

Opponent's Report on the Master's Thesis

Author of the Thesis: Václav Voráček

Title of the Thesis: Combinatorial Methods in the Study of Quantum Structures

The first part of the thesis introduces the reader into topical problems of quantum theory: Einstein–Podolski–Rosen paradox, Hidden variable hypothesis, Kochen–Specker theorem, etc. The author proves a great deal of mastery in analyzing these rather complex problems, he explains the mathematical aspects and contributes to them by several interesting observations and examples. This part, as much as the entire thesis, is written in a well-organized professional style with a concern for a scholar exposition. Also, the English of the thesis is very good.

The research contribution proper, at least in the opponent's opinion, lies in the sections 5 and 6. The problems pursued are twofold. First, the author concentrates on the topical questions of the theory of orthocomplemented difference lattices (abbr. ODLs). This theory tries to approach the quantum logics problems in an “almost Boolean” way, it allows for enriching quantum logics with an “abstract” symmetric difference (a kind of an XOR operation) though preserving a potentially high degree of non-compatibility.

Mr. Voráček asks and resolves one of the important problems in this theory: Do there exist ODLs without states? He shows by a highly non-trivial and inventive construction that they do. This, at first sight a rather exotic result, is enormously important for the further development of the theory - on the one hand, it is significant for the interpretation in theoretical physics, on the other hand, it is the first (most crucial) step in finding the characterization of the state space. The result is a mature piece of high-class mathematics; it stands the comparison with the theses of best students of the Charles University specialized in mathematics.

Second, the author of the thesis tackles the problem of a strong form of Kochen–Specker theorem. The problem is whether there is a \mathbb{Z}_2 -state, as a group-valued state, on the quantum logics of subspaces of \mathbb{R}^3 . As regards the recent development of this line of theoretical physics, this problem has a prominent position. Mr. Voráček managed to solve it. The solution was obtained by applying original ideas, there also was a certain help of a computer. The result was published, in extended form, in a renowned journal *Foundations of Physics*; the result should find its place in the monographs to be published.

Resume: The thesis under assessment meets the high standard of the world research in this field. Mr. Václav Voráček proved the courage and ability to solve difficult open problems. In doing so, he invented original techniques and procedures. He substantially contributed to the theory of quantum logics. I am impressed by his work and very much value the thesis. My assessment is

A (excellent)

and I would recommend the thesis for a special award.

7.6.2021

Prof. RNDr. Pavel Pták, DrSc.