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DOCTORAL THESIS STATEMENT

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DOCTORAL THESIS STATEMENT

NUMERICAL SIMULATIONS OF TURBINE BLADE FLUTTER

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Supervisor: doc. Ing. Jan Halama, Ph.D.

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Numerical Simulations of Turbine Blade Flutter

The doctoral thesis was produced during a full-time Ph.D. study in the Department of Technical Mathematics at the Faculty of Mechanical Engineering, Czech Technical University in Prague.

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doc. Ing. Jiří Fürst, Ph.D.
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Contents

Nomenclature

b	Blade span
C	Chord
$\tilde{C}_p$	Coeff. of instantaneous pressure fluctuation: $\tilde{C}_p(\boldsymbol{x},t) = \frac{p(\boldsymbol{x},t)-\overline{p}(\boldsymbol{x})}{\Delta\alpha(p_{01}-p_2)}$ for a pitching mode, $\tilde{C}_p(\boldsymbol{x},t) = \frac{p(\boldsymbol{x},t)-\overline{p}(\boldsymbol{x})}{h/C(p_{01}-p_2)}$ for a general mode
C_p	Pressure coeff.: $C_p = \frac{p-p_1}{p_{01}-p_1}$ for airfoils, $C_p = \frac{p-p_2}{p_{01}-p_2}$ for blade cascades
$\hat{C}_{p(1)}$	First harmonic of $\tilde{C}_p$
dS	Surface element
e_0	Total energy
$\boldsymbol{F}$	Flux
h	Max. displacement
k	Boundary-normal wavenumber
$\boldsymbol{l}_j$	j-th left eigenvector
m	Circumferential wavenumber
$\boldsymbol{n}$	Normal vector
p	Pressure
$\boldsymbol{q}$	Vector of primitive variables
$\boldsymbol{r}_j$	j-th right eigenvector
$\boldsymbol{s}$	Grid (ALE) velocity
t	Time
$\boldsymbol{u}$	Velocity vector
$\boldsymbol{W}$	Vector of conserved variables
W_{aero}	Work of aerodynamic forces
x, y	Coordinates
α	Angle of attack
Ξ	Aerodynamic damping coefficient
ρ	Density
σ	Interblade phase angle
ω	Angular frequency
Ω	Control volume
$_0$	Total
$_{1,2}$	Inlet, outlet
ALE	Arbitrary Lagrangian-Eulerian
AUSM	Advection upstream splitting method
BC	Boundary condition
CFD	Computational fluid dynamics
NRBC	Non-reflecting boundary condition
STCF	Standard configuration

1 Introduction

The design of a turbomachine needs to be executed with a careful consideration of aeroelastic effects in order to guarantee a long and safe operation. Arguably the greatest risk represent self-excited vibrations, known as flutter. Once initiated, their rapidly increasing magnitude threatens to induce a catastrophic failure within a very short time. The problem has been extensively studied and high-fidelity predictive tools have been introduced and refined over the years, but further research is still needed. This thesis focuses on the treatment of boundary conditions for the prevention of spurious wave reflections. The compact spatial arrangement of turbomachines causes that any perturbation formed at the inlet or outlet boundary impacts directly the near-blade flow solution. The issue is exacerbated by the recent trend of designing last-stage steam turbine rotors whose diameter is increased in pursuit of a higher power output. As a consequence, supersonic inflow conditions are encountered at higher spans. The upstream propagating bow shock, formed ahead of the blade leading edge, creates a particularly challenging environment for a reflection-free definition of inlet boundary conditions.

The present modelling approach is primarily based on the energy method with prescribed harmonic blade oscillations, but a fully coupled solution of fluid-structure interaction with two structural degrees of freedom is also provided. The solution of unsteady aerodynamics adopts Euler equations in two dimensions, cast in the arbitrary Lagrangian-Eulerian (ALE) formulation and discretised with a finite volume approach. The state-of-the-art Spectral Non-Reflecting Boundary Condition (NRBC) of Schlüß et al. (2016) is employed for inflow and outflow and compared with two other formulations. The numerical solution procedure is realised with an in-house solver written by the author in C++.

The computational model is first tested on the case of an oscillating isolated airfoil before proceeding to assessments of aeroelastic stability for three turbomachinery blade cascades. The test cases serve to validate the computational model and to analyse the performance of non-reflecting boundary conditions over a wide range of flow configurations, including a turbine blade cascade with a supersonic inflow.

2 Aeroelasticity in Turbomachinery

The concept of aeroelasticity in the modern sense was established by Collar (Collar, 1946). He defined it as an engineering discipline which studies the interaction between aerodynamic, elastic and inertial forces for a solid object submerged in a fluid flow. The present work is concerned with blade flutter, one of the most serious aeroelastic problems in turbomachinery. It is an instability of a structure surrounded by fluid flow, for which a small perturbation can trigger self-sustained vibrations with a fast growing amplitude. This phenomenon was first studied in the field of aeronautics, where a theory for wing flutter prediction was devised by Theodorsen (1935).

Blade Vibration Characteristics. Turbomachine blades bear a geometrical similarity with aircraft wings, but their flutter mechanisms are substantially different. They are characterised by a higher mass ratio and the aeroelastic behaviour of blades arranged on a wheel is influenced by a mechanical and an aerodynamic coupling. The former is realized by shrouds located at part-span or connecting the blade tips, either via an interlocking system or by welding the blades together. The aerodynamic coupling is formed as the motion of each blade influences the surrounding instantaneous flow field and affects directly the loads acting on its neighbours. Studies have indicated that the stability of an isolated blade and the coupling effects are equally important for flutter characteristics of a blade-row (Széchényi, 1985).

Computational Methods. The earliest attempts to solve aeroelastic problems employed severely simplified analytical theories (Lane, 1956; Whitehead, 1960). A significantly more realistic representation was achieved by transitioning to discrete numerical models, adopting at first time linearisation to reduce computational costs (Clark and Hall, 1999). A further improvement brought the nonlinear frequency domain methods (Ning and He, 1998), but only a time marching approach enables performing large-displacement aeroelastic computations with all types of nonlinear effects included. Aeroelastic analysis is typically performed by solving unsteady aerodynamics for precomputed modes of blade oscillation, in an approach referred to as the energy method. While this method is known to achieve satisfactory accuracy for conventional blades, a fully coupled technique may be needed for modern lightweight designs.

A discrete numerical solution requires the use of suitably formulated inflow and outflow boundary conditions to avoid spurious wave reflections. Most often, the issue is addressed either by adding a buffer layer to absorb the propagating waves (Hayder et al., 1999; Zhang et al., 2003), or by adopting a boundary condition designed to suppress the wave reflections. The latter strategy typically builds upon the theory of non-reflecting boundary conditions formulated by Giles (1988) for linearised Euler equations. Giles derived only an approximate method for unsteady flows, which spurred a further development of more accurate formulations. The Spectral NRBC, exact for linearised Euler equations, was introduced by Chassaing and Gerolymos (2007) and recently reimplemented by Schlüß et al. (2016) in a modification tailored to the needs of time-domain solvers.

3 Aims of the Thesis

The primary goal is to analyse the ability of non-reflecting boundary conditions to produce a disturbance free solution in complex flow conditions and to provide an accurate assessment of aeroelastic stability even on truncated domains. The subtasks defined for reaching the main objective are arranged in two groups:

Implement a numerical solver for the prediction of turbine blade flutter

- Devise a mesh motion strategy for flow solution on deforming domains
- Analyse techniques for gradient reconstruction and limiting
- Implement non-reflecting boundary conditions
- Validate the computational model

Investigate the performance of non-reflecting boundary conditions in flutter predictions

- Analyse reflection properties of the Spectral NRBC in nonlinear flows
- Quantify the impact of unsuppressed wave reflections on aeroelastic assessments
- Assess the sensitivity of flutter predictions to domain extent

4 Mathematical Model

Flow Model. The solution of unsteady aerodynamics adopts an inviscid compressible flow model, based on Euler equations in two dimensions. The system of partial differential equations is derived from conservation laws for mass, momentum and energy and it is cast in ALE formulation to facilitate solution on deforming domains (Smith, 1999):

$$\frac{\partial}{\partial t}\int_{\Omega(t)} \boldsymbol{W}\, d\Omega + \oint_{\partial\Omega(t)} \boldsymbol{F}^{ALE} dS = 0, \tag{1}$$

$$\boldsymbol{F}^{ALE} = \boldsymbol{F} - (\boldsymbol{s}\cdot\boldsymbol{n})\boldsymbol{W} = (\boldsymbol{u}-\boldsymbol{s})\cdot\boldsymbol{n}\begin{bmatrix}\rho\\ \rho\boldsymbol{u}\\ \rho e_0 + p\end{bmatrix} + \begin{bmatrix}0\\ p\boldsymbol{n}\\ p\boldsymbol{s}\cdot\boldsymbol{n}\end{bmatrix}. \tag{2}$$

A closure of the mathematical model is provided by the ideal gas law.

Boundary Conditions. The non-reflecting boundary conditions for inflow and outflow are based on the theory of Giles (1988). The boundary flowfield is reconstructed as a superposition of a mean state and perturbations, whose amplitude is assumed as sufficiently small to allow for a linearisation of the governing flow equations:

$$\frac{\partial\boldsymbol{q}}{\partial t} + A\frac{\partial\boldsymbol{q}}{\partial x} + B\frac{\partial\boldsymbol{q}}{\partial y} = 0, \tag{3}$$

where the matrices A, B are functions of the mean state only and the wave-like perturbations are considered in the form:

$$\boldsymbol{q} = \mathrm{Re}(\hat{\boldsymbol{q}}e^{\mathrm{i}(kx+my+\omega t)}). \tag{4}$$

One obtains an eigenvalue problem

$$(\omega A^{-1} + mA^{-1}B)\boldsymbol{r}_j = -k_j\boldsymbol{r}_j \quad \text{or} \quad \boldsymbol{l}_j(\omega A^{-1} + mA^{-1}B) = -k_j\boldsymbol{l}_j \tag{5}$$

with the right eigenvectors $\boldsymbol{r}_j$ representing waves that enter or leave the domain. The following boundary conditions are implemented in the solver for inflow and outflow:

- The *Spectral NRBC* (Schlüß et al., 2016) requires for any combination of m and ω, for each $\boldsymbol{l}_j$ corresponding to an incoming wave

$$\boldsymbol{l}_{(\omega,m)j}\cdot\hat{\boldsymbol{q}}_{(\omega,m)} = 0. \tag{6}$$

- The *Exact Steady NRBC* (Giles, 1988) considers only spatial perturbations and imposes the condition (6) only for modes with $\omega = 0$.
- The *Simple Turbomachinery BC* is a trivial formulation with no relation to Giles' theory of NRBC. It is derived by assuming an isentropic state change between the inflow stagnation conditions and the inlet boundary.

Aeroelastic Analysis. The assessment of aeroelastic stability relies primarily on the energy method. Predefined harmonic oscillations are imposed upon all blades in the cascade, with a constant interblade phase angle (IBPA) between each two neighbours. The energy transfer between fluid and structure is quantified with the aerodynamic damping coefficient, defined for torsion and for a general coupled mode respectively as

$$\Xi_{torsion} = \frac{-W_{aero}}{\pi b(\Delta\alpha C)^2(p_{01} - p_2)}, \qquad \Xi_{coupled} = \frac{-W_{aero}}{\pi b h^2(p_{01} - p_2)}. \qquad (7)$$

The lowest value of Ξ over all admissible IBPAs determines the overall aeroelastic stability.

As an alternative, a fully coupled fluid-structure interaction model for elastically mounted rigid bodies with two degrees of freedom is also implemented in the solver.

5 Numerical Solution

Domain Discretisation and Grid Motion. The solver features a functionality of importing and storing unstructured triangular, quadrilateral and mixed-element grids in the CGNS format. A computationally efficient procedure for grid motion is implemented, exploiting the representation of oscillating blades as rigid bodies. Their motion is interpolated onto the grid vertices, using coefficients calculated during initialization.

Unsteady Aerodynamics. The numerical solution uses a cell-centered finite volume discretisation of Euler equations in ALE formulation. The AUSM^{+}-up scheme (Liou, 2006) is selected for the approximation of inviscid fluxes thanks to its ability to provide a sharp shockwave resolution and applicability to all speed regimes. Second order spatial accuracy is achieved

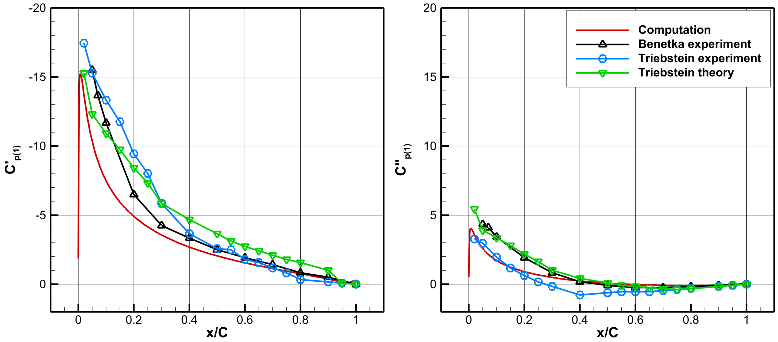

Figure 1: Distribution of the real ($C'_{p(1)}$, left) and imaginary ($C''_{p(1)}$, right) part of the 1^{st} unsteady pressure harmonic on NACA 0012 surface.

by performing gradient reconstruction with a weighted least squares method, followed by the application of a multidimensional limiter.

The flow equations are integrated in time with a second-order accurate dual-time implicit scheme (Eliasson et al., 1996), compliant with the Geometric Conservation Law. The techniques of local time-stepping and implicit residual smoothing are applied in the inner cycle to accelerate convergence.

6 Application and Analysis

NACA 0012 Airfoil. The computational model is first validated on the case of an isolated airfoil NACA 0012 submerged in a subsonic freestream flow, before proceeding to the more complex problem of blade cascades. First, a steady-state solution is shown to achieve a close agreement with experimental data in terms of the airfoil surface pressure distribution. Next, harmonic pitching oscillations are simulated to validate the solution of unsteady aerodynamics. The computed airfoil surface distribution of the first unsteady pressure harmonic is consistent with measurements and theoretical predictions (Fig. 1).

The study is concluded by simulating unsteady fluid-structure interaction for an airfoil with two degrees of freedom. As the freestream velocity is increases, the system undergoes a transition from damped oscillations through torsional divergence to flutter instability. The trends as well as the onset velocities are consistent with numerical results of other authors.

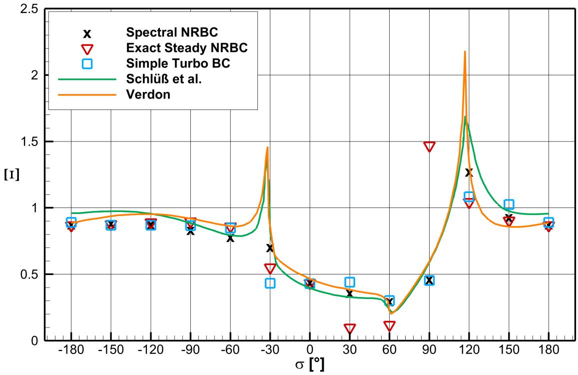

Figure 2: Aerodynamic damping curve of STCF10.

Tenth Standard Configuration. The subsonic compressor blade cascade STCF10 represents a challenging test case for the formulation of boundary conditions due to the sensitivity of unsteady flow solution to artificial wave reflections.

Steady-state predictions exhibited a close match with experimental data. Due to the moderate circumferential gradients and absence of shockwaves in the flow-field, nearly identical results were obtained with the Exact Steady NRBC and with the Simple Turbomachinery BC.

An analysis of aeroelastic stability was performed for a torsional mode of oscillation. The aerodynamic damping curve obtained with the Spectral NRBC was in a close agreement with numerical results of other authors, including the prediction of peaks near acoustic resonance conditions (Fig. 7). Here, the Exact Steady NRBC and the Simple Turbomachinery BC exhibited a considerable discrepancy, whereas they matched reasonably well in the cut-off range of inlet and outlet pressure waves, found between $\sigma \approx -180^\circ$ and $\sigma \approx -60^\circ$. Overall, they deviated from the Spectral NRBC predictions by up to $215\,\%$ and $39\,\%$ respectively.

The reflection properties of each boundary condition are well demonstrated by plotting contours of instantaneous unsteady pressure fluctuations (Fig. 3). While the Spectral NRBC handles the upstream cut-on as well as the downstream cut-off pressure waves without spurious reflections, the other two boundary conditions generate strong perturbations in the pressure field.

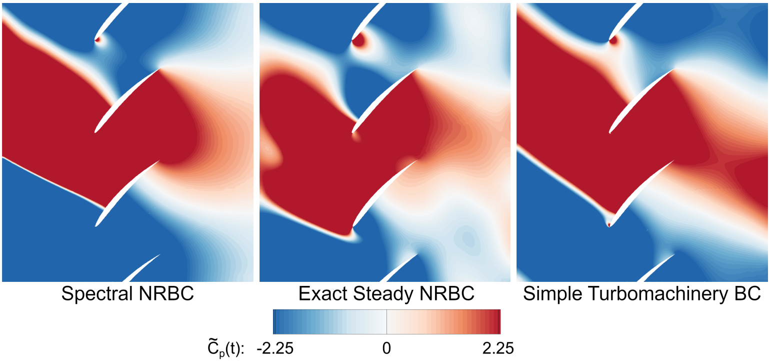
Spectral NRBC Exact Steady NRBC Simple Turbomachinery BC

Figure 3: Contours of the STCF10 instantaneous unsteady pressure fluctuations for $\sigma = 120°$.

Fourth Standard Configuration. The transonic turbine blade cascade STCF4 provides a suitable test case to validate the numerical model thanks to a public availability of experimental data. Results of steady-state computations matched well the measurements, including the shock impingement position on the blade surface. In a study comparing a set of six flux limiters, the van Albada-van Leer variant of the technique of Delis and Nikolos (2014) provided the best performance with a sharp shockwave resolution and low final residuals (Fig. 4).

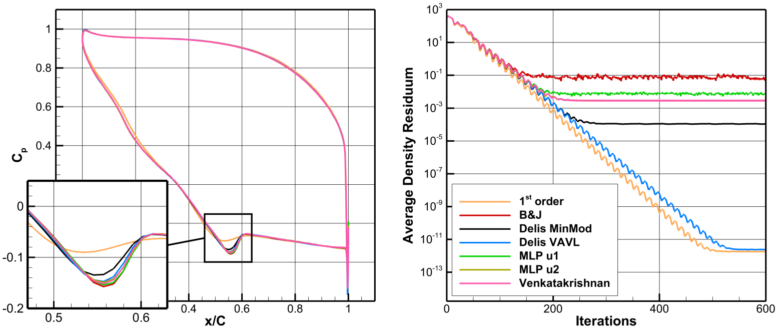

Figure 4: Pressure coefficient distributions (left) and convergence histories (right) obtained with different limiters on STCF4.

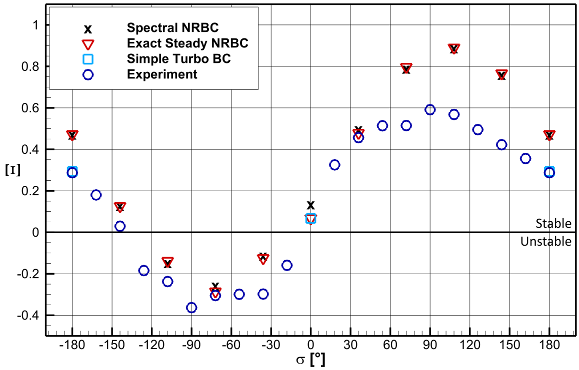

Figure 5: Aerodynamic damping curve of STCF4.

The steady-state solution obtained with the Exact Steady NRBC was nearly free of spurious perturbations, whereas the Simple Turbomachinery BC produced a strong reflection of the downstream propagating shockwave on the outlet boundary. Moreover, it exhibited a high sensitivity to the domain extent.

Simulations of bending mode blade oscillations with the Spectral NRBC were consistent with the measurements in terms of the aerodynamic damping curve shape and stability ranges (Fig. 5). Quantitatively, the numerical results were characterised by a positive offset in the damping coefficient, amounting up to $\Delta\Xi \approx 0.3$ for some IBPAs. The discrepancy can be partially accounted to the incorrect prediction of unsteady pressure phase downstream of shockwave impingement on the blade surface, due to the lack of viscous flow modelling. However, it also needs to be considered that the experimental damping coefficient was obtained with a large uncertainty due to a low number of pressure probes. The Exact Steady NRBC performed consistently with the Spectral NRBC, whereas the Simple Turbomachinery BC failed to converge in most cases.

The unsteady flow solution obtained with the two NRBCs exhibited negligible sensitivity to the upstream domain extent. The influence of the outlet boundary position was more notable, although still relatively minor in producing a maximum variation of Ξ by $9.7\,\%$ and $15.7\,\%$ with the Spectral and Exact Steady NRBCs respectively.

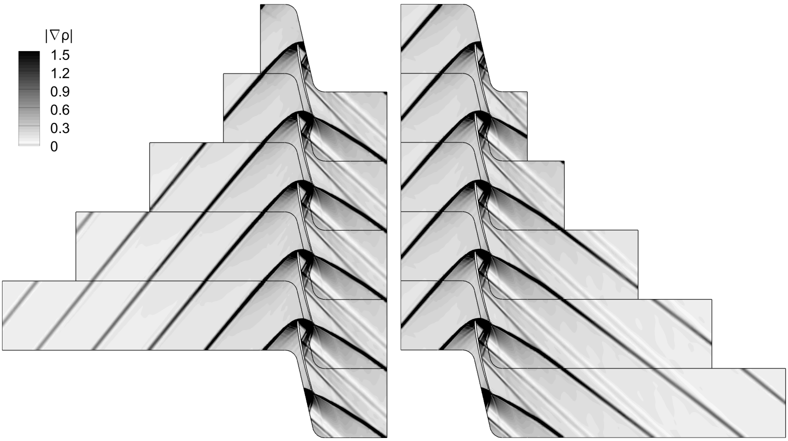

Figure 6: Contours of M8 density gradient with different upstream (left) and downstream (right) domain extents.

M8 Turbine Cascade. The blade cascade M8 is a $93\,\%$ span section of a last-stage steam turbine rotor with supersonic inflow conditions. A steady state solution using the Exact Steady NRBC exhibited low sensitivity to domain extent (Fig. 6), whereas solver divergence was encountered when employing the Simple Turbomachinery BC.

The aeroelastic analysis considered a coupled mode of blade oscillation and showed the cascade to be unstable in the whole IBPA range (Fig. 7). The Exact Steady NRBC performed consistently with the Spectral NRBC for some IBPAs, but deviated in the vicinity of acoustic resonance conditions and in the case of in-phase vibrations. The maximum discrepancy in the aerodynamic damping coefficient amounted to $38\,\%$.

The suboptimal reflection properties of the Exact Steady NRBC in application to unsteady flows resulted in a high sensitivity of the solution to domain extent. The aerodynamic damping predictions thus varied in function of the boundary position by over $300\,\%$. In contrast, the Spectral NRBC produced maximum variations of $12.9\,\%$ and $5.7\,\%$ with the inlet and outlet boundary positions respectively.

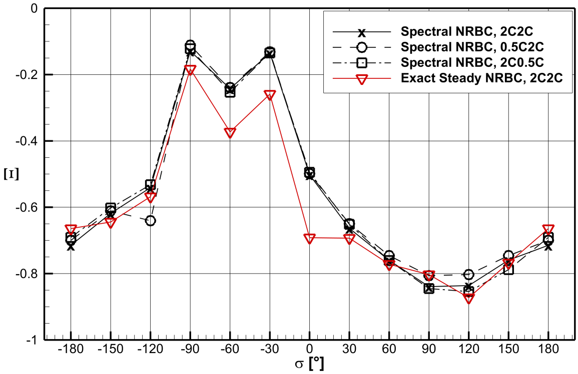

Figure 7: Aerodynamic damping curve of M8 using different domain sizes and boundary conditions.

7 Discussion and Conclusions

Achieved Results. A finite volume CFD solver was developed for the computation of unsteady flows on oscillating turbomachinery blade cascades. The flow model is based on Euler equations in ALE formulation and uses the AUSM^{+}-up scheme for the approximation of inviscid fluxes. Gradients are approximated with a weighted least squares method with a vertex-based stencil, for which the inverse distance weighting exponent set to $k = 1.0$ achieved an optimal combination of accuracy and convergence behaviour. Based on a comparison of six gradient limiting strategies, the face-based technique of Delis and Nikolos (2014) in combination with the van Albada-van Leer limiter was concluded to provide the best performance. It produced a sharp shockwave resolution and low final residuals.

An extensive analysis of non-reflecting boundary conditions in application to subsonic, transonic and supersonic blade cascades was performed. The Spectral NRBC was shown to be highly effective in preventing the generation of spurious wave reflections. An accurate prediction of aerodynamic damping can thus be obtained even in the presence of complex flow fields, such as on a blade cascade with a supersonic inflow. Although the wave reflections are not always completely suppressed on account of the underlying linearised model, their magnitude is low enough to have only a minor effect on the near-blade flow field. The solution exhibits only a very mild sensitivity to the inflow and outflow positions, which enables employing highly truncated domains without compromising the accuracy of the prediction. In spite of employing a phase-lagged updating procedure of Fourier coefficients, the Spectral NRBC did not exhibit a deteriorated convergence behaviour in comparison with the other two tested boundary conditions.

The study further demonstrated that using a boundary condition that fails to suppress the spurious wave reflections, such as the Exact Steady NRBC or the Simple Turbomachinery BC, can result in a fundamentally incorrect aeroelastic assessment.

Future Work Recommendations. The outcomes of this thesis can serve as a basis for future research, targeted primarily to improve the fidelity of the unsteady flow solver and to enhance the applicability of non-reflecting boundary conditions.

The present computational model describes inviscid two-dimensional flows. The inclusion of turbulence modelling could further improve the accuracy of aeroelastic assessments and an extension to three dimensions would open up the possibility to study a wide range of intriguing aeroelastic problems.

Prescribed boundary condition values need to be slightly altered when changing the domain extent, in order to maintain a constant operating point. The procedure for determining the corrected values could be further improved to reduce the variation of operating point with IBPA.

Although the Spectral NRBC exhibits good reflection properties even in strongly nonlinear unsteady flows, it cannot suppress the spurious wave reflections completely due to the linearised formulation. The magnitude of perturbations could be further reduced with the development of a higher order approximation.

The application of the Spectral NRBC is limited to flows periodic in both space and time. The construction of an effective non-reflecting boundary condition formulation for aperiodic flows remains an open challenge.

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Summary

This thesis deals with a numerical prediction of flutter in turbomachinery blade cascades and focuses primarily on the unsteady solution of flow around oscillating blades. The mathematical model is based on Euler equations in two dimensions, cast in the Arbitrary Lagrangian-Eulerian formulation to account for domain deformation in time. A particular emphasis is placed on the implementation of non-reflecting boundary conditions (NRBC), as the assessment of aeroelastic stability is highly sensitive to spurious wave reflections emanating from the inflow and outflow boundaries. Primarily, the energy method with prescribed harmonic oscillations of blades is considered, but a fully coupled model for elastically mounted bodies with two degrees of freedom is also implemented. The numerical solution procedure is realised with an in-house solver, written by the author in C++.

The computational model is employed to assess aeroelastic stability of an isolated airfoil NACA 0012 and of three blade cascades with subsonic (STCF10), transonic (STCF4) and supersonic (M8) flow conditions. An extensive validation is provided by comparing the results with experimental data and with numerical predictions of other authors.

The numerical solution adopts a gradient reconstruction and limiting procedure to increase the order of spatial accuracy. A set of six gradient limiting techniques is evaluated regarding the achieved resolution of discontinuities and convergence behaviour.

The primary objective is to analyse the ability of boundary conditions to yield unsteady flow solution without spurious wave reflections. The state-of-the art Spectral NRBC is compared with the Exact Steady NRBC, considering spatial modes only, and with the Simple Turbomachinery, lacking any non-reflecting treatment. The Spectral NRBC is shown to be highly effective in preventing the formation of spurious wave reflections and to exhibit low sensitivity of the solution to domain extent. In contrast, using a boundary condition with inferior reflection properties can result in a fundamentally incorrect aeroelastic assessment.

Keywords: Turbine blade flutter, Non-reflecting boundary conditions, Fluid-structure interaction, Arbitrary Lagrangian-Eulerian method, Computational fluid dynamics, Finite volume method

Resumé

Tato práce se zabývá numerickou predikcí samobuzeného kmitání (flutteru) lopatkových mříží turbostrojů a zaměřuje se především na řešení nestacionárního proudění okolo kmitajících lopatek. Výpočet je založen na Eulerových rovnicích ve dvou rozměrech, převedených do Arbitrary Lagrangian-Eulerian formulace pro zohlednění deformace oblasti v čase. Zvláštní pozornost je věnována implementaci bezodrazových okrajových podmínek, jelikož vyhodnocení aeroelastické stability je vysoce citlivé na rušivé odrazy vln od vstupní a výstupní hranice oblasti. Primárně je uvažována energetická metoda s předepsaným harmonickým pohybem lopatek, avšak současně je implementována i fully-coupled metoda pro kmitání elasticky uložených těles se dvěma stupni volnosti. Výpočty jsou provedeny ve vlastním řešiči, naprogramovaném autorem v jazyce C++.

Výpočetní model je použit pro posouzení aeroelastické stability osamoceného leteckého profilu NACA 0012 a tří lopatkových mříží se subsonickým (STCF10), transonickým (STCF4) a supersonickým (M8) režimem. Na základě srovnání výsledků s experimentálními daty a s numerickými predikcemi jiných autorů je provedena rozsáhlá validace řešiče.

Numerické řešení používá rekonstrukci gradientů s následným omezením nevazkých toků pro zvýšení řádu přesnosti v prostoru. Šest metod pro omezení toků je posouzeno na základě dosažené ostrosti zachycených nespojitostí a konvergence řešení.

Hlavním cílem práce je analýza schopnosti okrajových podmínek potlačit rušivé odrazy vln. Moderní spektrální bezodrazová okrajová podmínka (Spectral NRBC) je porovnána s Exact Steady NRBC, která uvažuje pouze prostorové módy vln, a se Simple Turbomachinery BC, v jejíž formulaci není zabránění odrazu vln zohledněno. Je ukázáno, že spektrální okrajová podmínka účinně zabraňuje rušivým odrazům vln a dosahuje nízké citlivosti řešení na rozsah oblasti. Oproti tomu použití okrajové podmínky s horšími bezodrazovými vlastnostmi může vést ke zcela nesprávnému vyhodnocení aeroelastické stability.

Klíčová slova: Flutter turbínových lopatek, Bezodrazové okrajové podmínky, Interakce proudění s elastickým tělesem, Metoda arbitrary Lagrange-Euler, Výpočetní mechanika tekutin, Metoda konečných objemů