Calculation of High Frequency 4-TP Impedance Standards

L. Vojáčková[1,2], J. Kučera[1,2], J. Hromádka[3] and J. Boháček[2]

[1] Czech Metrology Institute (CMI), Radiova 3, 120 00 Prague 10, Czech Republic
lvojackova@cmi.cz
[2] Czech Technical University in Prague (CTU), Faculty of Electrical Engineering, Department of Measurements
Technicka 2, 166 27 Prague 6, Czech Republic
[3] TechSoft Engineering, spol. s.r.o., Na Pankraci 26, 140 00 Prague 4, Czech Republic

***Abstract* — This paper deals with methods for calculating frequency characteristics of high frequency four-terminal-pair coaxial impedance standards. An analytical calculation for coaxial resistance and capacitance standards was performed at frequencies up to 100 MHz. For verification of the used method, numerical calculations based on domain FEM (Finite element method) in low and high frequency range are presently under investigation. The paper describes and summarizes first results, which will serve as a base for ongoing work on evaluating the effects of constructional limitations of four-terminal-pair impedance standards.**

***Index Terms* — analytical model, calculable standard, capacitance, impedance, numerical model, resistance.**

I. INTRODUCTION

Many NMIs base the traceability between low and high frequency impedance standards on Agilent 16380A standard capacitor set or General Radio 140x capacitance standards. One possible way how to evaluate frequency dependences (FDs) of these standards is to calculate them from the parameters of their equivalent circuits obtained as a result of measurement of ***Z*** or ***S*** matrices by means of network analyzers [1], [2]. A different approach is to calculate FDs directly from geometrical dimensions of the standards and from properties of the materials used in their construction [e.g. 3]. Such calculations were successfully made at the CTU for both coaxial capacitance and coaxial resistance standards and frequencies up to 1 MHz. For an extension to higher frequencies, a check by means of a suitable numerical method is essential to find out whether there is no limitation in Kelvin functions used in analytical calculations. Moreover, mastering of numerical calculations will make it possible to evaluate the effects of construction parts that are neither analytically calculable nor directly measurable.

At present, numerical modelling is used especially for characterization of one- terminal-pair (1TP) standards [e.g. 4]. In this paper, we present results of comparison of our recent analytical and numerical calculations of FDs of a four-terminal-pair (4TP) coaxial resistance standard in a frequency range from 10 kHz up to 200 MHz.

II. DESIGN OF CALCULABLE IMPEDANCE COAXIAL STANDARD

At the CTU, coaxial resistance (100 Ω and 1 kΩ) and capacitance (10 pF and 100 pF) standards were prepared and tested in a frequency range up to 1 MHz by means of an HF bridge [5, 6]. The below described calculation methods were newly used for characterizing the 100 Ω resistance standard (RC02) at frequencies up to 200 MHz. In this standard (Fig. 1), bare Nikrothal wire with a length of about 0.6 m and a diameter of 31 μm is used as the resistive element.

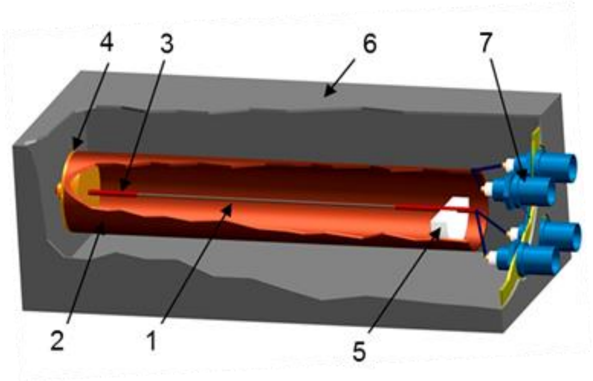

Fig. 1. Coaxial resistor design (1 – resistance wire, 2 – outer tube, 3 – supporting tube, 4 – shorting plate, 5 – PTFE support, 6 – shielding box, 7 – connectors) [6]

III. ANALYTICAL CALCULATION

The calculation of FD of the RC02 standard was based on a model with lumped parameters representing all parts of the resistor, each of them being divided into 100 subparts. Each subpart of the resistor model consists of a series impedance $\boldsymbol{Z}_i$, representing resistance and inductance of the inner wire and the outer tube, and a parallel admittance $\boldsymbol{Y}_i$ representing the capacitance distributed along the coaxial line, details in [6]. A calculus based on Kelvin functions was used for $\boldsymbol{Z}_i$ evaluation. The whole calculation was implemented in Wolfram Mathematica.

IV. NUMERICAL CALCULATION

Numerical calculation of FD of the RC02 standard was performed by ANSYS Maxwell and ANSYS HFSS software. ANSYS Maxwell offers quasi full-wave solver that uses **T**-Ω formulation of Maxwell equations in frequency domain to obtain **H** field and consequently derives **E** field (**T** – electric vector potential, Ω – magnetic scalar potential, **E** – electric

978-1-4673-9134-4/16/$31.00 ©2016 IEEE

field intensity, **H** – magnetic field strength). On the other hand, ANSYS HFSS solves wave equation for **E** field by FEM in frequency domain followed by derivation of **H** field. Whereas the approach of ANSYS Maxwell is suitable for low frequency problems since it neglects displacement current d**D**/dt in Ampere’s law (**D** – electric displacement field), ANSYS HFSS provides better results for the objects whose dimensions are comparable to the wavelength of electromagnetic excitations. An example of distribution of **E** and **H** fields is shown in Fig. 2.

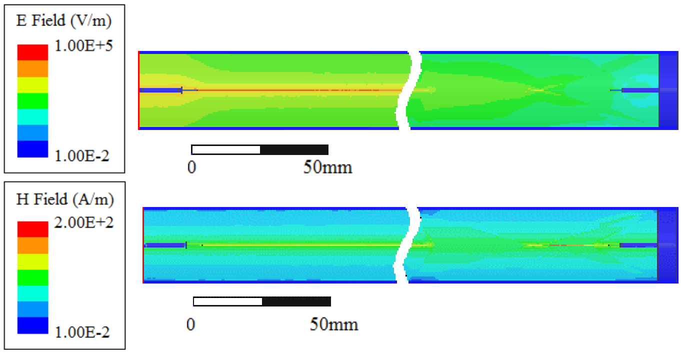

Fig. 2. Electric and magnetic field distribution along the coaxial resistor RC02 at frequency 10 MHz

V. RESULTS OF CALCULATION

Results of analytical and numerical modelling of resistance change and time constant behavior with frequency are shown in Fig. 3.

Comparison of the values of ac resistance calculated on the base of numerical and analytical models is given in Tab. 1. It is obvious that ANSYS Maxwell makes it possible to model FD of the RC02 standard with an accuracy better than 2 % up to 10 MHz and that ANSYS HFSS is well suitable for higher frequencies. It has to be noted that accuracy of numerical modelling depends on quality of adaptive meshing of the model (between 2.5 and 4.5 millions of elements were finally used).

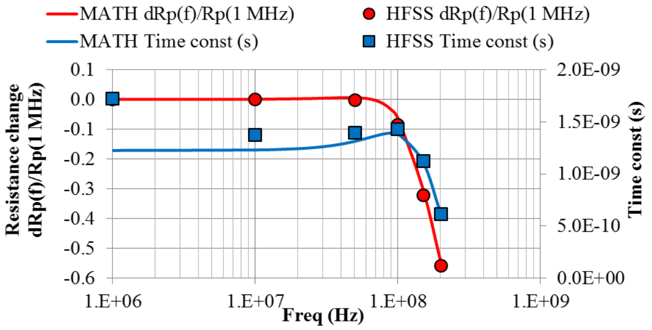

Fig. 3. Results of analytical (MATH) and numerical (HFSS) calculation at higher frequency range from 1 MHz to 200 MHz.

Tab. 1. Relative difference between numerically (num) and analytically (ana) modeled resistance of RC02

Frequency (MHz)	R_P(num)/R_P(ana) – 1	
	Maxwell (%)	HFSS (%)
0.01	1.2	(14)
1	1.1	2.6
10	1.4	2.5
50	(8.4)	1.8
100	(50)	0.005
150	-	-0.9
200	-	-0.7

VI. CONCLUSION

The results shown above correspond to numerical modelling of impedance standards geometries, which can also be evaluated by analytical methods. Even for modelling of the whole resistance standard RC02 consisting of objects with dimensions in wide scale (10^{-5} m to 10^{-1} m), agreement between numerical and analytical model better than 2 % from 0.01 MHz up to 200 MHz was achieved.

Further work will be focused mainly on modelling of parasitic effects due to connections between the coaxial structure and a connector interface of 4TP impedance standards.

REFERENCES

[1] K. Suzuki, “A new universal calibration method for four-terminal-pair admittance standards,” *IEEE Trans. Instrum. Meas.*, vol. 40, no. 2, pp. 420 – 422, 1991.

[2] L. Callegaro and F. Durbiano, “Four-terminal-pair impedances and scattering parameters,” *Meas. Sci. & Tech.*, vol. 14, no. 4, pp. 523 – 529, April 2003.

[3] S. A. Awan and B. P. Kibble, “Towards Accurate Measurement of the Frequency Dependence of Capacitance and Resistance Standards up to 10 MHz,” *IEEE Trans. Instrum. Meas.*, vol. 54, no. 2, pp. 516 – 520, 2005.

[4] F. Ziade, A. Poletaeff and D. Allal, D, “Primary Standard for S-Parameter Measurements at Intermediate Frequencies (IFs),” *IEEE Trans. Instrum. Meas* , vol. 62, no. 3, pp. 659 -666, 2013.

[5] J. Kucera, R. Sedlacek and J. Bohacek, “Improved calculable 4TP coaxial capacitance standards,” CPEM 2014 Conf. Digest, pp. 288 – 289, 2014.

[6] J. Kucera, R. Sedlacek and J. Bohacek, “An HF coaxial bridge for measuring impedance ratios up to 1 MHz,” *Meas. Sci. & Tech.*, vol. 23, no. 8, 085004, June 2012.