Report of an external referee

- Master thesis title: Information flows and their role in complex systems
- Candidate: Zlata Tabachová
- Institution: Department of Physics, Czech Technical University in Prague,
- Faculty of Nuclear Science and Physical Engineering, Břehová 7, 115 19, CZ
- Assessment: A (výborně)

In her master thesis, the candidate Zlata Tabachová treated in an organic way the subject of transfer entropies with a specific emphasis on Rényi entropies. The work presents a logically coherent framework, which also embeds various original contributions of the author.

The thesis deals with a specific information-theoretic approach to complex systems – the so-called transfer entropy (or information flow) approach – which is an active current research topic in data analysis and causal issues.

Transfer entropies and their estimators are of a great interest, both on theoretical and experimental level. The thesis centers around the issue of the Rényi entropy generalization of the transfer entropy, with specific applications in financial markets and chaotic dynamical systems. Transfer entropies based on Rényi entropy were pioneered by P. Jizba, (thesis advisor of Zlata Tabachová) and since their introduction they gained popularity in the financial community.

The candidate has followed in her thesis this line of reasoning by expanding the applicability Rényi's transfer entropies to other systems, such as coupled Rössler strange attractors (discussed in chapter 5.2). In this line, the results about the information-theoretic signatures of synchronization in couples Rössler systems appear quite intriguing and susceptible of further developments in the field of chaotic dynamical systems. As a matter of fact, already in this thesis are present few highlights on possible extensions and applications, e.g., connection with Pearl's causal model. These results are interesting and indeed the applicant has already published some of her findings.

One feature, which in could have been more emphasized, for the sake of better understanding, is the relation of the Rényi entropy (or Rényi entropy transfer) with the specific numerical estimators used. This is certainly important issue (already for Shannon entropies) and it would deserve more space, particularly in view of the numerical results presented in the thesis. Since the candidate published some work related to this topic, a more detailed discussion of this relevant issue could be given.

On the formal level, an aspect of the thesis which I have found particularly nice is the fact that each chapter ends with a brief summary of the key concepts there discusses. This is certainly of help for the reader to quickly get the main content of a given chapter.

I have no specific suggestions nor questions for the candidate. To me the work is well motivated, lucidly written and I could not spot any formal mistakes in it.

Overall, the work is excellent, because it presents in an organic and easy to follow fashion an interesting approach to the subject of transfer entropies with a balanced

admixture of theory of financial markets. The research line is treated coherently but developed in different interesting ramifications in both fundamental and applied directions. I think that this research can certainly provide in future numerous relevant contributions in theory of complex dynamical systems.

In conclusion, my judgment on the master thesis of Zlata Tabachova is A (excellent).

Fisciano, 2.2.2022

Prof. Massimo Blasone