Czech Technical University in Prague
Faculty of Electrical Engineering
Department of Cybernetics

Bachelor Thesis

Reconstruction of Spheres in 3D for Robot Calibration

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Study Program: Cybernetics and Robotics, Bachelor

Field of Study: Systems and Control

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BACHELOR‘S THESIS ASSIGNMENT

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Branch of study: **Systems and Control**

II. Bachelor’s thesis details

Bachelor’s thesis title in English:

Reconstruction of spheres in 3D for robot calibration

Bachelor’s thesis title in Czech:

Rekonstrukce koulí v prostoru pro kalibraci robotů

Guidelines:

1. Study principles of sphere projection to uncalibrated and calibrated images [1,2].
2. Suggest an approach to reconstructing a pair of touching spheres in 3D space.
3. Implement the approach and verify it on real data.

Bibliography / sources:

[1] Hui Zhang, Kwan-Yee K. Wong, and Guoqiang Zhang. Camera Calibration from Images of Spheres. IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 29, NO. 3, MARCH 2007 (http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=4069265)
[2] M. Agrawal and L. S. Davis, "Camera calibration using spheres: a semi-definite programming approach," Proceedings Ninth IEEE International Conference on Computer Vision, Nice, France, 2003, pp. 782-789 vol.2.
doi: 10.1109/ICCV.2003.1238428

Name and workplace of bachelor’s thesis supervisor:

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doc. Ing. Tomáš Pajdla, Ph.D. Supervisor’s signature	prof. Ing. Michael Šebek, DrSc. Head of department’s signature	prof. Ing. Pavel Ripka, CSc. Dean’s signature

III. Assignment receipt

The student acknowledges that the bachelor’s thesis is an individual work. The student must produce his thesis without the assistance of others, with the exception of provided consultations. Within the bachelor’s thesis, the author must state the names of consultants and include a list of references.

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Date of assignment receipt	Student’s signature

Aknowledgements

I would like to thank my advisor Tomáš Pajdla for introducing me into the world of research, showing me how to creatively solve the problems in projective geometry and for his guidance and advices that helped me to finish this work.

Declaration

I declare that the presented work was developed independently and that I have listed all sources of information used within it in accordance with the methodical instructions for observing the ethical principles in the preparation of university theses.

Prague, date ....

signature

Abstract

Computer vision is a fast and widely developing branch of science. One of the problems and tasks of computer vision is the determining of a particular object on the image and its position in space. In this work, we propose our algorithm, which allows us to uniquely determine the sphere in space. We concentrate on studying the theory, which directly relates to the theme of this work and apply this theory in practice to obtain a solution.

As a result, we have a working algorithm that gives a satisfactory result on real data. In the future, this algorithm can be used, for example, to calibrate the robot in such a way that determining the position of the sphere will determine the position of the last joint of robotic arm.

Keywords: computer vision, conic sections, quadrics, camera calibration

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Abstrakt

Po£íta£ové vid¥ní je rychle rozvijející se oblast v¥dy. Jedním z problem· a úkol· po£íta£ového vid¥ní je detekce konkretního objektu na obrázku a ur£ení jeho polohy v prostoru. V této práci navrhujeme algoritmus, který umoº¬uje jednozna£n¥ ur£it polohu sféry v prostoru. Soust°e¤ujeme se na teorii, která se p°ímo vztahuje k tématu této práce, a pouºíváme ji v praxi pro získání °e²ení.

Ve výsledku dostáváme funk£ní algoritmus, který dob°e funguje p°i zkou²ení na reálných datech. V budoucnu tento algoritmus m·ºe být pouºit, nap°íklad, pro kalibraci robota, a to tak, ºe ur£ení pozice sféry ur£í i pozici konce robotické ruky.

Klí£ová slova: po£íta£ové vid¥ní, kuºelose£ka, kvadrika, kalibrace kamery

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Chapter 1

Introduction

Computer vision is a fast and widely developing branch of science. In our time it is closely related to arti cial intelligence, namely, with the recognition and detection of objects. Extracting information from 2d images of 3D objects is also one of the most common branches of computer vision. It combines the solution of polynomial equations, linear algebra and projective geometry.

In this work, we focused our attention on recreating a sphere on the basis of its image. Our main task was to propose an algorithm that would allow unambiguous determination of the position of the sphere in space. Similar problem has been solved by Zisserman and Cross in [3]. They used two cameras to get stereo image and to determine the position of the object, called quadric.

The motivation for this work was a group of researchers who are engaged in calibrating the robot. They need to determine the position of the sphere in space, which is located on the end of the robotic arm, in order to accurately determine the coordinates of the last joint and to calibrate the robot.

This thesis can be divided to the following sections:

1. In the beginning we will conduct theoretical introduction in our problem, that is needed for better understanding the problem and that we needed to suggest the solution.
2. Then, we will move on with the main part of this thesis, namely we will suggest a solution to the given problem and discuss problems, that could show up.
3. After, we will test the solution on the real data and we will see the limitations of the algorithm.
4. Finally, we will summarize this work.

Chapter 2

Basic theory

In this chapter we will go through the theory, that we had to study to successfully tackle the issue. Theory consists of basic knowledge in computer vision and geometry, such as surfaces in 3D and di erent conic sections. We also studied the interaction between surfaces in 3D and its projections on the planes. Thus, we divide our theoretical introduction into the following sections

1. Perspective camera model
2. Conic sections
3. Quadrics
4. Projection of the sphere onto the image plane

2.1 Perspective camera model

There exist many types of cameras. We will concentrate ourselves on the perspective camera model, that maps points from 3D space onto the image plane. Basically, points on the image plane are given by the intersection between the vector, connecting the camera center with the point, and the image plane.

Chapters 6 and 7 in book [10] describe the basics of perspective camera model. In this section we will brie y summarize the main of these two chapters. We will use the notation of [10] and [5].

Let us consider following illustration 2.1

Figure 2.1: Coordinate systems of perspective camera [10].

Figure 2.1 shows the geometry of the perspective camera. We can see there the camera center C, the image plane and the point O, which is the origin for the main coordinate system.

To solve the problems of computer vision, such as extracting information about 3D object from 2D image, we will de ne some handful coordinate systems.

First, we introduce the world coordinate system $(O;)$, so we can express any point in the space with its help. This coordinate system consists of the origin O and orthonormal basis containing vectors $d_1; d_2$ and d_3, so every point can be written as

$$X = {}_1d_1 + {}_2d_2 + {}_3d_3 \tag{2.1.1}$$

World coordinate system allows us to de ne the camera projection center C.

Next we de ne the image coordinate system (o;) of the image plane . We use this coordinate system to express the points on the image plane through its origin o and two non-orthogonal basis vectors b_1 and b_2. So, any point on the image plane can be represented as vector in basis

$$u_i = u_i b_1 + v_i b_2 = \begin{matrix} u_i \\ v_i \end{matrix} \tag{2.1.2}$$

Finally, to associate the coordinates of points on the image with the camera center, we introduce the camera coordinate system (C;). Its origin is always placed in the projection center and its basis consists of basis with added vector b_3. We de ne vector b_3 as a vector connecting origins of two coordinate systems and . Thus, it is more convenient to write coordinates of the points in the image plane with respect to basis , because the third coordinate will always be equal to one. At this point, we can express above vector u in basis by simply adding one, i.e.

$$u_i = \begin{matrix} u_i \\ 1 \end{matrix} \tag{2.1.3}$$

Now let us take an arbitrary point X in the space. If we start drawing the line from the camera center C to point X, we will reach the image plane at the point x. We de ne such point x as the projection of point X onto the image plane and we can represent it with respect to as

$$x = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \tag{2.1.4}$$

If we want to represent point X with respect to the camera coordinate system, we can imagine, that point X in can be obtained from x as its multiple by some non-zero number . Thus, we can write

$$x = X \quad C \tag{2.1.5}$$

Now, suppose that we have the matrix A, that converts any vector from space, given with respect to the world coordinate system , to the vector represented by the camera coordinate system . We can write this as

$$x = Ax \tag{2.1.6}$$

The matrix A, as we will see later, contains the extrinsic and intrinsic parameters of the camera. We will give the de nition to these parameters a little bit later.
Using Equation 2.1.6 we can re-write Equation 2.1.5 as

$$x = A(X \quad C) \tag{2.1.7}$$

$$x = A \; I | \quad C \; \begin{matrix} X \\ 1 \end{matrix} \tag{2.1.8}$$

$$x = P \begin{matrix} X \\ 1 \end{matrix} \qquad (2.1.9)$$

We just have introduced the $3 \quad 4$ image projection matrix P.
With help of Equation 2.1.9 we can describe the relationship between the point in the space X and the point on the image plane u. If we know the value of and matrix A, we can obtain the coordinates of the point in space from Equation 2.1.7 as

$$X = A^{1}x + C \qquad (2.1.10)$$

Now we will look closer to the matrix A. We define it as

$$A = \frac{1}{f} KR \qquad (2.1.11)$$

where we introduce two new matrices K, R and parameter f, which is the focal length and is defined as the distance between the camera center and the image plane.

We have already told, that the matrix A contains parameters of the camera. We have mentioned extrinsic and intrinsic parameters of the camera. Matrix K determines the intrinsic parameters, that are given by camera construction and do not change when moving or rotating the camera. This matrix is given as follows

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ 0 & k_{22} & k_{23} \\ 0 & 0 & 1 \end{bmatrix} \qquad (2.1.12)$$

The elements of matrix K are defined as

$$k_{11} = \frac{f}{\|b_1\|} \qquad (2.1.13)$$

$$k_{12} = \frac{f \cos\angle(b_1; b_2)}{\|b_1\| \sin\angle(b_1; b_2)} \qquad (2.1.14)$$

$$k_{22} = \frac{f}{\|b_2\| \sin\angle(b_1; b_2)} \qquad (2.1.15)$$

Elements k_{13} and k_{23} determine the coordinates of the principal point in , that is given as intersection between vector c_3 and the image plane.

Now we will look at the second matrix introduced in Equation 2.1.11. This is $3 \quad 3$ rotation matrix, that determines the extrinsic parameters of the camera, i.e. its orientation. Matrix R and vector C uniquely define the position and orientation of the camera.

To continue, we will define new orthogonal camera coordinate system $(C; \ )$ with three basis vectors $c_1; c_2$ and c_3. These three vectors are chosen in such way, that $Span\{c_1; c_2\}$ defines the plane parallel to the image plane . Vector c_3 is orthogonal to both c_1 and c_2 and thus is orthogonal to the image plane and has the length equals to the focal length.

The basis holds

$$c_1 = k_{11} b_1 \qquad (2.1.16)$$

$$c_2 = k_{12} b_1 + k_{22} b_2 \qquad (2.1.17)$$