The Numerical Simulation of Fluid-Structure-Acoustic Interaction in Human Phonation (2/2)

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Summary

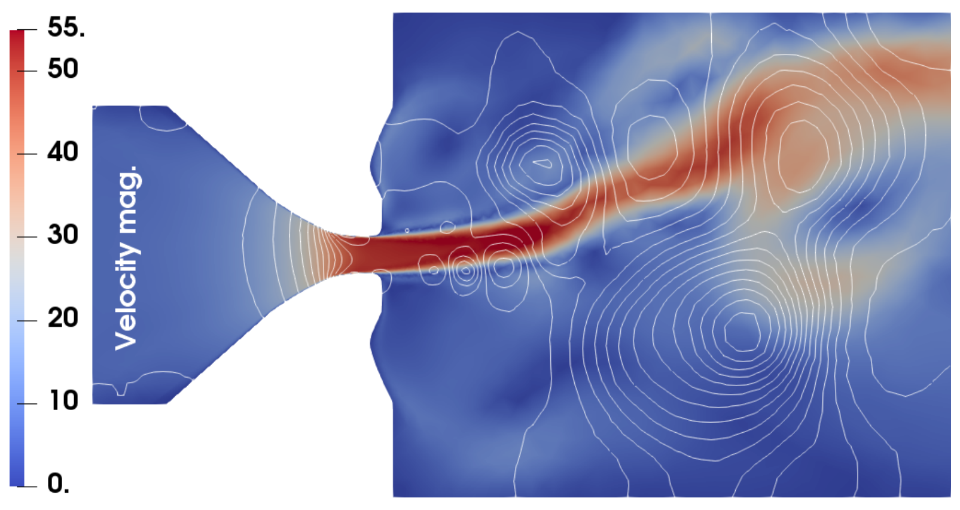
The human phonation is a complex phenomenon described by interaction of three physical fields – elastic body deformation, fluid flow and acoustics, and their mutual couplings. Therefore it is sometimes referred as fluid-structure-acoustic interaction (FSAI) problem, see e.g. [1]. The sound sources are produced by three main mechanisms – the modulated air stream emerged by repetitively opening and closing glottis, the eddy-induced sound of turbulent flow structures and the sound from the vibrating vocal folds (VFs).

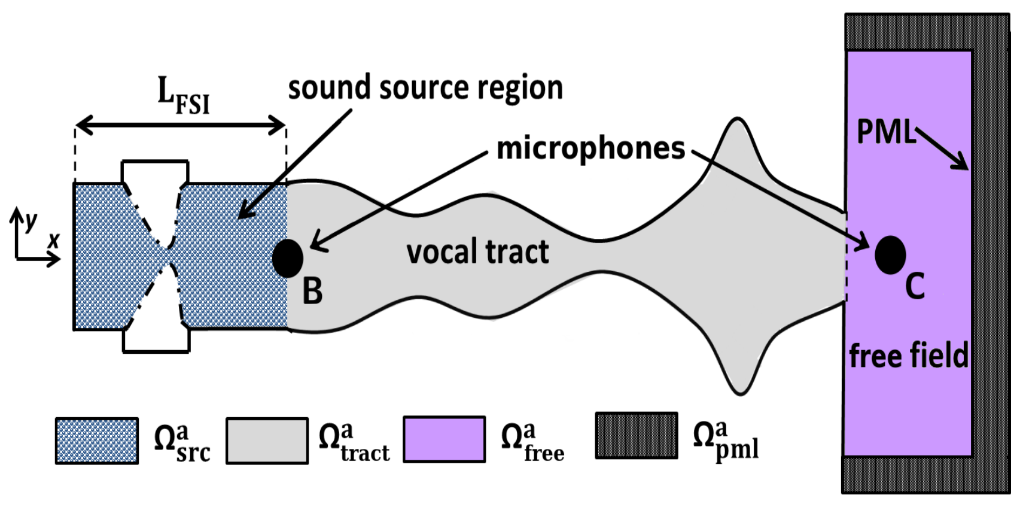
Here, due to small Mach number airflow in the glottis (≈ 0.2 Ma) the incompressible fluid flow model can be used and the sound of aerodynamic origin can be modelled with the aid of acoustic analogies. This approach compared to solution of compressible Navier-Stokes equations has many advantages, one of many is the problem formulation directly for unknown acoustic pressure what eliminates the magnitude disparity between acoustic pressure and overall hydrodynamic pressure, see [1]. Further the hybrid approach, when the FSI problem is solved first and later according to the chosen aeroacoustic analogy the sound propagation is computed, can benefit from problem-specific solvers and substantial lower computation demand, see [2]. The Lighthill acoustic analogy (LH) and acoustic wave equation (AWE) are described here.

The sound generated solely by VFs vibration is modelled by the wave equation with prescribed normal acceleration at the VF boundary. In order to obtain relevant acoustic results the aeroacoustic as well as the vibroacoustic simulations are performed in larger acoustic domain including vocal tract model. According to source-filter theory, see [3], it acts as a linear filter of sound waves generated in the larynx. Thus before acoustic simulations the resonant frequencies of used vocal tract models also called formants are determined, see [4]. For the modelling of open boundary problem at the vocal tract model outlet the perfectly matched layer (PML) technique is applied, see [1]. The numerical approximation of all three subproblems is based on the finite element method (FEM), particularly in the fluid flow case the advanced stabilization technique is used.

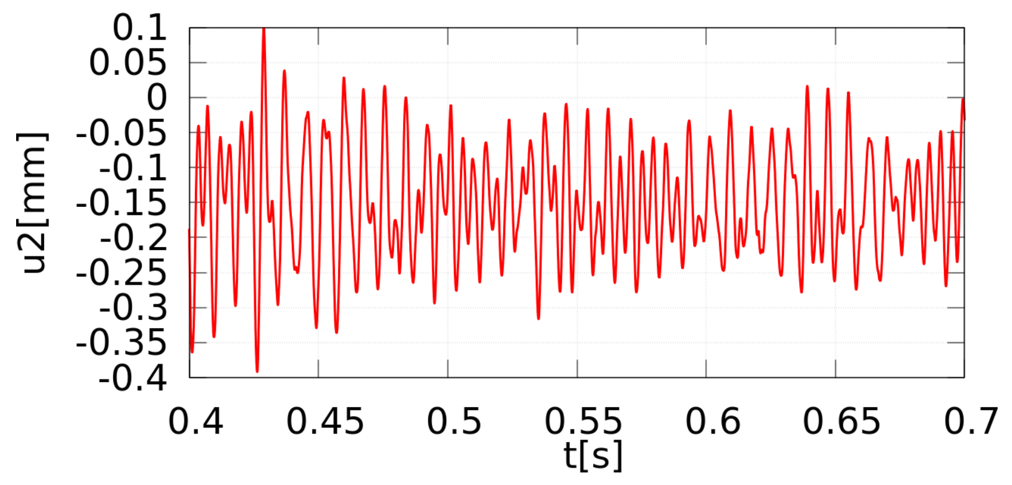
Fluid-structure interaction

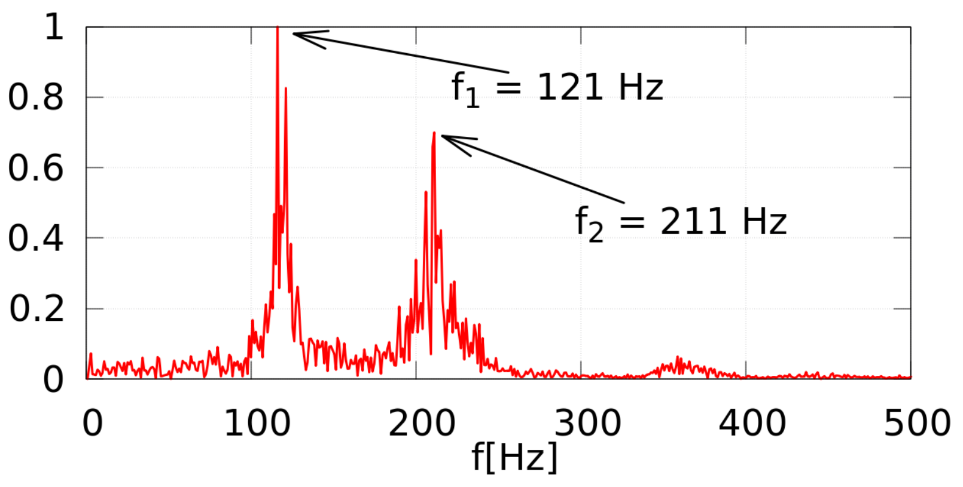
The FSI problem with prescribed pressure drop of $\Delta p = 1500$ Pa was numerically approximated in Ω_{src} by FEM using partitioned scheme approach. The typical distribution of **fluid velocity** in time $t = 0.6$ s is shown left and a scheme of acoustic domain on the right.





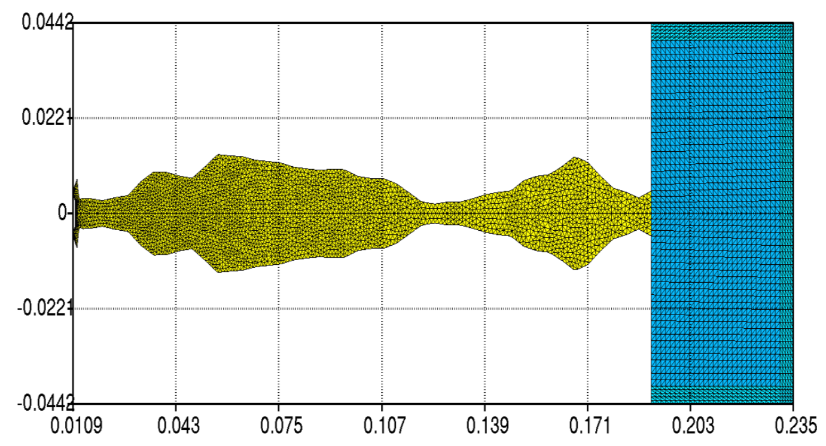
The stable VF oscillation with minimal gap approximately equal to 1.8 mm has developed. Stored **displacement** of the top point from bottom vocal fold in y-direction is shown left. Its **Fourier transform** is depicted right.

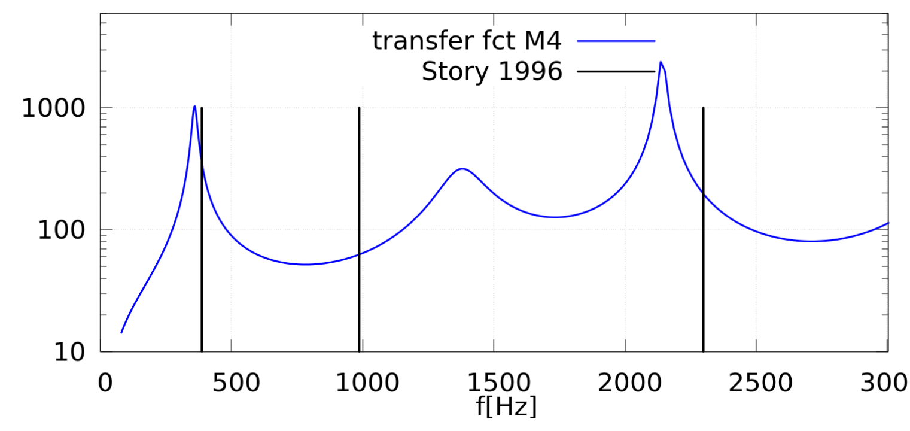


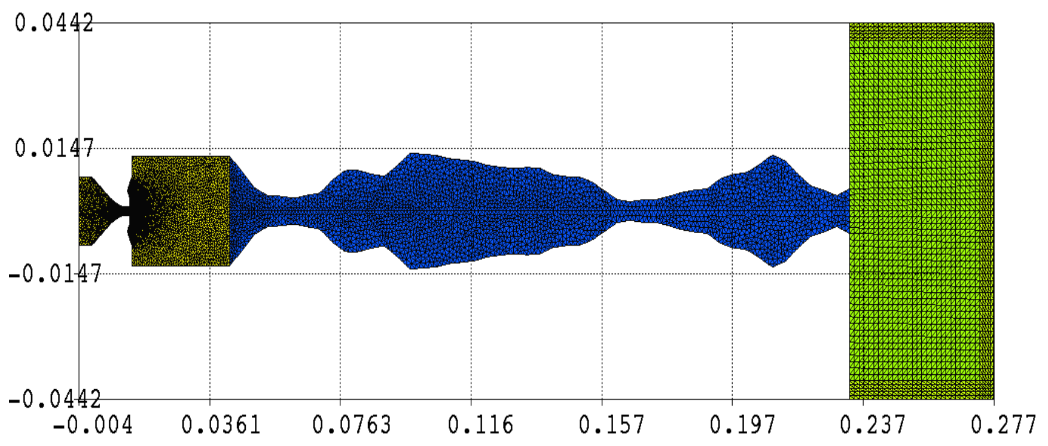


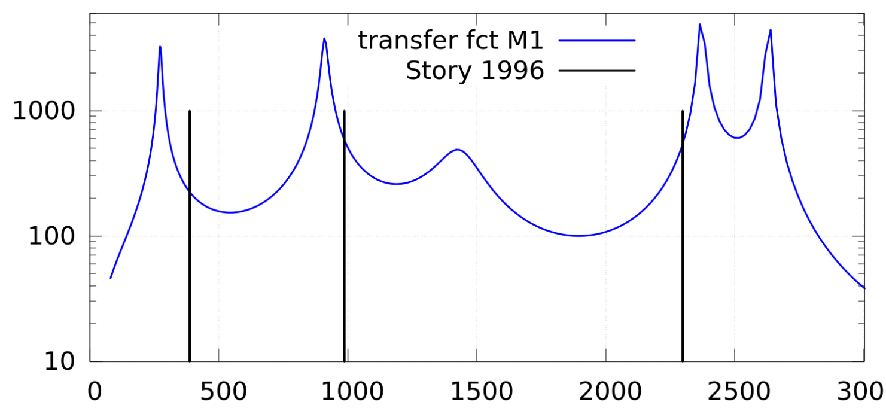
Acoustic resonances of vocal tract

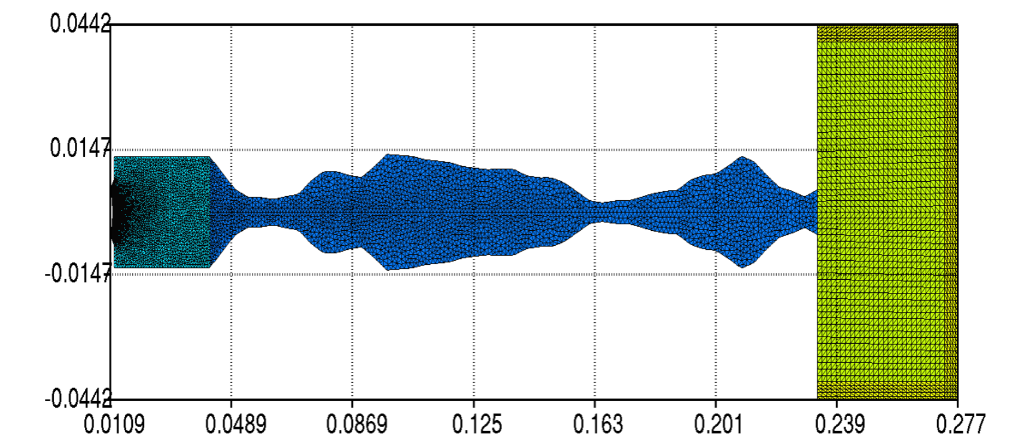
The 2D acoustic domain consists of sound source region (Ω^a_{src}), vocal tract (VT) region modelling vowel [u:] (Ω^a_{tract}), free field (Ω^a_{free}) and PML region (Ω^a_{pml}). The formants (i.e. resonant frequencies) of three VT models labeled as M4 and M1, M2 are analyzed using nonhomogeneous **Helmholtz equation**, see [4]. Comparison in terms of transfer functions of model M4 with resonances measured in vivo (black lines), see [3], is plotted below and the same comparison of models M1 and M2 are shown in the right column.

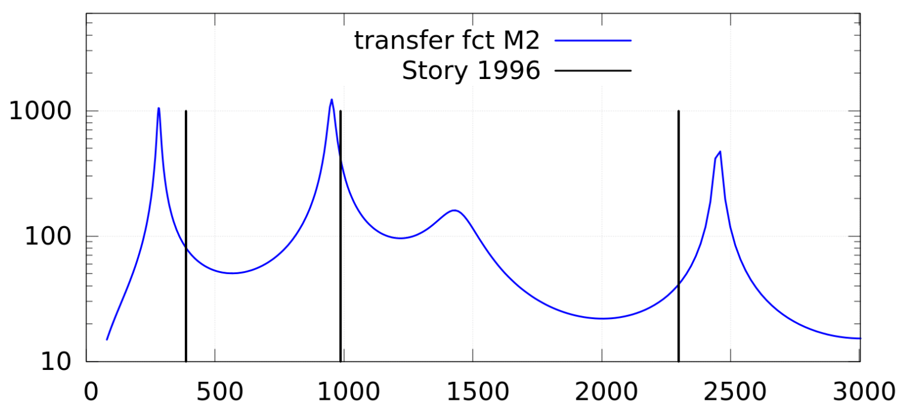










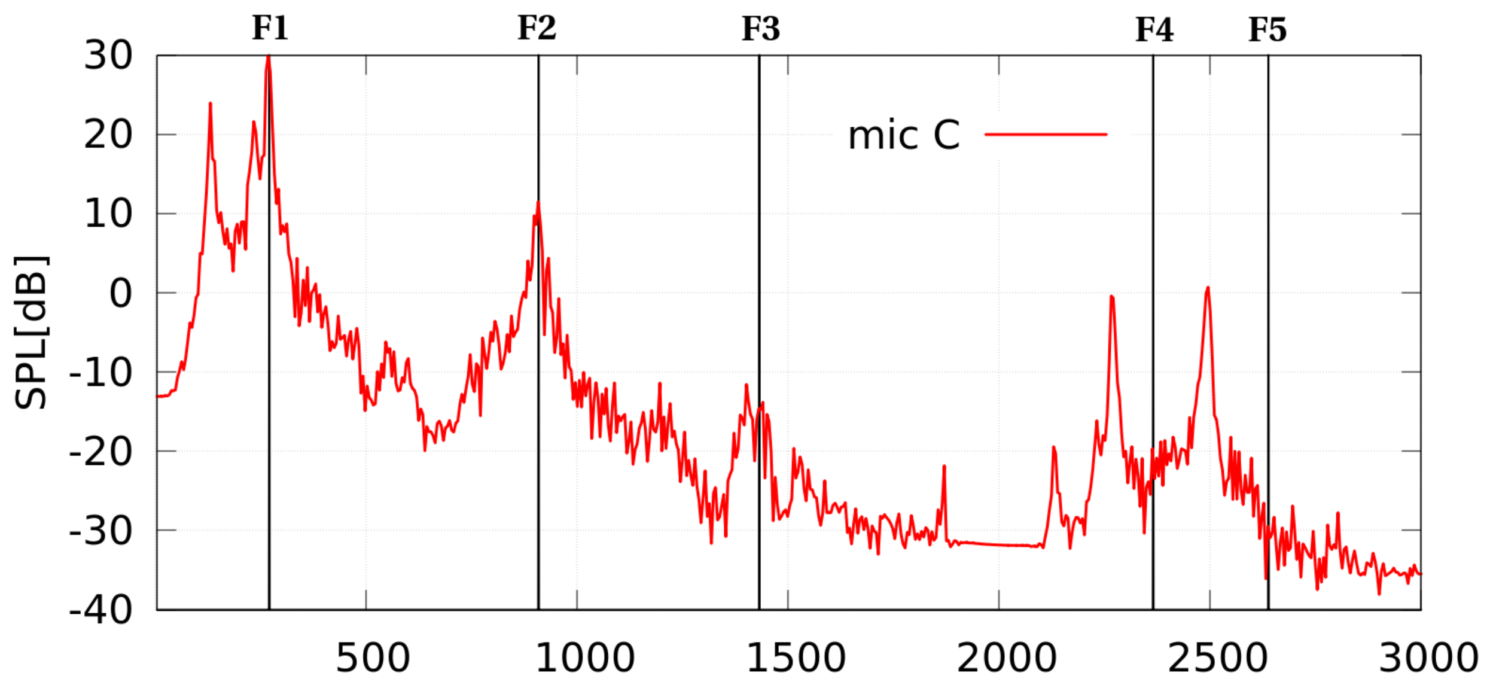


Vibroacoustic results

The sound propagation with omitted aeroacoustic sources is simulated, i.e. the sound emitted purely by the vibrating VF interface as obtained by the FSI simulation is considered. The vibroacoustic model is given by homogeneous **wave equation** for acoustic pressure p^{va} together with boundary condition with given VF displacement $\mathbf{u}$ and outer normal $\mathbf{n}^a$ to Γ_{W}

$$\frac{\partial p^{\text{va}}}{\partial \mathbf{n}^a}(x,t) = -\rho^f \frac{\partial^2 \mathbf{u}}{\partial t^2} \cdot \mathbf{n}^a, \qquad x \in \Gamma_{\text{W}_0},\ t \in (0,\text{T}).$$

Sound pressure level as computed from transient simulation at point C placed in front of mouth is displayed in the right column. Black vertical lines demonstrate first five formants of VT model M1. The first two VF eigenfrequencies (121 and 217 Hz) are also present.



Aeroacoustic results

Lighthill acoustic analogy. The propagation of pressure fluctuation p' is described by inhomogeneous wave equation

$$\frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} - \frac{\partial^2 p'}{\partial x_i^2} = \frac{\partial^2 L_{ij}}{\partial x_i \partial x_j},$$

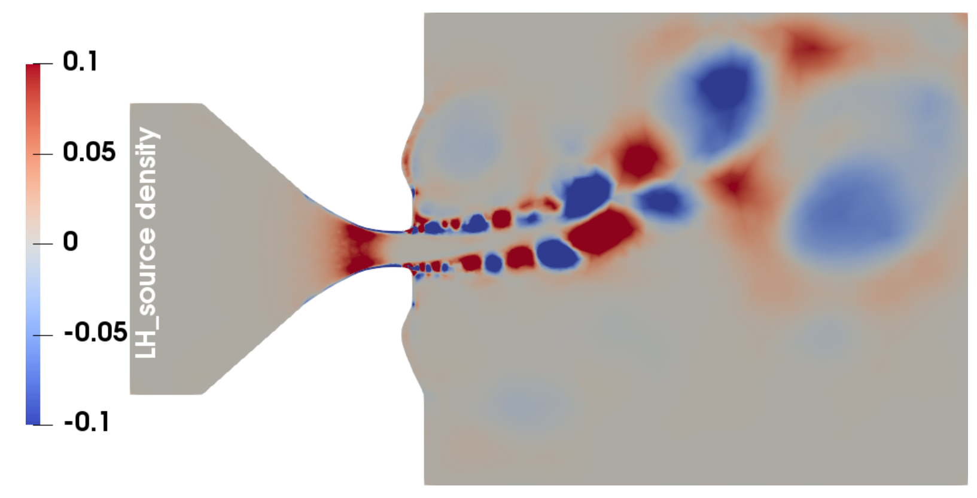
where Lighthill tensor is $L_{ij} = \rho^f v_i v_j + ((p') - c_0^2(\rho'^f))\delta_{ij} - \tau^f_{ij}$ determines the sound sources. Without considering of heat conduction it can be approximated as $L_{ij} \approx \rho^f v_i v_j$. The drawback of Lighthill analogy (LH) is bias of acoustic results in near field, see [1].

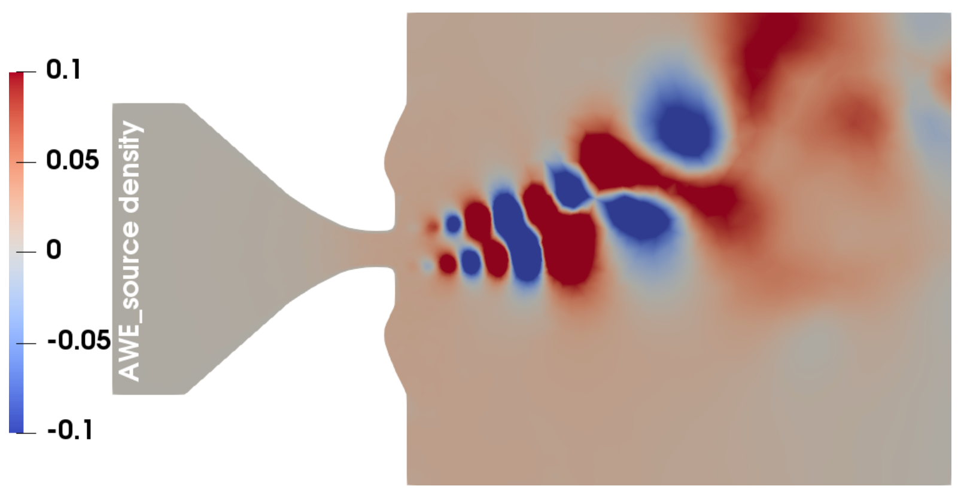
Acoustic wave equation analogy. The acoustic wave equation (AWE) approach is formulated directly for acoustic pressure p^a

$$\frac{1}{c_0^2}\frac{\partial^2 p^a}{\partial t^2} - \Delta p^a = -\frac{1}{\rho_0^f c_0^2}\frac{\partial^2 p_{ic}}{\partial t^2}.$$

It is derived based on splitting pressure into mean and fluctuating non-acoustic and acoustic parts, i.e. $p = \overline{p} + p^v + p^a$, where $p_{ic} = p^v + p^a$, and on the assumption of irrotational acoustic field, see [1]. Its advantage is acoustic source given by time derivative not space derivative.

Sound sources. The computed (normalized) sound source densities according to Lighthill analogy at time instant $t = 0.6$ s are shown left and for AWE analogy right, respectively.





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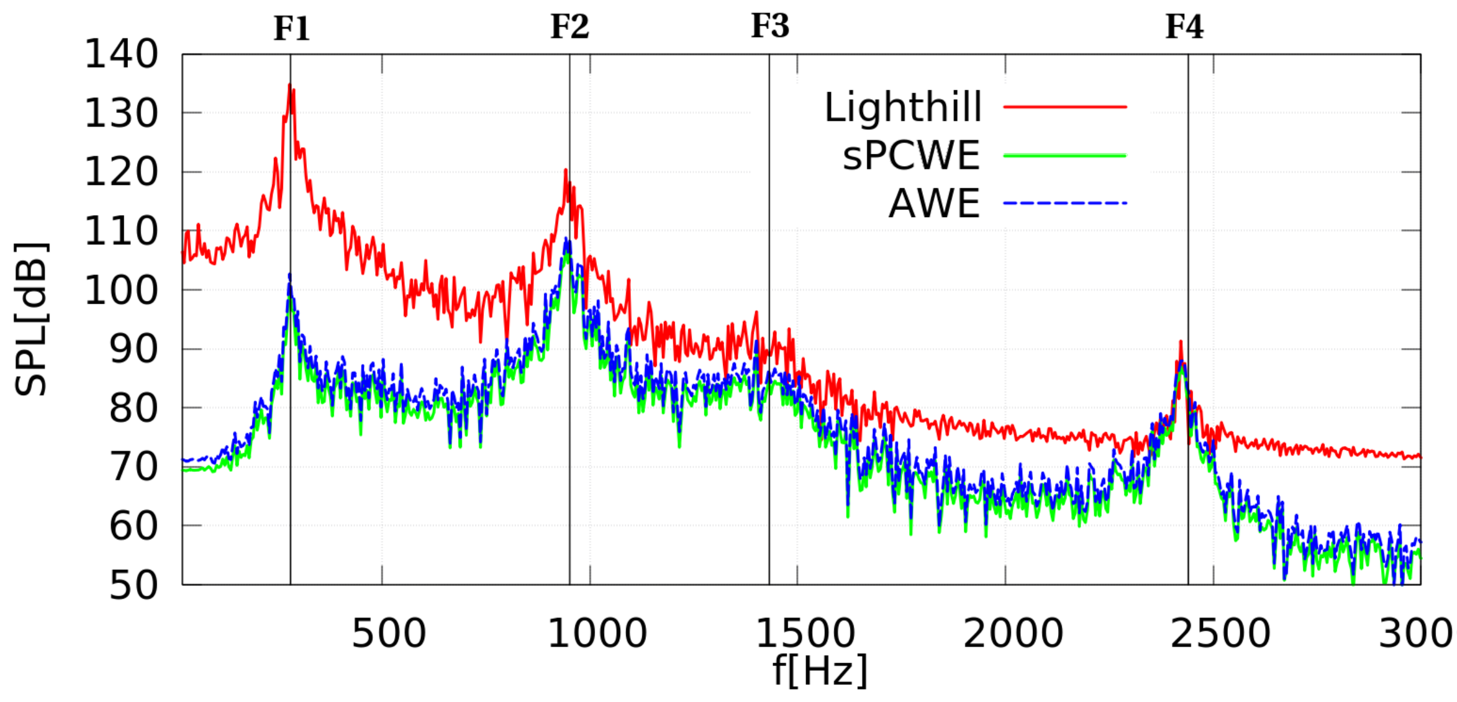
Sound propagation in the vocal tract model The transient aeroacoustic simulations were performed using interpolated sound sources in VT model M2. Sound pressure levels of acoustic pressure in frequency domain obtained by the LH analogy and the AWE approach at point C are compared with simplified perturbation convective wave equation (sPCWE) approach, see [5]. The black vertical lines mark the formants of VT model M2.



Conclusion.

- FSAI problem was mathematically formulated.
- All subproblems were numerically solved by FEM.
- Formants of different 2D vocal tract models of vowel [u:] were determined.
- Sound produced solely by VF vibration was simulated.
- Aeroacoustic sound sources based on FSI results were computed and interpolated.
- Numerical simulations with three different aeroacoustic approaches were compared.

References.

[1] M. Kaltenbacher, *Numerical simulation of mechatronic sensors and actuators*, Springer, 2015.

[2] P. Šidlof, S. Zörner, A. Hüppe, Biomechanics and Modeling in Mechanobiology, **14**(3), 473 (2014).

[3] B.H. Story, I.R. Titze, E.A. Hoffman, Journal of Acoustical Society of America, **100**(1), 537 (1996).

[4] J. Valášek, P. Sváček, J. Horáček, in *Topical problems of fluid mechanics 2018*, (2018), pp. 307–314.

[5] J. Valášek, M. Kaltenbacher, P. Sváček, Flow, Turbulence and Combustion **102**(1), 129 (2019).