Assessment of the master thesis by Oskar Krejčí

Minimal solvers with truncated normal forms

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Solving polynomial systems is one of the most fundamental computational techniques in applied mathematics and engineering. Historically, the dominant approach has been to use resultants or Gröbner-based symbolic–numeric procedures that transform the input system into an equivalent eigenvalue problem. While this general strategy can solve any consistent system, it is computationally very expensive. In engineering, however, many tasks lead to generic polynomial systems, making it possible to construct solvers that apply the same sequence of operations to every problem within a given class. Such specialized solvers are less general but far more efficient, and several methods for designing them have been developed. The PhD thesis of Simon Telen introduces an alternative transformation of polynomial systems into eigenvalue problems, offering additional flexibility for optimizing numerical performance. Consequently, a pressing question arises: How does Telen's approach compare with standard Gröbner-basis methods and with specialized solvers in terms of speed and accuracy?

The goal of the thesis was to understand the methods described in Telen's work and to compare his approach with standard Gröbner-based symbolic–numeric procedures and with specialized solvers on problems from computer vision. Let me point out the two most salient contributions of the thesis.

First, the setup, language, and methods described are rooted in algebraic geometry, a field not easily accessible to other branches of applied science. Although Telen's thesis included runnable code, using it efficiently requires understanding non-trivial concepts from algebraic geometry. Hence, considerable effort was devoted to interpreting and implementing Telen's ideas in order to test his methods.

Secondly, the experimental results show that Telen's methods occupy a middle ground between classical Gröbner-basis techniques and specialized solvers. Specialized solvers are still much more efficient than Telen's approach, yet general Gröbner-basis methods lag behind it. This is a significant and intriguing observation.

Oskar Krejčí has presented high-quality work that fulfills all the goals set in the assignment. He mastered non-trivial concepts from classical algebraic geometry as well as the new results in Telen's thesis. He demonstrated the ability to understand and apply advanced theory in an engineering context.

To conclude, the thesis employs non-trivial mathematical results to reveal relationships between different approaches to solving polynomial systems that had not been compared before. Therefore, I recommend awarding the thesis the grade of Excellent (A).

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