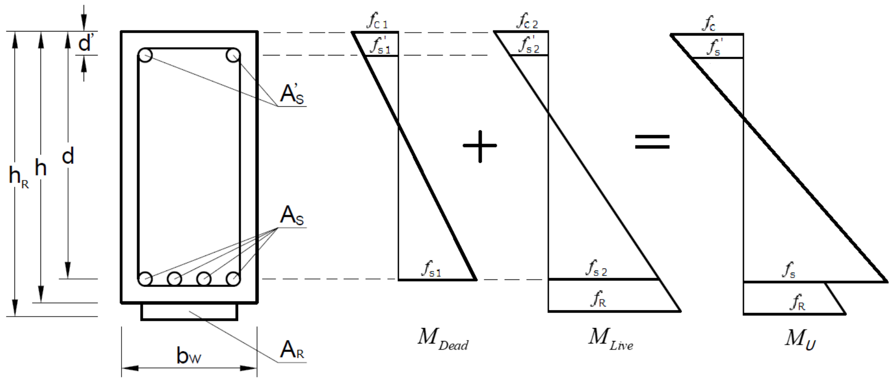

Figure 1. Stress diagram of a strengthened beam. Source: Adapted from Silveira [6].

assumed to have a linear elastic behavior; concrete tensile strength is neglected and stresses are limited to their respective elastic admissible values [6].

Thus, the strengthening plate area is obtained by equation 1:

$$A_s = \frac{\left(M_{Dead} + M_{Live}\right) + (f_{c1} + f_{c2})\, \frac{b \cdot y^2}{6} - (f_{s1} + f_{s2})\, A_s\, d}{f_R \cdot h} \quad (1)$$

As shown in Figure 1, design takes into account dead and live loads, referred, respectively, to the forces already acting in the section when the strengthening is carried out and after the rehabilitation.

2.1.2. Campagnolo Model

In this model, strengthening design is performed using classical formula for bending, considering composite beams theory [1]. Thus, the tensile stress calculation for the steel plate is obtained as follows in equation 2.

$$f_R = \frac{E_R}{E_c} \cdot \frac{M_u\,(h - x_{II})}{I_x} \quad (2)$$

2.1.3. Cánovas model

The design is performed on the basis that materials behave under their maximum allowable elastic stresses. Stress distribution before and after the strengthening execution are considered. Balance and design equations are obtained by superimposing the stress diagrams [1], as illustrated in Figure 1.

Thus, the necessary area of additional strengthening, AR, is obtained by balancing external and internal moments as in equation 3:

$$M_u = (A_s\, f_s + A_R\, f_R)\,(d - x') \quad (3)$$

2.1.4. Ziraba and Hussein model

The authors proposed a method in which the strengthening design is done at the ultimate limit state, following the specifications of ACI 318 [1, 7].

Stress resultants and the strengthening area are obtained from the balancing of moments in the section taken at the point of the compressive stress resultant in concrete, as reproduced in Figure 2, resulting in an iterative process, as shown in equation 4.

$$M_{sd} = R_s\left(d - \frac{y}{2}\right) + R_R\left(h_R - \frac{y}{2}\right) \quad (4)$$

2.2. Calculation models for carbon fiber reinforced polymer

Design of strengthening systems composed of carbon fibers, according to Machado [2] and the American code ACI 440 [8], may be done through a trial-and-error procedure, that gives the required area of strengthening at the ultimate limit state.

According to these methods, a position for the neutral axis (x) is initially arbitrated. Then, deformations (ε) and stresses (f) of materials are calculated using compatibility and equilibrium equations. It is then verified if the resulting forces are balanced (equation 5), and if the resistant bending moment gotten is greater than the external moment applied (equation 6). If any of the conditions is not satisfied, a new neutral axis depth is chosen, and the procedure is repeated.

$$N_{sd} = R_s + R'_s + R_R + R_C \quad (5)$$

$$M_{sd} = R_s\left(d - \frac{yx}{2}\right) + R'_s\left(\frac{yx}{2} - d'\right) + R_R\left(h_R - \frac{yx}{2}\right) \quad (6)$$

2.3. Proposed calculation model

This section contains the method proposed by the authors. It is a calculation guide used for structural strengthening design, addressing the association of thin-thickness materials adhered to the surface.

2.3.1. Calculation premise

The following hypotheses are adopted:

- The design is carried out at the ultimate limit state, disregarding the concrete tensile strength.
- Plane sections remain plane after loading, and the deformations are proportional to their distance to the neutral axis (Bernoulli criterion).

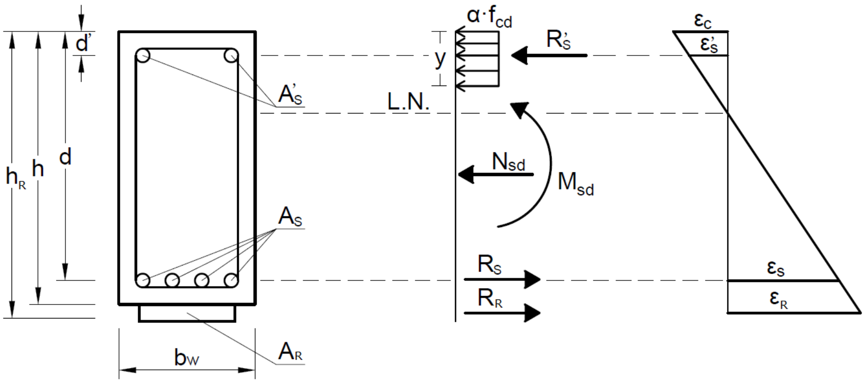

FIGURE 2. Stress and strain diagram for a section in the strengthened condition.

- Perfect adhesion exists between the concrete, the steel reinforcement, and the thin adhered material (Navier criterion).
- The cross-sectional failure is characterized when the specific deformation of any material reaches its limit (ultimate) value.

Figure 2 illustrates the behavior of a rectangular cross-section of a strengthened concrete element, submitted to simple normal flexure. The rectangular stress diagram presented in section 17.2.2 of the Brazilian standard NBR 6118: 2014 [5] is adopted. The compression stresses in concrete are distributed in a rectangle of depth y, with peak stress equal to $\alpha \cdot f_{cd}$, according to Figure 2.

The value of y is defined as (equation 7):

$$y = \lambda \cdot x \tag{7}$$

In which, λ is given by equation (8) for $f_{ck} \leq 50$ MPa, and λ is given by equation 9 for $f_{ck} > 50$ MPa:

$$y = 0.8 \tag{8}$$

$$y = 0.8 - \left(f_{ck} - 50\right)/400 \tag{9}$$

The α parameter is defined according to the concrete class as follows: For concrete classes up to C50:

$$\alpha = 0.85 \tag{10}$$

For concrete classes from C50 up to C90:

$$\alpha = 0.85\left[1 - \left(f_{ck} - 50\right)/200\right] \tag{11}$$

2.3.2. LONGITUDINAL STRAINS

Considering Bernoulli's hypothesis, the deformation of any point within the section is proportional to its distance to the neutral axis, given as a function only of its location (y) and the curvature radius (ρ).

Knowing that the strengthening material bonded to the element has a minimal thickness, about a few millimeters, it can be considered that its specific longitudinal deformation (ε_r) is equal to that of the concrete fiber substrate to which it is glued. Thus, the deformation of the material, taken as constant along its thickness, can be obtained by equation 12:

$$\frac{1}{\rho} = \frac{\varepsilon_c}{x} = \frac{\varepsilon_s}{(d-x)} = \frac{\varepsilon_{RF}}{(h-x)} = \frac{\varepsilon_s'}{(d-x)} \tag{12}$$

When the applied loads, including the dead load, are not eliminated before the strengthening application, the material added to the section is not stressed by all these loads. In this way, the original structure is subjected to pre-existing stresses and has already suffered deformations before strengthening. Under these conditions, it is necessary to know these stresses to determine the actual stress acting on the strengthening. Thus, strengthening deformations can be obtained using equation 13:

$$\varepsilon_R = \left(\varepsilon_{RF} - \varepsilon_{RI}\right) \leq \varepsilon_U \tag{13}$$

in which ε_{RF} is the specific deformation of the concrete substrate when all loads are applied, and ε_{RI} is the specific deformation in the section at the moment of strengthening application.

2.3.3. FAILURE CRITERIA

Analyzing the failure criteria, the ultimate concrete deformations are defined in section 8.2.10.1 of the Brazilian standard [5], by ε_{c2} and ε_{cu}, as shown in equations 14 to 17:

For concretes classes up to C50:

$$\varepsilon_{c2} = 0.2\,\% \tag{14}$$

$$\varepsilon_{cu} = 0.35\,\% \tag{15}$$

For concretes classes from C50 up to C90:

$$\varepsilon_{c2} = 0.2\,\% + 0.0085\,\%\left(f_{ck} - 50\right)^{0.53} \tag{16}$$

$$\varepsilon_{cu} = 0.35\,\% + 3.5\,\%\left[\left(90 - f_{ck}\right)/100\right]^4 \tag{17}$$

Considering Hooke's law valid up to yield stress (f_y) of any steel element, it is assumed that the

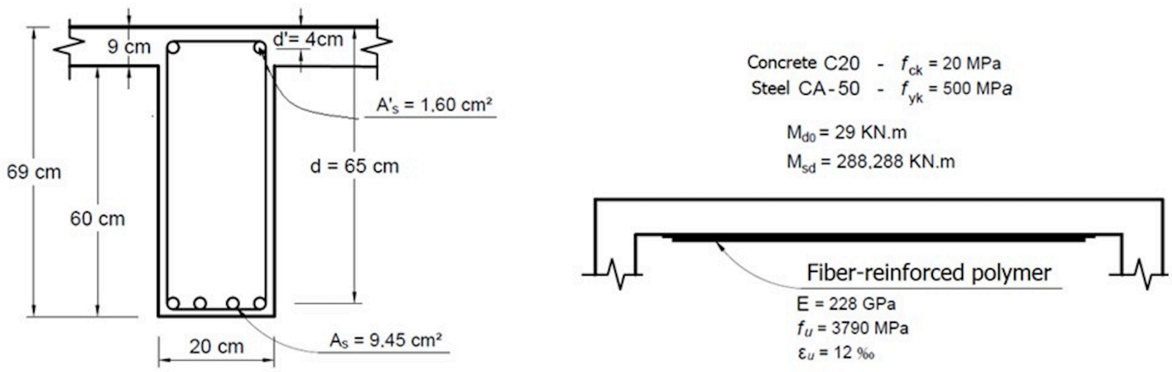

FIGURE 3. Strengthened beam scheme. Source: Adapted from Machado [2].

stresses are proportional to the deformations. In this way, the tensile resultant in the pre-existent steel reinforcement (R_s) and in the strengthening added to the element (R_R) can be done by equations equation 18 and 19.

$$R_s = A_s \cdot E_s \cdot \varepsilon_s \leq A_s \cdot f_{Sy} \tag{18}$$

$$R_R = A_R \cdot E_s \cdot \varepsilon_s \leq A_R \cdot f_{Ry} \tag{19}$$

It is important to note that, as the carbon fiber has high tensile strength, linear elastic behavior is expected for this type of material until failure of the critical section.

Finally, to prevent excessive plastic deformation in critical sections, the maximum allowable elongations are limited to = 1%.

2.3.4. Ductility criteria

In reinforced concrete structures, proper measures must be taken to ensure adequate ductility of all structural elements up to the ultimate limit state.

According to ACI 318-14 [7], if the tensile stress in the reinforcement is greater than ε_s = 0,5%, the section is characterized as ductile, presenting warnings about imminent rupture, such as the presence of large deformations and cracks. If the steel deformation is less than this value, a fragile rupture condition is expected, with few warnings of an imminent collapse.

Section 14.6.4.3 of the Brazilian standard NBR 6118: 2014 [5], in order to guarantee a ductile behavior for the slabs and beams, establishes that the depth of the neutral axis at the ultimate limit state must be limited to the following values: equation 20 for ≤ 50 MPa, and equation 21 for 50 MPa $< f_{ck} \leq 90$ MPa.

$$x \leq 0.45 \cdot d \tag{20}$$

$$x \leq 0.35 \cdot d \tag{21}$$

2.3.5. Strengthening equation

The equilibriums equation of strengthening supposes that dimensions of the cross-section, the reinforcement position, and the material properties are known. The procedure described below must consider the balance of the section, the compatibility of deformations, and its failure mode.

It was considered that the internal resisting bending moment must be at least equal to the external moment applied to the section. In this way, the equilibrium equation of moments can be carried out at the contact surface between concrete and the newly added strengthening. One of the variables is eliminated with this procedure. Therefore, the equilibrium of moments in a strengthened section is:

$$\begin{aligned} \sum M &= M_{sd} \\ M_{sd} &= R_c\,(h - x') + R_s\,(h - d) \\ &\quad + R'_s\,(h - d') \\ M_{sd} &= \alpha\, f_{cd}\, b_w\, y \left(h - \frac{y}{2}\right) + A_s\, f_s\,(h - d) \\ &\quad + A'_s\, f'_s\,(h - d') \end{aligned} \tag{22}$$

Knowing that the compressed concrete block is given as a function of equation 7, the position of the neutral axis can be obtained by as the positive root of equation 23, derived from equation 22 by algebraic manipulation:

$$\begin{aligned} &\left(-0.5\,\alpha\, f_{cd}\, b_w\, \lambda^2\right) x^2 + (\alpha\, f_{cd}\, b_w\, \lambda\, h)\, x \;\; + \\ &[A'_s\, f'_s\,(h - d') - A_s\, f_s\,(h - d) - M_{sd}] \;= 0 \end{aligned} \tag{23}$$

With the neutral axis position, equations 12 and 13 are used to identify the domain of ultimate limit state of the behavior of the section, and to determine the specific deformations along its height. The sectional stresses distribution from existing load action during the strengthening can be considered, taking

Variable	Iterative process [2]	Suggested model (Authors, 2021)
ε_{RI}	0.228 ‰	0.272 ‰
ε_{RF}	7.000 ‰	7.324 ‰
ε_{R}	6.772 ‰	7.052 ‰
L · N	0.228 cm	22.31 cm
f_R	1544.0 MPa	1607.9 MPa
Strengthening area	0.588 cm^2	0.563 cm^2

TABLE 1. Calculated carbon fiber strengthened area.

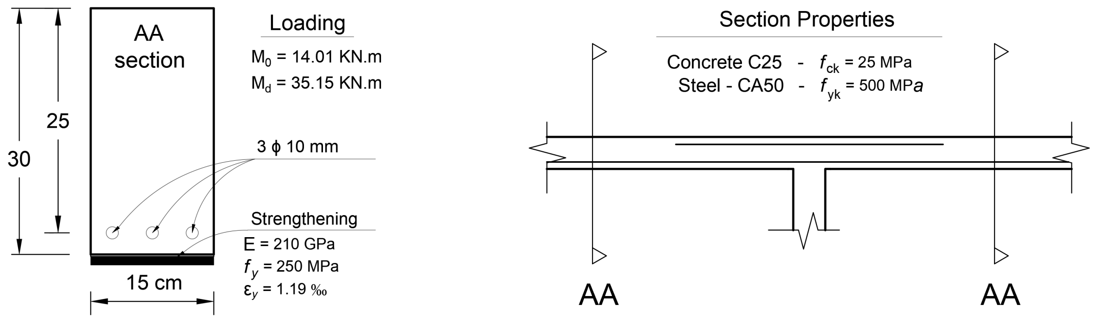

FIGURE 4. Representation of beam characteristics of the building.

Calculation model	Strengthening area	Difference
Suggested model	2.054 cm^2	-
J. Bresson	4.091 cm^2	99.17 %
Campagnolo	3.684 cm^2	79.36 %
Ziraba e Hussein	2.905 cm^2	41.43 %
Cánovas	2.286 cm^2	11.30 %

TABLE 2. Comparison of the calculated strengthening sheet area.

into account the cracked section properties, if necessary. Considering a cross-section submitted only to flexure forces, the resultant strengthening force is determined as presented in equation 24.

$$\begin{aligned} \sum F &= N_{sd} \\ N_{sd} &= R_c + R_s + R'_s + R_R \\ R_R &= N_{sd} - \alpha\, f_{cd}\, b_w\, \lambda\, x - A_s\, f_s - A'_s\, f'_s \end{aligned} \tag{24}$$

Once the resulting force and specific deformation have been determined, the strengthening area necessary to resist to the applied moment can be calculated using equation 19.

Finally, it is still necessary to assess whether the section mode of failure satisfies the ductility criteria set out in section 2.3.4. It is worth to mention that the proposed equation 24 also allows the direct application of the environmental reduction factors, as exposed in ACI 440 [8], to the strengthening resultant.

3. Results and discussions

A comparison between the design methods mentioned and the calculation model proposed in this paper was made. For this, some examples available in the literature and a study case were taken for. Analyzing example 4.1 of the Manual by Machado [2], a concrete beam was supposed to be strengthened through the association of carbon fiber sheets. All necessary data about the beam, the applied loads, and the material properties are shown in Figure 3.

The proposed calculation routine was compared with the iterative procedure used in Machado [2]. The design was carried out using the criteria of the Brazilian standard NBR 6118: 2014 [5], adopting the same parameters for the materials. The comparative results are shown in Table 1.

The actual strengthening provided had an effective area of 0.594 cm^2, composed of two carbon fiber layers, 0.165 mm thick and 18 cm width. It appears that the recommended procedure provides a value very close to the trial-and-error process for necessary carbon fiber strengthening area (less than 4.25%). This difference may be due to the tolerance criteria applied in the iterative process [2].

The second example is the case of a building, where the live load was foreseen to be increased. The additional strengthening was conceived through the association of metal sheets. Figure 4 shows the characteristics of one of the beams that needed to be strengthened.

Table 2 shows the section sizing of the steel plate obtained from the different methods.

It is essential to highlight that, although the proposed method presents similarities with that developed by Ziraba and Hussein - both consider the strengthening design at the ultimate limit state, their results are quite different. This fact is due, not only, but mainly because of criteria that govern ACI 318 [7] is different from those of NBR 6118 [5] and because of the consideration of pre-existing deformations considered in the proposed calculation.

4. Conclusions

This paper shows that it is possible to design fiber-reinforced polymer strengthening using a different approach from the trial and error method, frequently used in technical design. The proposed method leads to encouraging results with reasonable accuracy if compared with the classical ones. No iterative calculations are needed.

The method provides the depth of the neutral axis without evaluating the stress resultant of the internal reinforcement. Consequently, it is possible to consider its application for other thin materials bonded to the substrate, such as metal sheets.

There was a distinct difference in the results obtained with other literature models for sheet metal design. The authors attribute these divergences mainly to the difference between some calculation hypotheses adopted, such as the design based on ultimate limit state instead of the Allowable Stress Design methods, where material resistances are limited to its limit of elasticity. The proposed method also allows the usage of the Load and Resistance Factor Design, the consideration of pre-existing stresses, and the application of environmental reduction factors directly to the strengthening, which are not possible in some other methods.

Thus, the numerical analysis shown that the proposed method presents good efficiency. More exhaustive laboratory tests, experimental and/or numerical, may be foreseen to improve and calibrate the parameters of the method proposed herein.

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