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Assignment of bachelor’s thesis

Title:	Hydraulic Erosion
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Study program:	Informatics
Branch / specialization:	Web and Software Engineering, specialization Computer Graphics
Department:	Department of Software Engineering
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Instructions

Hydraulic erosion is a frequently occurring, natural phenomenon. Erosion simulation is essential for realistic terrain modification in computer graphics. The aim of this paper is to create a simulation and visualization of hydraulic erosion based on the presented articles.
1) Perform a search of articles on the topic of hydraulic erosion.
2) Focus on the representation of material using a voxel grid and fluid as Smoothed particle hydrodynamics, analyze the possibilities of this combination for your visualization.
3) Design and implement a prototype simulation and visualization of hydraulic erosion.
4) Use the C++ language and parallelism.
5) Test the prototype appropriately.

Electronically approved by Ing. Radek Richtr, Ph.D. on 23 June 2022 in Prague.

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Bachelor's thesis

Hydraulic erosion

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June 23, 2022

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Declaration

In Prague on June 23, 2022 . . . .

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Abstrakt

Tato práce se zaměřuje na studium a implementaci simulace hydraulické eroze v počítačové grafice. Výsledkem této práce je plně 3D aplikace, která simuluje a vizualizuje dopady způsobené kapalinou na terén. Pro simulaci tekutiny je použit Lagrangeův přístup nazvaný Smoothed Particle Hydrodynamics(SPH). Implementace využívá vícevláknové možnosti procesoru, aby se zvýšil výkon při výpočtech simulace tekutin.

Klíčová slova eroze, simulace tekutin, SPH, částice, řídký objem, Navier-Stokesovy rovnice, grafická aplikace, procedurální generování

Abstract

The current thesis focuses on the study and implementation of simulation of hydraulic erosion in computer graphics. The result of this work is a fully 3D application that simulates and visualizes the impact caused by fluid over terrain. For fluid simulation a Lagrangian approach named Smoothed Particle Hydrodynamics(SPH) is used. The implementation leverages the CPU's multithreading capabilities in order to boost the performance in fluid simulation calculations.

Keywords erosion, fluid simulation, SPH, particles, sparse volume, Navier-Stokes equations, graphical application, procedural generation

Contents

List of Figures

List of Tables

Introduction

Simulation of hydraulic erosion along with fluid simulation has been one of the hottest and actively researched topics in computer graphics over the course of the past decade. Hydraulic erosion is one of the phenomena that can be seen very often in nature. This phenomenon produces the most influential results when it comes to changes and deformations applied over terrain, which makes it a topic of particular interest in procedural generation.

The demand for such simulation may exist in the domains like engineering, scientific visualization, video game development, visual effects in film and television. While the first two i.e. engineering and scientific fields usually require a considerable degree of fidelity in order to produce practical results video games and films, on the other hand, are primarily focused on visually plausible outputs and thus tolerate the simulation to be approximated to look sufficiently realistic.

My motivation for choosing this topic is to develop and deepen my knowledge in the field of procedural generation as well as particle systems with their further application and integration in such sophisticated software systems as game engines and systems alike.

The theoretical part of this work describes the theory behind the Lagrangian approach of fluid simulation named Smoothed Particle Hydrodynamics(SPH) and the theory behind erosion and deposition of material.

The design part describes functional and non-functional requirements, use cases, the application structure and demonstrates the application control flow.

The implementation part is dedicated to the description of the implementation of fluid and erosion models from the theoretical part as well as generation of the terrain and how this is all visualized in a 3D scene using OpenGL.

After the implementation we will take a look at the results that the application produces.

This work is then concluded with tests and future work chapters.

Goals

The primary goal of the current thesis is to study, how hydraulic erosion can be simulated in computer graphics field, and implement a fully 3D working prototype that will run this simulation and visualize its results.

In order to simulate the erosion itself, the simulation of fluid must be done in the first place. The fluid is then poured onto the 3D terrain, where its particles interact with the boundary particles of the terrain located on its triangles. Pouring a small number of fluid particles will not give the most tangible results, therefore the preference is given to big volumes of water. To increase the efficiency of fluid simulation, its calculation will be parallelized.

It is important to note that the goal of this thesis is not to produce a fidelitous simulation suitable for engineering or scientific employment, but rather to achieve visually satisfactory results by using the given approximated models.

Overall the goals of this work can be broken down as follows:

1. Study SPH fluid simulation model and erosion simulation model
2. Implement simulation of the fluid in parallel fashion
3. Make collisions between fluid and a terrain generated from a heightmap
4. Implement the erosion model
5. Visualize the results by using OpenGL

Chapter 1

Theory and Background

In this chapter, we will study first, how fluids can be simulated using Lagrangian approach and then we will study, how the erosion is simulated leveraging this approach. In Section 1.1 we will familiarize with the famous Navier-Stokes equations. In Section 1.2 we will describe the Smoothed particle hydrodynamics solver and in Section 1.4 we will introduce, how these concepts can be used to simulate sparse volumes. In Section 1.5 we will examine the erosion computational model and how this model is coupled with SPH particles to achieve the material advection and deposition.

1.1 Navier-Stokes Equations

The state of a fluid in a given time can be determined by its physical quantities. The Navier-Stokes equations (NSE) are the equations that describe viscous fluids by taking into account those quantities. Mathematically they express conservation of momentum and conservation of mass for Newtonian fluids. The classical formulation for incompressible fluids is as follows:

$$\rho(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \mu \nabla \cdot (\nabla \mathbf{u}) + \mathbf{f}, \tag{1.1}$$

$$\nabla \cdot \mathbf{u} = 0, \tag{1.2}$$

where p is the pressure field, μ is the viscosity field, ρ is the density field, $\mathbf{u}$ is the velocity field and $\mathbf{f}$ is the sum of external forces acting on the fluid e.g. gravity.

Basically, this equation is the application of the Newton's second law for fluid motion. The right hand side describes all the forces that are applied on the fluid. The left part represents the product of mass and acceleration.

Equation (1.2) describes the conservation of mass for incompressible fluids.

1.2 Smoothed Particle Hydrodynamics

Smoothed particle hydrodynamics (SPH) is a computational method that originates from astrophysics. It is used in multiple research fields for simulating the mechanics of continuous systems. SPH is widely popular in simulation of meshfree Lagrangian fluid flows. We will be going through the Lagrangian approach in greater details in section 1.4. At its core SPH is an interpolation method to approximate values and derivatives of continuous field quantities by using discrete sample points. The sample points are identified as smoothed particles that carry concrete entities, e.g. mass, position, velocity, etc., but particles can also carry estimated physical field quantities dependent of the problem, e.g. mass-density, temperature, pressure, etc [1]. The SPH quantities are macroscopic and obtained as weighted averages from the adjacent particles [3].

Since it is an interpolation method any continuous field that we want to calculate can be represented as a quantity function A(r) and interpolated using integral. The integral interpolant is then defined over the space Ω by

$$A_I(\mathbf{r}) = \int_\Omega A(\mathbf{r}')W(\mathbf{r}-\mathbf{r}', h)\,d\mathbf{r}'. \tag{1.3}$$

In the above formula $\mathbf{r}$ is a point from Ω for which the quantity function A is being interpolated, W is smoothing kernel and h is the smoothing length, which controls the smoothness or roughness of the kernel. The numerical equivalent of the above integral is done by approximating it with summation:

$$A_S(\mathbf{r}) = \sum_j A_j \frac{m_j}{\rho_j} W(\mathbf{r}-\mathbf{r}_j, h), \tag{1.4}$$

where j iterates over all of the neighbouring particles, m_j is mass of particle j, ρ_j is its density and A_j is any quantity field of A at r_j that is being approximated. Equation (1.4) is the basis of the SPH method for interpolati any continuous quantities.

In NSE some fields contain first and second derivatives. In order to obtain a differentiable interpolant of (1.4) we only need to differentiate its kernel function. Thus, the first and second derivatives of (1.4) will be:

Gradient:

$$\nabla A_S(\mathbf{r}) = \sum_j A_j \frac{m_j}{\rho_j} \nabla W(\mathbf{r}-\mathbf{r}_j, h), \tag{1.5}$$

Laplacian:

$$\nabla^2 A_S(\mathbf{r}) = \sum_j A_j \frac{m_j}{\rho_j} \nabla^2 W(\mathbf{r}-\mathbf{r}_j, h), \tag{1.6}$$

1.3 Smoothing Kernels

The kernels of SPH interpolants have pivotal impact on how different quantities in NSE are calculated, which will influence the overall behavior of the fluid. Therefore, for each individual quantity field a suitable kernel function must be chosen. Because the interpolation occurs by iterating over the values of adjacent particles our kernel functions must meet a bunch of limitations.

As suggested in [4] suitable kernels require to have two properties:

$$\int_{\Omega} W(\mathbf{r}, h)\, dr = 1 \tag{1.7}$$

and

$$\lim_{h \to 0} W(\mathbf{r}, h) = \delta(\mathbf{r}), \tag{1.8}$$

where δ is Derac's delta function

$$\delta(\mathbf{r}) = \begin{cases} \infty, & \|\mathbf{r}\| = 0 \\ 0, & \text{otherwise} \end{cases} \tag{1.9}$$

The above conditions will not be enough for the SPH method additional requirements should also be fulfilled to ensure stable behavior:

the kernel must be positive

$$W(\mathbf{r}, h) \geq 0, \tag{1.10}$$

the kernel must be symmetric

$$W(\mathbf{r}, h) = W(-\mathbf{r}, h), \tag{1.11}$$

the kernel must ensure that any interactions outside of the smoothing radius h are omitted and not calculated

$$W(\mathbf{r}, h) = 0, \|r\| > h. \tag{1.12}$$

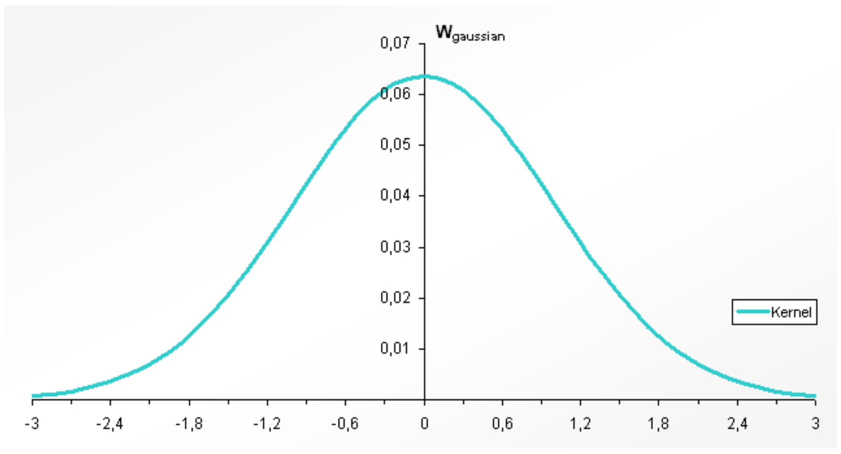


Figure 1.1: the isotropic Gaussian kernel. Taken from [1].

When there's a new interpolation of an SPH equation to be found then it is always best to assume the kernel as Gaussian [1]. The isotropic Gaussian kernel Figure 1.1 in n is given by

$$W_{gaussian}(\mathbf{r}, h) = \frac{1}{(2\pi h^2)^{\frac{3}{2}}} e^{-(\|\mathbf{r}\|^2/2h^2)}, h > 0, \tag{1.13}$$

Despite the fact that this kernel has good mathematical properties it is not constrained by (1.13), i.e. it calculates contribution of all particles rather than only the ones within the smooth radius, which makes it not the best option in practical implementation, this is why different kernels will be used. The examples of those kernels are listed below.

Standard Kernel

The standard kernel (Figure 1.2) is used for all the calculations by default unless specified differently. It has similar bell curve as the Gaussian kernel and it fulfills the constrain of (1.12). Its computation is faster, because it doesn't compute neither exponential function nor square root. The formulation of the standard kernel is

$$W_{default}(\mathbf{r}, h) = \frac{315}{64\pi h^9} \begin{cases} (h^2 - \|\mathbf{r}\|^2)^3 & 0 \leq \|\mathbf{r}\| \leq h \\ 0 & \|\mathbf{r}\| > h, \end{cases} \tag{1.14}$$

gradient

$$\nabla W_{default}(\mathbf{r}, h) = -\frac{945}{32\pi h^9} \mathbf{r}(h^2 - \|\mathbf{r}\|^2)^2, \tag{1.15}$$

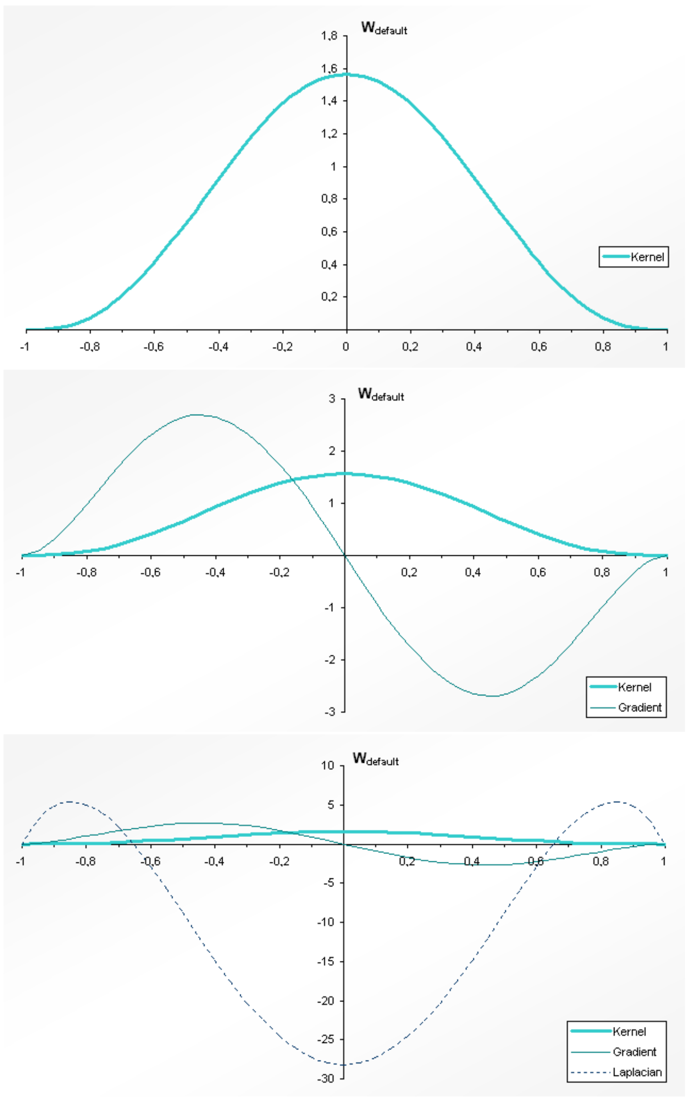


Figure 1.2: the standard kernel with gradient and Laplacian. Taken from [1].

Laplacian

$$\nabla^2 W_{default}(\mathbf{r}, h) = -\frac{945}{32\pi h^9}(h^2 - \|\mathbf{r}\|^2)(3h^2 - 7\|\mathbf{r}\|^2). \tag{1.16}$$

Figure 1.2 depicts the standard kernel, its gradient and Laplacian for smoothing radius h = 1.

Pressure Kernel

The pressure term in NSE requires the computation of derivation. We can see in Figure 1.2 that derivation for $r \to 0$ is 0. Due to that fact particles will build clusters. This makes the standard kernel unsuitable in the evaluation of the pressure force. According to [5] a good option for the pressure term will the spiky kernel

$$W_{pressure}(\mathbf{r}, h) = \frac{15}{\pi h^6} \begin{cases} (h - \|\mathbf{r}\|)^3 & 0 \leq \mathbf{r} \leq h \\ 0 & \|\mathbf{r}\| > h, \end{cases} \tag{1.17}$$

gradient

$$\begin{aligned} \nabla W_{pressure}(\mathbf{r}, h) &= -\frac{45}{\pi h^6} \frac{\mathbf{r}}{\|\mathbf{r}\|} (h - \|\mathbf{r}\|)^2, \\ \lim_{r \to 0^-} \nabla W_{pressure}(r, h) = \frac{45}{\pi h^6}, \lim_{r \to 0^+} \nabla W_{pressure}(r, h) &= -\frac{45}{\pi h^6}, \end{aligned} \tag{1.18}$$

Laplacian

$$\begin{aligned} \nabla^2 W_{pressure}(\mathbf{r}, h) &= -\frac{90}{\pi h^6} \frac{1}{\|\mathbf{r}\|} (h - \|\mathbf{r}\|)(h - 2\|\mathbf{r}\|), \\ \lim_{r \to 0} \nabla^2 W_{pressure}(r, h) &= -\infty \end{aligned} \tag{1.19}$$

Figure 1.3 depicts the pressure kernel for smoothing radius h = 1.

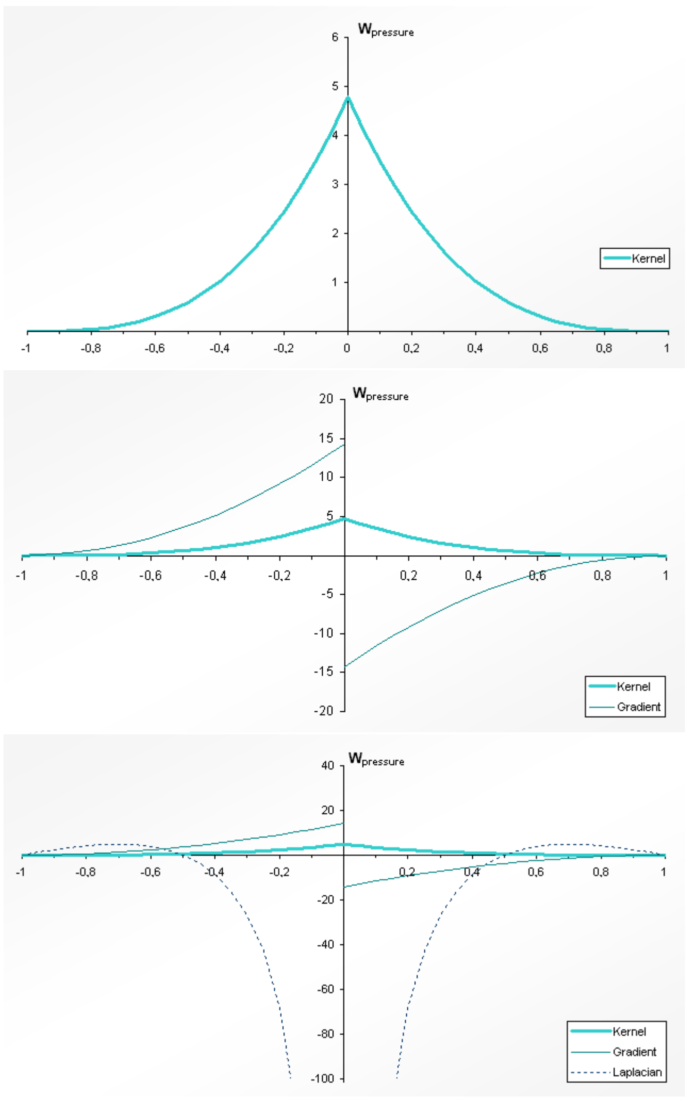


Figure 1.3: the pressure kernel with gradient and Laplacian. Taken from [1].

Viscosity Kernel

To calculate the viscosity term we need to evaluate its Laplacian, which implies that we will have to evaluate the Laplacian of its kernel. In order to maintain stability of the system the kernel has to be positive at any given time. As can be seen from the diagrams of two previous kernels they do not secure the Laplacian to be positive and thus cannot be used for the viscosity term. This is why we will need a different kernel. As proposed by [5] the kernel for viscosity will be

$$W_{viscosity}(\mathbf{r},h) = \frac{15}{2\pi h^3}\begin{cases} -\frac{\|\mathbf{r}\|^3}{2h^3} + \frac{\|\mathbf{r}\|^2}{h^2} + \frac{h}{2\|\mathbf{r}\|} - 1 & 0 < \|\mathbf{r}\| \leq h \\ 0 & \|\mathbf{r}\| > h, \end{cases} \quad (1.20)$$
$$\lim_{r \to 0} W_{viscosity}(r,h) = \infty,$$

gradient

$$\nabla W_{viscosity}(\mathbf{r},h) = \frac{15}{2\pi h^3}\mathbf{r}(-\frac{3\|\mathbf{r}\|}{2h^3} + \frac{2}{h^2} - \frac{h}{2\|\mathbf{r}\|^3}),$$
$$\lim_{r \to 0^-} \nabla W_{viscosity}(r,h) = +\infty, \lim_{r \to 0^+} \nabla W_{viscosity}(r,h) = -\infty, \quad (1.21)$$

Laplacian

$$\nabla^2 W_{viscosity}(\mathbf{r},h) = \frac{45}{\pi h^6}(h - \|\mathbf{r}\|), \quad (1.22)$$

Figure 1.4 depicts the viscosity kernel for smoothing radius h = 1.

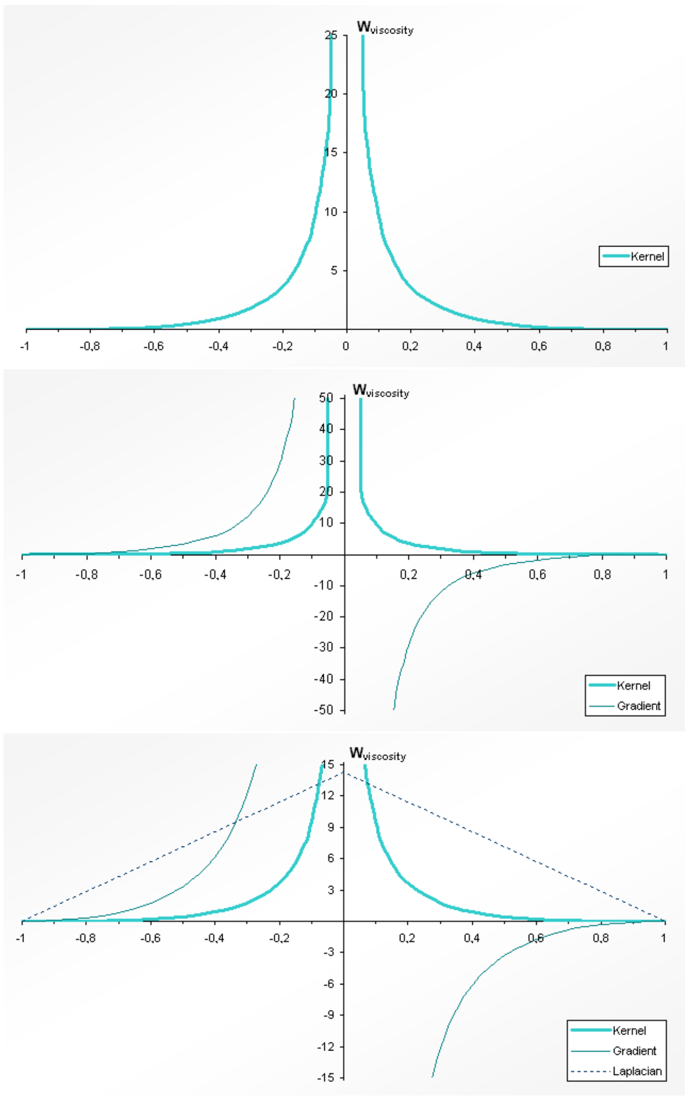


Figure 1.4: the viscosity kernel with gradient and Laplacian. Taken from [1].

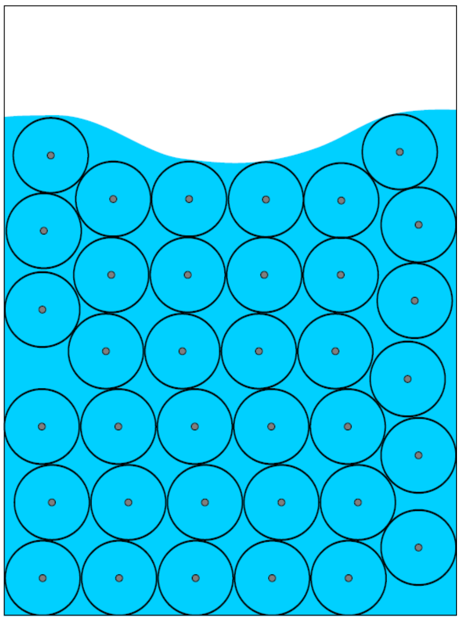


Figure 1.5: Lagrangian fluid particles. 2D projection. Taken from [2].

1.4 Lagrangian Fluid Dynamics

Compared to the classical (Eulerian) method of simulating fluids that uses 3D grid in its simulation the Lagrangian method doesn't use any grid, but uses particles instead (Figure 1.5). These particles contain physical values that are needed to represent the fluid. Each individual particle has velocity, position, mass and smoothed quantities obtained from the SPH approximation.

The NSE in the Lagrangian specification are significantly simplified. Equation (1.2) describes the mass preservation. If we assume that the number of particles is fixed and the mass of each particle is constant we can omit equation (1.2). In the Lagrangian method all particles freely move around the scene and thus it implies that any field quantity now depends on time only (t) rather than time and position (t and r) as it is in the Eulerian specification. The acceleration of a particle is then obtained by the ordinary time derivative $\frac{d}{dt}$ of its velocity $\mathbf{u}(t)$, which means that the equation (1.1) describing preservation of momentum will be simplified.

The formulation of the NSE for the Lagrangian specification has then the following formulation:

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}. \tag{1.23}$$

The right hand side describes the sum of forces acting on a particle. Where $-\nabla p$ term represents pressure, $\mu \nabla^2 \mathbf{u}$ the viscosity and $\mathbf{f}$ is the sum of all the external forces. The sum of these three fields $\mathbf{F} = -\nabla p + \mu\nabla^2\mathbf{u} + \mathbf{f}$ defines the acceleration of a Lagrangian particle

$$a_i = \frac{d\mathbf{u}_i}{dt} = \frac{\mathbf{F}_i}{\rho_i}, \tag{1.24}$$

where a_i is acceleration, $\mathbf{u}_i$ is velocity, ρ_i is density and $\mathbf{F}_i$ is the sum all forces of particle i.

1.4.1 Internal Forces

Density

In order to apply SPH to compute quantity fields of a particle we must know its mass and density. While mass is a user input constant density is a continuous field and has to be calculated. To compute its value for particle i we need to plug it into (1.4).

$$\begin{aligned} \rho_i &= \rho(\mathbf{r}_i) \\ &= \sum_j \rho_j \frac{m_j}{\rho_j} W(\mathbf{r}_i - \mathbf{r}_j, h) \\ &= \sum_j m_j W(\mathbf{r}_i - \mathbf{r}_j, h), \end{aligned} \tag{1.25}$$

where W is $W_{default}$ from (1.14).

Pressure

The first term on the right hand side of (1.23) is the negative gradient of pressure. It describes the tendency of particles to move to the areas with minimum pressure

$$\mathbf{f}_i^{press} = -\nabla p. \tag{1.26}$$

By applying SPH approximation (1.5) we get

$$\mathbf{f}_i^{press} = -\sum_{i \neq j} p_j \frac{m_j}{\rho_j} \nabla W(\mathbf{r}, h), \tag{1.27}$$

Equation (1.27) requires that the pressure of each particle participating in calculation is to be known. We can derive its value from the ideal gas formula

$$pV = nRT, \tag{1.28}$$

where p is pressure, $V = \frac{1}{\rho}$ is volume, n is number of particles per one mole, R is ideal gas constant, T is temperature. For the constant mass and constant temperature we can substitute the right hand side with gas stiffness constant k

$$pV = k \tag{1.29}$$

$$p\frac{1}{\rho} = k \tag{1.30}$$

$$p = k\rho \tag{1.31}$$

The above equation works well for ideal gas, however, for fluid it won't work as well. The particles will always have repulsive forces. That's the tendency of ideal gas to expand and occupy the space. In contrast, fluid must exercise cohesion and have constant density at rest. According to [6] we can use modified version of the ideal gas state equation, the final formula then will be

$$p = k(\rho - \rho_0), \tag{1.32}$$

where ρ_0 is the rest density constant.

The final thing is that the pressure force (1.27) is not symmetrical. This means that, when two particles with different pressure interact with each other the action-reaction law will not be conserved, because the pressure force will be asymmetric. We can symmetrize the pressure forces by modifying (1.27) as follows [1]

$$\mathbf{f}_i^{press} = -\rho_i \sum_{j \neq i} (\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2}) m_j \nabla W(\mathbf{r}_i, \mathbf{r}_j, h). \tag{1.33}$$

Because we want to avoid clustering of particles we cannot afford using the gradient of standard kernel here, since its value goes to zero with $\mathbf{r} \to 0$. Therefore in the above equation the special kernel for pressure $W_{pressure}$ (1.18) must be used.

Viscosity

When fluid flows its particles undergo friction. When friction happens kinetic energy transforms into heat. This friction between particles in terminology of fluid dynamics is called viscosity and is scaled by μ constant. By applying (1.6) to the second term in (1.23) we get the following SPH formulation

$$\begin{aligned} \mathbf{f}_i^{visc} &= \mu \nabla^2 \mathbf{u}(\mathbf{r}_i) \\ &= \mu \sum_{j \neq i} \mathbf{u}_j \frac{m_j}{\rho_j} \nabla^2 W(r_i - r_j, h). \end{aligned} \tag{1.34}$$

As in case with pressure the viscosity force is also asymmetric. As suggested by [5] we can symmetrize it as follows

$$\mathbf{f}_i^{visc} = \mu \sum_{j \neq i} (\mathbf{u}_j - \mathbf{u}_i) \frac{m_j}{\rho_j} \nabla^2 W(\mathbf{r}_i - \mathbf{r}_j, h). \tag{1.35}$$

Here the Laplacian is being calculated. The Laplacian of standard and pressure kernel produce negative outputs, which is not going to be suitable for viscosity term, because it will cause instabilities. Considering that fact a special kernel for viscosity W_{visc}(1.22) must be used.

1.4.2 External Forces

The third term on the right hand side of (1.23) represents the sum of all the external forces acting of a fluid particle

$$\mathbf{f}^{ext} = \sum_n \mathbf{f}^n,$$

where n represents each individual force. Some forces may be used directly on each particle, while others will require application of SPH on adjacent particles. Below are described two examples: the gravity, that acts directly on each particle and surface tension, which value is calculated from adjacent particles.

Gravity

The gravity force is applied for each particle regardless of their location or surroundings. It does not require SPH approximation to be used. For every particle gravity force will be

$$\mathbf{f}_i^{grav} = \rho_i \mathbf{g}, \tag{1.36}$$

where $\mathbf{g}$ is the gravitational acceleration constant.

Surface Tension

The surface tension force is not included in the NSE, therefore we need to consider it as an external force. All the attractive forces inside the fluid between adjacent particles are in perfect balance with each other, however, on the surface this balance is not preserved which causes surface tensions. The traction force of surface particles, oriented inside the fluid, is called the inward surface normal. For inner particles this normal is zero it only applies for the particles that are on or near the surface. The surface tension strength depends on the curvature of the surface, the greater the curvature is the greater the tension will be. As proposed by [5] we will use the following formulation for surface tension force

$$\mathbf{f}_i^{surf} = -\sigma \nabla^2 c_i \frac{\mathbf{n}_i}{\|\mathbf{n}_i\|}, \tag{1.37}$$

where σ is the tension coefficient that depends on the material of the fluid (e.g. water, air), c_i is the smoothed value of color field for particle i, $\mathbf{n}_i$ is the inward surface normal of particle i.

The color field term c is used to determine the surface of the fluid. Its value is $c = 1$ at particle location and $c = 0$ everywhere else [5]. In SPH formulation this term is calculated as follows

$$\begin{aligned} c_i &= c(\mathbf{r}_i) \\ &= \sum_j c_j \frac{m_j}{\rho_j} W(\mathbf{r}_i - \mathbf{r}_j, h) \\ &= \sum_j \frac{m_j}{\rho_j} W(\mathbf{r}_i - \mathbf{r}_j, h). \end{aligned} \tag{1.38}$$

The gradient of this color field is the inward surface normal

$$\begin{aligned} \mathbf{n}_i &= \nabla c(\mathbf{r}_i) \\ &= \sum_j \frac{m_j}{\rho_j} \nabla W(\mathbf{r}_i - \mathbf{r}_j, h). \end{aligned} \tag{1.39}$$

This normal is not zero only near the surface. The curvature of the surface is described by

$$\kappa = \frac{-\nabla^2 c}{\|\mathbf{n}\|}. \tag{1.40}$$

The final formula for surface tension is

$$\mathbf{f}^{surf} = \sigma \kappa \mathbf{n}, \tag{1.41}$$

which, applied to each particle, will give us (1.37). When $\|\mathbf{n}\| \to 0$ numerical instabilities may arise, thus we have to compute $\mathbf{f}_i^{surf}$ only when

$$\|\mathbf{n}_i\| \geq l,$$

where $l > 0$ is some threshold relating to the particle concentration.

1.4.3 Collision

Now that we've described the physics behind fluid's behaviour we need to describe how it is supposed to behave when it interacts with a solid. The collision algorithm is basically composed from two parts: collision detection and collision response. Let's describe them individually.

Collision Detection

In order for a particle to interact with surrounding objects (in our case that will be a heightmap) we need to know particle's position **p** and its velocity **v**. The particle's position prior to collision can be obtained by subtracting from the current position its velocity vector multiplied by Δt time step. The collision model with a heightmap can be observed on Figure 1.6.

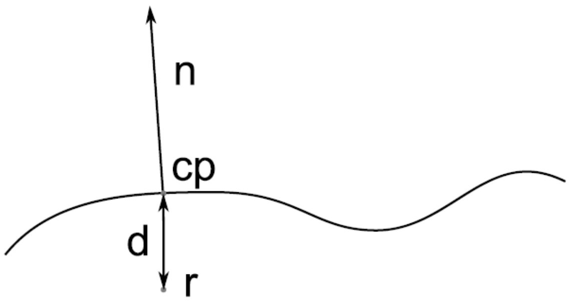


Figure 1.6: Collision with heightmap. Taken from [2].

To calculate collision response we will need to know the following data:

1. **cp** - contact point
2. d - penetration depth
3. **n** - normal vector at the contact point

As [1] put it the contact point **cp** does not necessarily have to be the intersection point on the line segment between the current position **p**(t) and the previous position **p**(t - Δt) we can use the closest point in the normal direction if it doesn't influence the result of collision too much. Before we talk about how we can use these data for calculating collision response we need to mention, how we detect collision with a heightmap fist.

A heightmap is represented as a matrix of heights. For each point its height z can be obtained from this matrix by providing its [x, y] coordinates. The implementation of terrain via heightmap will be discussed in more details in subsection 4.2. For any point **r** we can thus easily find height z and normal **n** of the surface. The penetration depth d and contact point **cp** are then easy to calculate

$$d = z(\mathbf{r}) - r_z,$$

$$\mathbf{cp} = \mathbf{r} + [0, 0, d].$$

Collision Response

To simulate collision response we need to project a particle onto the surface i.e. we must assign the contact point **cp** to particle's position **r**

$$\mathbf{r}_i = \mathbf{cp},$$

and adjust particle's velocity along the normal **n** direction [1]

$$\mathbf{u}_i = \mathbf{u}_i - 2(\mathbf{u}_i \cdot \mathbf{n})\mathbf{n} \tag{1.42}$$

Equation (1.42), describes a perfect elastic collision, which means that the kinetic energy will be conserved. That implies that a fluid particle will bounce off the surface, which fluids don't usually do. For that we need to control how much the kinetic energy will be conserved to achieve more realistic results. We will do this by introduction of the restitution constant [1]

$$\mathbf{u}_i = \mathbf{u}_i - (1 + c_R)(\mathbf{u}_i \cdot \mathbf{n})\mathbf{n}, \tag{1.43}$$

where $0 \leq c_R \leq 1$ is the coefficient of restitution.

However, this will not be enough, because the magnitude of c_R may be 1 and equation (1.43) will increase the kinetic energy of the particle, it will virtually behave no different from the equation (1.42). For that we need to constrain the outgoing energy to never exceed the incoming energy. We'll do this by introducing the ration of penetration depth to the distance between the last particle position and the penetrating position [1]

$$\mathbf{u}_i = \mathbf{u}_i - \left(1 + c_R \frac{d}{\Delta t \|\mathbf{u}_i\|}\right)(\mathbf{u}_i \cdot \mathbf{n})\mathbf{n}, \tag{1.44}$$

where we implicitly assume that $\|\mathbf{u}_i\| > 0$.

1.4.4 Time Integration

In order to actually move a fluid particle we need to advance its position through the time constant Δt that is set globally. Equation (1.24) is evaluated to obtain the particle's acceleration and then this acceleration is employed to advance the position.

In this work the Leap-Frog integrator will be used. The major peculiarity of the Leap-Frog integrator is that the position and velocity of a particle are not updated in parallel. It uses a half time step to compute the new value of particle's velocity and then uses that value to obtain the updated position of the particle. Figure 1.7 illustrates the concept. The integration scheme yields

$$\mathbf{u}_{t+1/2\Delta t} = \mathbf{u}_{t-1/2\Delta t} + \Delta t \mathbf{a}_t, \tag{1.45}$$

$$\mathbf{r}_{t+\Delta t} = \mathbf{r}_t + \Delta t \mathbf{u}_{t+1/2\Delta t}. \tag{1.46}$$

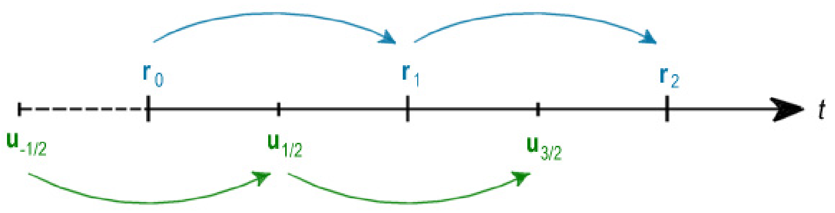


Figure 1.7: Leap-Frog scheme. Taken from [1].

with the initial velocity offset

$$\mathbf{u}_{-1/2\Delta t} = \mathbf{u}_0 - \frac{1}{2}\Delta t \mathbf{a}_0. \tag{1.47}$$

The velocity in time t can be obtained by following approximation

$$\mathbf{v}_t \approx \frac{\mathbf{v}_{t-\frac{1}{2}\Delta t} + \mathbf{v}_{t+\frac{1}{2}\Delta t}}{2}. \tag{1.48}$$

1.5 Hydraulic Erosion Model

In this section we will describe the hydraulic erosion model proposed by [3] that couples the SPH fluid simulation model, discussed in section 1.4 and the Eulerian physically based erosion model.

Simulation of hydraulic erosion has been a tough challenge in Computer Graphics. Any hydraulic erosion simulation is based on the simulation of fluid flows. The classical way is to use the Eulerian method, but its major disadvantage is the usage of 3D grid that is not easy to scale dynamically, that makes it unsuitable to simulate sparse volume. It will rapidly become memory costly, when applied for large domains. This is why in their work [3] proposed to employ SPH fluid model to simulate erosion phenomenon. Its memory efficiency enables to simulate large scale terrains, since it focuses its computations only in regions, where fluid particles physically present.

SPH particles will be used to carry material sediment and thus no additional particles are needed to represent it. The sediment will be contained directly within a SPH particle and represented as percentage of the volume it occupies. The movement of sediment is driven by both implicit and explicit advection. The implicit advection follows the flow of SPH particles, whereas the explicit is the donor-acceptor advection scheme, where sediment only advects from donor to acceptor. The donor-acceptor scheme will be discussed more in details in subsection 1.5.4.

1.5.1 Boundary Particles and External Forces

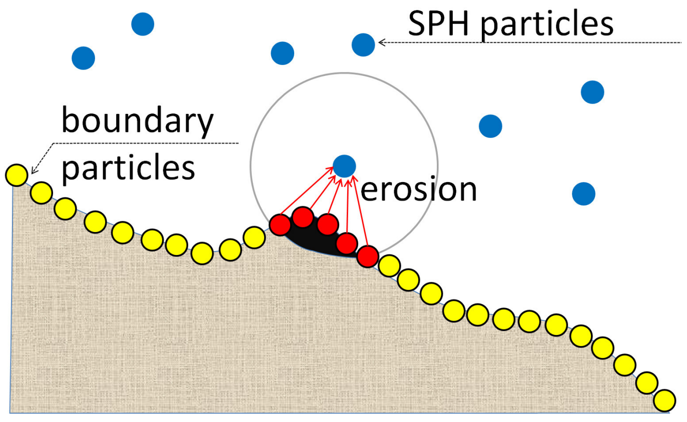


Figure 1.8: Boundary particles is the means for interacting SPH particles with the terrain. Taken from [3].

The fluid-terrain interaction will take place via boundary particles that will be seeded, with the in-between distance Δs, over the triangles of the terrain that is represented as a heightmap Figure 1.8. This interaction will involve three things: friction, erosion and deposition.

In subsection 1.4.2 we've broken down and described the external forces used in the equation (1.23) that act upon fluid. We concluded that we would be taking into account only gravity $\mathbf{f}^{grav}$ and surface tension $\mathbf{f}^{surf}$. Now, when boundary particles come into play, we will extend the external forces term $\mathbf{f}^{ext}$ by appending the force due to solid boundary $\mathbf{f}^{bound}$. The resulting external force is going to be the sum of the following three components [3]

$$\mathbf{f}^{ext} = \mathbf{f}^{grav} + \mathbf{f}^{surf} + \mathbf{f}^{bound}. \tag{1.49}$$

In [3] the boundary force $\mathbf{f}^{bound}$ is represented as the sum of no-slip $\mathbf{f}^{ns}$ and no-penetration conditions $\mathbf{f}^{np}$ i.e.

$$\mathbf{f}^{bound} = \mathbf{f}^{ns} + \mathbf{f}^{np}. \tag{1.50}$$

The no-slip condition states that the fluid cannot penetrate the surface and the no-slip condition sets the fluid's relative speed on the boundary to zero, which represents friction. However, we already described, how fluid particles will collide with the terrain in subsection 1.4.3. That collision method will substitute the no-penetration condition. Therefore, we will drop the $\mathbf{f}^{np}$ term and the equation (1.50) will take the following form

$$\mathbf{f}^{bound} = \mathbf{f}^{ns}, \tag{1.51}$$

where we only calculate friction in a fluid-boundary interaction. The no-slip $\mathbf{f}^{ns}$ is given as [3]

$$\mathbf{f}^{ns}(\mathbf{r}) = \sum_b L_b^2 \tau^{visc}(|\mathbf{r} - \mathbf{r}_b|), \tag{1.52}$$

where b is the boundary particles, $L_b = \Delta s$ is the distance between the boundary particles and τ^{visc} is traction. The traction is expressed as [3]

$$\tau^{visc}(r) = -\mu_{bf}\mathbf{v}\nabla^2 W_{visc}(r, h), \tag{1.53}$$

where μ_{bf} is the boundary friction constant and W_{visc} is the viscosity kernel (1.22).

1.5.2 Erosion

One important thing here is that fluid flows produce shear stress on boundary particles. This is a force caused by parallel fluid forces. To apply it on boundary particles [7] proposed to give them non-Newtonian fluid characteristics via a power-law model

$$\tau = K\theta^n, \tag{1.54}$$

where τ is the shear stress, K is the shear stress constant, θ is the shear rate (a measure of shear deformation). The shear rate can be approximated as follows [3]

$$\theta = \frac{v_{rel}}{l}, \tag{1.55}$$

where v_{rel} is the velocity of the fluid relative to solid surface and l is the distance over which the shear rate is applied.

Next we need to the erosion rate. The erosion rate is related with shear stress. It was formulated by [8] as follows

$$\varepsilon = K_{\varepsilon}(\tau - \tau_c), \tag{1.56}$$

where K_{ε} is the erosion strength and τ_c is the critical shear stress (material erosion resistance). The change of mass of particle b by the SPH particle j within the distance of smooth radius h is [3]

$$\frac{dM_b}{dt} = -\sum_j L_b^2 \varepsilon(j). \tag{1.57}$$

1.5.3 Sediment Transportation

The process of sediment transportation is described by the general equation [3]

$$C(\mathbf{x}, t) = P(C) + J(C, \mathbf{x}, t), \tag{1.58}$$

where C is percentage of local volume of SPH particle occupied by sediment particles, P represents the physical redistribution processes and J is the sources and sinks that reflect erosion and deposition. In order to apply equation (1.58) in SPH we need to use the diffusion equation for SPH proposed by [4]

$$\frac{dC}{dt} = \frac{1}{\rho}\nabla(D\nabla C) + J, \tag{1.59}$$

where $\frac{dC}{dt}$ is the total derivative, denoting the time rate of change following particles at velocity $\mathbf{u}$ and D is the molecular diffusivity. In our simulation we use $\mathbf{u}$ to describe the total velocity of a sediment particle, which is composed from the sum of fluid velocity $\mathbf{v}$ and settling velocity $\mathbf{v}_s$ [3].

$$\mathbf{u} = \mathbf{v} + \mathbf{v}_s. \tag{1.60}$$

In section 8.3 we addressed that the NSE for the Lagrangian fluid dynamics are simplified, due to the fact that this approach describes fluid flows as a set of particles freely moving around in the space and that any quantity field depends on time only rather than time and position. As a result on the left-hand side we changed the acceleration term by substituting $\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla$ with total time derivative $\frac{d}{dt}$. In our simulation we use the implicit sediment particles i.e. sediment particles follow SPH particles and contained within them. In order to switch from the framework of explicit particles into implicit we first switch into a space-fixed Eulerian frame of reference. We do so by substituting the total derivative in (1.59) with $\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla$ and plugin (1.60) into the velocity term, which yields [3]

$$\frac{\partial C}{\partial t} + (\mathbf{v} + \mathbf{v}_s) \cdot \nabla C = \frac{1}{\rho}\nabla(D\nabla C) + J. \tag{1.61}$$

Opening the brackets yields

$$\frac{\partial C}{\partial t} + \mathbf{v} \cdot \nabla C + \mathbf{v}_s \cdot \nabla C = \frac{1}{\rho}\nabla(D\nabla C) + J. \tag{1.62}$$

By replacing the term $\frac{\partial C}{\partial t} + \mathbf{v} \cdot \nabla C$ back by the total derivative $\frac{\partial C}{\partial t}$ we'll get the advection-diffusion equation for sediment tranport in SPH

$$\frac{dC}{dt} = -\mathbf{v}_s \cdot \nabla C + \frac{1}{\rho}\nabla(D\nabla C) + J. \tag{1.63}$$

In the following section we describe the numerical implementation of the advection term $-\mathbf{v}_s \cdot \nabla C$ and the diffusion term $\frac{1}{\rho}\nabla(D\nabla C)$ in the SPH context.

1.5.4 Advection (donor-acceptor scheme)

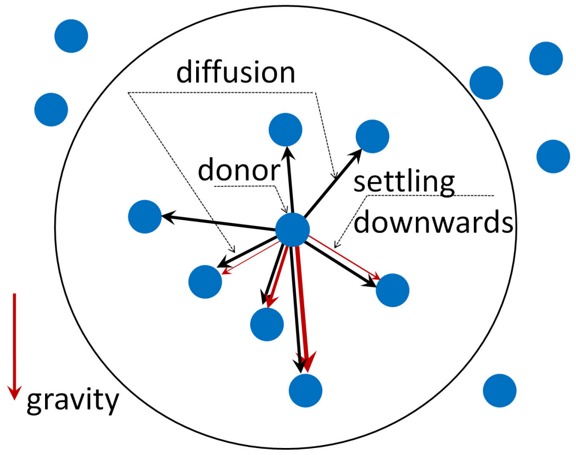


Figure 1.9: The donor-acceptor scheme. Donor particles pass the sediment to acceptor particles, whose relative position is with respect to direction of the settling velocity. Taken from [3].

The term $-\mathbf{v}_s \cdot \nabla C$, from equation (1.63), represents advection of sediment between SPH particles in the direction of the settling velocity. To implement it [3] introduced the donor-acceptor scheme that describes interaction between particles. A SPH particles i thus is either a donor or an acceptor in every $i-j$ particle interaction. It is being an acceptor, when its relative position with respect to the donor particle corresponds the direction of the settling velocity vector Figure 1.9. For SPH the advection term is expressed as

$$-\mathbf{v}_s \cdot \nabla C = -\sum_j \begin{cases} m_j \frac{C_j}{\rho_j}(\mathbf{v}_s \cdot \hat{\mathbf{r}}_{ij})F(|\mathbf{r}_{ij}|, h), & \mathbf{v}_s \cdot \mathbf{r}_{ij} \geq 0 \\ m_i \frac{C_i}{\rho_i}(\mathbf{v}_s \cdot \hat{\mathbf{r}}_{ij})F(|\mathbf{r}_{ij}|, h), & \mathbf{v}_s \cdot \mathbf{r}_{ij} < 0, \end{cases} \tag{1.64}$$

where

$$\nabla W(\mathbf{r}_{ij}, h) = \hat{\mathbf{r}}_{ij} F(|\mathbf{r}_{ij}|, h),$$

$$\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j,$$

$$\hat{\mathbf{r}}_{ij} = \mathbf{r}_{ij}/|\mathbf{r}_{ij}|$$

and F is the derivative of the cubic spline kernel W taken with respect to $|\mathbf{r}_{ij}|$. Particle i acts as the acceptor, when $\mathbf{v}_s \cdot \mathbf{r}_{ij} \geq 0$ and as a donor otherwise. The hindered settling velocity $\mathbf{v}_s$ is formulated as

$$\mathbf{v}_s = \frac{2}{9} r_s^2 \frac{\rho_s - \rho_f}{\mu} \mathbf{g} f(C), \tag{1.65}$$

where ρ_s, ρ_f are the sediment and fluid densities, $\mathbf{g}$ is the gravity acceleration constant r_s is the radius of a sediment particle, μ is the viscosity of the fluid and C is the solid volume fraction at the acceptor particle. The function $f(C)$ is the hindering settling function approximating decreasing advection rate with higher sediment concentration. It is approximated using the Richardson-Zaki relation [9]

$$f(C) = 1 - (C/C_{max})^e, \tag{1.66}$$

where C_{max} is the maximum solid volume fraction in a fluid particle and e is an exponent 4 < e < 5.5. For the case of $C > C_{max}$ we set $f(C) = 0$. When $C > C_{max}$ the saturated particle do not receive the sediment. Figure 1.10 depicts the transportation of the material along the settling velocity vector.

1.5.5 Diffusion

Diffusion is the tendency of the material to flow from a high concentration area to low concentration area Figure 1.11. As suggested by [4] the diffusion can be expressed as

$$\frac{dC}{dt} = \frac{1}{\rho}(D \nabla C), \tag{1.67}$$

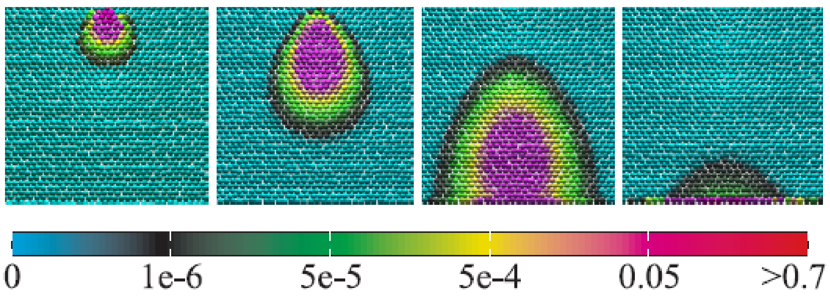


Figure 1.10: Material advection. The sediment advects with respect to direction of the settling velocity that corresponds with gravity. Taken from [3].

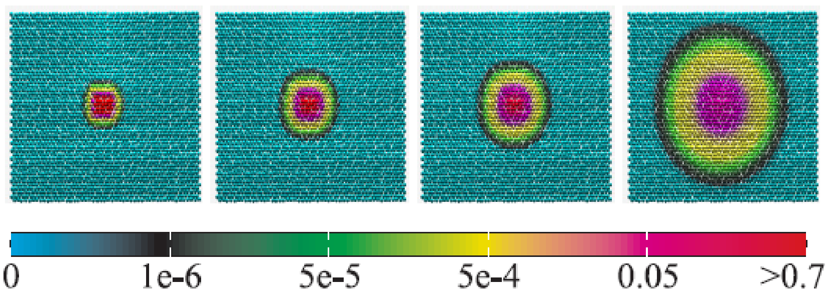


Figure 1.11: Material diffusion. Blue represents low concentrated areas while red is highly concentrated. Taken from [3].

where D is the coefficient of diffusion. The diffusion equation for SPH will be [3]

$$\frac{dC_i}{dt} = \sum_j \frac{m_j}{\rho_i \rho_j} D(C_i - C_j) F(|\mathbf{r}_i - \mathbf{r}_j|, h), \tag{1.68}$$

where F is the gradient function of a cubic spline kernel. This equation shows a positive rate of change for values of $C_i < C_j$.

1.5.6 Deposition

Deposition is the process of moving the material contained within a fluid particle onto a boundary particle once the fluid particle gets close to the ground. This process is calculated in two steps: the SPH particles communicate with boundary particles and boundary particles communicate with the terrain. Figure 1.12 depicts this process. The change of mass of a boundary particle b due

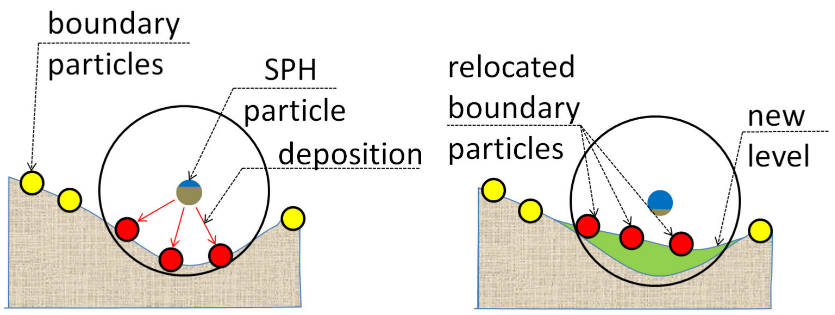


Figure 1.12: Material deposition. The sediments flows from SPH particle to boundary particles (left). The terrain height is changed and new boundary particles generated (right). Taken from [3].

to exchange of C from fluid particle j is [3]

$$\frac{dM_b}{dt} = \sum_j \rho_s \frac{m_j}{\rho_j} \frac{dC(j)}{dt} \tag{1.69}$$

where $dC(j)/dt$ is the advection term from (1.64) with fluid particles strictly set as donors and boundary particles as acceptors. For the case of $(\mathbf{v}_s \cdot \mathbf{r}_{ij}) < 0$, there is no material deposition.

1.5.7 Terrain Modification

SPH fluid particles interact with the terrain via boundary particles that are located on the triangles of the terrain. The terrain is represented as a regularly sampled heightmap. The change of mass of a triangle p is related with the change of mass of all its boundary particles and is expressed as follows [3]

$$\frac{dM_p}{dt} = \sum_b \frac{dM_b}{dt}. \tag{1.70}$$

Because we use a regular heightmap the deformation of the terrain corresponds to changing the height of the triangle's vertices. The equation for computing the total height change H of a triangle is [3]

$$H = \frac{3}{6} \frac{m}{\rho_s} \frac{1}{A_b}, \tag{1.71}$$

where m is the triangle's mass, the term $(1/6)$ reflects the fact that changing one vertex results in a change of six attached triangles to an inner vertex on the heightmap, the term A_b is the area of the vertically projected triangle. For the deposition case i.e. $H > 0$ we first distribute H to the lowest triangle vertices, when their height are even we then distribute H uniformly. The same principle applies for the erosion case ($H < 0$). We subtract the height from the heightest vertices first and once they're all even we subtract uniformly.

Chapter 2

Method Anylysis

This chapter describes which algorithms and methods will be used and eventually implemented in the implementation section 4.

2.1 Algorithms

The fluid will be represented and simulated using the SPH that employs the Lagrangian fluid dynamics approach. The advantage of this method is that each individual fluid particle is represented as a 3D object freely moving around in a 3D space. Besides its physical parameters like *pressure*, *density*, *viscosity* we will have to keep track of its current *position* and *velocity*. The *position* and the *velocity* of the same particle in the next iteration can then be easily calculated via the Leap-Frog integrator described in the subsection 1.4.4. This approach enables us to simulate sparse volumes, where particles are able to get anywhere in the world and are not fixed by a grid, that drastically reduces memory costs of the entire simulation. The simulation implemented in such fashion is much easier to scale. On the other hand the trade-off of this approach is that a fluid volume would need to have more particles to deliver realistic behaviour.

The erosion model proposed by [3] efficiently couples SPH fluid with physically-base erosion model. It has a couple of game-changing advantages. First, it uses a similar interpolation techniques with boundary particles to calculate erosion of material. Second, it stores sediment inside water particles, so we don't have to introduce a new kind of particles to represent it. Third, we can generate boundary particles only at places where fluid particles are to avoid memory costs.

2.2 Terrain Representation

The assignment of this work states that the terrain will be represented as a 3D voxel grid, however in the end I decided to give up on this idea.

The voxel grid has several complications. First, the grid is preallocated and consumes a lot of memory, if we want to expand the terrain it will end up being memory costly. Second, it is an intrinsically different kind of approach. The voxel grid is related with the Eulerian approach. We have the fluid implemented as the SPH i.e. the Lagrangian approach that calculates its physical forces by interpolating among the adjacent particles. If we couple a fluid system that employs an interpolation methods with a voxel grid we will run into the issue of performing excessive calculations. Specifically, when a SPH particle approaches the surface besides dealing with boundary voxels it will have to reach the voxels underneath the surface and count them in as well. This will produce additional calculations. When a surface voxel is washed out it reveals up to 5 new voxels underneath it that must be processed in the same iteration. Rather than bothering what should and should not be counted in underneath the surface it would be much better, if fluid particles only have to deal with what's on the surface and based on these interactions adjust the terrain.

A better option to represent the terrain is by using a heightfield. The heightfield represents only the surface of the terrain. We can then efficiently seed boundary particles on its triangles only at places where fluid particles are. The fluid particles will only interact with the boundary particles on the surface and the terrain will be adjusted correspondingly.

2.3 Analysis Summary

In this chapter we've talked about methods and algorithms that we will stick with and implemented in the implementation chapter 4. The fluid will be represented as SPH. We opted to give up on voxel grid and stick with the heightfield to represent the terrain.

SPH method

- ⊕ freely moving particles
- ⊕ memory efficient
- ⊕ no redundant computations
- ⊕ fast sparse fluid simulation
- ⊖ requires big amount of particles to deliver a realistic behavior
- ⊖ might be imprecise on boundary conditions

Erosion model

- $\oplus$ sediment is stored within SPH particles
- $\oplus$ boundary particles are used communicate with the fluid and the terrain 1.5.1
- $\oplus$ simulates erosion 1.5
- $\oplus$ simulates deposition 1.5.6
- $\oplus$ simulates advection (1.5.4) and diffusion (1.11)
- $\oplus$ uses heightfield
- $\ominus$ less accurate, heightfield only moves vertically

Chapter 3

Software Design

In this chapter we will talk about, how the application will be designed in terms of software design. We will talk about functional and non-functional requirements (section 3.1), use cases (section 3.2), take a look at the diagrams that describe the architecture of the application (section 3.3) and discuss the user interface of the application in more details (section 3.4).

The current application serves primarily as a visualization of hydraulic erosion applied on a terrain. A user of the application can upload a terrain map and watch, how a water volume will approximately change it.

3.1 Functional and Non-functional Requirements

In this section we list specific functional and non-functional requirements of the application. The functional requirements are listed in table 3.1. The non-functional requirements are listed in table 3.2.

Id	Requirement	Description
F1	Simulation of water volume	The application must simulate a water volume with given number of particles
F2	Simulation of erosion	The water must erode the terrain
F3	Water-terrain collision	The water must collide with the terrain and not go through it
F4	Boundary particles on the surface	The surface of the terrain must be seeded with boundary particles
F5	Terrain as a heightfield	The map is generated based on the bitmap image
F6	Illumination	The simulation must use basic lighting model (like Phong) to make the objects visible in the scene
F7	Mouse and keyboards control	User must control the camera in the scene with mouse and keyboard
F8	Adjusting the simulation parameters	User must be able to change the simulation parameters (e.g. set the number of particles, change fluid forces, change map, etc.)
F9	Fluid particles represented as 3D spheres	Each individual particle must be rendered as a 3D sphere
F10	Graphical interface to interact with the simulation	The application must provide a graphical user interface to give a user an opportunity to change the set the parameters

Table 3.1: Functional requirements.

Id	**Requirement**	**Description**
N1	Boundary particles generated dynamically	The boundary particles must be generated only in places where fluid particles reach
N2	Terrain is grid-accelerated	The terrain represents the grid of cells from top-view perspective, each grid is composed out of two triangles
N3	Parallelization of the water simulation	The water simulation must leverage the CPU multithreading to perform calculations
N4	Usage of C++ programming language	The implementation of the application must be written in C++
N5	Usage of OpenGL	To render all graphics OpenGL graphical API must be used
N6	Application is designed for Windows platform	The application must be defined to run on Windows 10 platform

Table 3.2: Non-functional requirements.

3.2 Use Cases

This section elaborates on different use cases. The individual use cases are described in Table 3.3, Table 3.4, Table 3.5.

Use Case 1	Run the simulation without changing anything
Actor	User
Use case overview	User runs the application. The scene is loaded and the simulation is paused by default. User sees the Help Window with the information about how to use the application. User proceeds to close the Help Window and unpauses the simulation. The water volume begins to pour onto the terrain.
Success scenario	The particles of the water volume disappear after their lifetime is expired. User observes the deformed terrain.

Table 3.3: Use case: simulation run.

Use Case 2	Initialization parameters set up
Actor	User
Precondition 1	User ran the application
Precondition 2	Simulation is paused by default
Precondition 3	User closed the Help Window
Use case overview	User opens up the graphical interface window and edits the initialization parameters (e.g. number of particles, particle lifetime, terrain map) in the section labeled as Initialization. User then closes the interface window and unpauses the simulation. The water volume begins to pour onto the terrain.
Success scenario	The particles of the water volume disappear after their lifetime is expired. The user observes the deformed terrain.

Table 3.4: Use case: initialization parameters set up.

Use Case 3	Runtime parameters editing
Actor	User
Precondition 1	User ran the application
Precondition 2	User closed the Help Window
Precondition 3	User unpaused the simulation
Use case overview	User opens up the graphical interface window and edits the runtime parameters of the simulation (e.g. light direction, terrain color, water density, gravity, etc.)
Success scenario	The the simulation and the scene components are influenced by the edited parameters

Table 3.5: Use case: runtime parameters editing.

Table 3.6: The requirements fulfillment table

	Requirements									
Use Cases	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
UC1	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$		$\oplus$	$\oplus$
UC2	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$
UC3	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$

3.3 Application Design

In this section we will talk about the structure of the application as well as demonstrate the sequence of internal processes.

3.3.1 Control Flow and Processes

The application begins with the initialization of multiple components that are required for its running. This happens before the render loop begins. Here, OpenGL is initialized and a view port window is created, terrain map is loaded, shaders are loaded, fluid origin and camera placement are set and finally the fluid system is initialized with given number of particles.

After all the components are set up and ready to be used the application proceeds to the render loop. Within the render loop use input is read, graphical interface is drawn, the simulation is calculated and the scene is rendered. The fluid system has to communicate with the terrain, therefore the pointer to the terrain is passed to it.

The fluid system calculate the fluid forces first. Right after the fluid is calculated boundary particles are seeded over the surface of the terrain and required boundary forces are calculated. While seeding the boundary particles the fluid system communicated with the terrain to give it the information where fluid particles are and to obtain the generated list of boundary particles. After that the hydraulic erosion is simulated (i.e. sediment transfer, deposition, erosion). Next the terrain is updated, again fluid module communicates with the terrain module. Finally the fluid particles are advanced and the collisions with the terrain are resolved. Figure 3.1 demonstrates the described control flow.

3.3.2 Structure and Architecture

Fluid system The fluid system is implemented in $fluid\_system.h$. Class $FluidSystemSPH$ contains the parameters necessary to describe fluid state and the containers that store the particles. Method $Initialize()$ is called at the beginning of the application. It creates a water volume with given number of particles and sets each particle with initial values. Method $Run()$ performs the simulation steps, fluid and erosion are calculated here. Method $Draw()$ is responsible fore drawing fluid and boundary particles. The class provides the following containers:

- *m_Particles* - a vector with that stores fluid particles
- *m_ParticlesSpatialHash* - the spacial hashing data structure that divides the space on cells and stores indexes of fluid particles from $m_Particles$ in those cells

- *m_ParticleNeighbors* - a 2D vector that stores neighbour lists of each fluid particle
- *m_BParticles* - a vector that stores all the boundary particles generated throughout the terrain
- *m_NearestBParticles* - an unordered set that stores neared boundary particles that were detected by fluid particles in current iteration
- *m_FluidsOfBoundary* - an unordered map that assigns to each boundary particle indices of the water particles that it was detected by

Terrain The terrain is implemented in *grid.h*. Class Grid is responsible for representation and management of the terrain. It builds up the heightfield from the uploaded bitmap, generates and stores boundary particles and resolves the collisions. The class is called grid, because the terrain is accelerated with grid topology (we will discuss it in more details in section 4.2). The map is uploaded and initialized in the *constructor* of the class. Method $HFUpdate()$ is responsible for adjusting the geometry of the terrain. Method $collision()$ resolves the collision of fluid particles with the heightfiled. Method $SeedCell()$

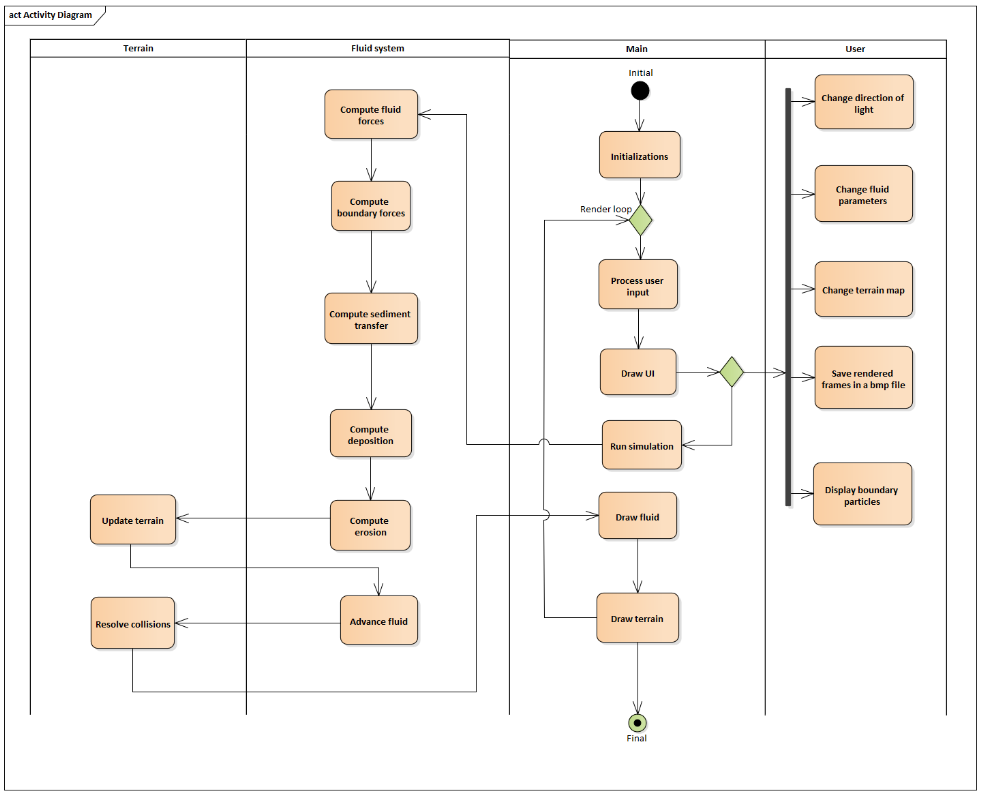


Figure 3.1: Action diagram of the application.

is called for each fluid particle in the fluid system and seeds boundary particles over the cell, where current fluid particle is located. The Grid class uses kd-tree data structure that stores boundary particles of each grid (kd-tree will be discussed in more details in subsection 1.5.1). The class implemented its own drawing method $Draw()$ that renders the terrain.

Camera The camera is abstracted away in *camera.h.* It encapsulates the logic of camera computations. The class processes input from keyboard and mouse and produces the $viewmatrix$.

Shader The shader class is implemented in *shader.h.* It provides a handy functionality to read and compile the code of vertex and fragment shader files. Its interface provides the abilities to pass uniform arguments and bind the shader program.

3.4 User Interface

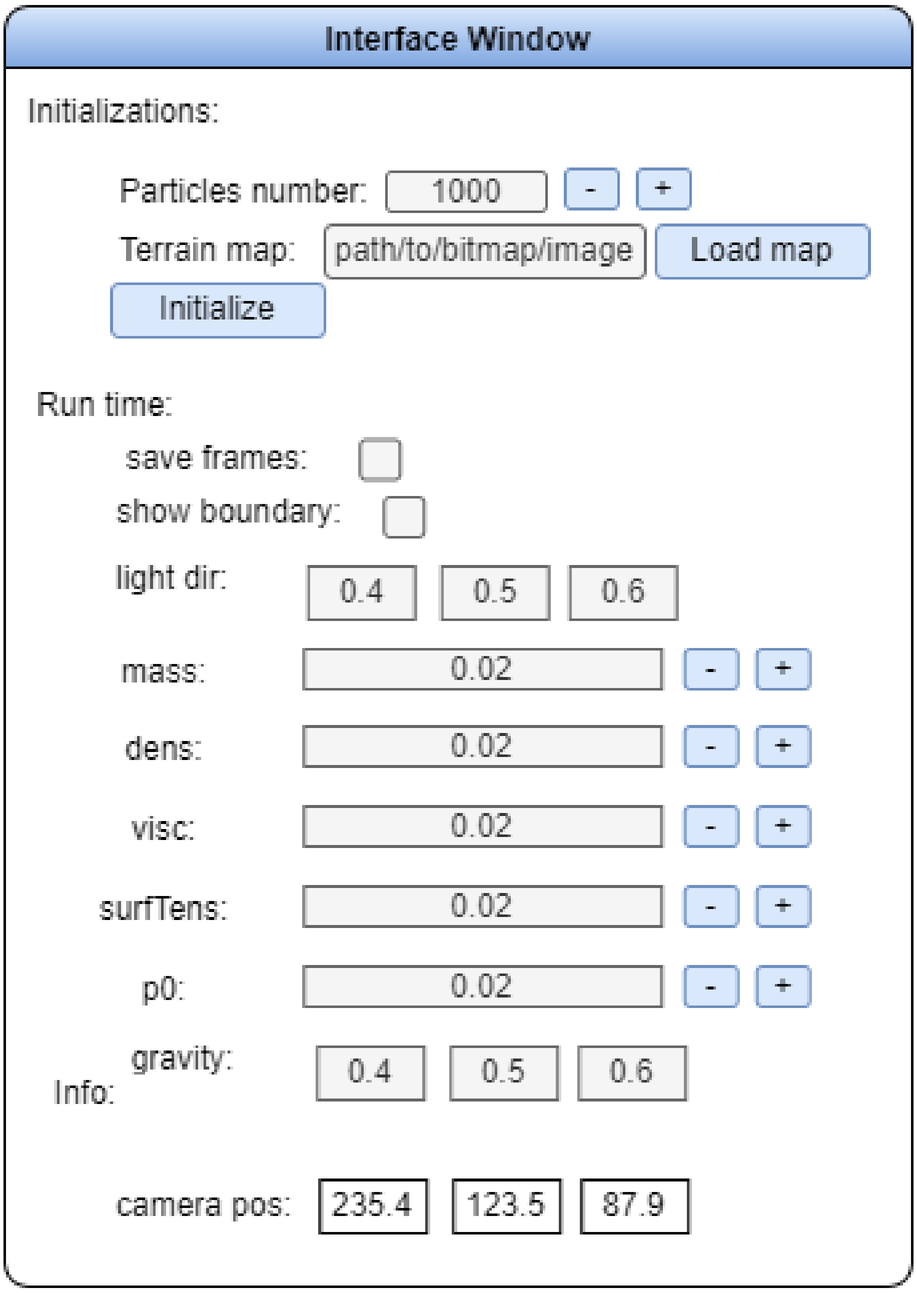
A graphical user interface provides a handy way to interact with any software. This program itself is a graphical application that provides visual outcomes, however when we want to change some settings we might end up having hard times doing so. That's why it is a good idea to have some UI that will enable us to manipulate the program in run time.

The graphical interface in this work will provide functionalities to adjust the simulation parameters, as well as set up the scene. It will be organized in three sections:

- Initialization - set up the initial parameters
- Run time - adjust the parameters during run time(e.g. fluid density, pressure)
- Info - information about the state of the application (e.g. camera world position)

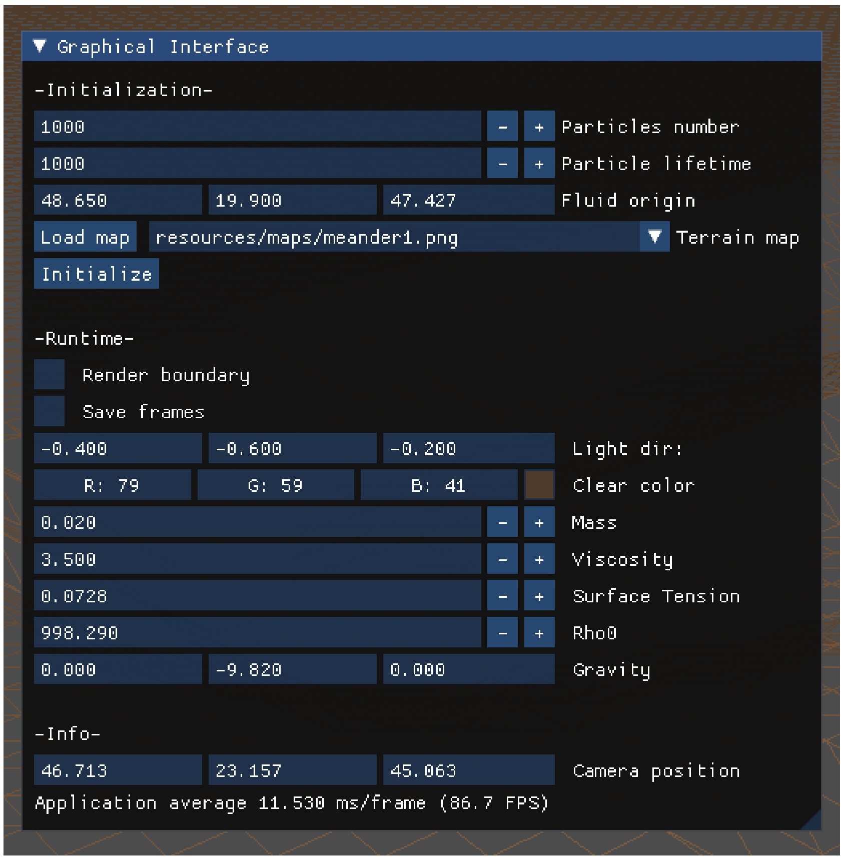
It is important to note that interaction with the UI might be very slow while simulating huge volumes of water. In this work the Dear ImGui library is used to deliver a graphical interface.

The wireframe of the user interface is pictured on Figure 3.2. The actual user interface in the final program is pictured on Figure 3.3. In the final version I added $Fluidorigin$, $Particlelifetime$ fields in the *Initialization* section, as well as *Clearcolor* in the *Runtime* section.



Particles number: 1000
Terrain map: path/to/bitmap/image
save frames: ☐
show boundary: ☐
light dir: 0.4 0.5 0.6
mass: 0.02
dens: 0.02
visc: 0.02
surfTens: 0.02
p0: 0.02
gravity: 0.4 0.5 0.6
camera pos: 235.4 123.5 87.9

Figure 3.2: The graphical interface wireframe.



▼ Graphical Interface

-Initialization-

```
1000                          - + Particles number
1000                          - + Particle lifetime
48.650    19.900    47.427        Fluid origin
Load map  resources/maps/meander1.png ▼ Terrain map
Initialize
```

-Runtime-

```
☐ Render boundary
☐ Save frames
-0.400    -0.600    -0.200        Light dir:
R: 79     G: 59     B: 41         Clear color
0.020                         - + Mass
3.500                         - + Viscosity
0.0728                        - + Surface Tension
998.290                       - + Rho0
0.000     -9.820    0.000         Gravity
```

-Info-

```
46.713    23.157    45.063        Camera position
Application average 11.530 ms/frame (86.7 FPS)
```

Figure 3.3: A screenshot with implemented graphical user interface.

Chapter 4

Implementation

The previous section 1 describes the theoretical part of fluid and erosion model. In this section we talk about how we can put this theory in a practical use. The entire simulation is run on CPU using OpenMP API to boost the calculation by leveraging multithreading. The logic of the simulation is illustrated on Figure 4.1. We will talk in details about each step down below in this chapter.

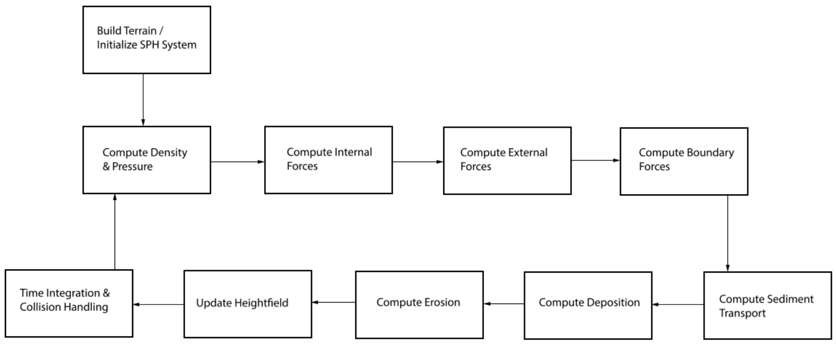

Build Terrain / Initialize SPH System
Compute Density & Pressure
Compute Internal Forces
Compute External Forces
Compute Boundary Forces
Compute Sediment Transport
Compute Deposition
Compute Erosion
Update Heightfield
Time Integration & Collision Handling

Figure 4.1: The simulation algorithm scheme.

Our simulation uses a lot of particles (fluid + boundary). We need to know the adjacent particles of each SPH particle to calculate its physical quantities and to pass the carrying sediment between them. And also, we need to know which boundary particles are in range of each fluid particle in order to erode/deposit material from/onto them. That implies the involvement of lots of iterations until we find the particles we need. This part of simulation can very quickly lead to performance issues and significantly increase the simulation time. Luckily, it can be improved algorithmically.

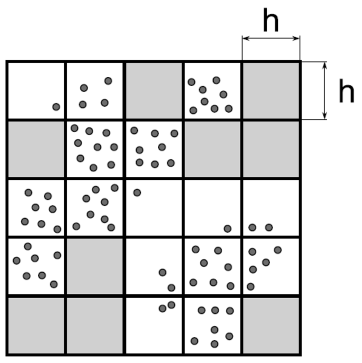


Figure 4.2: Spatial Hashing data structure to accelerate particles' neighbours search. The cell size h is the smooth radius of SPH particles. Taken from [2].

4.1 Fast Nearest Neighbor Search

Iterating over all particles in the scene and checking if a given particle is adjacent to the current particle will have $O(n^2)$ time complexity. This complexity can be reduced down to $O(mn)$ by employing the *spatial hashing* method.

4.1.1 Spatial Hashing

The spatial hashing method is an algorithm that enables us to efficiently find nearest neighbors Figure 4.2. Basically, it is a way of using a hash table to subdivide 3D space. Each particle position goes through a hash function, which maps a 3D position to 1D by generating a hash key for each grid cell. The hash function proposed by [10] is as follows

$$hash(\hat{\mathbf{r}}) = (\hat{\mathbf{r}}_x p_1 \ xor \ \hat{\mathbf{r}}_y p_2 \ xor \ \hat{\mathbf{r}}_z p_3) \ mod \ n_H, \tag{4.1}$$

where n_H is the size of the table and

$$\hat{\mathbf{r}}(\mathbf{r}) = (\lfloor \mathbf{r}_x/l \rfloor, \lfloor \mathbf{r}_y/l \rfloor, \lfloor \mathbf{r}_z/l \rfloor)^T \tag{4.2}$$

is the position of a point in 3D space, with cell size l. The three unknowns in (4.1) are

$$p_1 = 73\ 856\ 094$$

$$p_2 = 19\ 349\ 663$$

$$p_3 = 83\ 492\ 791$$

According to analyses from [10] the most efficient table size n_H is

$$n_H = prime(2n), \tag{4.3}$$

where $prime(x)$ is the function that returns a prime number which is $\geq x$ and n is the amount of particles in the simulation. For the cell size we use

$$l = h, \tag{4.4}$$

which is the most optimal, since we want to only include particles that are within the smoothing radius i.e. $\|\mathbf{r}\| > h$.

4.1.2 Spatial Queries

Before being able to quickly access adjacent particles we need to build up the hash table itself. We can do this by iterating twice through the whole set of particles.

The first pass creates a hash key for each particle and puts it in appropriate position in the hash table

$$hash\_table[hash(\hat{\mathbf{r}}(\mathbf{r}_i))] = Particle_i, \tag{4.5}$$

where $\mathbf{r}_i$ is the position of particle i. Each hash_table value has to be a dynamic list, where we will store the adjacent particles with particle i.

The second pass performs the actual particle queries. For a given particle position $\mathbf{r}_Q$ we calculate the bounding box around it as follows

$$BB_{min} = \hat{\mathbf{r}}(\mathbf{r}_Q - (h, h, h)^T), \quad BB_{max} = \hat{\mathbf{r}}(\mathbf{r}_Q + (h, h, h)^T). \tag{4.6}$$

Then we iterate from BB_{min} to BB_{max} in each three axis, creating unique discrete positions pos_D and retrieve a dynamic list L for each such position

$$L = hash\_table[hash(pos_D)]. \tag{4.7}$$

Finally we iterate over L and check for each particle j if it is within the smoothing radius of the query particle i.e. $\|\mathbf{r}_Q - \mathbf{r}_j\| \leq h$ and add it to a resulting particle container.

4.2 Terrain

We implement the terrain as a heightmap. Because the water will only interact with the surface there's no need to import a 3D mesh with a complex geometry that might cause additional performance issues. It is sufficient to have only the surface of the terrain. That's what a heightmap (or heightfield) is designed for. We import a grayscale bitmap and build the terrain from the values of its

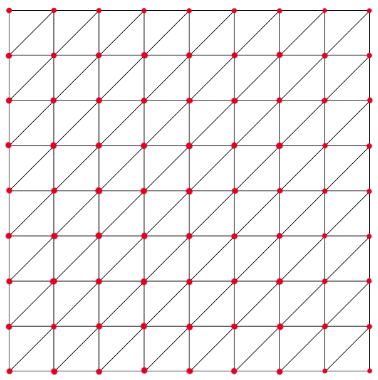


Figure 4.3: Top-view projection on grid accelerated terrain. Red dots represent geometry points in 3D space.

pixels. Each pixel in the bitmap has a value in range from 0 to 255. This value will serve as a height for a 3D point of our terrain. The bitmap is essentially a 2D matrix of pixels. Thus, we can easily build the entire terrain geometry by iterating over the entire bitmap extracting the height on each position. For a terrain point (x, y, z) we obtain its coordinates as follows

$$(x, y, z) = (i, j, h(i, j)), \tag{4.8}$$

where

$$h(i, j) - \textit{height value at } i, j \textit{ position}, \tag{4.9}$$

i is the index that iterates from 0 to Dim_x of the bitmap, j is the index that iterates from 0 to Dim_y.

Looking from top-view projection at our terrain we can see that all points are evenly distanced between one another giving us a uniformly distributed grid (Figure 4.3). We are going to leverage this property to efficiently access any triangle. The distance between points in both directions will be represented by a 2D vector called *cell_size* and is going to be a user input value. Each cell is built up from 4 points and 2 triangles. We can map a 3D point (x, y, z) to a grid cell with index (x_i, y_i) as follows

$$(x_i, y_i) = (\lfloor \frac{x}{cell\_size} \rfloor, \lfloor \frac{y}{cell\_size} \rfloor). \tag{4.10}$$

In the equation (4.10) we don't need the height coordinate z, because we want to figure out where the point is on XY plane.

We can reverse the equation (4.10) as follows

$$(x_w, y_w) = (\lfloor x_i * cell\_size \rfloor, \lfloor y_i * cell\_size \rfloor). \tag{4.11}$$

Equation (4.11) converts a cell index into world space coordinates. The coordinates of vector (x_w, y_w) will always correspond to coordinates of "top-left" point of the (x_i, y_i) cell in XY projection. By extracting the height of this position we obtain the coordinates of the top-left point in 3D space

$$(x_w, y_w, h(x_w, y_w)). \tag{4.12}$$

By leveraging these properties we can efficiently determine where exactly a SPH particle is located on the grid and access all the appropriate vertices. We initialize the terrain by iterating over the bitmap once and for each (i, j) pair we compose its 3D version by using (4.8) and storing these geometry data in a dedicated buffer.

4.3 Collision

In section 1.4.3 we talked about how a SPH particle will detect and react on a collision with a heightfield. Further, in section 1.4.4 we described how we can employ the Leap-Frog time integration scheme to advance particles based on delta time step. In this section we are going to use them in collision handling algorithm.

Our goal is to register an intersection with a triangle or multiple triangles in 3D space. When a particle intersects the terrain it means that it de facto goes through the surface and ends up underneath it. Because all the triangles are located arbitrarily we have to implement a robust algorithm that will successfully detect any collision with any triangle at any point on the terrain. We can detect whether a particle i intersected a triangle by computing its next position in Δt time step

To detect whether the particle intersected with a given triangle we employ a ray casting technique. The idea is to cast a ray from the particle position along the directional vector. We can easily calculate the directional vector of a particle by subtracting its next position from the current

$$\mathbf{dir} = \mathbf{r}_{t+\Delta t} - \mathbf{r}_t, \tag{4.13}$$

where $\mathbf{r}_{t+\Delta t}$ is obtained by (1.46)

In order to detect a ray-triangle intersection we need to know three things: position, directional vector and the vertices of the triangle. We can efficiently obtain them by employing equations (4.10) and (4.11) to convert particle's position (x, y, z) into a cell index (x_i, y_i) and then obtain the top-left (in XY plane) point coordinates (x_w, y_w). To form a triangle we need to get the adjacent points within the cell. We get them by adding *cell_size* to each coordinate

$$(x_w, y_w)$$

$$(x_w + cell\_size, y_w)$$

$$(x_w, y_w + cell\_size)$$

$$(x_w + cell\_size, y_w + cell\_size)$$

By extracting the height value from the heightfield and assigning it to z coordinate we get 3D positions of these points

$$A = (x_w, y_w, h(x_w, y_w))$$

$$B = (x_w + cell\_size, y_w, h(x_w, y_w))$$

$$C = (x_w, y_w + cell\_size, h(x_w, y_w))$$

$$AA = (x_w + cell\_size, y_w + cell\_size, h(x_w, y_w))$$

We use Moller-Trumbore algorithm to detect a ray-triangle intersection. As an input we have to provide a position, a direction and three triangle's vertices. The output is a boolean value whether the collision has happened or not and the parameter t. This parameter determines the intersection point on the ray in the ray equation

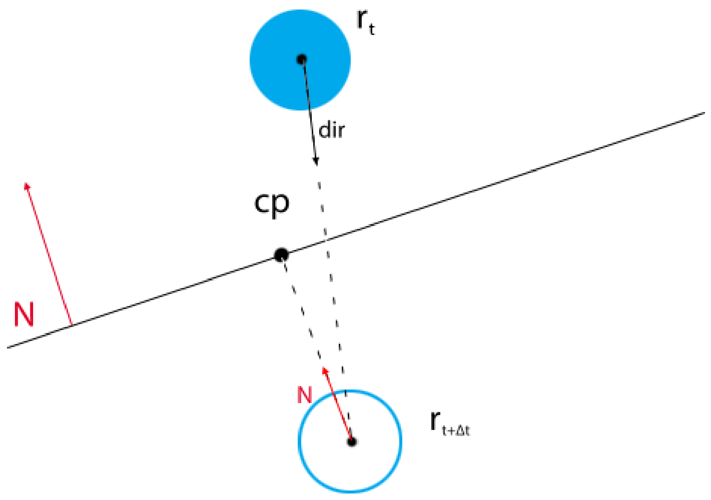


Figure 4.4: Particle is projected back onto the triangle it intersects with.

$$\mathbf{R} = \mathbf{O} + t * \mathbf{d}, \tag{4.14}$$

where **O** is the ray's origin and **d** is the direction vector

In our case, we want to determine the collision once the particle is underneath the surface, so we need to provide the next position $\mathbf{r}_{t+\Delta t}$, reversed direction **-dir** and three vertices of a triangle

$$\textbf{bool}\ intersection = MollerTrumbore(\mathbf{r}_{t+\Delta t}, -\mathbf{dir}, \mathbf{A}, \mathbf{B}, \mathbf{C}, \&t) \tag{4.15}$$

If particle intersects a triangle it means that a collision happened and the particle must be projected back onto the surface (triangle). We calculate the contact point **cp** by applying the Moller-Trumbore one more time, but this time as a directional vector we specify the triangle's normal vector

$$\textbf{bool}\ intersection = MollerTrumbore(\mathbf{r}_{t+\Delta t}, \mathbf{n}_{ABC}, \mathbf{A}, \mathbf{B}, \mathbf{C}, \&t). \tag{4.16}$$

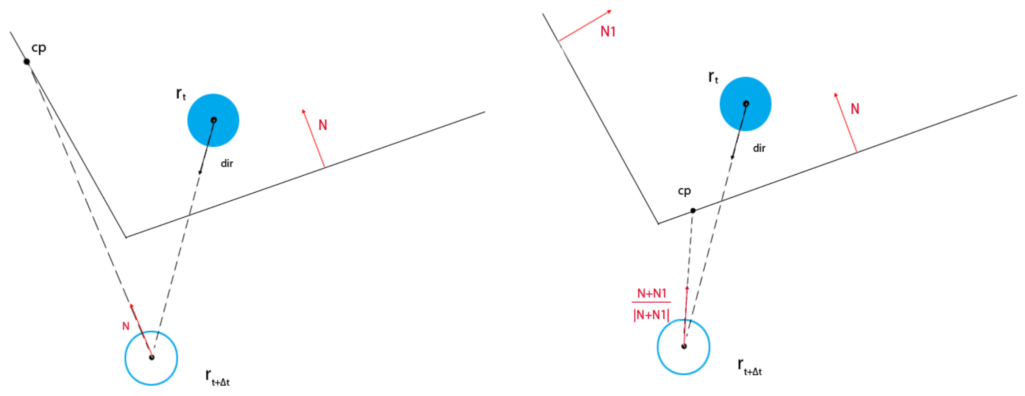
The contact point is then obtained as follows

$$\mathbf{cp} = \mathbf{r}_{t+\Delta t} + t * \mathbf{n}_{ABC}. \tag{4.17}$$

The penetration depth d can be easily calculated

$$d = \|\mathbf{cp} - \mathbf{r}_{t+\Delta t}\| \tag{4.18}$$

Figure 4.4 depicts how SPH particle is projected back onto triangle.



(a) Particles intersects with a triangle, but failed to be projected on it along the triangle's normal **N**.

(b) Particles is projected on the triangle by calculating the interpolation of normal vectors of the triangle it intersects with and the adjacent triangle.

Figure 4.5: Edge case, when SPH particle is projected by means of interpolation of normal vectors.

The last thing to solve is edge cases. When the particle intersects the triangle somewhere on the edge we can have hard time projecting it back as shown on Figure 4.5a). We solve this problem by taking the both triangle's normal, calculating their interpolation and then trying to project the particle on either of those triangles Figure 4.5b).

The second edge case is when the particle intersects the triangle on its corner. Here we adhere to the same principle only this time we want to interpolate the normal vectors of six triangles and try to project the particle on each of them.

Let's wrap up the section with pseudo code of the entire algorithm

```
function COLLISION($posCur$, $posNext$, &$cp$, &$norm$) -> bool
    $dir \leftarrow posNext - posCur$
    $cellIdx \leftarrow toCellIdx(posCur)$
    $cellIdxNext \leftarrow toCellIdx(posNext)$
    if $posCur = posNext$ then                                ▷ if in the same cell
        $cellTriangles[] \leftarrow getCellTrinagles(cellIdx)$
        if $MollerTrumbore(posNext, -dir, cellTriangles[0], \&t)$ then
            $norm \leftarrow cellTriangles[0].norm$
            if $MollerTrumbore(posNext, norm, cellTriangles[0], \&t)$ then
                $cp \leftarrow posNext + t * norm$
                return true
            else if edge case within the same cell then
                $mapBetweenTrianglesOfTheSameCell(\&t, \&norm)$
                $cp \leftarrow posNext + t * norm$
                return true
            else if edge case between cells then
                $mapBetweenTrianglesWithAdjacentCell(\&t, \&norm)$
                $cp = posNext + t * norm$
                return true
            else if corner case then
                $mapBetweenSixTrianglesAtTheCorner(\&t, \&norm)$
                $cp \leftarrow posNext + t * norm$
                return true
            end if
        end if
        if $MollerTrumbore(posNext, -dir, cellTriangles[1], \&t)$ then
            $norm \leftarrow cellTriangles[1].norm$
            if $MollerTrumbore(posNext, norm, cellTriangles[1], \&t)$ then
                $cp \leftarrow posNext + t * norm$
                return true
            else if edge case within the same cell then
                $mapBetweenTrianglesOfTheSameCell(\&t, \&norm)$
                $cp \leftarrow posNext + t * norm$
                **return** true
            **else if** edge case between cells **then**
                $mapBetweenTrianglesWithAdjacentCell(\&t, \&norm)$
                $cp = posNext + t * norm$
                **return** true
            **else if** corner case **then**
                $mapBetweenSixTrianglesAtTheCorner(\&t, \&norm)$
                $cp \leftarrow posNext + t * norm$
                **return** true
            **end if**
        **end if**
    **else**                                                ▷ If in different cells
        **if** corner case **then**
            $mapBetweenSixTrianglesAtTheCorner(\&t, \&norm)$
            $cp \leftarrow posNext + t * norm$
            **return** true
        **else**
            $cellTriangles[] \leftarrow getCellTriangles(cellIdxNext)$
            **if** $MollerTrumbore(posNext, -dir, cellTrinagles[0], \&t)$ **then**
                $norm \leftarrow cellTriangles[0]$
                **if** $MollerTrumbore(posNext, norm, cellTrinagles[0], \&t)$ **then**
                    $cp \leftarrow posNext + t * norm$
                    **return** true
                **else**
                    $mapBetweenTrianglesWithAdjacentCell(\&t, \&norm)$
                    $cp \leftarrow posNext + t * norm$
                    **return** true
                **end if**
            **end if**
            **if** $MollerTrumbore(posNext, -dir, cellTrinagles[1], \&t)$ **then**
                $norm \leftarrow cellTriangles[0]$
                **if** $MollerTrumbore(posNext, norm, cellTrinagles[1], \&t)$ **then**
                    $cp \leftarrow posNext + t * norm$
                    **return** true
                **else**
                    $mapBetweenTrianglesWithAdjacentCell(\&t, \&norm)$
                    $cp \leftarrow posNext + t * norm$
                    **return** true
                **end if**
            **end if**
        **end if**
        **return** false
    **end if**
**end function**
```

4.4 Hydraulic Erosion Model

In this section we will take a look on a high level, how an individual step of erosion is implemented. Here we will give definitions of different containers and functions that will be later used in Subsection 4.8 The algorithm of hydraulic erosion simulation can be broken down, according to [3] into the following steps:

1. Calculate fluid and boundary forces.
2. Calculate sediment transfer among SPH particles.
3. Calculate erosion and deposition exchange between SPH and boundary particles.
4. Update terrain height according to the change of sediment in boundary particles.

Let's elaborate on each step individually.

4.4.1 Boundary Forces

In this step the following things must be carried out:

1. Seed the cell in which a SPH particle i is located.
2. Detect the nearest boundary particles to the SPH particle i.
3. Calculate friction for particle i.
4. Add particle i to the neighbour list of all the boundary particles that it detected.
5. Append the detected boundary particles by SPH particle i to the container with all the detected boundary particles in this iteration.
6. Allocate buffer for each sph-boundary advection terms.

As we discussed in section 4.2 the position of a SPH particle can be translated into cell index to figure out where exactly on the terrain it is located. Once we figure out which cell it is we want to seed boundary particles over the cell's triangles (in subsection 4.6.3 we will talk about how we do this). The set of seeded particles are stored in a kd-tree (subsection 4.6.2). We use a hash table named *m_SeededCells* to store the boundary particles of each cell. The key of this table is a cell index and the value is kd-tree with boundary particles.

After we detected nearest boundary particles of a given SPH particle i we calculate friction(no-slip), as described in subsection 1.5.1.

Next we want to remember all the nearest boundary particles of the current particle i. We store all the detected boundary particles in current iteration in an unordered set $m\_NearestBoundaryParticles$. So, we easily merge the boundaries of particle i into $m\_NearestBoundaryParticles$.

Also we want to remember, what SPH particle is associated with what boundary particle, so we create a hash table, where we store a list of fluid particles for each boundary particles $m\_FluidsOfBoundary$. The key represents the id (while seeding each boundary particles is assigned one) and the value is the list of fluid particles.

Knowing how many boundary particles were detected and with what fluid particles each of them is associated we can allocate a container for each fluid-boundary interaction pairs dC_BP. This container will store the advection terms of each such pair from equation (1.69).

4.4.2 Sediment Transfer

SPH particles pass material to each other before it is settled back onto the surface. This process is described by the advection and diffusion equations (1.64) and (1.68). Both of them require to know the gradient of cubic spline $F(r,h)$, which according to [4] is

$$F(q,h) = \begin{cases} \frac{1}{4\pi h^3} * (12 * (1-q)^2 - 3 * (2-q)^2) & q < 1, \\ -\frac{1}{4\pi h^3} * 3 * (2-q)^2 & q \geq 1, \end{cases} \tag{4.19}$$

where

$$q = \|\mathbf{r}_{ij}\| * \frac{2}{h}. \tag{4.20}$$

Similarly as in Subsection 4.4.1 we store the advection term of each $i - j$ interaction pairs between all fluid particles. We name this collecion dC.

4.4.3 Deposition

Computing the deposition we store the advection term for each fluid-boundary interaction in $dC\_BP[ij]$.

The gradient of the cubic spline is the same as for sediment transfer (4.19). The part prior to the advection term in the equation (1.69) is responsible for converting the percentage of occupied volume C into the actual mass. This enables us to define the C to mass conversion as follows

$$C\_TO\_MASS(C) = \rho_s \frac{m_j}{\rho_j}. \tag{4.21}$$

We've already familiarized with these variables in Subsection 1.5.4

4.4.4 Erosion

The equation of erosion requires to know the relative velocity of current fluid particle to the surface, as it is stated in equation (1.55). We can obtain this velocity via dot product of the particle's velocity and the normal vector of the triangle

$$v_{rel} = \|\mathbf{v}\| - abs(\mathbf{v} \cdot \mathbf{n}), \tag{4.22}$$

where $\mathbf{v}$ is the SPH particles velocity and $\mathbf{n}$ is the normal vector of the surface triangle.

Because the equation (1.57) only describes the change of mass on boundary particle b we must update the sediment ration of SPH particle j manually. We do so by converting the eroded mass produced by (1.57) for each SPH particle j and convert it to C as follows

$$MASS\_TO\_C(m) = \frac{1}{\rho_s * m_j/\rho_j} * m, \tag{4.23}$$

where m is the output of equation (1.57).

4.4.5 Terrain Modification

At this point all the detected boundary particles have their delta mass dM calculated. Each such particle is located on some triangle, this means that we can calculate the total mass for each triangle. We store the total masses of both triangles of a cell in a hash table named $m\_TriangleMass$, where key is cell index and value is pair of two floats representing the masses of each triangle individually. Also we keep track only of those cells whose boundary particles were detected and stored in $m\_NearestBoundaryParticles$ container. We store those cells in a vector named $cellsToUpdate$. After all the boundary particles have been iterated over and masses accumulated we update the geometry of cells from the $cellsToUpdate$ container.

4.5 Fluid Particles

In this subsection we'll talk about what fields a fluid particle structure must have. Unlike a regular SPH particle we need to extend ours due to the fact that we implicitly represent sediment particles. We append $float\ Sedim$ that represents how much volume is occupied by sediment in per cents and $float\ Sedim\_delta$ that represents the change of occupied volume in the current iteration. The structure below defines a SPH particle

```
struct FluidParticle
{
    Vec3f Position;
    Vec3f Velocity;
    Vec3f Acceleration;
    float Density;
    float Pressure;
    Vec3f PressureForce;
    Vec3f ViscocityForce;
    Vec3f GravityForce;
    Vec3f SurfaceForce;
    Vec3f SurfaceNormal;
    Vec3f BoundaryForce;
    float Sedim;
    float Sedim_delta;
}
```

4.6 Boundary particles

In Subsection 1.5.1 we discussed that our erosion model uses boundary particles as a mediator between the fluid and the terrain. SPH particles first cause impact over boundary particles and then this impact is transformed into the terrain's changes. With that in mind we need to have them seeded throughout the whole terrain, because SPH particles must access them a any give position on the terrain. Having all of them everywhere at once will be inefficient that's why we will seed them only at places where SPH particles are.

4.6.1 Definition

Similarly as with fluid particles let's define the structure for boundary particles.

```
struct BoundaryParticle
{
    Vec3f Position;
    Vec3f TriangleNormal;
    char Triangle;
    float dM;
}
```

The *char Triangle* field denotes on which of the two triangles of a cell a given boundary particle located. In this implementation we use either $'A'$ or $'B'$. The *float dM* denotes the change of mass of the current particle described by (1.69).

4.6.2 Kd-tree

In section 4.2 we defined our terrain as a heightfield. We made it grid-accelerated and established that each cell consists of two triangles. When a SPH particle approaches the surface it detects all the boundary particles within its smooth radius as depicted on Figure 1.8. In order to efficiently find the closest boundaries we store them in a kd-tree. For each cell we build a new kd-tree and store its particles in there. Each kd-tree is then stored in a hash table, where key is a cell index and value is a pointer to allocated kd-tree. When a fluid particle enters the cell we apply nearest neighbours search algorithm and detect, which particles are within the smooth radius.

4.6.3 Seeding

Before we store any particles in a kd-tree we need to seed them. The principle of this seeding algorithm was derived from famous *scan line algorithm.* We seed the particles in scan line fashion for a triangle in 3D space. Consider the Figure 4.6. We have a cell composed of two triangles **ABC** and **DBC**. Let's say we want to seed particles over the triangle **ABC**. We do so by moving from **C** to **A** along the **AC** vector with Δs step and at the same time we move along the hypotenuse **BC**. Thus, we obtain two points to compose a line segment that is parallel with the cathetus **AB**. Finally, we move along this line segment with the same Δs step from left to right i.e. from cathetus **AC** to hypotenuse **BC** and place particles. Because all the points of both triangles are 3D points we virtually fill the triangle in 3D space. The same principle applies for the second triangle only now we move along the **DC** vector.

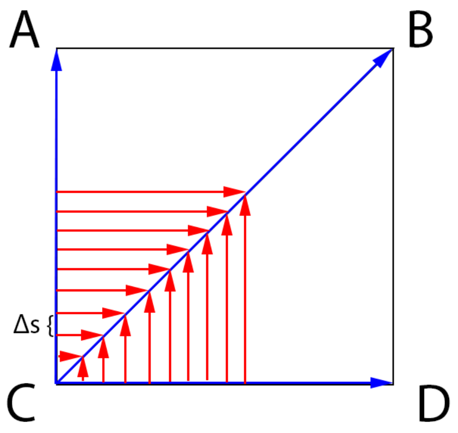


Figure 4.6: Seeding triangles of a cell in 3D space.

4.7 Physical Parameters

In this section we describe what actual quantities we use in the simulation.

4.7.1 Fluid

To achieve the most stable and reliable behaviour of fluid its physical parameters must be carefully chosen. The search of such values that enable to achieve a high level of realism and at the same time keep the simulation stable might be a tough challenge. Below is the Table 4.1 for water material taken from [1] with physical parameters of fluid quantities in the standard System International (SI) units, that provide good simulation results in terms of both realism and stability. In this thesis we focus only on water material.

4.7.2 Erosion

Throughout the Section 1.5 a number of constants are used that weren't defined so far. Most of them were taken from [3], however the boundary friction constant μ_{bf} was determined experimentally. The value that gives the most realistic behaviour varies between 0.1 and 0.25. In this work 0.11 was chosen. Table 4.2 presents the constants used in erosion model.

Description	**Symbol**	**Value**	**Unit**
Density (rest)	ρ_0	998.29	$\frac{kg}{m^3}$
Mass (particle)	m	0.02	kg
Viscosity	μ	3.5	$Pa \cdot s$
Surface tension	σ	0.0728	$\frac{N}{m}$
Gas stiffness	k	3	J
Restitution	c_R	0	n/a
Smooth radius	h	0.0457	m

Table 4.1: Physical parameter for water simulation

4.8 The Simulation Algorithm

4.8.1 Build the Terrain

1. Read the bitmap with the dimensions Dim_x, Dim_y and store it in a 1D array $m\_Heightfield$.
2. For z from 0 to Dim_x do:

    a) For x from 0 to Dim_y do:

        i. Get height at (x, z) using (4.9): $y = h(x, z)$.
        ii. Add the coordinates of the 3D point into *vertexData* buffer considering the cell size:
        iii. $vertexData.append(x * cell\_size)$.
        iv. $vertexData.append(y)$.
        v. $vertexData.append(z * cell\_size)$.

The *vetexData* buffer is regenerated each time the terrain gets updated and is only used for passing the geometry data to OpenGL pipelines to render the visual output. The origin of the terrain is at $(0, 0)$, which theoretically can be changed, but is not necessary in this simulation. When a particle interacts

Description	Symbol	Value	Unit
Distance between boundary particles	Δs	0.02285	m
Boundary friction constant	μ_{bf}	0.11	n/a
Erosion strength	K_{ε}	0.1	n/a
Critical shear stress	τ_c	3	Pa
Sediment density	ρ_s	3	$\frac{kg}{m^3}$
Maximum solid volume fraction	C_{max}	0.7	n/a
Exponent (Richardson-Zaki)	e	4.5	n/a
Coefficient of diffusion	D	0.1	n/a
Sediment particle radius	r_s	0.00001	m

Table 4.2: Physical parameter for erosion simulation

with the terrain it translates its 3D position into a cell index, in which it is currently located by using (4.10). Knowing the cell index coordinates we access the heightfield buffer and extract a corresponding height of each point of the cell. Thus, we don't need to access *vertexData* buffer to obtain point's coordinates.

4.8.2 Initialize SPH system

1. Initialize fluid constants from the Table 4.1.

2. Allocate n particles and set them positions, velocities, $acceleration = 0$, $sediment = 0$, $sediment\_delta = 0$.

3. Create the spatial hashing data structure using (4.3) and (4.4).

4. Create an empty **unordered_set** $m\_NearestBoundaryParticles$, where we will store all the detected boundary particles in a given iteration that are within the smooth radius of fluid particles.

5. Create an empty **unordered_map** $m\_FluidsOfBoundary$ that will store a list of fluid particles that detected a given boundary particle.

6. Create an empty vector dC defined in subsection 4.4.2, where we will store all the interaction pairs between fluid particles.

7. Create an empty vector dC_BP defined in section 4.4.3, where we will store all the interaction pairs between a fluid and a boundary particle.

We can omit the initialization of the leap-frog integrator for each particle here. It will be done automatically while calculating the acceleration term in further subsection (Subsection 4.8.11) in the very first iteration of simulation.

4.8.3 Compute Density and Pressure

Before we begin iterating through the particles we insert them into the spatial hashing data structure and build a list of neighbours for each of them using the spacial query from Subsection 4.1.2. Let's call this procedure $RebuildTable()$. It clears the table from the previous iteration and builds it back for the current one.

$RebuildTable()$
(The following for-loop is parallelised with OpenMP)
For each particle i do:

1. Iterate through each of its neighbour j from the neighbourhood N_i and compute its density ρ_i using (1.25)

2. Compute pressure using (1.32).

4.8.4 Compute Internal Forces

(The following for-loop is parallelised with OpenMP)
For each particle i do:

1. Iterate through each of its neighbour j from the neighbourhood N_i.
2. Compute the pressure force using (1.33).
3. Compute the viscosity force using (1.35).
4. Compute the surface normal using (1.39).
5. $f^{internal} = f^{pressure} + f^{viscosity}$

4.8.5 Compute External Forces

(The following for-loop is parallelised with OpenMP)
For each particle i do:

1. Compute the gravity force using (1.36).
2. Compute the color field using (1.38).
3. Compute the surface curvature using (1.40).
4. Compute the surface tension force using (1.41).
5. $f^{external} = f^{gravity} + f^{surface}$

4.8.6 Compute Boundary Forces

(The following for-loop is parallelised with OpenMP)
For each particle i do:

1. Seed the cells that are in range of smooth radius of the particle i, which are not seeded, using the technique described in the Subsection 4.6.3.
2. $f^{boundary} = 0$.
3. Find nearest boundary particles to the particle i using the $nearestneighboursearch$ algorithm of kd-tree for each detected cell around particle i and put them to a temporary collection **unordered_set** *nearest_boudary*.
4. If *nearest_boundary* is not empty:
   a) For each boundary particle bp in *nearest_boundary*:
      i. Compute the no-slip using (1.52) and add it to $f^{boundary}$
5. For each boundary particle bp in *nearest_boundary* push back the current fluid particle i into the $m\_FluidsOfBoundary[bp]$.
6. Merge *nearest_boundary* into $m\_NearestBoundaryParticles$.

4.8.7 Compute Sediment Transfer

For each fluid particle i do:

1. For each neighbour particle j from neighbourhood N_i:

   a) Compute the advection term of $i - j$ interaction using (1.64) and set the result to dC.

   b) Compute the diffusion term of $i - j$ interatction using (1.68) and add the result to dC.

   c) If dC of current $i - j$ pair is ≤ 0 then: $sedim\_delta_i$ += $dC[ij]$ else: $sedim\_delta_j$ -= $dC[ij]$.

The size of dC is the number of interaction pairs between fluid particles in a given iteration that can be computed from the neighbours list of each particle from the spatial hashing data structure.

4.8.8 Compute Deposition

For each boundary particle *bp* in $m\_NearestBoundaryParticles$:

1. For each fluid particle fp in $m\_FluidsOfBoundary[bp]$:

   a) If $fp.sedim \leq 0$ then *continue*.

   b) Compute the term $\frac{dC(j)}{dt} from(1.69)$ and put the result to dC_BP.

   c) If $dC\_BP < 0$ for the current $bp - fp$ pair:

      i. Add dC_BP to sediment delta of fp.

      ii. Compute the deposition of material using (1.69) and subtract the result from the mass of the current boundary particle *bp*.

4.8.9 Compute Erosion

For each boundary particle *bp* from $m\_NearestBoundaryParticles$:

1. For each fluid particle fp from $m\_FluidsOfBoundary[bp]$:

   a) Compute the erosion of material using (1.57) and subtract the result from mass of current boundary particle *bp*.

   b) Convert the result of erosion from the previous step into the sediment percentage C and add it to sediment delta of current fluid particle fp.

4.8.10 Update Heightfield

For each boundary particle *bp* in $m\_NearestBoundaryParticles$:

1. Convert *bp.Position* to $cell\_index$.

2. If $cell\_index$ was not discovered before then create $triangleMass[cell\_index] = pair(0.0, 0.0)$ and $cellsToUpdate.append(cell\_index)$.

3. Depending to which triangle the boundary particle *bp* belongs add *bp.dM* to either $triangleMass[cell\_index].first$ or $triangleMass[cell\_index].second$.

4. For each $cell\_index$ in $cellsToUpdate$:

   a) Get geometry of cell triangles $getCellTriangles(cell\_index)$.
   b) Update vertices of both triangles from the previous step in the heightfield using (1.70) and (1.71).
   c) Erase all boundary particles in the cell $cell\_index$ (they will be seeded again in Step 1 of 4.8.6).

4.8.11 Time Integration and Collision Handling

(The following for-loop is parallelised using OpenMP)
For each fluid particle i:

1. $f^{external} += f^{boundary}$.

2. $F_i = f^{internal} + f^{external}$.

3. Compute particle's acceleration a_i using (1.24).

4. Compute $\mathbf{u}_{t+1/2\Delta t}$ and $\mathbf{r}_{t+\Delta t}$ using (1.45) and (1.46).

5. Compute collision using the algorithm from Subsection 4.3.

6. If a collision occurred

   a) Compute penetration depth d using (4.18).
   b) Update the velocity using (1.44).

7. If sediment of current particle i is $sedim_i < 0$ then $sedim_i = 0$

8. $sedim_i += \Delta t * sedim\_delta_i$.

9. If $sedim_i < 0$ then $sedim_i = 0$.

10. Zero out delta sediment of the current particle: $sedim\_delta_i = 0$.

4.8.12 Render

1. Render the terrain using *vertexData* buffer.
2. For each fluid particle i do:
    a) Render a sphere with its center at $\mathbf{r}_i$ and radius r = 0.001.

We do not render the boundary particles, because we need them only to perform the computations.

Chapter 5

Results

In this chapter we will examine what outcomes we obtain when we put to use our implementation from Chapter 4. We will test the fluid individually in order to demonstrate how it behaves in the first place and then we add a terrain into the scene and pour some volume on it and simulate erosion.

5.1 Fluid

In this section we will test, how the Lagrangian solver works. First we will examine how it behaves with different parameter values and then we execute performance tests.

5.1.1 Fluid Parameters

Below is the illustration of a water volume with 1000 particles dropping in the invisible box Figure 5.1. The properties of the fluid are the exact much of those described in Table 4.1.

The water material has very low viscosity as a result its particles are very mobile and agile. When a water volume hits a solid the particles splash all over. If we risen viscosity i.e. risen inner friction between particles the volume will bear a more resemblance with honey. Below is the same volume with higher viscosity Figure 5.2.

Compared to the previous animation the particles don't move around so freely and are less splashy. Their motion is slowed down by friction with each other.

If we try to increase the particle's mass the density field will increase, hence the pressure will increase too. This implies that the particles will repel more from each other and have more space in between them Figure 5.3. They are splashy enough, but too heavy to bounce off too high.

The above tests demonstrate that our Lagrangian solver produces the expected behaviours and has a pretty high degree of realism.

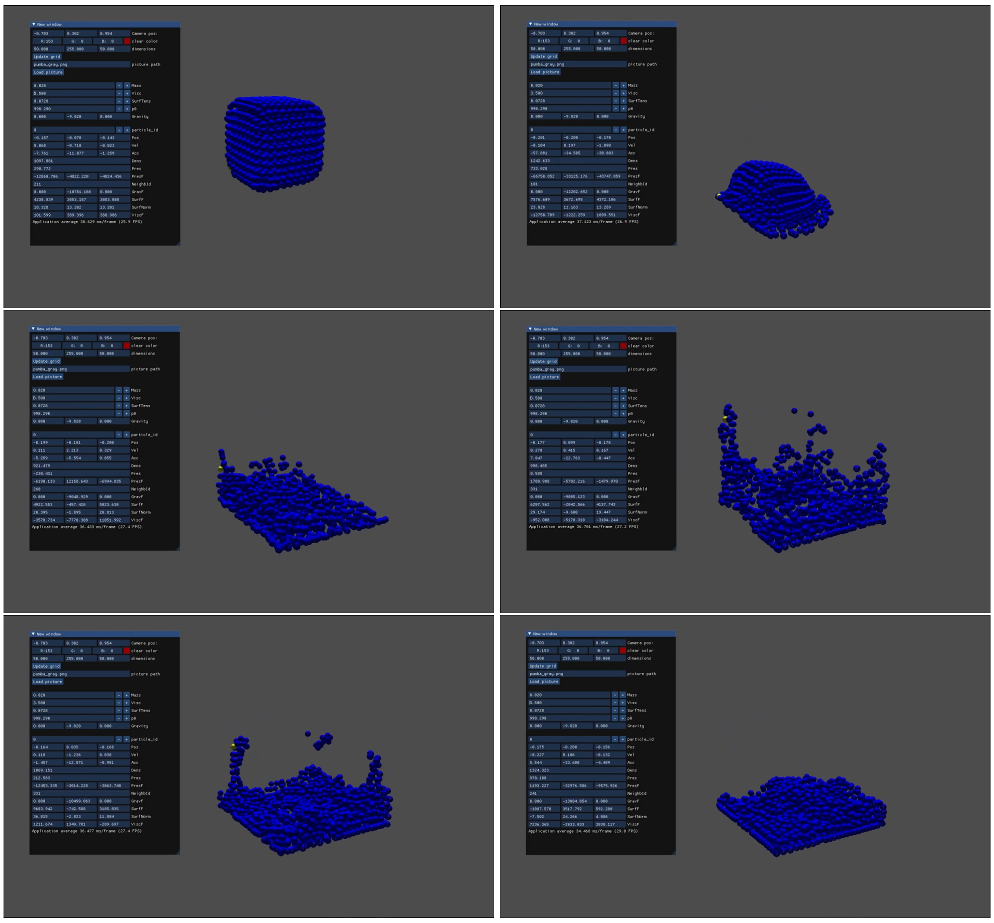


Figure 5.1: 1000 SPH particles with the parameters of water.

5.2 Erosion

Now that we have our water tested let's pour it onto an actual terrain and have the erosion algorithm to do its job.

5.2.1 Ditch

Figure 5.4 features a ditch with a ridge in the middle. The volume of 200 000 SPH particles is poured onto it. The radius of each particle is $r = 0.001m$. The cell size is $cell\_size = 0.1m$. Particles life time is 3000 frames.

The ridge in the middle seeks to being washed out and leveled off with the surface. The material from the ridge is taken away by water particles and deposited towards the area on left from it. The longer the particles exist the more the ridge gets leveled off with the surface.

The ridge in the middle is the spot that gives the most pronounced visual results in fact the entire terrain gets eroded and the surface level throughout

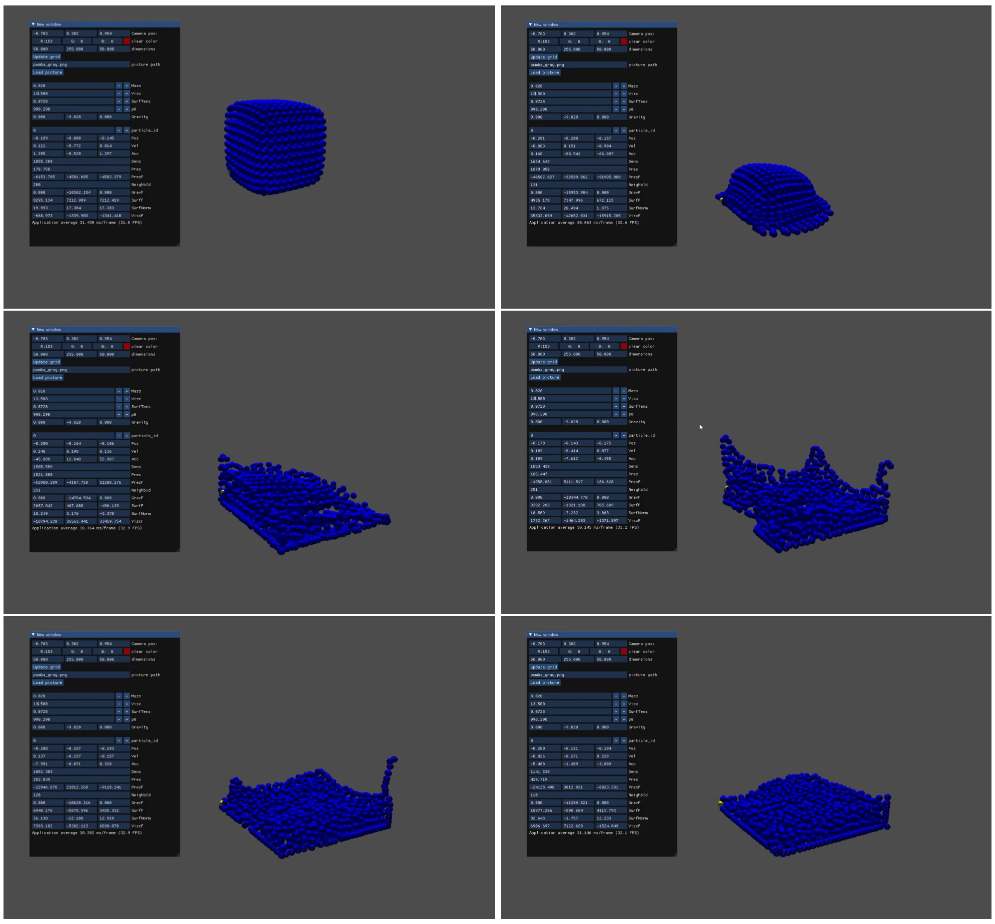
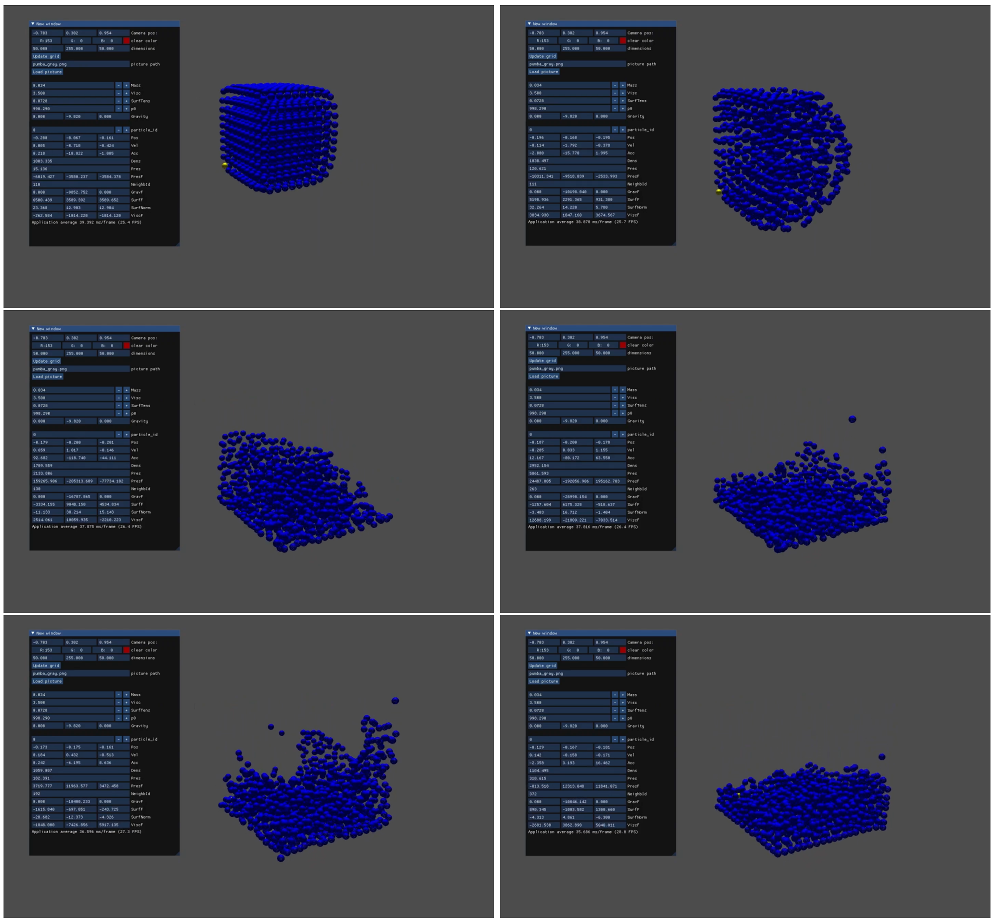


Figure 5.2: 1000 SPH particles water particles with increased viscosity (visc=17.5).

the scene lowers a little. The comparison before/after is illustrated on Figure 5.5.

5.2.2 Meander

The next scene is a meander. Similarly as in the previous test a volume of water with 200 000 particles is poured on. The particles lifetime is 2000 frames here. As can be seen of Figure 5.6 the material reached by fluid gets eroded while the part of the meander where fluid didn't reach remains untouched.



```
▼ New window
-0.703      0.302       0.954      Camera pos:
 R:153       G:  0       B:  0     clear color
50.000      255.000     50.000     dimensions
Update grid
pumba_gray.png                     picture path
Load picture

0.034                       - +    Mass
3.500                       - +    Visc
0.0728                      - +    SurfTens
998.290                     - +    p0
0.000       -9.820      0.000      Gravity

0                           - +    particle_id
-0.280      -0.067      -0.161     Pos
0.005       -0.718      -0.424     Vel
0.218       -10.822     -1.005     Acc
1003.335                           Dens
15.136                             Pres
-6019.427   -3580.237   -3584.378  PresF
110                                NeighbId
0.000       -9852.752   0.000      GravF
6500.439    3589.392    3589.652   SurfF
23.368      12.903      12.904     SurfNorm
-262.584    -1014.220   -1014.120  ViscF
Application average 39.392 ms/frame (25.4 FPS)
```

```
▼ New window
-0.703      0.302       0.954      Camera pos:
 R:153       G:  0       B:  0     clear color
50.000      255.000     50.000     dimensions
Update grid
pumba_gray.png                     picture path
Load picture

0.034                       - +    Mass
3.500                       - +    Visc
0.0728                      - +    SurfTens
998.290                     - +    p0
0.000       -9.820      0.000      Gravity

0                           - +    particle_id
-0.196      -0.168      -0.195     Pos
-0.114      -1.792      -0.370     Vel
-2.080      -15.770     1.995      Acc
1038.497                           Dens
120.621                            Pres
-10311.341  -9510.839   -2533.993  PresF
111                                NeighbId
0.000       -10198.840  0.000      GravF
5198.936    2291.365    931.380    SurfF
32.264      14.220      5.780      SurfNorm
3034.930    1847.168    3674.567   ViscF
Application average 38.870 ms/frame (25.7 FPS)
```

```
▼ New window
-0.703      0.302       0.954      Camera pos:
 R:153       G:  0       B:  0     clear color
50.000      255.000     50.000     dimensions
Update grid
pumba_gray.png                     picture path
Load picture

0.034                       - +    Mass
3.500                       - +    Visc
0.0728                      - +    SurfTens
998.290                     - +    p0
0.000       -9.820      0.000      Gravity

0                           - +    particle_id
-0.179      -0.200      -0.201     Pos
0.659       1.017       -0.146     Vel
92.602      -118.740    -44.111    Acc
1709.559                           Dens
2133.806                           Pres
159265.906  -205313.609 -77734.102 PresF
130                                NeighbId
0.000       -16787.865  0.000      GravF
-3334.155   9048.150    4534.834   SurfF
-11.133     30.214      15.143     SurfNorm
2514.061    10059.935   -2210.223  ViscF
Application average 37.875 ms/frame (26.4 FPS)
```

```
▼ New window
-0.703      0.302       0.954      Camera pos:
 R:153       G:  0       B:  0     clear color
50.000      255.000     50.000     dimensions
Update grid
pumba_gray.png                     picture path
Load picture

0.034                       - +    Mass
3.500                       - +    Visc
0.0728                      - +    SurfTens
998.290                     - +    p0
0.000       -9.820      0.000      Gravity

0                           - +    particle_id
-0.187      -0.200      -0.170     Pos
-0.205      0.033       1.155      Vel
12.167      -80.172     63.550     Acc
2952.154                           Dens
5861.593                           Pres
24487.805   -192056.906 195162.703 PresF
263                                NeighbId
0.000       -28990.154  0.000      GravF
-1257.604   6175.328    -518.637   SurfF
-3.403      16.712      -1.404     SurfNorm
12688.199   -21809.221  -7833.514  ViscF
Application average 37.816 ms/frame (26.4 FPS)
```

```
▼ New window
-0.703      0.302       0.954      Camera pos:
 R:153       G:  0       B:  0     clear color
50.000      255.000     50.000     dimensions
Update grid
pumba_gray.png                     picture path
Load picture

0.034                       - +    Mass
3.500                       - +    Visc
0.0728                      - +    SurfTens
998.290                     - +    p0
0.000       -9.820      0.000      Gravity

0                           - +    particle_id
-0.173      -0.175      -0.161     Pos
0.184       0.432       -0.513     Vel
0.242       -6.195      8.636      Acc
1059.087                           Dens
182.391                            Pres
3719.777    11963.577   3472.458   PresF
192                                NeighbId
0.000       -10400.233  0.000      GravF
-1615.840   -697.051    -243.725   SurfF
-28.682     -12.373     -4.326     SurfNorm
-1848.000   -7426.856   5917.135   ViscF
Application average 36.596 ms/frame (27.3 FPS)
```

```
▼ New window
-0.703      0.302       0.954      Camera pos:
 R:153       G:  0       B:  0     clear color
50.000      255.000     50.000     dimensions
Update grid
pumba_gray.png                     picture path
Load picture

0.034                       - +    Mass
3.500                       - +    Visc
0.0728                      - +    SurfTens
998.290                     - +    p0
0.000       -9.820      0.000      Gravity

0                           - +    particle_id
-0.129      -0.167      -0.181     Pos
0.142       -0.158      -0.171     Vel
-2.358      3.193       16.462     Acc
1104.495                           Dens
318.615                            Pres
-813.518    12313.048   11841.071  PresF
372                                NeighbId
0.000       -10846.142  0.000      GravF
890.345     -1003.582   1300.660   SurfF
-4.313      4.861       -6.300     SurfNorm
-2681.530   3862.898    5040.811   ViscF
Application average 35.686 ms/frame (28.0 FPS)
```

Figure 5.3: 1000 SPH water particles with increased mass and default viscosity (mass=0.034)

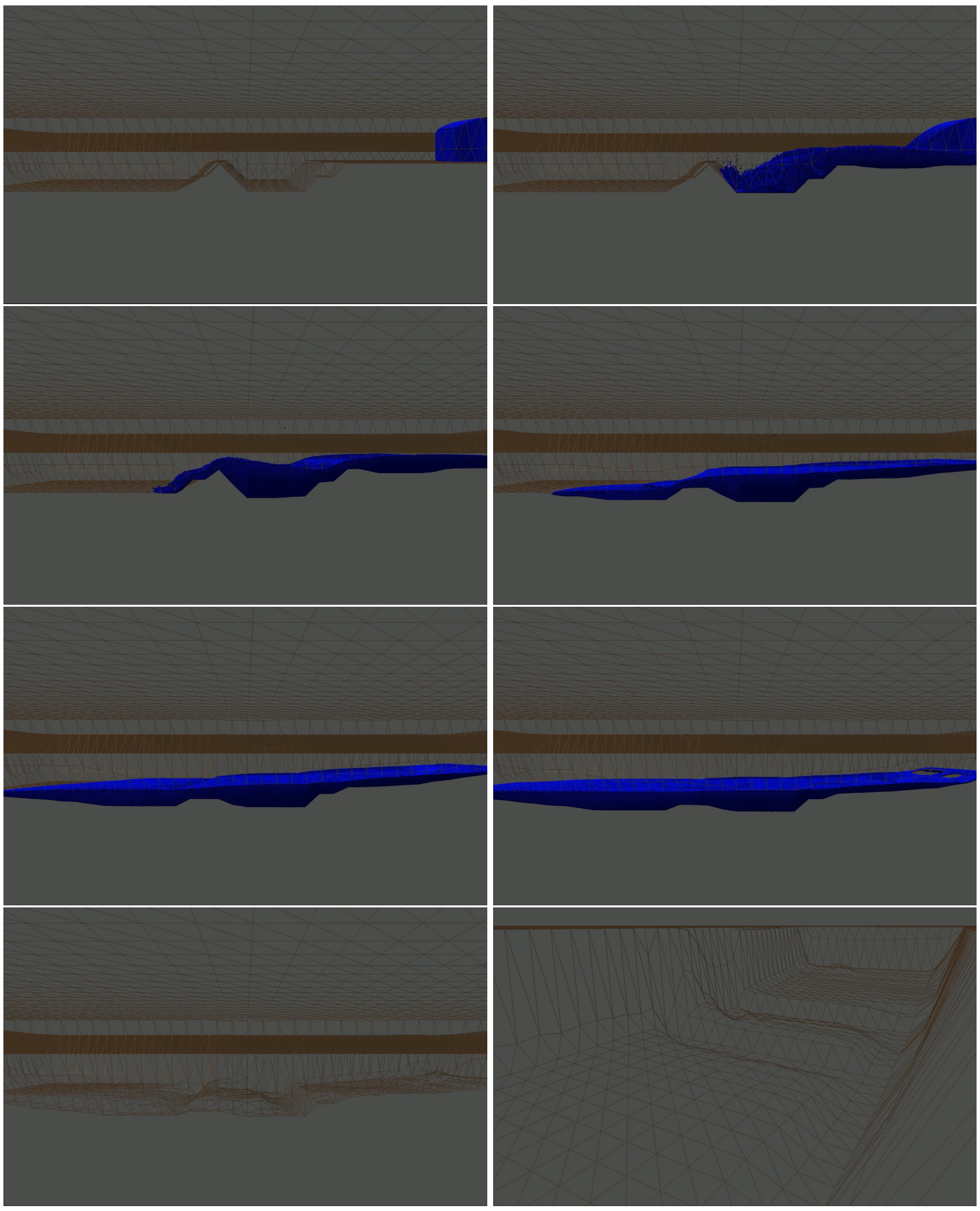


Figure 5.4: A ditch with a ridge. 200 000 SPH particles.

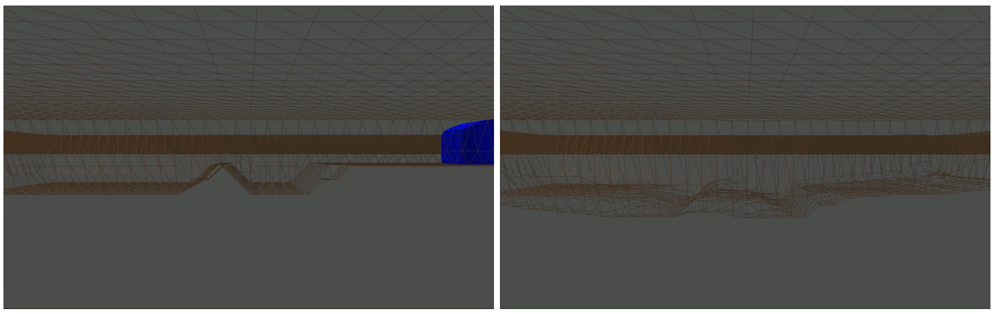


Figure 5.5: Before/after. The ditch with the ridge. 200 000 SPH particles.

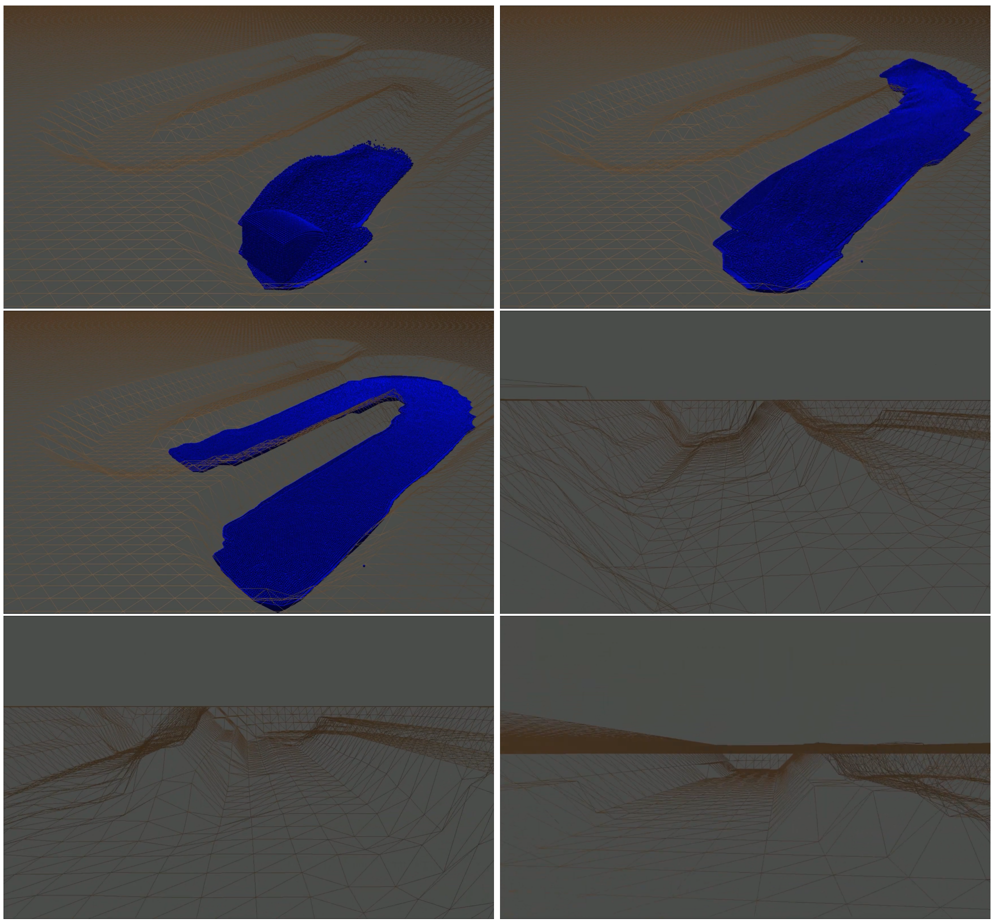


Figure 5.6: A meander. 200 000 SPH particles.

Chapter 6

Tests

6.1 User tests

Script

1. Run the simulation
2. Pause the simulation and reset it
3. Set the number of particles to 100000 and run the simulation
4. Set the number of particles to 1000 and particles lifetime to 100
5. Upload a new map and change the fluid volume origin
6. Run the simulation and change the fluid parameters in real time
7. Save simulation frames

Questionnaire

1. When I asked to change something in the program was it clear how to do it?
2. What are your thoughts about the graphical interface? Was it clear to use?
3. What would you call the worst experience while using the program?
4. What did you like the most while using the program?

Test 1

At the very beginning the user was intuitively clicking on mouse and pressing arrows. The first question that popped up was ”How to enable the cursor?”. Next ”How to make the app to be full screen?”.

Turning the simulation on for the first time wasn’t an issue, because the user had familiarized with the Help Window. Neither was the pause and reset (item 2 in the script).

When asked to change the number of particles (item 3), after doing so the user didn’t figure out that he needs to press the Initialize button to apply the changes. At this point I had to explain that whenever a change is performed in the Initialization section it has to be applied with the Initialize button. When asked to change particles lifetime (item 4) the user applied the changes without additional hints. From that point on every time when the user changed something in the Initialization section he used the Initialize button to confirm the changes.

Uploading a new map and changing the fluid origin (item 5) wasn’t a big issue, neither was changing the parameters of the fluid in real time (item 6).

When the user was asked to save the simulation frames he knew how to do so from the Help Window, however struggled to find the location, where the results were saved (item 7).

While using the program the user tried to close the UI windows by pressing ESC button that caused exiting the program.

Test 1 Survey

Q: When I asked to change something in the program was it clear how to do it?
A: Yes. I couldn’t figure out how to change the map at first, because the button name was ”Load picture” and the label name was ”Picture path”. It would’ve been better if at least the button was named ”Load map”.

Q: What are your thoughts about the graphical interface? Was it clear to use?
A: Yes, overall it’s pretty clear and self-explanatory. The issue was only with map loading. And the section name labels are unusually at the right, I am used to see them on the left.

Q: What would you call the worst experience while using the program?
A: The program works slowly with big amounts of particles. The controls are slightly non-intuitive. The pause button is F and the interface button is R, I’d rather expect them to be P and I. You have F1 and F2 keys that toggle the view of the polygons, but you forgot to include them in the Help Window.

The same story with the Shift and Space buttons that slow down/accelerate the camera movement.

Q: What did you like the most while using the program?
A: I liked how interactive it was. I liked that I could upload a map in real time, change the volume placement and adjust number of particles.

Test 2

The user opened up the application and right off the bat figured out, how to navigate in it without reading anything so far. He then went ahead to read the Help Window and ran this simulation (item 1).

I instructed him to pause and reset the simulation (item 2), he easily did so.

Then I told the user to change the number of particles (item 3). Without no additional hints he immediately went ahead and opened up the UI window and quickly found the filed which is responsible for the number of particles. He set the appropriate value and immediately ran the simulation omitting the Initialize button. After a short time he asked me what was wrong. I told him the he did not apply the changes. He went back to the UI window and the very first thing he did was clicking on the Initialize button. I revealed that he must use this button every time when something is changed in the Initialization section.

When I asked to change the particles lifetime (item 4) he did so without any hesitation and this time did not forget to click the Initialize button.

When asked to change the map (item 5) the user hesitated slightly, but eventually got to the appropriate field. After changing the map with my help the user asked me how he could move the water volume. I said that it could be done in the UI window. He found the volume origin field and figured out that he can assign it the camera coordinates from the Info section without me explaining anything.

Then I asked him to change the terrain color in the Run time section (item 6). The user didn't figure out that he could click on the color icon and choose a new color via the color picker window. Instead, he went ahead to search for an RGB value on the web.

Finally, I told the user that he could save the frames of simulation (item 7) by checking the appropriate checkbox in the UI window. The user checked the checkbox and went ahead to run the simulation to see what would happen. He then asked me, where he could find the saved frames.

Test 2 Survey

Q: When I asked to change something in the program was it clear how to do it?

A: Yes, because there is only one interface window it was clear that I had to do something in there. Everything is clear except for the map change.

Q: What are your thoughts about the graphical interface? Was it clear to use?
A: Yes, it is very intuitive. I didn't like the shortenings of fluid parameters. For instance I did not understand that "Visc" means viscosity and "Dens" density. Also the section labels are on the left, but I think it would be good to have them in the middle.

Q: What would you call the worst experience while using the program?
A: I didn't like the hotkeys. I think that having pause on the Space bar is better then F key, it would be easier to remember.

Q: What did you like the most while using the program?
A: I liked that the navigation is very intuitive like in computer games. I like how the UI shows you all the parameters and you reach anything with ease.

6.2 Performance Test

In this section the results of performance tests are presented. The frequencies are measured in frames per second. The simulation was run on Intel(R) Xeon(R) W-2245 CPU @ 3.90GHz 3.91GHz with 128GB of RAM on Windows 10.

First, the SPH system without erosion is tested. Table 6.1 demonstrates the results with different amount of particles **n**.

While simulating erosion SPH particles will get to interact with boundary particles that they detect. Unlike in the case with calculations of fluid forces interaction between boundary and fluid particles cannot be parallelized and is calculated sequentially. Table 6.2 illustrates the results of erosion simulation with different amount of particles. The choice of map doesn't influence the performance very much, since SPH particles detect boundary particles within their smooth radius. On any map a SPH particle encompasses on average the same amount of boundary particles.

n	**fps**
8000	45.7
10000	36.2
20000	17.9
50000	6.1
100000	3.5

Table 6.1: SPH system performance test.

n	**fps**
5000	13.6
10000	6.1
25000	2.3
50000	1.3
100000	0.7
200000	0.4

Table 6.2: Results from performance test of SPH system + erosion.

Chapter 7

Future Work

This work can be extended to leverage GPU power to parallelize fluid simulation. Although the paralleliztion via CPU was used here it doesn't compare with the GPU parallelization. This will enable to create greater scale simulations with more particles. Such boost in performance will allow to achieve the same results as in this work in much shorter time. A very popular tool for general-purpose computing on GPU is CUDA platform from NVIDIA.

Next thing that might be done is improving the visual outputs. As of right now the water is rendered as a set of spheres. This is good, because we can easily keep track of every particle and immediately spot if somethings goes wrong. It serves a good purpose in developing the system, however once we are confident that the system is stable enough and will want to deliver a graphically pleasant result then improving the visual part sounds like a good idea. Particularly the technique named *Screen Space rendering*[11] might be a good option to render translucent surface.

Conclusion

In this work the simulation as well as visualization of hydraulic erosion was implemented. It first went through the theory behind fluid simulation and then hydraulic erosion model. The implemented the design in terms of software development. The implementation section of this work put in practice theoretical concepts of both fluid system and erosion model and demonstrated how these concepts can be incorporated. To improve the performance of the entire simulation the fluid system was designed to leverage the parallelism of CPU. In the end the preference to heightmap over voxel grid was given due to the improvement in efficiency.

Bibliography

[1] Kelager, M. Lagrangian fluid dynamics using smoothed particle hydrodynamics. *University of Copenhagen: Department of Computer Science*, volume 2, 2006.

[2] Novák, O. Simulace viskózních kapalin. *Prague, 2007. Master thesis. CTU in Prague, Faculty of Electrical Engineering, Department of computing.*

[3] Krištof, P.; Beneš, B.; et al. Hydraulic erosion using smoothed particle hydrodynamics. In *Computer Graphics Forum*, volume 28, Wiley Online Library, 2009, pp. 219–228.

[4] Monaghan, J. J. Smoothed particle hydrodynamics. *Annual review of astronomy and astrophysics*, volume 30, no. 1, 1992: pp. 543–574.

[5] Müller, M.; Charypar, D.; et al. Particle-based fluid simulation for interactive applications. In *Symposium on Computer animation*, volume 2, 2003.

[6] Desbrun, M.; Gascuel, M.-P. Smoothed particles: A new paradigm for animating highly deformable bodies. In *Computer Animation and Simulation'96*, Springer, 1996, pp. 61–76.

[7] Wojtan, C.; Carlson, M.; et al. Animating Corrosion and Erosion. In *NPH*, Citeseer, 2007, pp. 15–22.

[8] Partheniades, E. Erosion and deposition of cohesive soils. *Journal of the Hydraulics Division*, volume 91, no. 1, 1965: pp. 105–139.

[9] Richardson, J.; Zaki, W. Sedimentation and fluidisation: Part I. *Chemical Engineering Research and Design*, volume 75, 1997: pp. S82–S100.

[10] Teschner, M.; Heidelberger, B.; et al. Optimized spatial hashing for collision detection of deformable objects. In *Vmv*, volume 3, 2003, pp. 47–54.

[11] van der Laan, W. J.; Green, S.; et al. Screen space fluid rendering with curvature flow. In *Proceedings of the 2009 symposium on Interactive 3D graphics and games*, 2009, pp. 91–98.

Appendix A

Acronyms

SPH	Smoothed Particle Hydrodynamics
NSE	Navier-Stokes equations
UI	User Interface

Appendix **B**

Contents of enclosed CD

```
│ readme.txt .................. the file with CD contents description
├── exe ............................ the directory with executables
├── src ............................ the directory of source codes
│   ├── wbdcm ............................. implementation sources
│   └── thesis ......... the directory of LaTeX source codes of the thesis
└── text .............................. the thesis text directory
    └── thesis.pdf ................... the thesis text in PDF format
```