Investigation of Flow and Agitation of non-Newtonian Fluids

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Introduction

The present study is aimed to investigate the flow and agitation of purely viscous non-Newtonian fluids in the laminar flow regime. A new method is proposed for the determination of the shear viscosity of the power-law fluids in non-circular channels. The provided method is then validated by experimental and numerical methods. It is found that the proposed method is successful for the determination of the shear viscosity. Then, the provided method is utilized for the prediction of friction factor of the flow of power-law fluids in non-circular channels using the Reynolds number and a simple method is suggested for the rapid calculation of the friction factor of power-law fluids in laminar regime particularly for the engineering calculations. Finally, the power and flow characteristics of a newly designed in-line rotor-stator mixer are investigated experimentally and numerically for the Herschel–Bulkley model. The power draw of the mixer is measured experimentally and then obtained power draw values are validated by numerical simulations. The power draw and Metzner-Otto coefficients are determined from the experimentally and numerically obtained power draw results and a new slope method is suggested based on the Rieger-Novak method for mixing of viscoplastic fluids in the laminar regime. The shear and velocity profile in the mixer analyzed via numerical methods and the effect of geometrical configuration on velocity, shear, and power consumption are discussed.

Objectives

•To show that rectangular channels and concentric annulus can be used for the determination of rheological parameters of the power-law fluids as capillary and slit rheometers. An alternative, one parameter correlation will be suggested for the estimation of the rheological parameters.

•To propose a new and very simple correlation for the prediction of the pressure drop for the laminar flow power-law fluids through non-circular channels by using geometrically independent Reynolds number and expressing friction factor-Reynolds number relationship by a simple linear equation.

•Analyzing power characteristics and flow profiles of an in-line rotor-stator mixer experimentally and numerically using shear-thinning viscoplastic fluid under the laminar regime. And then, suggesting expression to specify the effect of yield stress on the power demand of a mixer. Finally, proposing a practical method for the determination of the Metzner-Otto coefficient for the Herschel–Bulkley fluids.

Literature Review

1- Flow of power-law fluids in non-circular channels

The relationship between wall shear rate and wall shear stress is (Kozicki et. al, 1966)

$$\tau_w = K\left[(b+\frac{a}{n})\frac{8\bar{u}}{D_h}\right]^n$$

- **The friction factor –Reynolds number relationship for non−circular channels**

$$\lambda Re_G = 16$$

Two parameter model Reynolds number (Re_G)

$$Re_G = \frac{\rho \bar{u}^{\,2-n} {D_h}^n}{8^{n-1}K(b+a/n)^n}$$

- Two parameter model (a and b)
- Determination of rheological properties K and n depends on the geometrical parameters a and b

Function of shape of channel and geometric ratios

(a=0.25, b=0.75) –circular channel

(a=0.5, b=1)-Parallel plate

2- Agitation of viscoplastic fluids in a rotor-stator mixer in laminar regime

Power number-Reynolds number relationship in laminar mixing (Rieger and Novak, 1973) **:**

$$PoRe = C$$

Power number : $Po = \frac{P}{\rho N^3 D^5}$ **Reynolds number :** $Re = \frac{\rho N D^2}{\eta_a}$

Effective shear rate (Metzner and Otto, 1957)**:**

$$\dot{\gamma}_{eff} = k_s N$$

Effective viscosity:

$$\eta_a = \frac{\tau}{\dot{\gamma}_{eff}}$$

Effective viscosity for Herschel-Bulkley model (Archard et. al., 2016)

$$\eta_a = \frac{\tau_o}{k_s N} + K(k_s N)^{n-1} \longrightarrow Re = \frac{\tau_o}{k_s N} + K(k_s N)^{n-1}$$

C and k_s depend upon type and geometry of the impeller

1-Rectangular channel rheometer and a method for the prediction of friction factor of power-law fluids

Rectangular rheometers

Suggested method: (Ayas et al.2019; Ayas et.al, 2020)

$$\alpha = (b + a/n)^n$$

The ratio of α to $(0.75 + 0.25/n)^n$ is ε

$$\varepsilon = \left(\frac{4(bn+a)}{3n+1}\right)^n$$

For Newtonian case (n=1)

$(a + b) = C/16$

Assumption: For 0<n<1, ε can be assumed as a linear function of n and C

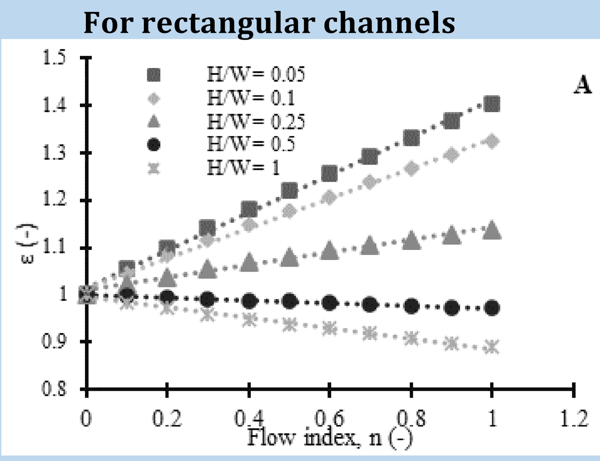
$$\varepsilon = \left(\frac{4(bn+a)}{3n+1}\right)^n = Mn + N$$

$$M = \frac{C}{16} - 1 \qquad N = 1$$

One parameter suggested equation:

$$\tau_w = K((\frac{C}{16}-1)n+1)\left[\left(\frac{3n+1}{4n}\right)\frac{8u}{D_h}\right]^n$$

For rectangular channels



Alternative method: Using the parameters a=0.5 and b=1 (Ayas et. al, 2021)

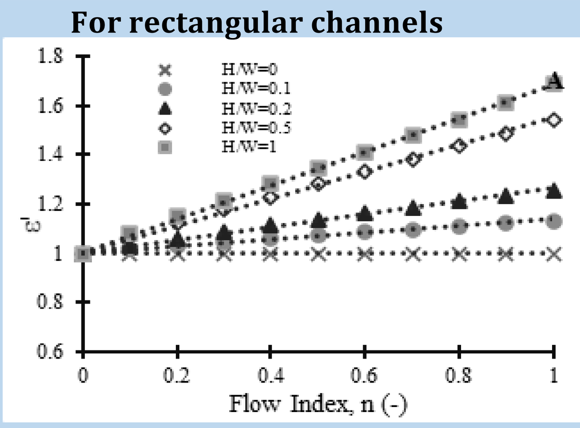
$$\varepsilon' = \frac{(1+\frac{0.5}{n})^n}{(b+\frac{a}{n})^n} = M'n + N'$$

$$M' = \frac{24}{C} - 1 \qquad N' = 1$$

Alternative one parameter equation:

$$\tau_w = \frac{K}{M'n+1}\left[\left(\frac{2n+1}{2n}\right)\frac{8u}{D_h}\right]^n$$

For rectangular channels



Friction factor-Reynolds number

The friction factor is defined as the ratio of wall shear stress to the flux of inertial forces

$$\lambda = \frac{2\tau_w}{\rho \bar{u}^2}$$

Substituting suggested one parameter equations

$$\lambda Re_M = (C-16)n + 16$$

Alternatively

$$\lambda Re'_M = \frac{16C}{(24-C)n + C}$$

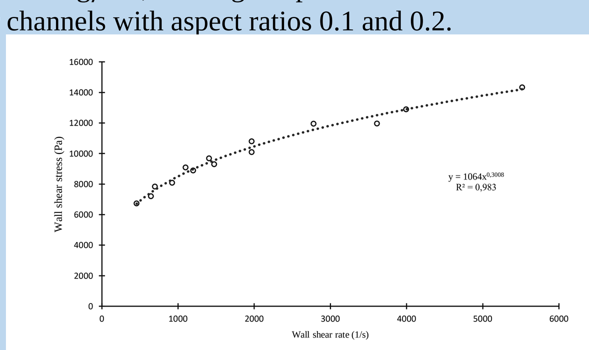
Re_M is the Reynolds number suggested by Metzner and Reed (Metzner and Reed, 1955)

$$Re_M = \frac{\rho u^{2-n} {D_h}^n}{8^{n-1}K\left(\frac{3n+1}{4n}\right)^n} \qquad Re'_M = \frac{\rho \bar{u}^{\,2-n} {D_h}^n}{8^{n-1}K\left(1+\frac{1}{2n}\right)^n}$$

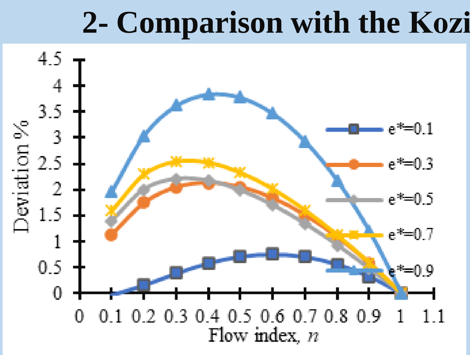
Validation

1- Validation by experiment: Using experimental data in the literature (Skocilas et al, 2017). Using bovine collagen as a fluid with a density of 1100 kg/m³, rheological parameters were determined in the rectangular channels with aspect ratios 0.1 and 0.2.

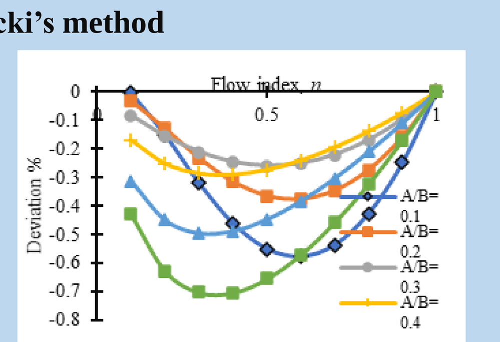
Result



2- Comparison with the Kozicki's method



L-shape



The square duct with a centered cylindrical core

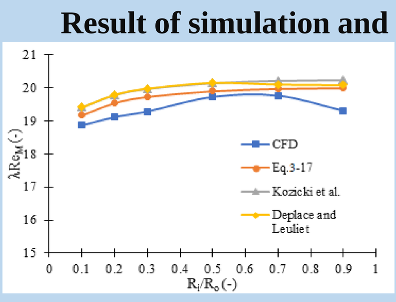
3-Validation by simulations

- Suggested correlation verified numerically by means of ANSYS FLUENT 15 for rectangular, elliptical, L-shape and concentric annulus cross-sections.
- For a fluid with a density of 1000 kg/m³ and for K=0.5 Pa.sⁿ and n=0.5 under the laminar regime

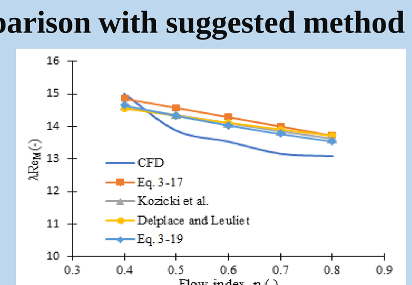
- Boundary conditions:
  Inlet: Velocity inlet (1.5 m/s)
  Outlet: Pressure outlet
  Wall: Stationary wall
  Convergence criteria: 10^{-6} for continuity

The result of comparisons indicates that the maximum deviation between the Kozicki's method and suggested correlations is less than 10 %.

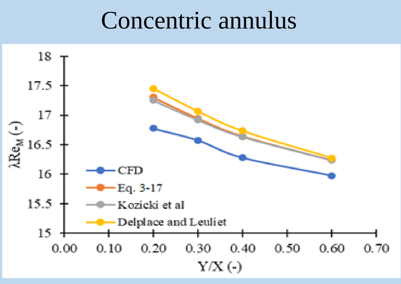
Result of simulation and comparison with suggested method



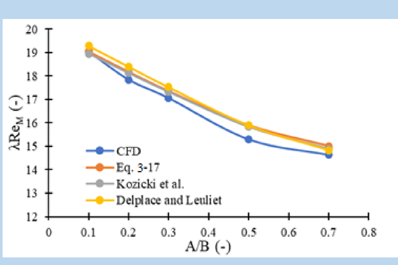
Concentric annulus



Isosceles triangle



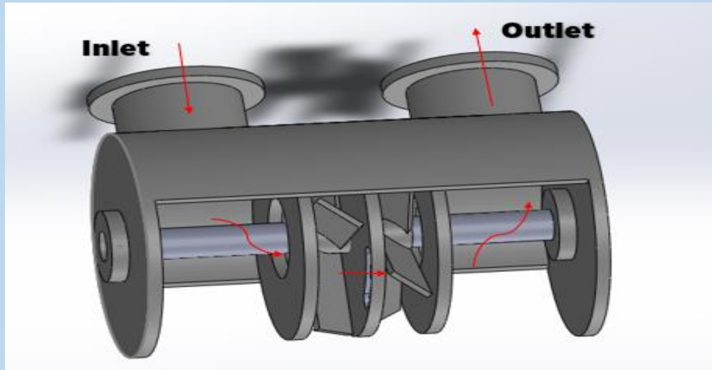
Elliptical channel



L-shape

2-Agitation of viscoplastic fluid in in-line rotor-stator mixer

Agitation of viscoplastic fluids gives rise to the formation of the well-mixed region (cavern) in the vicinity of the impeller, and dead zones are generated next to the wall of the mixing vessel and which leads to poor mixing. In such cases, a more efficient mixing operation can be achieved by agitation viscoplastic fluids in rotor-stator mixers. In order to provide an efficient mixing process, a in-line rotor stator mixer designed to achieve efficient mixing process for the fluids with high yield stress and power characteristics and flow profile of the mixer have been investigated experimentally and numerically.



. The newly designed in-line rotor-stator mixer consists of two serial mixing heads which are installed in a cylindrical barrel and two impellers were mounted on the same shaft. The agitation of the mixer is provided by 45° four-pitch blade impellers with a diameter (D) of 194 mm and the diameter of the stator (Z) is 200 mm.

- Power number-Reynolds number relationship for the agitation of viscoplastic fluids (Ayas et. al, 2020).

$$PoRe_{RN} = \frac{C}{k_s}Bi^* + C(k_s)^{n-1}$$

$$Bi^* = \frac{\tau_0}{K(N)^n} \qquad Re_{RN} = \frac{\rho(N)^{2-n}D^2}{K} \qquad Po_R = Po_Y + Po_S$$

The plot of $PoRe_{RN}$ versus Bi^* for the same fluid at different velocities should be linear and the slope of that curve is equal to C/k_s, hence k_s the value can be determined directly. Moreover, power number for viscoplastic fluids is

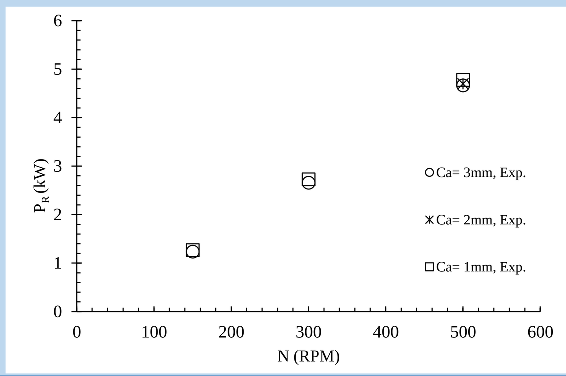
$$Po_Y = \frac{CBi^*}{Re_{RN}k_s} \qquad Po_S = \frac{C(k_s)^{n-1}}{Re_{RN}}$$

Shear efficiency, X ⟶ $X = \frac{Po_S}{Po_Y + Po_S}$

Experiment and simulation

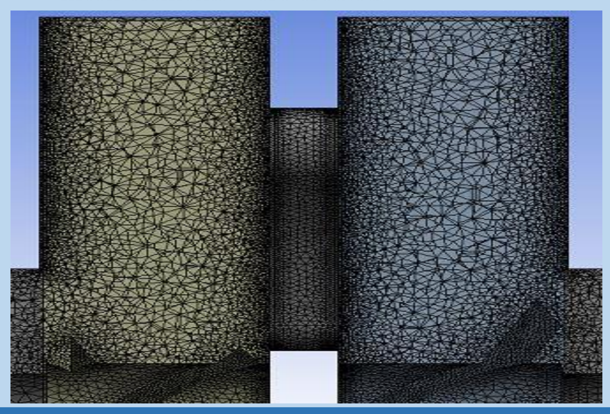
1-Experiment

Experiments were performed using collagen in order to obtain power characteristics of the designed mixer for axial clearances of 1 mm, 2 mm and 3 mm and for the flow rate of 6 kg/min (τ_0 = 4600 Pa, K=420 Pa.sⁿ , n=0.34).



2-Simulation:

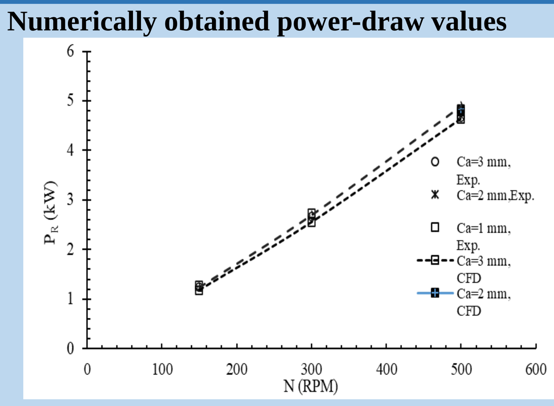
- Simulations were carried out to verify experimental results and in order to obtain design parameter C and analyzing flow profiles in the investigated mixer using MRF approach for rotational speed of impellers of 150, 300 and 500 RPM. For simulation 2.5 millions of mesh elements were created.



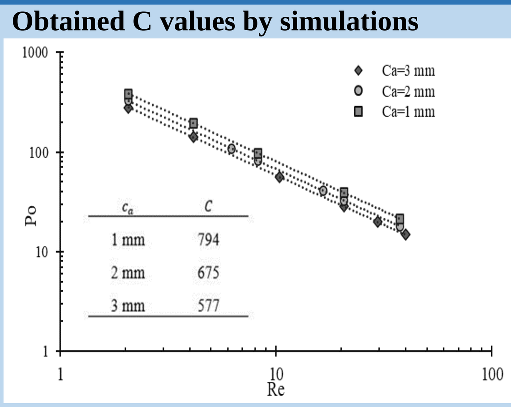
- Boundary conditions:
  Inlet: Mass flow inlet (6 kg /min)
  Outlet: Pressure outlet
  Impellers: Rotational wall
  Wall of the mixer: Stationary wall
  Convergence criteria: 10^{-9} for continuity

Results

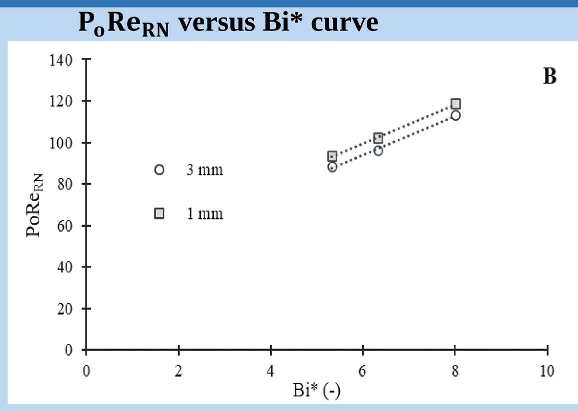
Numerically obtained power-draw values



Obtained C values by simulations



P_oRe_{RN} versus Bi* curve

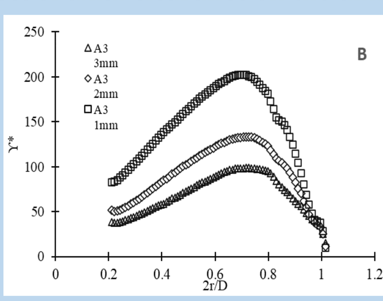


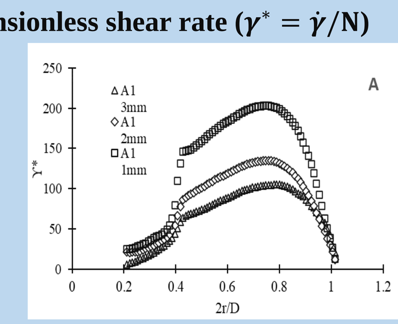
Numerically and experimentally obtained k_s values

N	c_a=1 mm		c_a=2 mm		c_a=3 mm	
RPM	k_s (Exp.)	k_s (CFD)	k_s (Exp.)	k_s (CFD)	k_s (Exp.)	k_s (CFD)
150	82.5	84.0	*	*	57.5	61.6
300	84.4	84.5	*	*	59.2	62.5
500	86.2	83.7	72.5	70.2	60.8	61.3
Average	84.4	84.1	72.5	70.2	59.2	61.8

- The deviation between numerically and experimentally obtained power draw values is less than 6%.
- Determined k_s values from the experimental data slightly deviate with rotor speed, whereas obtained k_s values from the simulations are approximately constant.
- From the P_oRe_{RN} versus Bi* curve, It has been found that obtained dimensionless shear rate values increases with decreasing values of axial clearances.
- The efficiency X increases with the increasing speed of the impellerand decreases with descending values of the axial clearance. Howeverthe clearance has a weak effect on the defined efficiency and significantly varies with rotor speed

Effect of axial clearance on the dimensionless shear rate ($\gamma^* = \dot{\gamma}/N$)





Efficiency, X values

c_a (mm)	N (RPM)	P_o (Exp.)	Po_y (Exp.)	Po_S (Exp.)	X (Exp.)	Po (CFD)	Po_y (CFD)	Po_S (CFD)	X (CFD)
3	150	266	178.3	88.2	0.33	250.8	166.5	84.3	0.34
3	300	70.4	43.3	27.4	0.39	67.5	41.1	26.4	0.39
3	500	26.7	15.2	11.5	0.43	26.5	15.1	11.5	0.43
2	500	26.9	14.9	12.0	0.45	27.6	14.9	12.0	0.43
1	150	266.78	171.1	95.6	0.36	262.6	168.1	94.5	0.36
1	300	71.5	41.7	30.5	0.41	71.6	41.8	29.8	0.42
1	500	27.3	14.7	12.5	0.46	28.0	15.2	12.8	0.46

Conclusion

- This work deals with the measurement of rheological properties of the purely viscous non-Newtonian fluid, prediction of friction factor, and power and flow characteristics of an in-line rotor-stator mixer. Firstly, a method is suggested for the evaluation of the rheological parameters for the power-law fluids using the rectangular channel and concentric annuli.
- According to the method, the relationship between wall shear rate and wall shear stress can be represented by one geometrical parameter for any aspect ratios. Then, the obtained simplified correlation has been used to re-cast friction factor-Reynolds number relationship for the rapid engineering calculations. The provided method is validated by comparing the most frequently used methods and through numerical simulations and it was found that the suggested method can predict friction factor accurately.
- Finally, the power characteristics and flow field of a newly designed in-line rotor-stator mixer have been analyzed experimentally and numerically. Firstly, the power draw of the mixer has been measured experimentally for the three rotational speeds of the impeller and three different axial clearances of the mixer, and then obtained power draw results have been validated by numerical simulation, and a good agreement was found between the numerically and experimentally obtained power values. The Newtonian power draw coefficient has been calculated by numerical simulations and then, Metzner-Otto constants have been determined from the experimentally and numerically obtained power draw results.
- It was found that determined Metzner-Otto coefficients from the experimental and numerical methods are in good agreement. Besides, the methodology for the determination of Metzner-Otto constant for power-law fluids suggested by Rieger and Novak is extended to the Herschel–Bulkley model. A slope method was proposed for the determination of the Metzner-Otto coefficient for the Herschel–Bulkley model and it was shown that the introduced method is successful for the prediction of the Metzner-Otto coefficient. In the final step, the effect of axial clearance on velocity and shear profile is discussed and it was found that axial clearance has a remarkable effect on flow profile of the agitated fluid in the mixer.

Literature and publications

Literature

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Autor's Publication

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[2] Ayas M., Skocilas J., and Jirout T., Friction factor of shear thinning fluids in non-circular ducts – a simplified approach for rapid engineering calculation, Chem. Eng. Commun. doi.org/10.1080/00986445.2020.1770232, (2020). (Published online-article in press)

[3] Ayas M., Skocilas J. and Jirout T., Analysis of Power Input of an In-Line Rotor-Stator Mixer for Viscoplastic Fluids, Processes, **8**, 1-15, (2020).

[4] Ayas M., Skocilas J. and Jirout T., A method for the determination of shear viscosity of power-law fluids in a rectangular duct and concentric annulus, Asia-Pac. J. Chem. Eng. (Submitted).