Ing. MASt. Petr Jizba, PhD
Department of Physics, FNSPE
Czech Technical University in Prague
Břehová 7, 115 19, Czech Republic
E-mail: p.jizba@fjfi.cvut.cz

August 29, 2021

REPORT OF A THESIS ADVISOR

Master's thesis title: "Role of Weyl symmetry in the early Universe cosmology"
Candidate: Kamil Mudruňka
Institution: Department of Physics, Faculty of Nuclear Science and Physical Engineering, Czech Technical University in Prague, Břehová 7, 115 19
Assessment: A (výborně)

It is a pleasure to report on Bachelors's Thesis of Kamil Mudruňka. Mr. Mudruňka worked under my supervision throughout the period September, 2020 – August, 2021. During that time he focused his attention on the understanding of classical aspects of *Weyl gravity* in cosmology and astrophysics.

The concept of Weyl's gravity was originally introduced in a series of papers by H. Weyl in 1918-20, even though the incentives can be traced back to A. Einstens' 1915 General Relativity. Broadly speaking, Weyl's gravity is a pure metric theory that possesses general coordinate invariance, which augments standard General Relativity with the additional Weyl scale symmetry. Weyl gravity is due to its power-counting renormalizability and asymptotic freedom considered currently as one of the serious contenders for consistent Quantum Gravity.

In his work was Mr. Mudruňka inspired mainly by papers of P.D. Mannheim *at al.* [1,2,3] (Weyl gravity and its applications) as well as by various recent papers from phenomenological cosmology [4,5,6]. He approached the subject of his Thesis with a wide knowledge of prerequisite mathematical concepts. The work itself is well structured and basic concepts are easy to grasp. The Thesis material also undoubtedly benefited from an expertise that Mr. Mudruňka acquired during our informal online "Quantum gravity seminars", where among guests speakers was also Prof. Mannheim who delivered 3 interactive lectures on Weyl gravity.

Chapters 1 and 2 provide some conceptual background on Weyl symmetry. Beside a standard material Mr. Mudruňka also discusses such key topics as connection between conformal symmetry and Weyl symmetry or the way how various tensor quantities transform under Weyl transformation. In Chapter 3 he constructively formulates Weyl gravity and derives ensuing equation of motion (Bach equations) and compares it with Einstein's gravity. Chapter 4 is devoted to various important solutions of Bach equation and their interpretation. Particularly important is the discussion connected to Schwarzschild-like spherically symmetric vacuum solutions (Mannheim–Kazanas solution) and its various generalizations including gravitational source with non-zero electric charge and/or angular momenta (i.e., analogues of Reissner–Nordström, Kerr and Kerr–Newman solutions). I have also found particularly illuminating the discussion of the Mach principle in Weyl gravity. Chapter 5 is dedicated to cosmological implications of classical Weyl gravity. Here Mr. Mudruňka discusses Weyl gravity analogue of conventional Friedmann–Lemaître–Robertson–Walker model and related cosmological implications. The role of Weyl gravity as an alternative to cosmological inflation is discussed at the end of Chapter 5. It is expected that this issue will be explored in more detail during Mr. Mudruňka research project. The Thesis ends with two appendices which: a) give necessary background on the differential geometry and topology that is used in the main body of the text and b) provide codes for computations of relevant tensor quantities in Python language.

All in all, Bachelor Thesis of Mr. Mudruňka has in my opinion a very high quality. It offers a thorough and in many respects original selection and corresponding discussion of topics that are indispensable for a full-fledged research in the field of Weyl (and more generally "higher-order") gravity. Key aspects of classical Weyl gravity, such as ensuing cosmological and astrophysical implications are worked out logically and clearly. Last but not least, Weyl gravity is conceptually and numerically demanding endeavor within the steadily growing field of Quantum Gravity and I am sure that expertise gained by Mr. Mudruňka will be beneficial to him in the years to come.

[1] P.D. Mannheim and D Kazanas, *Exact vacuum solution to conformal Weyl gravity and galactic rotation curves*, Astrophysical Journal **342** (1989) 635.

[2] e.g., P.D. Mannheim and J. O'Brien, *Impact of a Global Quadratic Potential on Galactic Rotation Curves*, Phys. Rev. Lett. **106** (2011) 121101.

[3] P.D. Mannheim, *Is dark matter fact or fantasy? — Clues from the data*, International Journal of Modern Physics D **28** (2019) 1944022.

[4] A. Diaferio, L. Ostorero, and V. Cardone, *Gamma-ray bursts as cosmological probes: ΛCDM vs. conformal gravity*, Journal of Cosmology and Astroparticle Physics **10** (2011) 008.

[5] G. Singh and D. Lohiya, *Inhomogeneous nucleosynthesis in linearly coasting cosmology*, Monthly Notices of the Royal Astronomical Society *473* (2017) 14.

[6] C. Roberts, et al, *Tests of ΛCDM and Conformal Gravity using GRB and Quasars as Standard Candles out to $z \sim 8$*, arXiv:1711.10369.

Dr. Petr Jizba