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REPORT ON THE DIPLOMA THESIS OF BC. ROMANA KVASNIČKOVÁ ENTITLED "EFFECTIVE QUANTUM HAMILTONIAN IN THIN DOMAINS WITH NON-HOMOGENEITY"

Effective models for partial differential operators in thin domains is a topic of constant interest in mathematical physics. In the diploma thesis under consideration a divergence-type second-order elliptic partial differential operator in a thin shell of arbitrary dimension is considered. Neumann boundary condition is imposed and the coefficient in the differential equation describing the non-homogeneity is assumed to be positive and bounded. The shell is built over a hypersurface and the aim of the thesis is to study the convergence of the elliptic operators on the shells to an effective operator on the underlying hypersurface as the thickness of the shells tends to zero.

This model is inspired by a recent paper: T. Yachimura, Two-phase eigenvalue problem on thin domains with Neumann boundary condition. Differential and Integral Equations 31, 735 (2018). In the diploma thesis by Romana Kvasničková the results of Yachimura are generalized and improved in several ways. First, much more general coefficients in the differential expression are covered and the coefficient appearing in the effective operator is a proper averaging of the coefficient in the thin shell. Second, besides convergence to the spectral data of the effective operator proved also by Yachimura the results on strong and norm-resolvent convergence in a generalized sense are obtained. Finally, a very delicate estimate of the rate of the norm resolvent convergence is also derived. The analysis of the resolvent

convergence is indeed rather difficult due to the fact that the underlying Hilbert space is varying.

The proofs in thesis combine non-trivial facts from operator theory, partial differential equations, and differential geometry. This combination of different areas of mathematics clearly required from the author additional efforts. The proofs are outlined in detail and the presentation of the material is of the quality sufficient to follow the arguments. The results of the thesis substantially generalize a paper published in an international mathematical journal. Therefore, these new results are also potentially publishable in a journal. For the possible publication it might be of interest to analyse whether the order of the norm-resolvent convergence can be in some cases indeed slower than linear in thickness.

Finally, I recommend the mark

A (EXCELLENT).

Dr. Vladimir Lotoreichik, 07.06.2021.