The Numerical Simulation of Fluid-Structure-Acoustic Interaction in Human Phonation (1/2)

J. Valášek

Czech Technical University, Faculty of Mechanical Engineering, Department of Technical Mathematics, Karlovo náměstí 13, 121 35 Praha 2

Summary

The flow driven vibration of elastic bodies is a problem investigated and solved in many technical applications. This contribution deals with biomechanics of human vocal folds (VFs). Human voice as one of basic human being's characteristics originates as sound produced during complex phenomenon of flow-induced VFs vibration and it further propagates through vocal tract which in rough approximation functions as acoustic linear filter. It is finally articulated into human speech in mouth. Here we focus purely on the numerical simulation of fluid-structure interactions.

Due to relatively small airflow velocities incompressible fluid flow model is used. The two-dimensional Navier-Stokes equations are coupled with a structure modelled by the linear elasticity model. In order to incorporate the effect of time dependent fluid domain the ALE method is applied, see [1]. For the numerical approximation of both subproblems the finite element method (FEM). Due to high Reynolds number fluid flow the FEM needs to be stabilized.

This contribution compares three different inlet boundary conditions (BCs). The two frequently used possibilities are to prescribe the inlet velocity or the inlet pressure however their numerical behaviour do not correspond with experimental data, see cp. [2] and [3]. A remedy for this situation seems to be newly proposed penalization approach published in [3]. where inlet velocity is weakly enforced with the help of penalization parameter ϵ. Numerical results contain parametric study of FSI simulation behaviour, see [4].

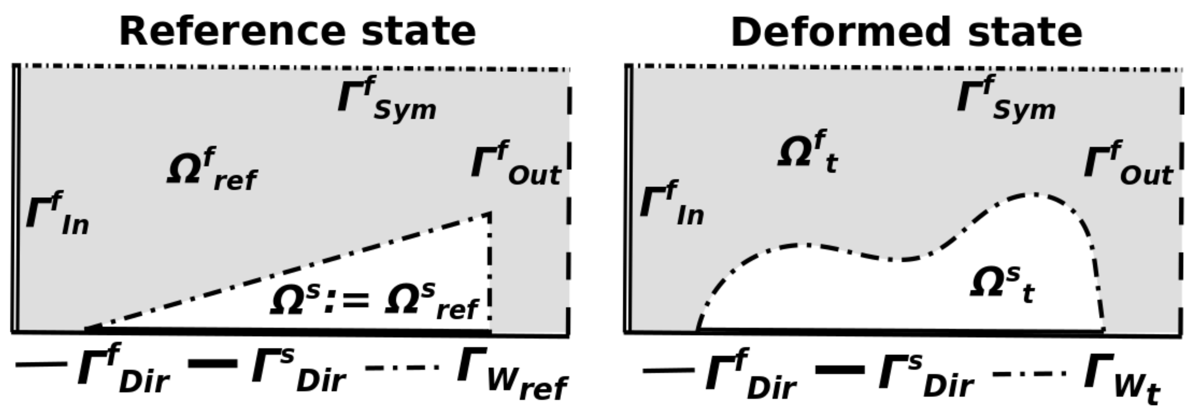
Mathematical model

The displacement of **elastic body** is governed by equations

$$\rho^s \frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{\partial \tau^s_{ij}(\mathbf{u})}{\partial x_j} = \mathbf{0} \quad \text{in } \Omega^s \times (0, \mathrm{T}), \tag{1}$$

where the Cauchy stress tensor $\boldsymbol{\tau}^s$ under assumptions of isotropic body and small displacements can be expressed as $\tau^s_{ij} = \lambda^s \mathrm{div}\, \mathbf{u} + \mu^s(\nabla \mathbf{u} + \nabla^T \mathbf{u})$ with λ^s, μ^s denoting Lamé coefficients. The elastic body is firmly clamped at wall Γ^s_{Dir}, i.e. $\mathbf{u} = \mathbf{0}$ on Γ^s_{Dir}, and it is loaded by aerodynamic forces (q^s_1, q^s_2) at interface $\Gamma_{\mathrm{W_{ref}}}$, i.e. $\tau^s_{ij}\, n^s_j = q^s_i$.

Schema of vocal fold model before and after deformation with denoted boundaries: inlet Γ^f_{In}, outlet Γ^f_{Out}, symmetric boundary Γ^f_{Sym}, walls Γ^f_{Dir}, Γ^s_{Dir} and interface Γ_{W}.



The **viscous incompressible fluid flow** in the time dependent domain Ω^f_t is described by the Navier-Stokes equations in the ALE form

$$\frac{D^A \mathbf{v}}{Dt} + ((\mathbf{v} - \mathbf{w}_D) \cdot \nabla)\mathbf{v} - \nu^f \Delta \mathbf{v} + \nabla p = \mathbf{0}, \qquad \mathrm{div}\, \mathbf{v} = 0 \quad \text{in } \Omega^f_t, \tag{2}$$

where $\mathbf{v}$ denotes the fluid velocity and p is the kinematic pressure, $\mathbf{w}_D(x,t)$ stays for the domain deformation velocity, see [1] and ν^f is the kinematic viscosity. Equations (2) are equipped with no-slip BC at Γ^f_{Dir} and directional do-nothing BC at Γ^s_{Out}, see [1].
The three different BCs at the inlet $\Gamma^f_{\mathrm{In}} = \Gamma^f_{\mathrm{In,dir}} \cup \Gamma^f_{\mathrm{In,p}} \cup \Gamma^f_{\mathrm{In},\epsilon}$ are considered, see [4]:

$$\begin{array}{llll} \text{a)} & \mathbf{v}(x,t) = \mathbf{v}_{\mathrm{Dir}}(x,t), & & \text{for } x \in \Gamma^f_{\mathrm{In,dir}}, \\ \text{b)} & (p(x,t) - p_{\mathrm{in}})\mathbf{n}^f - \nu^f \dfrac{\partial \mathbf{v}}{\partial \mathbf{n}^f}(x,t) = -\dfrac{1}{2}\mathbf{v}(\mathbf{v}\cdot\mathbf{n}^f)^-, & & \text{for } x \in \Gamma^f_{\mathrm{In,p}}, \\ \text{c)} & (p(x,t) - p_{\mathrm{in}})\mathbf{n}^f - \nu^f \dfrac{\partial \mathbf{v}}{\partial \mathbf{n}^f}(x,t) = -\dfrac{1}{2}\mathbf{v}(\mathbf{v}\cdot\mathbf{n}^f)^- + \dfrac{1}{\epsilon}(\mathbf{v} - \mathbf{v}_{\mathrm{Dir}}), & & \text{for } x \in \Gamma^f_{\mathrm{In},\epsilon}. \end{array} \tag{3}$$

The fluid flow and structure motion is coupled at the interface Γ_{W} via **coupling conditions**:

- Kinematic BC prescribed for fluid: $\mathbf{v}(x,t) = \mathbf{w}_D(x,t)$ for $x \in \Gamma_{\mathrm{W_t}}$.
- Dynamic BC prescribed for structure: $q^s_i(X,t) = \sum_{j=1}^2 \sigma^f_{ij}(x)\, n^s_j(x),\ x \in \Gamma_{\mathrm{W_t}}$, where $\boldsymbol{\sigma}^f = (\sigma^f_{ij})$ is the fluid stress tensor and $\mathbf{n}^s = (n^s_j)$ is the unit outer normal to $\Gamma^s_{\mathrm{W_t}}$.

Numerical approximation

Elastic body.
The standard FE approximation of elasticity problem is applied leading to the ODE system

$$\mathbb{M}\ddot{\mathbf{u}} + \mathbb{C}\dot{u} + \mathbb{K}\mathbf{u} = \mathbf{b}, \tag{4}$$

where $\mathbb{M}, \mathbb{C}$ and $\mathbb{K}$ are mass, damping and stiffness matrices, resp., and vector $\mathbf{b} = (b_i)$ is given by $b_i(t) = (\mathbf{f}^s, \boldsymbol{\Phi}_i)_{\Omega^s} + (\mathbf{q}^s, \boldsymbol{\Phi}_i)_{\Gamma_{\mathrm{W}}}$. The problem (4) is discretized in time by the Newmark method and the linear Lagrange FEs are used for practical computation.

Fluid flow.
Time discretization. The time derivative is approximated by the BDF2 scheme

$$\frac{D^A \mathbf{v}}{Dt}(t_{n+1}) \approx \frac{3\mathbf{v}^{n+1} - 4\overline{\mathbf{v}}^n + \overline{\mathbf{v}}^{n-1}}{2\Delta t}. \tag{5}$$

Fluid flow.
Weak formulation of problem (2) reads: Find such functions $V = (\mathbf{v}, p) \in \mathbf{H}^1(\Omega^f) \times L^2(\Omega^f)$, that

$$\begin{aligned} &\left(\frac{3\mathbf{v}}{2\Delta t}, \boldsymbol{\varphi}\right)_{\Omega^f} + \nu^f(\nabla \mathbf{v}, \nabla \boldsymbol{\varphi})_{\Omega^f} - (p, \mathrm{div}\, \boldsymbol{\varphi})_{\Omega^f} + (q, \mathrm{div}\, \mathbf{v})_{\Omega^f} + \frac{1}{\epsilon}(\mathbf{v}, \boldsymbol{\varphi})_{\Gamma^f_{\mathrm{In},\epsilon}} + \\ &+\frac{1}{2}(((\mathbf{v}^* - 2\mathbf{w}^{n+1}_D) \cdot \nabla)\mathbf{v}, \boldsymbol{\varphi})_{\Omega^f} - \frac{1}{2}((\mathbf{v}^* \cdot \nabla)\boldsymbol{\varphi}, \mathbf{v})_{\Omega^f} + \frac{1}{2}((\mathbf{v}^* \cdot \mathbf{n})^+ \mathbf{v}, \boldsymbol{\varphi})_{\Gamma^f_{\mathrm{In}} \cup \Gamma^f_{\mathrm{Out}}} = \\ &= \left(\frac{4\overline{\mathbf{v}}^n - \overline{\mathbf{v}}^{n-1}}{2\Delta t}, \boldsymbol{\varphi}\right)_{\Omega^f} + \frac{1}{\epsilon}(\mathbf{v}_{\mathrm{Dir}}, \boldsymbol{\varphi})_{\Gamma^f_{\mathrm{In},\epsilon}} + (p_{\mathrm{ref}} \mathbf{n}^f, \boldsymbol{\varphi})_{\Gamma^f_{\mathrm{Out}}} + (p_{\mathrm{in}}\, \mathbf{n}^f, \boldsymbol{\varphi})_{\Gamma^f_{\mathrm{In,p}}}. \end{aligned} \tag{6}$$

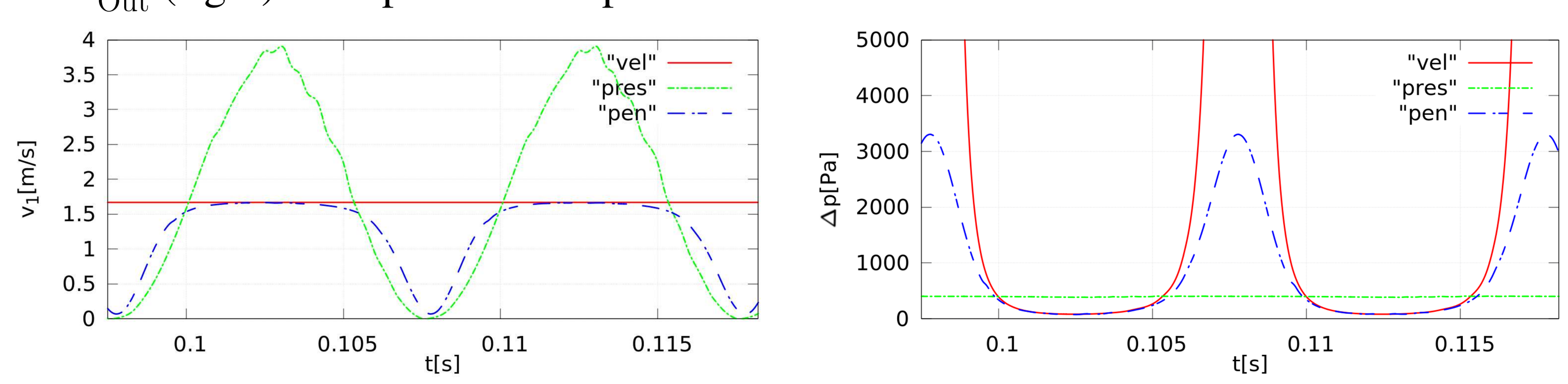
is satisfied for any functions $\Phi = (\boldsymbol{\varphi}, q) \in \mathbf{X} \times M$, see details in [4].

Spatial discretization. For practical computation the chosen P1-bubble/P1 finite elements satisfy the Babuška–Brezzi condition, nevertheless the FE solution is further stabilized by a combination of the SUPG method, PSPG method together with *div-div* stabilization, see [1].

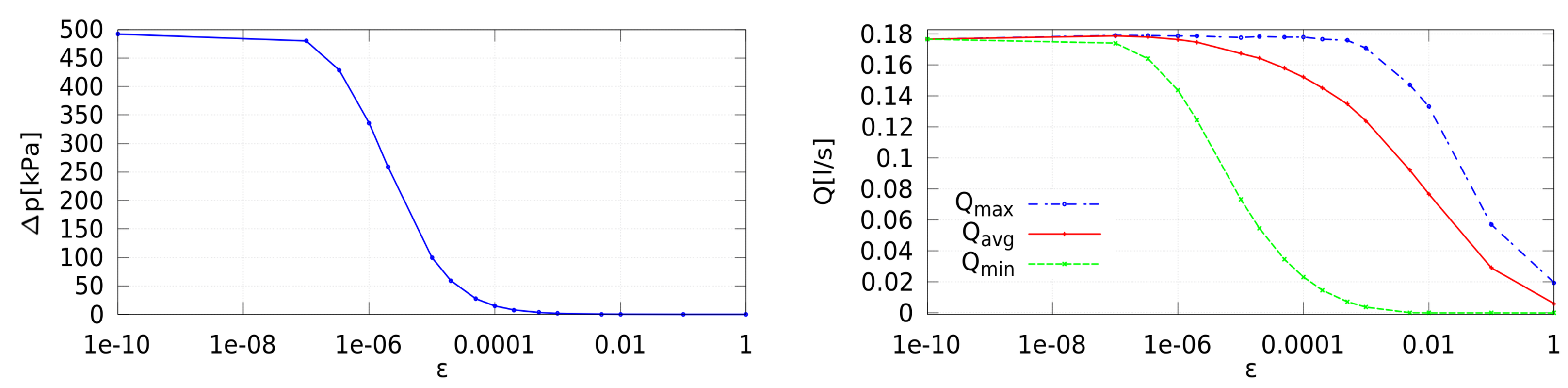
Numerical results

Prescribed motion of structure.
The periodical VF motion with the minimal half-gap $g_{0,min} = 0.0114$ mm is prescribed and three simulations are considered: 1) **vel** - Dirichlet boundary condition (3a) with $v_{\mathrm{Dir},1} = 1.7$ m/s, 2) **pres** - prescribed condition (3b) with $p_{\mathrm{in}} = 400$ Pa, 3) **pen** - penalization BC (3c) with $v_{\mathrm{Dir},1}$ and $\epsilon = 5 \cdot 10^{-4}$ s/m.

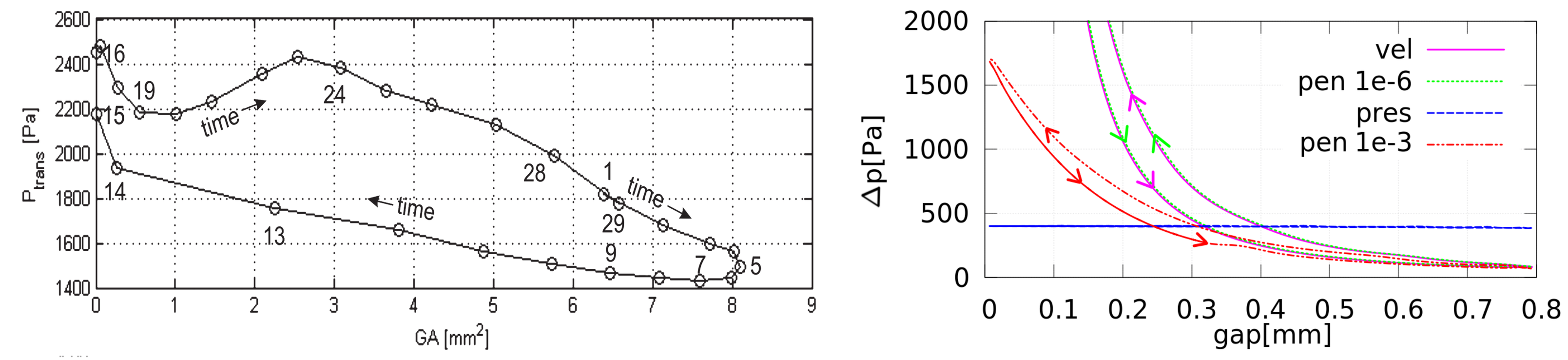
The time variation of inlet velocity monitored at Γ^f_{In} (left) and the pressure drop between Γ^f_{In} and Γ^f_{Out} (right). The pressure drop in the case of **vel** has unrealistic maximum of 472 kPa.



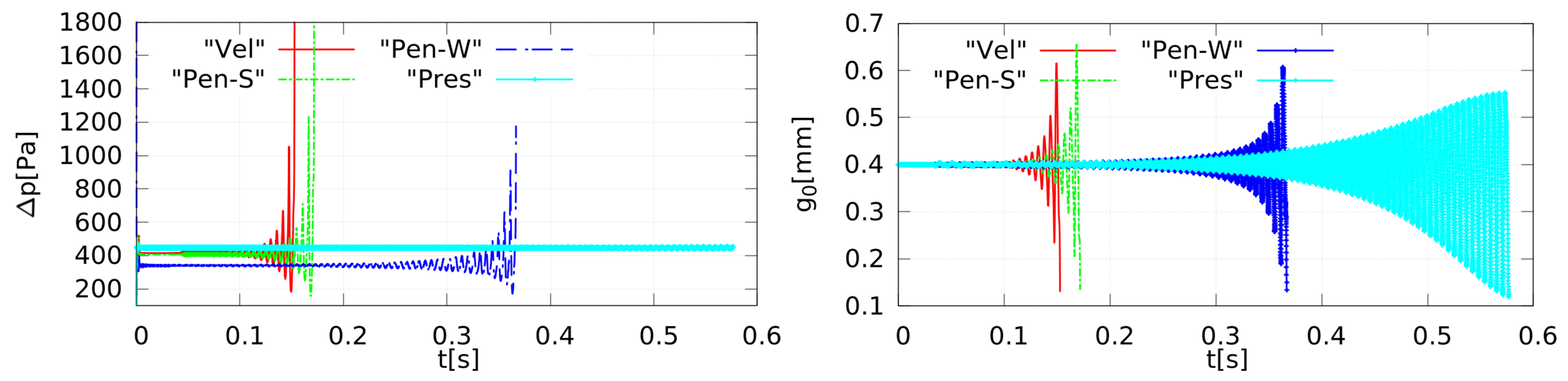
The sensitivity of the maximal pressure drop and the inlet flow rate on the parameter ϵ is plotted on right and on left, respectively.



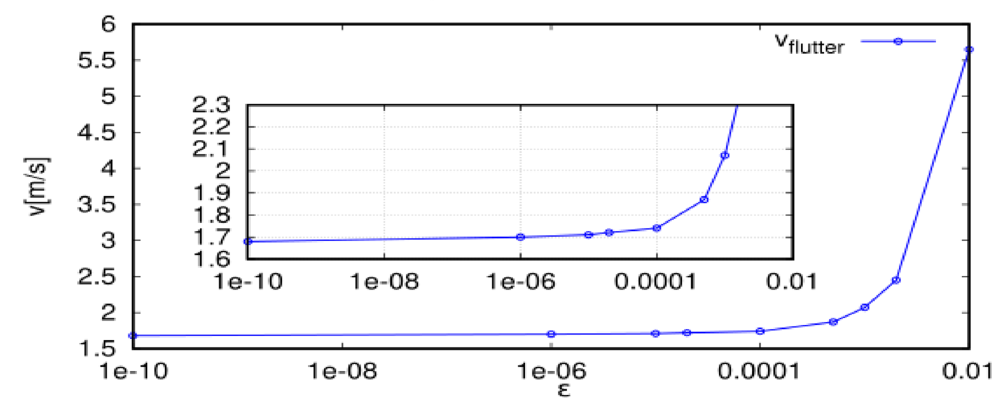
Typical behaviour of the transglottal pressure in the dependence on the glottal gap area (GA) during one oscillation cycle as measured by [2] (left). The dependence of the transglottal pressure on the gap for four simulation cases is shown right.



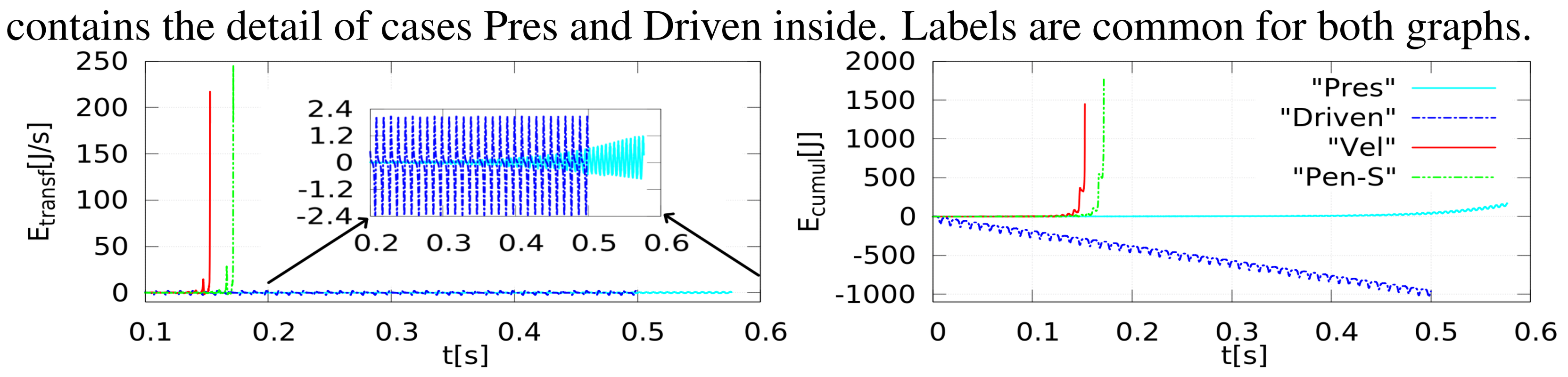
Fluid-structure interaction of the hemi-larynx configuration.
VF motion is now apriori unknown. Pressure drop between the inlet and the outlet (left) and time development of the half-gap (right) are shown for cases Vel, Pen-S, Pen-W and Pres.



Dependence of the critical flutter velocity on the penalization parameter.



Energetic considerations related to the VF flutter Energy transfer function (left) and energy cumulative function (right) for cases Vel, Pen-S, Pres and Driven. The top graph contains the detail of cases Pres and Driven inside. Labels are common for both graphs.



Conclusion.

- FSI problem was mathematically formulated.
- Three different inlet boundary conditions were analyzed.
- Numerical approximation based on the FEM was implemented.
- Numerical results showed penalization BC as most suitable for human phonation problem.
- Sensitivity of pressure drop and critical velocity to penalization parameter were determined.

References.

[1] M. Feistauer, P. Sváček, J. Horáček, in *Fluid-structure Interaction and Biomedical Applications*, Birkhauser, 2014.

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[3] P. Sváček, J. Horáček, in *Applied Mathematics and Computation*, **319**, 178 (2018).

[4] J. Valášek, P. Sváček, J. Horáček, in *Applications of Mathematics*, **64**(2), 225 (2019).

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