Acta Polytechnica **61**(1):242–252, 2021
https://doi.org/10.14311/AP.2021.61.0242

APPLICATION OF GLOBAL OPTIMIZATION TO PREDICT STRAINS IN RC COMPRESSED COLUMNS

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Abstract. In this paper, the results of an application of global and local optimization methods to solve a problem of determination of strains in RC compressed structure members are presented. Solutions of appropriate sets of nonlinear equations in the presence of box constraints have to be found. The use of the least squares method leads to finding global solutions of optimization problems with box constraints. Numerical examples illustrate the effects of the loading value and the loading eccentricity on the strains in concrete and reinforcing steel in the cross-section.

Three different minimization methods were applied to compute them: trust region reflective, genetic algorithm tailored to problems with real double variables and particle swarm method. Numerical results on practical data are presented. In some cases, several solutions were found. Their existence has been detected by the local search with multistart, while the genetic and particle swarm methods failed to recognize their presence.

Keywords: Global optimization, nonlinear equations, least squares method, RC compressed structure members.

1. Introduction

Our problem is to determine the normal strains in the cross-sections of reinforced concrete structure members subjected to compression. Mathematically, it may be formulated as a task of solving sets of equations with box constraints. The unknown variables are: $\epsilon^{'}$ – maximum strain in the cross section and ξ – coordinate describing the location of the neutral axis.

The presence of the box constraints makes a direct use of numerical methods for solving sets of nonlinear equations impractical. Therefore, our task is reformulated by means of the frequently used least squares method. It leads to a nonlinear, nonconvex optimization problem of finding a minimum of a nonlinear function with the restricted scope of variables.

1.1. Motivation to study the strains in RC compressed structure members

Reinforced concrete structure members subjected to the compression are frequently encountered in the engineering practice (columns, pillars, tower-like structures etc.). The determination of strains is very important in the safety assessment of existing RC structures. In order to solve this problem analytically, several physical models of materials and methods were proposed. Lechman and Lewiński [1] considered a generalized linear section model. A simplified approach based on the rectangular stress distribution for concrete was used by Knauff [2] and Knauff et al. [3]. Nieser and Engel [4] and CICIND [5] applied the parabola-rectangle diagram for the design of cross-sections.

For reinforcing steel itself, both linear and nonlinear models are used, see for instance Lechman and Stachurski [6], Lechman [7–10], where the ring sections were investigated. The results of FE (finite element) modelling of failure behaviour of RC compressed columns were presented by Majewski et al. [11] and Rodriguez et al. [12]. In Kim and Lee [13], a numerical method for predicting the behaviour of RC columns subjected to axial force and biaxial bending is proposed and verified in tests. Campione et al. [14] experimentally investigated the behaviour of compressed concrete columns subjected to the overcoring technique, see also Campione et al. [15]. The list of researchers working in various directions could be continued. Let's mention some of them: Lloyd and Rangan [16], Bonet et al. [17], Ye et al. [18], Xu et al. [19], Trapko and Musiał [20], Trapko [21], Hadi and Le [22], El Maddawy et al. [23], Csuka and Kollar [24], Elwan and Rashed [25], Sadeghian et al. [26], Eid and Paultre [27], Wu and Jiang [28], Quiertant and Clement [29], Lee et al. [30], Kumar and Patel [31] and many others. Of course, the list is not complete.

Despite the variety of calculation methods and experimental investigations concerning this problem, there are not any appropriate analytical solutions based on the nonlinear material laws for determining the strains in RC externally compressed structure members that considers concrete softening.

The aim of our paper is twofold. Firstly, to formulate equilibrium equations allowing to calculate the strains. Secondly, to investigate the usefulness of some

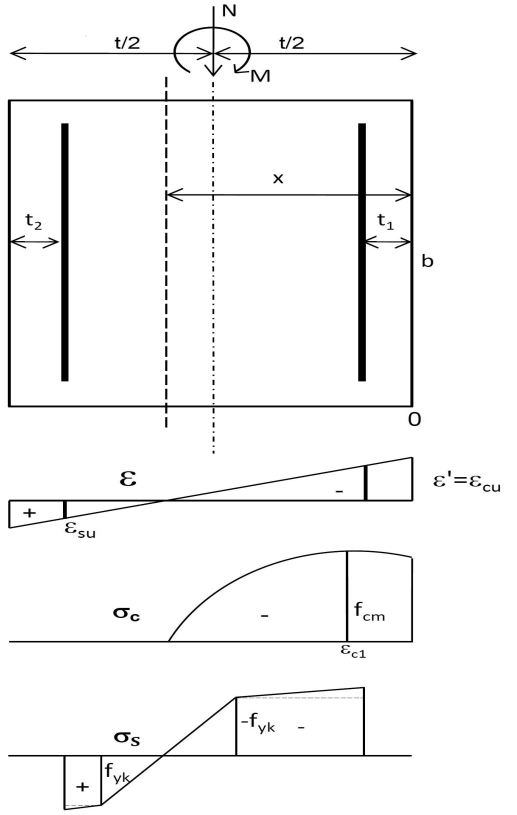

FIGURE 1. Distribution of strain ϵ, stresses in concrete σ_c and stresses in steel σ_s across the section

global optimization methods to solve the problem numerically.

2. Formulation of the equilibrium equations

To get the required equations, we started with the integral equilibrium equations and integrated them. The rectangular RC cross-section is subjected to the axial force N and the bending moment M (see Fig. 1). The content of the current section is an extension of that presented in Lechman and Stachurski [32]. The detailed way of deriving the formulas for the section wholly in compression is included.

In the derivation of the governing equations, the following assumptions are made:

- plane cross-sections remain plane,
- elasto-plastic stress/strain relationships for concrete and reinforcing steel are used,
- the tensile strength of concrete is ignored,
- the ultimate strains for concrete are determined as ϵ_{cu} and for reinforcing steel as ϵ_{su}.

In Fig. 1, the following notation is used: t, b - the thickness and the width of the cross-section, respectively, t_1, t_2 - coordinates describing the locations of rebars, x, $x^{'}$ - coordinates describing the location of the neutral axis and the location of any point of the section, respectively. In accordance with the Eurocode 2 [33], the stress-strain relation for concrete $\sigma_c - \epsilon_c$ in compression for a short term uniaxial loading is assumed as

$$\sigma_c = \frac{k\eta_c - \eta_c^2}{1 + (k-2)\eta_c} f_{cm}, \qquad (1)$$

where: $\eta_c = \epsilon_c/\epsilon_{c1}$, ϵ_{c1} – the strain at peak stress on the $\sigma_c - \epsilon_c$ diagram, $k = 1.05\ E_{cm}|\epsilon_{c1}|/f_{cm}$, f_{cm} – the mean compressive strength of concrete, E_{cm} – secant modulus of elasticity of concrete, ϵ_{cu} (ϵ_{cu1}) – the ultimate strain for concrete.

The reinforcing steel is characterized by yield stress f_{yk}, E_s – modulus of elasticity and E_h – coefficient of steel hardening (linear elastic model with hardening).

2.1. Equations for strains in the rectangular sections

In further considerations, the corresponding dimensionless coordinates are used:

$$\xi = x/t,\ \ \xi^{'} = x^{'}/t,\ \ \xi_1 = t_1/t,\ \ \xi_2 = t_2/t. \qquad (2)$$

2.1.1. Equations for sections wholly in compression

Let us consider the section wholly in compression. The strain distribution can be expressed in the form

$$\epsilon = \epsilon_1 + (\epsilon_2 - \epsilon_1)\xi^{'}, \qquad (3)$$

where:

ϵ_1 – maximum compressive strain in the cross section,

ϵ_2 – minimum compressive strain in the cross section.

Thus, η_c occuring in (1) assumes the form

$$\eta_c = k_2\xi^{'} + k_1, \qquad (4)$$

after including in (3) the following assignements: $k_1 = \dfrac{\epsilon_1}{\epsilon_{c1}}$ and $k_2 = \dfrac{\epsilon_2 - \epsilon_1}{\epsilon_{c1}}$.

The equilibrium equation of the axial forces in the cross-section takes the following form

$$\int_{A_c} \sigma_c dA_c\ + \sigma_{s1}F_{a1} + \sigma_{s2}F_{a2} + N = 0, \qquad (5)$$

where: dA_c - element of the concrete area A_c, F_{a1}, F_{a2} - areas of the steel in compression and in tension, respectively.

The sectional equilibrium of the bending moments about the symmetry axis of the rectangle can be expressed in the form

$$\int_{A_c} \sigma_c(0.5t - x^{'})dA_c\ + \sigma_{s1}F_{a1}(0.5t - t_1) + \sigma_{s2}F_{a2}(0.5t - t_2) - M = 0. \qquad (6)$$

In order to obtain the final form of the equilibrium equations, we integrated formulas in (5) and (6). The most difficult part was to find the antiderivatives of the functions in the integral expressions in (5) and (6). After substituting relations (3) and (4) in relation (1), the function to be integrated in (5) is

$$f_N(\xi') = \frac{k(k_2\xi^{'} + k_1) - (k_2\xi^{'} + k_1)^2}{1 + (k-2)(k_2\xi^{'} + k_1)}, \tag{7}$$

and in (6) is

$$f_M(\xi') = \frac{k(k_2\xi^{'} + k_1) - (k_2\xi^{'} + k_1)^2}{1 + (k-2)(k_2\xi^{'} + k_1)}(0.5 - \xi^{'}). \tag{8}$$

Finally, the following equilibrium equations for strains in the rectangular sections are found. The first one concerns the equilibrium equation of the axial forces

$$\begin{aligned} & n + \tfrac{1}{k-2}\left\{W_1 + 0.5k_2 + \tfrac{W_2}{W_3}(\ln W_5 - \ln W_6)\right\} + \\ & \mu_1 \tfrac{f_{yk}}{f_{cm}}\left\{\delta_{i1}\left[-1 + \tfrac{E_h}{f_{yk}}\left(((\epsilon_2 - \epsilon_1)\,\xi_1 + \epsilon_1\right) + \right.\right. \\ & \left.\left.\epsilon_{ss})\right] + \delta_{i1+1}\frac{(\epsilon_2 - \epsilon_1)\xi_1 + \epsilon_1}{\epsilon_{ss}}\right\} + \\ & \mu_2 \tfrac{f_{yk}}{f_{cm}}\left\{\delta_{i2}\left[1 + \tfrac{E_h}{f_{yk}}\left[((\epsilon_2 - \epsilon_1)\,(1-\xi_2) + \epsilon_1\right) - \right.\right. \\ & \left.\epsilon_{ss}]\right] \\ & \left. + \,\delta_{i2+1}\frac{(\epsilon_2 - \epsilon_1)(1-\xi_2) + \epsilon_1}{\epsilon_{ss}}\right\} = 0 \end{aligned} \tag{9}$$

and the second one represents the sectional equilibrium of the bending moments

$$\begin{aligned} & -m + \tfrac{1}{k-2}\left\{-\tfrac{k_2}{12} + 0.5\tfrac{W_2}{W_6}(\ln W_5 - \ln W_6)\right. \\ & \left. - \tfrac{W_2}{W_3}\left[1 - \tfrac{W_6}{W_4}(\ln W_5 - \ln W_6)\right]\right\} + \\ & \mu_1 \tfrac{f_{yk}}{f_{cm}}(0.5 - \xi_1)\left\{\delta_{i1}\left[1 + \tfrac{E_h}{f_{yk}}\left(((\epsilon_2 - \epsilon_1)\,\xi_1 + \epsilon_1\right) + \right.\right. \\ & \left.\left.\epsilon_{ss})\right] + \delta_{i1+1}\frac{(\epsilon_2 - \epsilon_1)\xi_1 + \epsilon_1}{\epsilon_{ss}}\right\} + \\ & \mu_2 \tfrac{f_{yk}}{f_{cm}}(0.5 - \xi_2)\left\{\delta_{i2}\left[1 + \tfrac{E_h}{f_{yk}}\left(((\epsilon_2 - \epsilon_1)\,(1-\xi_2) + \right.\right.\right. \\ & \left.\left.\epsilon_1) - \epsilon_{ss})\right] + \delta_{i2+1}\frac{(\epsilon_2 - \epsilon_1)(1-\xi_2) + \epsilon_1}{\epsilon_{ss}}\right\} = 0 \end{aligned} \tag{10}$$

where:

$$\begin{array}{l|l} W_1 = k_1 - k - \dfrac{1}{k-2} & W_2 = k(k-2) + 1 \\ W_3 = (k-2)(k-2)k_2 & W_4 = (k-2)k_2 \\ W_5 = 1 + \dfrac{k-2}{k_2 + k_1} & n = \dfrac{N}{btf_{cm}} \\ W_6 = 1 + (k-2)(k_2 + k_1) & m = \dfrac{M}{bt^2 f_{cm}} \\ \delta_i = 0.5((-1^i) + 1), \quad i = 1,2 & \end{array}$$

and: μ_1 – the reinforcement ratio of steel in compression, μ_2 – the reinforcement ratio of steel in tension. The unknown variables are: ϵ_1, ϵ_2.

2.1.2. Section under combined compression with bending

Let us consider the section under combined compression and bending. Due to the Bernoulli assumption, one obtains (see Fig. 1)

$$\epsilon = \left(1 - \frac{\xi^{'}}{\xi}\right)\epsilon^{'}, \tag{11}$$

where: $\epsilon^{'}$ – the maximum compressive strain in concrete.

The resulting formulas are given below. Equation (12) (for axial forces)

$$\begin{aligned} & n + \tfrac{1}{k-2}\left\{W_1\xi + 0.5k_2\xi^2 - \tfrac{1}{k-2}\left[\tfrac{W_2}{W_3}\ln W - \xi\right]\right\} + \\ & \mu_1 \frac{f_{yk}}{f_{cm}}\left\{\delta_{i1}\left[-1 + \frac{E_h}{f_{yk}}\left(\left(1 - \frac{\xi_1}{\xi}\right)\epsilon^{'} + \epsilon_{ss}\right)\right]\right. \\ & \left. + \,\delta_{i1+1}\frac{\epsilon^{'}}{\epsilon_{ss}}\left(1 - \frac{\xi_1}{\xi}\right)\right\} + \\ & \mu_2 \frac{f_{yk}}{f_{cm}}\left\{\delta_{i2}\left[+1 + \frac{E_h}{f_{yk}}\left(\left(1 - \frac{1-\xi_2}{\xi}\right)\epsilon^{'} - \epsilon_{ss}\right)\right]\right. \\ & \left. + \,\delta_{i2+1}\frac{\epsilon^{'}}{\epsilon_{ss}}\left(1 - \frac{1-\xi_2}{\xi}\right)\right\} = 0 \end{aligned} \tag{12}$$

and equation (13), representing the sectional equilibrium of the bending moments

$$\begin{aligned} & -m + \tfrac{1}{k-2}\left\{0.5\left(W_1 + \tfrac{1}{k-2}\right)\xi + \right. \\ & 0.5\left[-W_1 + 0.5k_2 - \tfrac{1}{k-2}\right]\xi^2 - \\ & \left.\tfrac{1}{3}k_2\xi^3 - \tfrac{W_2}{(k-2)W_3}\left[0.5\ln W + \xi - \tfrac{W}{W_3}\ln W\right]\right\} + \\ & \mu_1 \tfrac{f_{yk}}{f_{cm}}(0.5 - \xi_1)\left\{\delta_{i1}\left[1 + \tfrac{E_h}{f_{yk}}\left(\left(1 - \tfrac{\xi_1}{\xi}\right)\epsilon^{'} + \epsilon_{ss}\right)\right]\right. \\ & \left. + \,\delta_{i1+1}\frac{\epsilon^{'}}{\epsilon_{ss}}\left(1 - \frac{\xi_1}{\xi}\right)\right\} + \\ & \mu_2 \tfrac{f_{yk}}{f_{cm}}(0.5 - \xi_2)\left\{\delta_{i2}\left[+1 + \tfrac{E_h}{f_{yk}}\left(\left(1 - \tfrac{1-\xi_2}{\xi}\right)\epsilon^{'} - \right.\right.\right. \\ & \left.\left.\epsilon_{ss})\right] + \delta_{i2+1}\frac{\epsilon^{'}}{\epsilon_{ss}}\left(1 - \frac{1-\xi_2}{\xi}\right)\right\} \\ & = 0, \end{aligned} \tag{13}$$

where:

$$\begin{array}{l|l} W_1 = k - k_2\xi & W_2 = k(k-2) + 1 \\ W_3 = (k-2)k_2 & W = 1 + (k-2)k_2\xi \\ \delta_i = 0.5((-1)^i + 1), \quad i = 1,2 & k_2 = \dfrac{\epsilon^{'}}{\epsilon_{c1}\xi} \end{array}$$

The unknown variables are:

- $\epsilon^{'}$ – maximum strain in the cross section,
- ξ – coordinate describing the location of the neutral axis.

3. Computational solution and numerical results

It is not our first work with models of the processes in the RC structure members. We have already got some experience with the circular RC structure members [6]. This experience suggests that we have to expect many global and local solutions of the least squares problem. Therefore, we decided to compare three different algorithms: local search method (trust region reflective) started many times from all points from a net of points equally distributed on the feasible box and genetic and particle swarm algorithms designed for searching a global optimum.

For the verification of the obtained formulae, two rectangular cross-sections 0.3 m × 0.3 m under the compression have been considered: the unreinforced one and that of reinforced ($\mu f_{yk}/f_{cm} = 0.1$). Both sections had the following characteristics: the concrete grade $C20/25$, the yield stress of steel $f_{yk} = 500MPa$ (reinforced), reinforcement ratios of the steel in compression and in tension $\mu_1 = \mu_2 = \mu$, $t_1/t = t_2/t = 0.1$, $E_h = 0$. It is assumed that the resistance of the cross-section is reached when the compressive strain in concrete $\epsilon_{cu} = -3.5‰$ or the ultimate strain in the reinforcing steel equals $\epsilon_{su} = 10‰$. After some rearrangements and substituting $\epsilon^{'} = x$ and $\xi = y$, the set of equations (12–13) takes the forms (14–15) for the unreinforced and (16–17) for the reinforced cross-sections, respectively.

Due to the appearance of the term $y - x$ in the denominator in some sets of equations, a danger of the division by 0 occurs. For this reason, fmincon has been finally applied with the algorithm option set to "interior point method", that allowed to include special constraints eliminating this danger. Moreover, the existence of multiple minima may not be avoided. However, the least squares formulation of the problem itself may, in general, involve extra local solutions (such a counterexample may be found in Stachurski [34]). This has been confirmed through computational results. Many local minima and sometimes several global minima that resulted from the numerical properties of the optimization problem were encountered. Therefore, the clusterization idea imported from clusterization the methods of the global optimization was incorporated (see, for instance, Thorn and Żilinstas [35]). The size of the problem and computation time were of secondary importance. We have also tested the genetic and particle swarm algorithms from the global optimization Matlab's toolbox, comparing them with a local search method started from all points of the net covering the whole set Ω of feasible points. For testing purposes, the sets of equations were used that described the reinforced or unreinforced concrete sections subjected to the compression.

The equations for the concrete without the reinforcement – the subject to the compression with bending are

$$\begin{aligned} r_1(x,y) = &-a_1 + (2.25 + 0.5x)y- \\ &0.25xy- \\ &4\left[-12.5\frac{y}{x}\ln(1-0.125x) - y\right] = 0, \end{aligned} \tag{14}$$

$$\begin{aligned} r_2(x,y) = &-a_2 + (3.125 + 0.25x)y+ \\ &\left[-3.125 + 0.25x - 0.125\frac{x}{y}\right]y^2+ \\ &0.16667xy^2+ \\ &50\frac{y}{x}\Big[0.5\ln(1-0.125x) + y + \\ &8\frac{(1-0.125x)y}{x}\ln(1-0.125x)\Big] \\ &= 0, \end{aligned} \tag{15}$$

where:

x – maximum compressive strain in concrete	$x \in [-5, -10^{-10}]$
y – coordinate specifying location of the neutral axis of the cross-section	$y \in [10^{-10}, 1]$

Different values of constants a_1 and a_2 correspond to different axial forces N and bending moments M. Parameters **a** are collected in table 1

The corresponding equations for the reinforced concrete section subjected to the compression with bending are given below

$$\begin{aligned} r_1(x,y) = &(2.25 + 0.5x)y - 0.25xy- \\ &4\left[-12.5\frac{y}{x}\ln(1-0.125x) - y\right]+ \\ &0.01x\left(1-\frac{0.9}{y}\right) - a_1 = 0, \end{aligned} \tag{16}$$

$$\begin{aligned} r_2(x,y) = &(3.125 + 0.25x)y- \\ &\left[3.125 + 0.25x + 0.125\frac{x}{y}\right]y^2+ \\ &0.16667xy^2+ \\ &50\frac{y}{x}\Big[0.5\ln(1-0.125x) + y + \\ &8(1-0.125x)\frac{y}{x}\ln(1-0.125x)\Big]+ \\ &0.004x(1-\frac{0.9}{y}) - a_2 = 0, \end{aligned} \tag{17}$$

where x and y have the same meaning and scope as in equations (14) and (15).

We used two sets of constant parameters a_1 and a_2 for that case specified below.

Set No.	a_1	a_2
1	0.143445	0.0292155
2	0.129182	0.0348055

We have to solve sets of two nonlinear equations with two unknowns x and y specified above

$$\begin{cases} r_1(x,y) &= 0 \\ r_2(x,y) &= 0 \end{cases} \quad \text{where} \begin{bmatrix} x \\ y \end{bmatrix} \in \Omega$$
$$\Omega = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in R^2 \;\middle|\; \begin{matrix} x^L \le x \le x^U \\ y^L \le y \le y^U \end{matrix} \right\} \tag{18}$$

x^L, x^U are the lower bound and upper bound on variable x and similarly y^L, y^U are the lower bound and upper bound on variable y. Due to their presence, a direct use of numerical methods for solving sets of nonlinear equations seems to be impractical.

Therefore, our task was reformulated by means of the frequently used the least squares method. It has

Set No.	a_1	a_2
1	0.17157	0.01717
2	0.13345	0.02522
3	0.10918	0.026805
4	0.08579	0.02574
5	0.07065	0.02369
6	0.04448	0.01763
7	0.02571	0.01138

TABLE 1. Sets of parameters for unreinforced concrete subjected to compression with bending

lead to a nonlinear, nonconvex optimization problem

$$\begin{aligned} \min_{(x,y)} \quad & f(x,y) = \tfrac{1}{2}\left(r_1^2(x,y) + r_2^2(x,y)\right) \\ \text{s.t.} \quad & x^L \le x \le x^U \\ & y^L \le y \le y^U \end{aligned} \tag{19}$$

Below are given the results of the local search with multiple starting points, genetic algorithm and the particle swarm optimization method.

In the first approach, we selected the fmincon function from the MATLAB Optimization Toolbox as a tool to solve the least squares problem (19), because it allows the introduction of of the box constraints.

The steps of the procedure may be summarized as follows:

Set S – set of solution clusters to be an empty set.

while (there are non used points in the net covering Ω)

- take a new point $\mathbf{x}^0 \in \Omega$,
- solve the least squares problem by means of the fmincon function from the MATLAB's toolbox OPTIMIZATION starting from the point $\mathbf{x}^0$,
- denote the found solution by x,
- `if f(x) < resTOL`

```
if x belongs to some cluster in S
 compare function value f(x) with the best
 in the cluster and save the better of the
 two points as the seed of the cluster
else
 save the current point as the seed of
 a new cluster
endif
endif
```

We assumed the threshold value $resTOL = 1.0e-20$. The only exception was the set of sample problems for the reinforced concrete section subjected to the compression with bending where $resTOL = 1.0e-10$. In the clusterization, we treated a new point as a structure member of the cluster if the following inequality was verified

$$\|\hat{\mathbf{x}} - \mathbf{x}_{seed}\| \le distTOL$$

where $\mathbf{x}_{seed}$ is the seed point of the current cluster. We assumed $distTOL = 1.0e^{-10}$.

The need of the clusterization is fully justified by the table 2 presented below.

We can evidently observe four different clusters in the table.

First seven examples are connected with the concrete sections without reinforcement. They are subjected to compression and bending. Parameters **a** are collected in table 1 and the calculated solutions are put to table 3.

Consecutive table 4 contains the solutions for two sets associated with the situation, when the sections are reinforced.

The results of calculations with the genetic algorithm are summarized in table 5 (for the sections without reinforcement) and in table 6 (for sections with reinforcement). Unfortunately, the implementation of the genetic algorithm from the Matlab's global optimization toolbox has found only one global solution, even for sets where the local minimizer detected more global solutions. Furthermore, the accuracy of the ga solution is definitely poorer compared with that found by the local minimizer.

Tables 7 and 8 summarize the results obtained by means of the particle swarm algorithm implementation in the Matlab's global optimization toolbox. The same comment as for the ga Matlab function is valid for the particle swarm one.

4. COMPARISON OF EXPERIMENTAL AND NUMERICAL RESULTS

In order to verify the calculated results, 175 mm × 175 mm × 1680 mm (the height) column specimens under eccentric compression were considered, the results of which were presented in detail by Lloyd at al. [16]. The longitudinal steel reinforcement of the columns consisted of three rebars ϕ12 mm, $f_{yk} = 430$ MPa, $E_s = 200$ GPa and they were made of concrete $f_{cm} = 44.78$ MPa, $E_{cm} = 32$ GPa. The static diagram and test specimen are shown in Fig. 2. In the above mentioned tests, the following failure loads and corresponding eccentricities were measured: $P_1 = 1476$ kN, $e_1 = 15$ mm; $P_2 = 830$ kN, $e_2 = 50$ mm; $P_3 = 660$ kN, $e_3 = 65$ mm. The ultimate strain in concrete at failure was assumed in calculations as -2.4 ‰, which corresponds to the peak stress on the $\sigma_c - \epsilon_c$ diagram (Fig. 1). The values collected in table 9 confirm a good

x(1)	x(2)	f(x)
-2.7790923390672915e+00	-3.0371456600817055e+00	9.8607613152626476e-32
-2.7790923390376046e+00	-3.0371456601087723e+00	2.2186712959340957e-29
-3.5000157890703703e+00	-4.9999510298477368e-01	1.2325951644078309e-29
-2.7790923390051390e+00	-3.0371456601383064e+00	9.1507865005637369e-29
-2.7790923391531197e+00	-3.0371456600033153e+00	1.9721522630525295e-29
-2.7790923392382747e+00	-3.0371456599104514e+00	8.4692264556708872e-25
-1.2763043306571440e+00	-1.0595130305186657e+00	4.9303806576313238e-31
-1.2763043306472746e+00	-1.0595130305290790e+00	6.9364539396083568e-27
-2.7790923389334106e+00	-3.0371456602237172e+00	5.7569619331116098e-25
-1.2763043307376327e+00	-1.0595130304452089e+00	4.7302072029314920e-28
-1.2763043305691497e+00	-1.0595130306000431e+00	2.0523202525456148e-27
-6.2452876553920056e-01	-3.6491369858491312e+00	3.6484816866471796e-30

Table 2. Sample table of results without clusterization

x(1)	x(2)	f
Set 1		
a(1) = 1.7157000000000000e-01	a(2) = 1.7170000000000001e-02	
-3.5001760272980009e+00	8.9999746818148496e-01	1.6308774769071113e-29
Set 2		
a(1) = 1.3345000000000001e-01	a(2) = 2.5219999999999999e-02	
-3.4992749633586557e+00	6.9999666861370502e-01	1.9772367181057118e-29
-1.7538910914819157e+00	8.3132002867829691e-01	4.3364238627823002e-29
Set 3		
a(1) = 1.0918000000000000e-01	a(2) = 2.6804999999999999e-02	
-3.5000264028206489e+00	5.7271595174704082e-01	8.1807341061747740e-29
-1.7532806514095229e+00	6.8025870700809510e-01	2.0954117794933126e-29
Set 4		
a(1) = 8.5790000000000005e-02	a(2) = 2.5739999999999999e-02	
-3.4999421856987043e+00	4.5001889527019823e-01	2.6437933681383566e-28
-1.7533490610068840e+00	5.3451336316461950e-01	2.9496002284279395e-29
Set 5		
a(1) = 7.0650000000000004e-02	a(2) = 2.3689999999999999e-02	
-3.5002461261714624e+00	3.7060720525856033e-01	6.1800472650662032e-28
-1.7531021761012342e+00	4.4021717424283136e-01	4.8915539099524771e-29
Set 6		
a(1) = 4.4479999999999999e-02	a(2) = 1.7630000000000000e-02	
-3.4981410371676795e+00	2.3329954055348942e-01	3.9372571632525573e-27
-1.7548124428383809e+00	2.7700765058540189e-01	8.0396019598500773e-29
Set 7		
a(1) = 2.5710000000000000e-02	a(2) = 1.1379999999999999e-02	
-3.2265678368900650e+00	1.3352149291135310e-01	4.2148274638856933e-26
-1.9823327248432530e+00	1.5060554010027433e-01	2.5005850693336105e-28

Table 3. Results for non-reinforced concrete subjected to compression with bending

x(1)	x(2)	f
	Set 1	
a(1) = 1.4344499999999999e-01	a(2) = 2.9215499999999998e-02	
-2.2782425251491150e+00	7.7232501367591999e-01	2.2709743058168710e-29
-3.4999345396782120e+00	6.9999699585013297e-01	3.2106687592596898e-29
-3.4997710707333067e+00	6.9999415656504049e-01	6.6164575189148840e-13
	Set 2	
a(1) = 1.2918199999999999e-01	a(2) = 3.4805500000000003e-02	
-2.8855483058111950e+00	5.9743506676707159e-01	5.7336006495255961e-30
-3.4999204514197872e+00	5.7272335751221370e-01	3.6503587461192280e-28
-2.8894696731050842e+00	5.9713482908132576e-01	4.5482057578604896e-11
-2.8850298075074248e+00	5.9747474535630740e-01	8.0528227557234545e-13
-2.8819290341073800e+00	5.9771169963840232e-01	3.9597006753181514e-11

Table 4. Results for reinforced concrete subjected to compression with bending

x(1)	x(2)	f
	Set 1	
a(1) = 1.7157000000000000e-01	a(2) = 1.7170000000000001e-02	
-2.0055257755736533e+00	9.9999993722360547e-01	4.9367829047643648e-06
	Set 2	
a(1) = 1.3345000000000001e-01	a(2) = 2.5219999999999999e-02	
-1.7533927171625709e+00	8.3145002503412480e-01	1.1683502858156642e-11
	Set 3	
a(1) = 1.0918000000000000e-01	a(2) = 2.6804999999999999e-02	
-3.5000168287943323e+00	5.7271604685964617e-01	4.2733490746564677e-15
	Set 4	
a(1) = 8.5790000000000005e-02	a(2) = 2.5739999999999999e-02	
-3.5000605612775848e+00	4.5002358354890987e-01	1.0830715796115672e-13
	Set 5	
a(1) = 7.0650000000000004e-02	a(2) = 2.3689999999999999e-02	
-1.7531661968763803e+00	4.4020811931145998e-01	1.4944588897123600e-14
	Set 6	
a(1) = 4.4479999999999999e-02	a(2) = 1.7630000000000000e-02	
-3.4974023122097648e+00	2.3328811178613765e-01	1.9405016478273012e-13
	Set 7	
a(1) = 2.5710000000000000e-02	a(2) = 1.1379999999999999e-02	
-3.8846165545827720e+00	1.3993640821435832e-01	1.8945631674275027e-08

Table 5. Results for non-reinforced concrete subjected to compression with bending obtained by Matlab's genetic algorithm function

x(1)	x(2)	f
	Set 1	
a(1) = 1.4344499999999999e-01	a(2) = 2.9215499999999998e-02	
-2.2782188146226554e+00	7.7232945260092078e-01	1.9240836428670515e-14
	Set 2	
a(1) = 1.2918199999999999e-01	a(2) = 3.4805500000000003e-02	
-2.8837525833446813e+00	5.9758208926874357e-01	1.0366597163520163e-11

Table 6. Results for reinforced concrete subjected to compression with bending obtained by Matlab's genetic algorithm function

x(1)	x(2)	f
	Set 1	
a(1) = 1.7157000000000000e-01		a(2) = 1.7170000000000001e-02
-3.5002095622539371e+00	8.9992251419894265e-01	1.1469551926718323e-10
	Set 2	
a(1) = 1.3345000000000001e-01		a(2) = 2.5219999999999999e-02
-3.4988243017354601e+00	6.9999134440783828e-01	8.0833419633913090e-12
	Set 3	
a(1) = 1.0918000000000000e-01		a(2) = 2.6804999999999999e-02
-3.5005599263496445e+00	5.7271548379815385e-01	9.7034824624575985e-12
	Set 4	
a(1) = 8.5790000000000005e-02		a(2) = 2.5739999999999999e-02
-3.4996043294305919e+00	4.5005795509525537e-01	4.3542554256757773e-11
	Set 5	
a(1) = 7.0650000000000004e-02		a(2) = 2.3689999999999999e-02
-3.5287486517409783e+00	3.7098837709011562e-01	3.6828714611703899e-09
	Set 6	
a(1) = 4.4479999999999999e-02		a(2) = 1.7630000000000000e-02
-1.8974393623306212e+00	2.6652417435303338e-01	1.4855280597896296e-08
	Set 7	
a(1) = 2.5710000000000000e-02		a(2) = 1.1379999999999999e-02
-3.2244553544557881e+00	1.3346506124150329e-01	5.6375712128260839e-11

TABLE 7. Results for non-reinforced concrete subjected to compression with bending obtained by Matlab's particle swarm function

x(1)	x(2)	f
	Set 1	
a(1) = 1.4344499999999999e-01	a(2) = 2.9215499999999998e-02	
-3.5011205692965111e+00	6.9993319470774185e-01	1.0180508409446182e-10
	Set 2	
a(1) = 1.2918199999999999e-01	a(2) = 3.4805500000000003e-02	
-3.5003741045884293e+00	5.7271843442355697e-01	8.6633784199345278e-13

TABLE 8. Results for reinforced concrete subjected to compression with bending obtained by Matlab's particle swarm function

Experimental		Numerical				
Failure load P_i [kN]	eccentricity e_i [mm]	$\epsilon_{cu} = \epsilon_{c1}$ [‰]	$\epsilon^{'}$ [‰]	ξ	strain in steel ϵ_s [‰]	ϵ_1/ϵ_2 [‰]
$P_1 = 1476$;	$e_1 = 15$	-2.4	-2.20			-2.20 / -0.35
$P_2 = 830$;	$e_2 = 50$	-2.4	-2.20	0.63	0.92	
$P_3 = 660$;	$e_3 = 65$	-2.4	-2.20	0.49	1.85	

TABLE 9. Comparison of the experimental and numerical results – 1

Experimental		Numerical		
Failure load P_i [kN]	eccentricity e_i [mm]	$\epsilon_{cu} = \epsilon_{c1}$ [‰]	ϵ_1 [‰]	ϵ_2 [‰]
$P_1 = 1548$;	$e_1 = 0$	-2.1	-2.1	-2.1
$P_2 = 1386$;	$e_2 = 16$	-2.1	-1.95	-0.31
$P_3 = 1098$;	$e_3 = 32$	-2.1	-1.95	-0.12

Table 10. Comparison of the experimental and numerical results

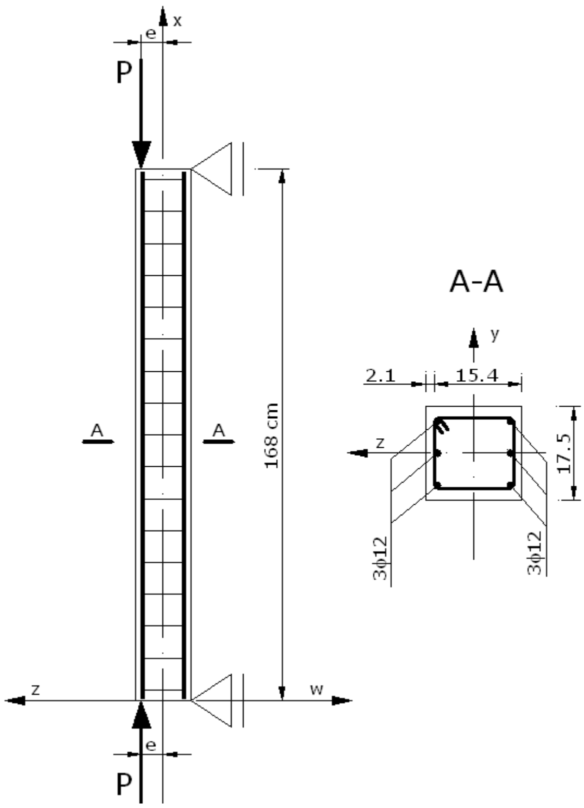

Figure 2. Static diagram and test specimen

conformity between the numerical solution and the experimental data given by Lloyd and Rangan [16]. It is worth noting that the theoretical values are lower than those obtained from the experiment due to neglecting the effect of confinement of the column.

As the next example, the results of tests conducted by Trapko at al. [20] on unstrenghtened column specimens 200 mm × 200 mm × 1500 mm (the height) under eccentric compression were analyzed. The longitudinal reinforcement of the column consisted of two rebars $\phi 12$ mm, steel grade A-IIIN, $f_{yk} = 608$ MPa, $E_s = 224$ GPa and the transverse reinforcement consisted of stirrups $\phi 6$ mm, steel grade A-I. The columns were made of concrete $f_{cm} = 31.9$ MPa. $E_{cm} = 31$ GPa. The failure loads and the corresponding eccentricities were determined in these tests as: $P_1 = 1548$ kN, $e_1 = 0$ mm; $P_2 = 1386$ kN, $e_2 = 16$ mm; $P_3 = 1098$ kN, $e_3 = 32$ mm. The ultimate strain in concrete at failure was assumed in calculations as -2.1 ‰. The characteristic failure mechanisms of the tested specimens occurred in the form of crushing the concrete in the upper part of the structure members and yielding the longitudinal reinforcing steel. A good conformity between the calculated and experimental results are confirmed by the values collected in Table 10. In author's opinion, further experimental work is needed concerning the post-critical behaviour of RC columns under eccentric compression.

5. Conclusions and comments

Our numerical results have confirmed that the elaborated analytical deformation model (taking into account the effect of concrete softening) may be used to determine the strains in rectangular cross-sections of RC compressed structure members. It can be applied to predict the behaviour of such structure members.

The current Matlab's implementations of the global optimization algorithms (ga and particle swarm) do not seem to be suitable for our application. In genetic algorithm (ga), elitism is used (part of the previous population survives to the next one). But it does not ensure finding the correct solution. The particle swarm procedure also does not guarantee the computation of the correct solution. Of course, we may tune some of their parameters, but we do not expect to gain much from that. Our experiments with the local search method have frequently shown the existence of several global minima. All of them are almost equally good from the point of view of a numerical calculations specialist. We decided to select them by means of the Hamilton minimum energy principle. In our opinion, from the existing global optimization methods, the most promising may be the clusterization methods (see for instance Törn and Żilinskas [35]). The calculated results conform to the experimental ones. The proposed approach enables to evaluate the structural safety of tower-like structures with rectangular sections without testing the drilling-out cores taken from the structure. It may also be useful in the structural design and maintenance.

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