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Generalized Relay Shifting Method for System Identification

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Abstract: The paper describes an improved relay identification method, which enables the estimation of the parameters of a transfer function with various structures (stable, unstable, non-minimum phase, integrating, time delay systems, etc.) from a single relay feedback test. This identification approach follows the original shifting method and offers new possibilities. The proposed identification is demonstrated on a very versatile second-order time-delayed model and on systems with different structures. The proposed approach can be used for the tuning of PID controllers. The shifting method was implemented into a PLC Tecomat Foxtrot and used to control the laboratory apparatuses called "Air Aggregate", "Water Levitation" and "Air Levitation"

Keywords: System identification, relay control, parameter estimation, frequency characteristic, auto-tuning, PID control.

1. INTRODUCTION

The relay identification method is a widely used technique in control engineering. Rotac (1961) originally used that technique in control engineering. Tsypkin (1974) used this approach for process identification. Åström and Hägglund (1984) applied this method to automatic tuning of PID controllers, where the tuning is performed in a short time and in a controllable manner. A process under a relay feedback is shown in Fig. 1, where w denotes the desired variable, y is the controlled variable, u is the manipulated variable and e is the control error. Successful practical results obtained by the relay feedback identification have generated interest in this approach and led to the development of new relay identification methods, such as Yu (1999), Liu and Gao (2012), Liu, Wang, and Huang (2013), Chidambaram and Sathe (2014), Kalpana and Thyagarajan (2018), Ruderman (2019), Dharmalingam and Thangavelu (2019), Ghorai et al. (2019). The presented methods usually assume linear low order models with a low number of parameters, which are sufficient for modelling many industrial processes. Only a few mentioned relay methods are able to obtain all parameters of the model using just a single relay test without a priori information. In addition, some relay identification methods do not take into account problems with the effects of static load, measurement noise and nonzero initial process conditions, which are often in practical applications.

The goal of the paper is to present the generalized relay shifting method, expanding the possibilities of the original relay shifting method (Hofreiter, 2015) allowing estimating more points of a system frequency response from a single relay feedback test. The paper shows that the introduced method allows the parameter estimation of even more complex models with various structures. The proposed method is demonstrated on a second-order time-delayed model.

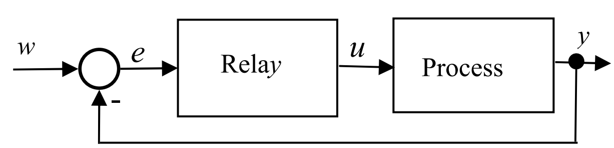

Fig. 1. Configuration of a process under a relay feedback.

2. RELAY SHIFTING METHOD - GENERALIZATION

2.1 Specifications

Consider a process described around its operating point by the frequency transfer function $G_P(j\omega)$, where ω is the angular frequency and j is the imaginary unit. The process is controlled by a two-position biased relay, see Fig. 1. The task is to determine a process model from a single relay feedback test.

2.2 Shifting technique for relay feedback identification

A recently published method called "shifting method" (Hofreiter, 2015) can be applied for this task. A biased (asymmetric) relay with hysteresis is used for this purpose, see Fig. 2. The hysteresis reduces the effects of measurement noise and increases the period of oscillation. By this approach, it is possible to determine two points $G_P(j\omega_0)$ and $G_P(j2\omega_0)$ obtained from the measured courses $y(t)$ and $u(t)$ and the calculated auxiliary variables $y_a(t)$ and $u_a(t)$ within the stable oscillation with the period T_p ($T_p=T_1+T_2$, $T_1 \neq T_2$) during the relay feedback test, see Fig. 3, where

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$$u_a(t) = u(t) + u\left(t - \frac{T_p}{2}\right) \tag{1}$$

$$y_a(t) = y(t) + y\left(t - \frac{T_p}{2}\right) \tag{2}$$

and

$$\omega_0 = \frac{2\pi}{T_p} \tag{3}$$

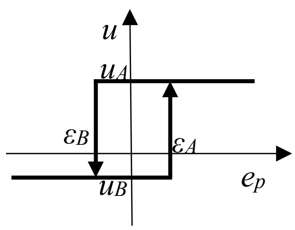

Fig. 2. The steady state characteristic of the biased relay with hysteresis.

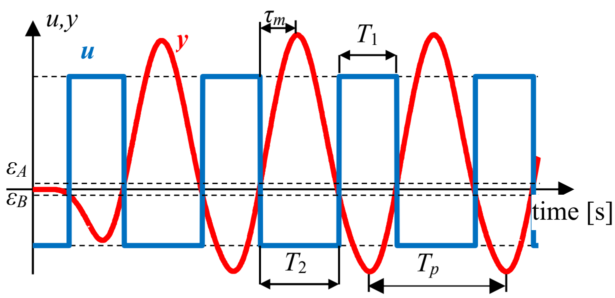

Fig. 3. The time courses of u and y during the relay feedback test.

In the next part, we generalize the shifting method to obtain the points $G_P(j\cdot n\cdot\omega_O)$, $n=1,2,\ldots,N$; $N<T_p/T_1$ from the measured courses $y(t)$ and $u(t)$ and the calculated auxiliary variables $y_n(t)$ and $u_n(t)$, $n=2,3,..,N$ within the stable oscillation, where

$$u_n(t) = \sum_{k=0}^{n-1} u\left(t - k\frac{T_P}{n}\right), \tag{4}$$

$$y_n(t) = \sum_{k=0}^{n-1} y\left(t - k\frac{T_P}{n}\right). \tag{5}$$

The calculated variables $y_n(t)$ and $u_n(t)$, $n=2,3,..,N$ are periodic with the period T_P/n if the measured variables $u(t)$ and $y(t)$ are periodic with the period T_P, which we assume in our case for $t>t_L$, where t_L is the time when the stable oscillation is achieved. In Fig. 4, the time courses of the measured variables u and y are depicted together with the calculated variables $y_n(t)$ and $u_n(t)$, $n=2,3$ for illustration. The relationships (4) and (5) describe the shifting filter with the frequency transfer function

$$F_n(j\omega) = \sum_{k=0}^{n-1} e^{-j\omega\cdot k\cdot\frac{T_p}{n}} \tag{6}$$

and it holds

$$F_n(j\omega) = \begin{cases} n & \text{for } \omega = n\cdot 2\pi/T_P \\ 0 & \text{for } \omega = k\cdot 2\pi/T_P,\, k=1,2,\cdots,n-1 \end{cases} \tag{7}$$

The shifting filter F_n filters out the lower harmonics and amplifies the n-th harmonic.

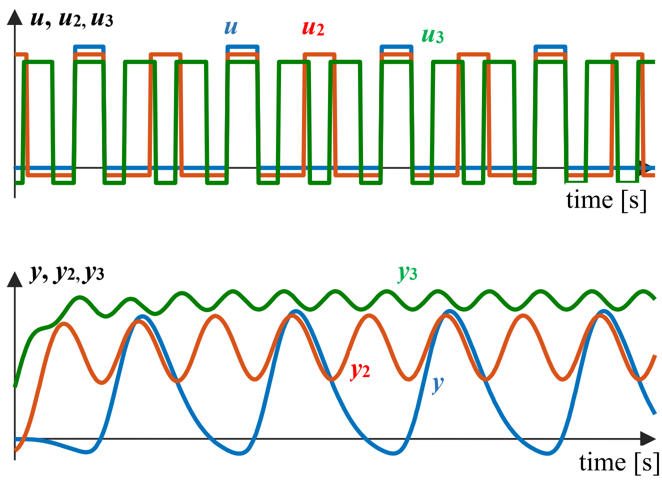

Fig.4. The time courses of u and y during the relay feedback test together with the calculated u_2,y_2,u_3,y_3.

To simplify writing, we denote

$$u_1(t) = u(t), \quad y_1(t) = y(t) \tag{8}$$

and then the frequency response points $G(j\omega_k)$,$k=1,2,..,N$ can be determined by

$$G(j\omega_k) = \frac{\int_t^{t+T_P} y_k(\tau)e^{-j\omega_k\tau}d\tau}{\int_t^{t+T_P} u_k(\tau)e^{-j\omega_k\tau}d\tau}, t > t_L, k = 1,2,\cdots,N \tag{9}$$

and

$$\omega_k = k\cdot\frac{2\pi}{T_P}. \tag{10}$$

The integrals in the numerators are computed by numerical integration.

The position $G(j\omega_k)$ can also be determined **without utilization of relation (9)** using the approximation of the periodic signals $u_k(t)$ and $y_k(t)$ for $k=1,2,\ldots,N$, $t>t_L$ by harmonic functions, see e.g. Hofreiter (2015).

The great advantage of the shifting method is that without assuming any model structure, it enables to estimate the frequency response points $G(j\omega_k)$,$k=1,2,..,N$ from a single relay test. The next advantage of this approach is that the presence of a static load disturbance with a magnitude of d_A does not have any influence on the calculation of $G(j\omega_k)$,$k=1,2,..,N$ as it holds

$$\int_t^{t+T_p} d_A\cdot e^{-j\omega_k\tau}d\tau = d_A\int_t^{t+T_p} e^{-j\omega_k\tau}d\tau = 0. \tag{11}$$

If we know the working point (u_0,y_0), then the static gain K of a proportional system can also be estimated from a single relay test by the following formula (see Shen, Wu, and Yu (1996) or Berner, Hägglund, and Åström (2016))

$$K = G(0) = \frac{\int_t^{t+T_p}\left(y(\tau)-y_0\right)d\tau}{\int_t^{t+T_p}\left(u(\tau)-u_0\right)d\tau}, t > t_L \tag{12}$$

as the shifting method uses the biased relay.

3. MODEL FITTING

The frequency response points $G(j\omega_k),k=1,2,..,N$ can be used for model fitting. For this purpose, it is necessary to select the structure of the model. A linear model with a low number of parameters is suitable, because the goal is to obtain a model for control design. For the model fitting, we can use, e.g. the criterion

$$Kr(\theta)=\sum_{i=1}^{N}\left(G\left(j\omega_i\right)-M\left(j\omega_i,\theta\right)\right)^2 \tag{13}$$

where θ is the vector containing the parameters of model M described by the frequency transfer function. The model parameters can be estimated by

$$\hat{\theta}=\arg\min_{\theta\in\Theta} Kr(\theta), \tag{14}$$

where Θ is the set of acceptable values of the model parameters. This approach can be used to estimate the parameters of linear models with various structures (stable, unstable, integrating systems with or without a transport delay).

The introduced approach is shown for a second order time delayed (SOTD) model, with a transfer function

$$M\left(s\right)=\frac{K\cdot e^{-s\cdot T_d}}{a_2 s^2+a_1 s+1}, \tag{15}$$

where s is the complex variable in the Laplace transform. The SOTD model is very versatile and can be used to describe most non-oscillatory or oscillatory proportional systems with or without a transport delay, see Skogestad (2003), Rodriguex et al. (2017). This model has only four parameters a_2, a_1, K, T_d which can be estimated from the frequency response points $G(j\omega_k),k=1,2,..,N$. For this purpose, we can use criterion (14), where the vector of model parameters

$$\theta=[K\ a_2\ a_1\ \tau]^T \tag{16}$$

and "T" denotes the transpose operator.

For a stable system, the value of vector θ, that minimises the criterion (13), can be determined by (14), where

$$\Theta=\{(K,a_2,a_1,\tau): K>0, a_2>0, a_1>0, \tau\in\langle 0,\tau_m\rangle\} \tag{17}$$

and τ_m is the maximal possible transport delay (see Fig. 3).

By denoting the real and imaginary part of the complex values $G(j\omega_i)$

$$G\left(j\omega_i\right)=R_i+I_i\cdot j,\ \text{for } i=1,2,\cdots,N \tag{18}$$

then

$$\hat{\theta}=\arg\min_{\substack{\tau=0,\Delta\tau,\cdots,\tau_m\\ K,a_2,a_1>0}} Kr\left(\begin{bmatrix}\left(Z^TZ\right)^{-1}\cdot Z^Tp\\ \tau\end{bmatrix}\right) \tag{19}$$

where $\Delta\tau$ is the chosen precision of the estimation τ, the matrix Z is assumed with the maximum column rank and

$$Z=\begin{bmatrix}\cos\omega_1\tau & R_1\omega_1^2 & I_1\omega_1\\ -\sin\omega_1\tau & I_1\omega_1^2 & -R_1\omega_1\\ \cos\omega_2\tau & R_2\omega_2^2 & I_2\omega_2\\ -\sin\omega_2\tau & I_2\omega_2^2 & -R_2\omega_2\\ \vdots & \vdots & \vdots\\ \cos\omega_N\tau & R_N\omega_N^2 & I_N\omega_N\\ -\sin\omega_N\tau & I_N\omega_N^2 & -R_N\omega_N\end{bmatrix}, p=\begin{bmatrix}R_1\\ I_1\\ R_2\\ I_2\\ \vdots\\ R_N\\ I_N\end{bmatrix}. \tag{20}$$

This way, we can estimate the model parameters from up to $2N$ equations (generalization in comparison with Hofreiter (2019)). The similar approach can also be used for fitting linear models with more parameters.

Model (15) has four parameters and therefore only two frequency response points are sufficient for the parameter estimation (N=2). In the case that we know the values of some parameters, for example, the static gain or the transport delay, the number of estimated parameters will be reduced, and a similar procedure may be used.

4. SHIFTING METHOD WITH ADDITIONAL INTEGRATOR OR DELAY

Using the shifting method, we obtain the frequency response points for higher frequencies. To get the frequency response points for lower frequencies, we can use an additional integrator or a delay, see Fig. 5.

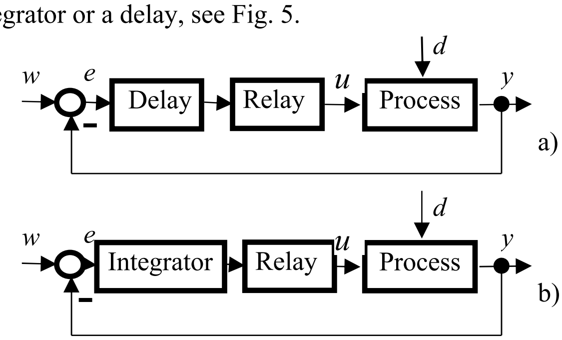

Fig. 5. The relay feedback with the additional delay (a) and with the additional integrator (b)

The additional integrator or the delay can be used only if a steady-state oscillation can be achieved with the relay feedback control.

5. EXAMPLES

The relay shifting method is demonstrated on proportional processes under various assumptions. Proportional process models are selected in the form (15) and their parameters are estimated from points $G(j\omega_1)$, $G(j\omega_2)$. In all simulated examples, the process was initially in a steady state and the controlled variable y was contaminated by a Gaussian white noise with the standard deviation of 0.1. T_P was calculated as the average cycle time. In all examples, the biased relay had the following parameters, see Fig. 2

$$u_A=2,\ u_B=-1,\ \varepsilon_A=0.1,\ \varepsilon_B=-0.1. \tag{21}$$

Example #1

The ***non-oscillatory*** process described by the transfer function

$$P_1(s)=\frac{1}{(s+1)^5}\,. \tag{22}$$

is controlled by a relay ***with the additional delay*** D=5 s, see Fig. 5a. The relay output u and the process output y affected by noise are depicted in Fig. 6.

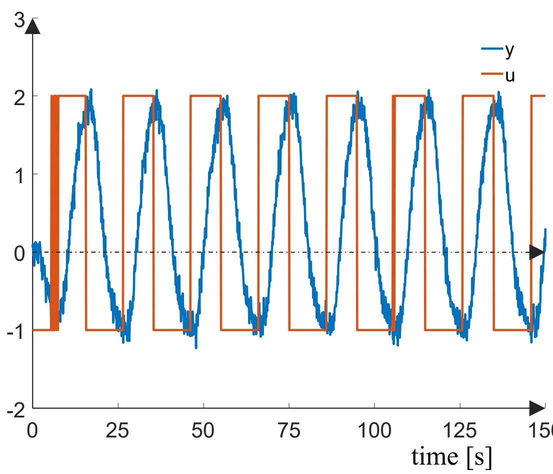

Fig. 6. The relay output u and the observed output y of the process $P_1(s)$ controlled by the relay with the additional delay.

Using the shifting method, we can determine from the observed stable courses u and y, that

$$T_P = 20.1\ \text{s} \tag{23}$$

$$\omega_1=\frac{2\pi}{T_p}=0.3126\ \text{rad}\cdot\text{s}^{-1},\ \omega_2=\frac{4\pi}{T_p}=0.6252\ \ \text{rad}\cdot\text{s}^{-1} \tag{24}$$

$$G(j\omega_1)=0.016-0.781j\,,\ G(j\omega_2)=-0.384-0.201j \tag{25}$$

$$\tau_m = 2.5\,\text{s} \tag{26}$$

and the SOTD model, estimated according to Section 2, is

$$M_{1D}(s)=\frac{1.087\cdot\exp(-1.47s)}{3.649s^2+3.951s+1}\,. \tag{27}$$

The Nyquist frequency characteristics of the process $P_1(s)$ and the model $M_{1D}(s)$ are shown in Fig. 7. In the same figure, the points $G(j\omega_1)$ and $G(j\omega_2)$ are depicted as well.

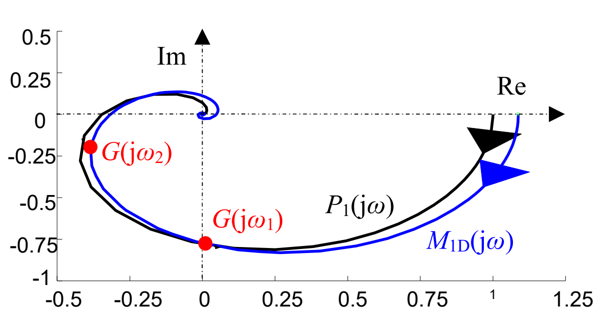

Fig. 7 The Nyquist frequency plots of the transfer functions $P_1(s)$ and $M_{1D}(s)$.

Example #2

The ***non-oscillatory*** process described by the transfer function (22) is controlled by a relay ***with the additional integrator***, see Fig. 5b. The time courses of the relay output u and the process output y affected by noise are depicted in Fig. 8.

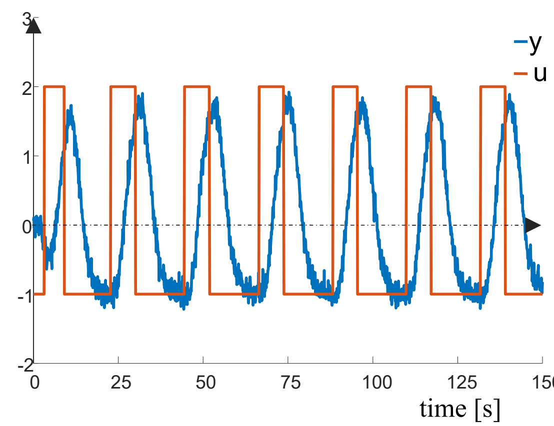

Fig. 8. The relay output u and the observed output y of the process $P_1(s)$ controlled by a relay with the additional integrator.

Using the shifting method, we can determine from the observed stable courses u and y, that

$$T_P = 21.9\ \text{s} \tag{28}$$

$$\omega_1=\frac{2\pi}{T_p}=0.2869\ \text{rad}\cdot\text{s}^{-1},\ \omega_2=\frac{4\pi}{T_p}=0.5738\ \ \text{rad}\cdot\text{s}^{-1} \tag{29}$$

$$G(j\omega_1)=0.132-0.805j\,,\ G(j\omega_2)=-0.430-0.243j \tag{30}$$

$$\tau_m = 2.5\,\text{s} \tag{31}$$

and the SOTD model, estimated according to Section 2, is

$$M_{1I}(j\omega)=\frac{0.958\cdot\exp(-1.58s)}{3.029s^2+3.345s+1} \tag{32}$$

The Nyquist frequency characteristics of the process $P_1(s)$ and the model $M_{1I}(s)$ are shown in Fig. 9. In the same figure, the points $G(j\omega_1)$ and $G(j\omega_2)$ are also depicted.

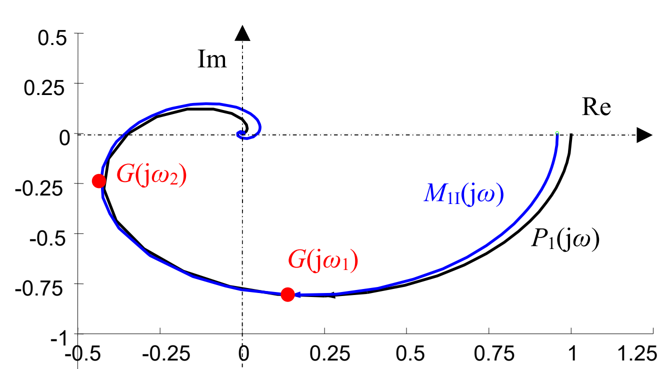

Fig. 9 The Nyquist frequency plots of the transfer functions $P_1(s)$ and $M_{1I}(s)$.

Example #3

The ***non-oscillatory*** process described by the transfer function (22) is controlled by a relay ***without any additional part***, see Fig. 1. The relay output u and the process output y affected by noise are depicted in Fig. 10.

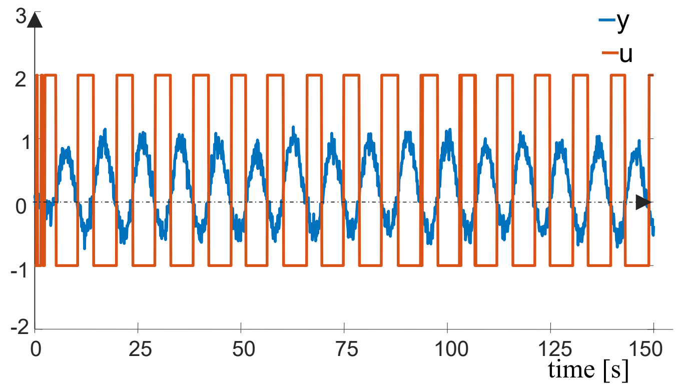

Fig. 10. . The relay output u and the observed output y of the process $P_1(s)$ controlled by a relay without any additional part.

Using the shifting method, we can determine from the observed stable courses u and y, that

$$T_P = 9.1 \text{ s} \tag{33}$$

$$\omega_1 = \frac{2\pi}{T_p} = 0.6095 \text{ rad}\cdot\text{s}^{-1},\ \omega_2 = \frac{4\pi}{T_p} = 1.3810 \text{ rad}\cdot\text{s}^{-1} \tag{34}$$

$$G(j\omega_1) = -0.371 - 0.029j,\ G(j\omega_2) = -0.006 + 0.078j \tag{35}$$

$$\tau_m = 2.5 \text{ s} \tag{36}$$

and the SOTD model, estimated according to Section 2, is

$$M_{10}(s) = \frac{0.4791 \cdot \exp(-1.35s)}{3.533s^2 + 1.583s + 1} \tag{37}$$

The Nyquist frequency characteristics of the process $P_1(s)$ and the model $M_{10}(s)$ are shown in Fig. 11. In the same figure, the points $G(j\omega_1)$ and $G(j\omega_2)$ are also depicted. The previous result can be improved if we know the static gain K (e.g., from a static process characteristic) or we can estimate the value of K using formula (12).

According to formula (12), the static gain K is determined from the observed stable courses u and y depicted in Fig. 10, as

$$K = G(0) = 0.9397. \tag{38}$$

The SOTD model $M_{1K}(s)$ derived from the values $G(0)$, $G(j\omega_1)$ and $G(j\omega_2)$ given by (38) and (35) is

$$M_{1K}(s) = \frac{0.9397 \exp(-1.35s)}{5.537s^2 + 3.105s + 1}. \tag{39}$$

The Nyquist frequency plots of the models $M_{10}(s)$, $M_{1K}(s)$ and the process $P_1(s)$ are in Fig. 11.

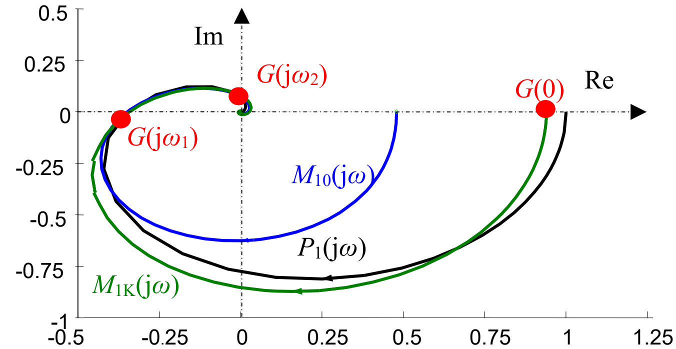

Fig. 11. The Nyquist frequency diagrams of the models $M_{10}(s)$, $M_{1K}(s)$ and the process $P_1(s)$.

Example #4

The ***oscillatory*** process ***with time-delay*** described by the transfer function

$$P_2(s) = \frac{\exp(-0.5s)}{s^3 + 2s^2 + 2s + 1}. \tag{40}$$

is controlled by a relay ***with the additional integrator***, see Fig. 5b. The relay output u and the observed output y of the process $P_2(s)$ are depicted in Fig. 12. In this example

$$T_P = 12.3 \text{ s} \tag{41}$$

$$\omega_1 = \frac{2\pi}{T_p} = 0.5108 \text{ rad}\cdot\text{s}^{-1},\ \omega_2 = \frac{4\pi}{T_p} = 1.0216 \text{ rad}\cdot\text{s}^{-1} \tag{42}$$

$$G(j\omega_1) = 0.209 - 0.969j,\ G(j\omega_2) = -0.683 - 0.130j \tag{43}$$

$$\tau_m = 1.5 \text{ s}$$

and the SOTD model

$$M_2(s) = \frac{0.9404 \exp(-1.14s)}{1.239s^2 + 1.295s + 1}. \tag{44}$$

The Nyquist frequency responses of the process $P_2(s)$ and the model $M_2(s)$ are in Fig. 13.

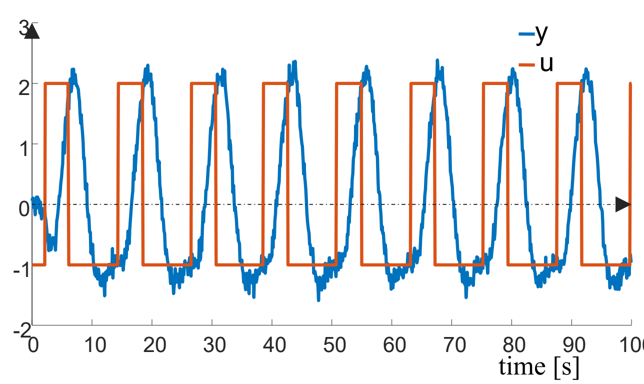

Fig. 12. The relay output u and the observed output y of the process $P_2(s)$ controlled by a relay with the additional integrator.

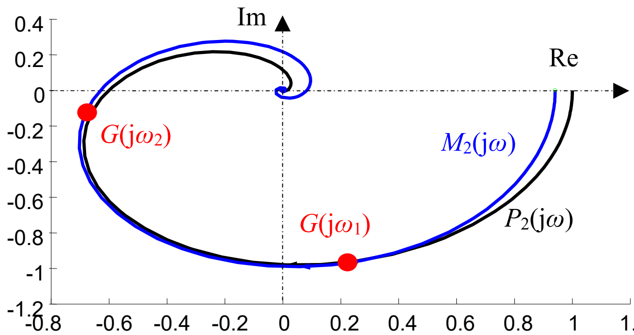

Fig. 13. The Nyquist frequency responses of the model $M_2(s)$ and the process $P_2(s)$.

6. IMPLEMENTATION

The relay shifting method was implemented into PLC code using the Mosaic programming environment for a PLC Tecomat Foxtrot. This identification approach enables to estimate parameters of the SOTD model, which is used for the PLC automatic tuning. This PLC was used to control the laboratory apparatuses called "Air Aggregate", "Water Levitation" and "Air Levitation", see Hornychová and Hofreiter (2019), Vaněk and Slabý (2020).

7. CONCLUSIONS

The basic properties of the generalized relay shifting method are as follows:

- The generalized relay shifting method enables to estimate frequency response points $G(j\omega_i)$, $i=1,2,\ldots,N$ from a single relay test without any prior knowledge of the model.
- The real and imaginary parts of the points $G(j\omega_i)$, $i=1,2,\ldots,N$ define $2N$ conditions, which can be used for model fitting.
- The position of the points $G(j\omega_i)$, $i=1,2,\ldots,N$ significantly affects the estimation of the model parameters. An additional integrator or delay help to improve the parameter estimation.
- The shifting filter described by (4) and (5) filters out the lower harmonics and amplifies the n-th harmonic and the frequency response point $G(j\omega_n)$ can also be determined without utilization of relation (9).
- The shifting method can be applied to model fitting for stable/integrating processes regardless of whether they are overdamped/underdamped processes or with a time-delay if there are stable oscillations for the relay feedback test.
- The relay shifting method is appropriate only for processes describable by linear and time-invariant models.

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