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CONSTRAINING SPACETIME TORSION WITH THE MOON, MERCURY AND LAGEOS

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Abstract. We consider an extension of Einstein General Relativity where, beside the Riemann curvature tensor, we suppose the presence of a torsion tensor. Using a parametrized theory based on symmetry arguments, we report on some results concerning the constraints that can be put on torsion parameters by studying the orbits of a test body in the solar system.

Keywords: Riemann-Cartan spacetime, torsion, autoparallel trajectories, geodetic precession, perihelion advance, frame dragging, lunar laser ranging, planetary radar ranging.

1. Introduction

One of the various generalizations of the Einstein theory of General Relativity is the so-called Einstein–Cartan theory [11]: it consists of a spacetime endowed with a locally Lorentzian metric $g_{\lambda\mu}$ and with a *compatible* connection ${\Gamma^{\nu}}_{\lambda\mu}$, not assumed to be symmetric, so that ${\Gamma^{\nu}}_{\lambda\mu}$ is not equal, in general, to ${\Gamma^{\nu}}_{\mu\lambda}$. The connection ${\Gamma^{\nu}}_{\lambda\mu}$ has a curvature tensor, and its lack of symmetry amounts to the additional presence of a torsion tensor [4][1] ${S_{\lambda\mu}}^{\nu} := \frac{1}{2}\left({\Gamma^{\nu}}_{\lambda\mu} - {\Gamma^{\nu}}_{\mu\lambda}\right)$. ${\Gamma^{\nu}}_{\mu\nu}$ is determined uniquely by $g_{\mu\nu}$ and by the torsion tensor, as follows: ${\Gamma^{\nu}}_{\lambda\mu} = \left\{ \begin{matrix} \nu \\ \lambda\mu \end{matrix} \right\} + {S_{\lambda\mu}}^{\nu} + {S^{\nu}}_{\mu\lambda} + {S^{\nu}}_{\lambda\mu}$, where $\{\cdot\}$ is the Levi-Civita connection[2].

Usually, the torsion tensor is related to the intrinsic spin of matter [10, 11][3]. Since the spins of elementary particles in macroscopic matter are usually oriented in a random way, such theories predict a negligible amount of torsion generated by massive bodies. As a consequence, spacetime torsion would be observationally negligible in the solar system. On the other hand, as pointed out in [15], the existence or nonexistence of a torsion tensor in the solar system should be tested experimentally[4]: for this reason the authors of [15] developed a parametrized theory based on symmetry arguments and, computing the precessions of gyroscopes, put constraints on torsion with the Gravity Probe B experiment.

The aim of this short paper is to report on the results of [16, 17] concerning the constraints on torsion that can be put by studying the orbits of a test body in the solar system, following the nonstandard parametrized approach of [15]. The computations concern torsion corrections to: (i) the orbital geodetic (or de Sitter) precession, (ii) the precession of the pericenter of a body (a planet) orbiting around a central mass, (iii) the orbital frame-dragging (or Lense–Thirring) effect. We then use the measured Moon geodetic precession, Mercury's perihelion advance and the data from the measurements of LAGEOS satellites[5], to put contraints on torsion, looking at the secular perturbations of the orbits.

2. Working assumptions and autoparallel trajectories

Our main working hypotheses are the following: (i) Weak field approximation and slow motion of the test bodies, assumptions probably sufficiently accurate for solar system experiments. (ii) Spherical or axial symmetry, depending on the situation at hand (in the case of the Lense–Thirring effect, we suppose also Earth uniformly rotating, while in the computation of the de Sitter effect Earth and Sun are supposed to be nonrotating). For example, if m is the mass of the body, the torsion tensor around a spherically symmetric body (Sun/Earth) in spherical coordinates (t, r, θ, ϕ) can be parametrized to second order

[1] Explicit examples of torsion of a connection compatible with a Riemannian metric in $\mathbb{R}^2$, $\mathbb{R}^3$ or in a surface embedded in $\mathbb{R}^3$ can be found for instance in [1, 2, 20]. See also [21] for a discussion on nonsymmetric connections.

[2] We recall that for a connection, it is possible to choose coordinates so that ${\Gamma^{\nu}}_{\lambda\mu} + {\Gamma^{\nu}}_{\mu\lambda} = 0$ [13, Prop. 8.4] at a point.

[3] In [10] the reader can find an interesting analogy between torsion and the density of dislocations in crystals.

[4] In the teleparallel theory of Hayashi and Shirafuji [9] a massive body generates a torsion field. Gravitational forces are due entirely to spacetime torsion and not to curvature.

[5] Remember that in General Relativity we have that the secular precession of the perihelion of Mercury is 43″/century, the geodetic precession is 19.2 mas/yr, and the Lense–Thirring effect on the longitude of the nodes of the LAGEOS satellites is around 31 mas/yr.

in $m/r \ll 1$ as

$$S_{tr}{}^{t} = t_1 \frac{m}{2r^2} + t_3 \frac{m^2}{r^3}, \quad S_{r\theta}{}^{\theta} = S_{r\phi}{}^{\phi} = t_2 \frac{m}{2r^2} + t_4 \frac{m^2}{r^3},$$

where t_1, t_2, t_3, t_4 are dimensionless *torsion parameters* (all other components of the torsion tensor vanish. Moreover, it turns out that t_4 will not enter into our computations[6]). In case of a uniformly rotating spherical body, the expression of the torsion tensor reads (to second order in m/r) as

$$\begin{aligned} S_{tr}{}^{t} &= t_1 \frac{m}{2r^2}, & S_{r\phi}{}^{t} &= w_1 \frac{J}{2r^2} \sin^2\theta, \\ S_{t\phi}{}^{r} &= w_3 \frac{J}{2r^2} \sin^2\theta, & S_{tr}{}^{\phi} &= w_5 \frac{J}{2r^4}, \\ S_{r\theta}{}^{\theta} = S_{r\phi}{}^{\phi} &= t_2 \frac{m}{2r^2}, & S_{\theta\phi}{}^{t} &= w_2 \frac{J}{2r} \sin\theta\cos\theta, \\ S_{t\phi}{}^{\theta} &= w_4 \frac{J}{2r^3} \sin\theta\cos\theta, & S_{t\theta}{}^{\phi} &= -w_4 \frac{J}{2r^3} \frac{\cos\theta}{\sin\theta}, \end{aligned}$$

where $w_1, \ldots, w_5$ are other *torsion parameters*, m is the mass of the rotating body and J its angular momentum. (iii) Bodies move along (causal) autoparallel trajectories, namely they satisfy

$$\frac{\mathrm{d}^2 x^\nu}{\mathrm{d}\tau^2} + \Gamma^\nu{}_{\lambda\mu} \frac{\mathrm{d}x^\lambda}{\mathrm{d}\tau} \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = 0,$$

and not along geodesics. This latter assumption is more questionable; on the other hand, assuming that the test bodies move along geodesics does not give any constraint on the torsion parameters. For example, indicating with $m_\odot$ the mass of the Sun, the system of autoparallel trajectories (of the Moon) in the case of computation of the geodetic precession reads as

$$\begin{aligned} \frac{d^2 x^\alpha}{dt^2} + m_\odot \Big(\frac{X^\alpha}{\Delta^3} - \frac{\xi^\alpha}{\rho^3} \Big) = \\ 2(\beta - t_3) m_\odot^2 \Big(\frac{X^\alpha}{\Delta^4} - \frac{\xi^\alpha}{\rho^4} \Big) \\ + (t_2 + 2) m_\odot \Big(\frac{\dot\Delta \dot X^\alpha}{\Delta^2} - \frac{\dot\rho \dot\xi^\alpha}{\rho^2} \Big) \\ + 3\gamma m_\odot \Big(\frac{X^\alpha \dot\Delta^2}{\Delta^3} - \frac{\xi^\alpha \dot\rho^2}{\rho^3} \Big) \\ - (2\gamma + t_2) m_\odot \Big(\frac{X^\alpha \sum_\sigma (\dot X^\sigma)^2}{\Delta^3} - \frac{\xi^\alpha \sum_\sigma (\dot\xi^\sigma)^2}{\rho^3} \Big), \end{aligned}$$

where ξ^α, ρ are the heliocentric rectangular coordinates of the Earth, X^α, Δ are the heliocentric rectangular coordinates of the Moon, and x^α, r are the geocentric rectangular coordinates of the Moon. In the case of the Lense–Thirring effect, the system of autoparallel (of a satellite) reads as

$$\begin{aligned} \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} &= -\frac{m_\oplus}{r^3} x + \frac{J}{r^5} \Big[(D + A) xy \frac{\mathrm{d}x}{\mathrm{d}t} \\ &\quad + \big(-Dx^2 + Ay^2 + Bz^2 \big) \frac{\mathrm{d}y}{\mathrm{d}t} + (A - B) yz \frac{\mathrm{d}z}{\mathrm{d}t} \Big], \\ \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} &= -\frac{m_\oplus}{r^3} y - \frac{J}{r^5} \Big[(D + A) xy \frac{\mathrm{d}y}{\mathrm{d}t} \\ &\quad + \big(Ax^2 - Dy^2 + Bz^2 \big) \frac{\mathrm{d}x}{\mathrm{d}t} + (A - B) xz \frac{\mathrm{d}z}{\mathrm{d}t} \Big], \\ \frac{\mathrm{d}^2 z}{\mathrm{d}t^2} &= -\frac{m_\oplus}{r^3} z + \frac{J}{r^5} (D + B) z \Big[y \frac{\mathrm{d}x}{\mathrm{d}t} - x \frac{\mathrm{d}y}{\mathrm{d}t} \Big], \end{aligned}$$

where $m_\oplus$ is the mass of the Earth, J its angular momentum, $A = 1 + \gamma + \frac{\alpha_1}{4} + w_1 - w_3$, $B = -2\left(1 + \gamma + \frac{\alpha_1}{4}\right) + w_2 - w_4$, $D = -\left(1 + \gamma + \frac{\alpha_1}{4}\right) - w_1 - w_5$. It should be noted that from assumption (iii) it follows that the antisymmetric part of the torsion tensor cannot be measured (the torsion tensors that we will consider are not totally antisymmetric). (iv) In the computation of the geodetic precession we assume that we can superimpose linearly the metric and torsion fields of Sun and Earth to obtain the global fields. (v) All computations have been performed by Taylor expanding in m/r at the required order[7].

3. Description of the results

Using perturbative methods in Celestial Mechanics, in the case of the three-body problem we can constrain the torsion parameters with the Moon as follows. The secular precession of node Ω of the satellite orbiting around Earth turns out to be:

$$(\delta\Omega)_{\mathrm{sec}} = \frac{1}{2} \frac{m_\odot \nu_0}{\rho} \left(1 + 2\gamma + \frac{3}{2} t_2 \right) t, \tag{1}$$

where ν_0 is the angular velocity of the Earth and t is time. We observe that $(\delta\Omega)_{\mathrm{sec}}$ is independent of the details of the satellite[8]. The same right hand side of Eq. 1 is obtained for the secular precession of the lunar perigee $(\delta\tilde\omega)_{\mathrm{sec}}$. From these computations, we find

$$b \equiv \frac{\text{geodetic precession with torsion}}{\text{geodetic precession in GR}} = \frac{1}{3}(1 + 2\gamma) + \frac{t_2}{2}.$$

Using the Lunar Laser Ranging data giving the relative deviation from GR [23] we find $|b - 1| < 0.0064$. Using the Cassini measurement $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$ [3] gives

$$|t_2| < 0.0128. \tag{2}$$

[6] All torsion parameters are independent of the PPN parameters appearing in the expression of the metric.

[7] Other assumptions that we make are the following: (vi) Existence of the Newtonian limit, which fixes $t_1 = 0$. (vii) All PPN parameters different from γ, β (and α_1 in the case of the Lense–Thirring effect) are negligible. (viii) Test bodies, such as planets, are supposed to be pointwise, in particular structureless. (ix) In the computation of secular effects, we perform time averages over suitably chosen time intervals.

[8] This remark is important, since it allows to use the result also in the case of LAGEOS satellites, and to decouple the geodetic precession from the Lense–Thirring precession.

Using a two-body computation, we can now constrain the torsion parameters with Mercury as follows. If $\widetilde{\omega}$ is the longitude of the pericenter, the secular contribution $(\delta\widetilde{\omega})_{\text{sec}}$ reads as

$$(\delta\widetilde{\omega})_{\text{sec}} = (2+2\gamma-\beta+2t_2+t_3)\,\frac{m_\odot}{a(1-e^2)}v,$$

where a is the semimajor axis of the orbit, e is the eccentricity and v the true anomaly. Then

$$\begin{aligned} B \equiv& \frac{\text{perihelion precession with torsion}}{\text{perihelion precession in GR}} \\ =& \frac{1}{3}(2+2\gamma-\beta+2t_2+t_3). \end{aligned}$$

Using the planetary radar ranging data giving the relative deviation from GR of 10^{-3} [22], we find $|B-1| < 0.001$. Using the Cassini measurement one gets $|1-\beta+t_3| < 0.0286$. If in addition we assume $\beta = 1 + (1.2 \pm 1.1) \times 10^{-4}$ [23], then

$$|t_3| < 0.0286. \tag{3}$$

In the case of the Lense–Thirring effect, the precession of the node Ω of LAGEOS reads as

$$(\delta\Omega)_{\text{sec}} = \frac{J}{a^3(1-e^2)^{3/2}}\Big(1+\gamma+\frac{\alpha_1}{4}-\frac{w_2-w_4}{2}\Big)t,$$

where a the semimajor axis of the satellite, e the eccentricity of the orbit and t is time. We then have

$$\begin{aligned} b_\Omega \equiv& \frac{\text{precession of the node with torsion}}{\text{precession of the node in GR}} \\ =& \frac{1}{2}\Big(1+\gamma+\frac{\alpha_1}{4}\Big)-\frac{w_2-w_4}{4}. \end{aligned}$$

Using $|\alpha_1| < 10^{-4}$ [24] and the measurements of [5], we get $|b_\Omega - 0.99| < 0.10$ and

$$-0.36 < w_2 - w_4 < 0.44. \tag{4}$$

Reasoning similarly for the perigee, we eventually have

$$\begin{aligned} -0.22 <& 0.11w_1 - 0.20w_2 - 0.06w_3 \\ &+ 0.20w_4 + 0.06w_5 < 0.42. \end{aligned} \tag{5}$$

It is worthnothing that Eqs. 4 and 5 are obtained making use also of the previously obtained estimates Eqs. 2 and 3.

4. Discussion

Beside the assumptions listed in Section 2, the bounds Eqs. 2, 3, 4 and 5 rely on the combination of a certain number of experimental estimates on the PPN parameters and on the various precessions. They can be improved as far as the experimental estimates improve.

Estimates Eqs. 2–5 should be coupled with the estimates obtained in [15] obtained analyzing the precession of gyroscopes of GPB. These latter estimates turn out to be different from ours, see [16, 17] for the details.

5. Future prospects

Before the end of the decade, robotic missions on the lunar surface could deploy new scientific payloads which include laser retroreflectors and thus extend the Lunar Laser Ranging reach for new physics (and possibly for torsion). In particular, the single, large, fused-silica retroreflector design developed by the University of Maryland and INFN-LNF [6] could improve on the performance of current Apollo arrays by a factor 100 or more.

After the end of this decade, results from the BepiColombo Mercury orbiter are expected to improve the classical test of the perihelion advance [19]. The latter measurement can be cross-checked by new planetary radar ranging data taken simultaneously with BepiColombo's ranging data. Mercury's special role in the search for new physics effects, and for spacetime torsion in particular, is due to the relatively large value of its eccentricity and to its short distance to the Sun.

Eventually we observe that the recently approved JUNO mission to Jupiter [18] will make it possible, in principle, to attempt a measurement of the Lense–Thirring effect through JUNO's node. Hence such a mission may yield an opportunity to improve of the constraints on torsion parameters.

6. Conclusions

Estimates Eqs. 2–5 give an order of magnitude of the torsion tensor; they neither prove nor disprove the existence of a non-vanishing torsion tensor in the solar system. A more definite answer could be given by refining these estimates, taking advantage of future missions.

Acknowledgements

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