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F3

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Network Channel State Estimation for WPNC Radio Networks

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Acknowledgements

Declaration

I declare that I completed the presented thesis independently and that all used sources are quoted in accordance with the Methodological instructions that cover the ethical principles for writing an academic thesis.

In Prague, 21.5.2021

Prohlašuji, že jsem předloženou práci vypracoval samostatně a že jsem uvedl veškeré použité informační zdroje v souladu s Metodickým pokynem o dodržování etických principů při přípravě vysokoškolských závěrečných prací.

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Abstract

The topic of this thesis is statistical signal processing in the MAC stage of simple WPNC radio networks. The readers will first get acquainted with the fundamentals of digital communication, estimation and Wireless physical layer network coding (WPNC).

Then the work focus on finding algorithms for estimating amplitude, phase and delay in BPSK two-source channel, using the Least Square estimator.

Part of the thesis is also computer simulations, verifying given results.

Keywords: WPNC, hierarchical MAC channel, channel estimator

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Abstrakt

Tématem této práce je zpracování stochastických signálů MAC fáze jednoduchých WPNC rádiových sítí. Čtenáři se nejprve seznámí se základy digitální komunikace, estimace a kódování na fyzické vrstvě sítě (WPNC).

Práce je dále zaměřena na hledání algoritmů pro estimaci amplitudy, fáze a zpoždění ve dvouzdrojovém kanálu s modulací BPSK za použití estimátoru nejmenších čtverců.

Součástí práce jsou také počítačové simulace, potvrzující dané výsledky.

Klíčová slova: WPNC, hierarchický MAC kanál, estimátor kanálu

Překlad názvu: Estimace stavu síťového kanálu pro WPNC rádiově sítě

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Chapter 1

Introduction

Wireless physical layer coding (WPNC) is a concept providing a faster form of communication than traditional ways. On the opposite of them, in WPNC, one node receives messages from multiple sources simultaneously. The relays typically are not able to decode the initial messages, but it is not a problem because, in destinations, enough information for decoding them is provided.

One of the biggest problems we are dealing with in WPNC is interference, negatively affecting observation, WPNC is hence still under research.

Chapter 2

Digital Communication

In this chapter, fundamentals of digital communication will be introduced. Modulator is block, which discrete data (in either time and values) transforms into continuous signal. Input-output relation for modulated signal is

$$s(t) = \sum_n g(q_n, t - nT_s), \tag{2.1}$$

where $s(t)$ is signal, g denotes modulation, q_n is channel signal and T_s is symbol period.

If we want to have linear modulation, we have to use modulation pulse with unit energy,

$$\int_{-\infty}^{\infty} |g(t)|^2 \, dt = 1, \tag{2.2}$$

and the input-output relation then is

$$s(t) = \sum_n q_n(d_n, \sigma_n) g(t - nT_s), \tag{2.3}$$

where q_n depends on data d_n and modulator state σ_n.

For modulation without memory this relation becomes simpler, because there is only one state of modulator, so we get

$$s(t) = \sum_n q_n g(t - nT_s), \tag{2.4}$$

where $g_n = d_n$. Memoryless modulation has to fulfill Nyquist condition.

Definition 2.0.1. (Nyquist condition) Pulses g_1 and g_2 are Nyquist, if they hold

$$\int_{-\infty}^{\infty} g_1(t + mT_s) g_2^*(t) \, dt = 0, \ \forall m \neq 0, \tag{2.5}$$

where * denotes comlex conjugate. [4]
In other words, two pulses are Nyquist, if they are orthogonal for their every nonzero integer shift.

Pulse is Nyquist, if holds

$$\int_{-\infty}^{\infty} g(t + mT_s) g^*(t) \, dt = 0, \ \forall m \neq 0. \tag{2.6}$$

2.1 Constellation space

Definition 2.1.1. (Constellation space) Constellation space is a orthonormal signal space representation of modulated signal. The basis of constellation space is

$$\{\zeta_{n,i}\}_{n,i} = \{\zeta_i(t - nT_s\}_{i=1}^{N_s}, \tag{2.7}$$

where n denotes sequence number and i is index of modulation dimension. [4]

Definition 2.1.2. (Constellation points) For every nth symbol there exists vector $\mathbf{s}_n$ called constellation point. [4] His components $s_{n,i}$ are defined as

$$s_{n,i} = \int_{-\infty}^{\infty} s(t)\zeta_i^*(t - nT_s)\,dt = 0. \tag{2.8}$$

If pulses are orthonormal and Nyquist, we can write

$$\zeta_i(t - nT_s) = g_i(t - nT_s), \tag{2.9}$$

$$s_{n,i} = q_{n,i},\ \ i \in \{1, ...N_s\}. \tag{2.10}$$

If in addition modulation is linear ($N_s = 1$), then

$$\zeta(t - nT_s) = g(t - nT_s), \tag{2.11}$$

$$s_{n,i} = s_n = q_n. \tag{2.12}$$

2.1.1 PSK modulation

Phase Shift Keying (PSK) modulation is linear modulation with all constellation points on unit circle, thus all points has same symbol energy and the difference between them is only in phase shifting. Constellation points q_n then are

$$q_n \in \{e^{j\frac{2\pi}{M_q}i}\}_{i=0}^{M_q-1}, \tag{2.13}$$

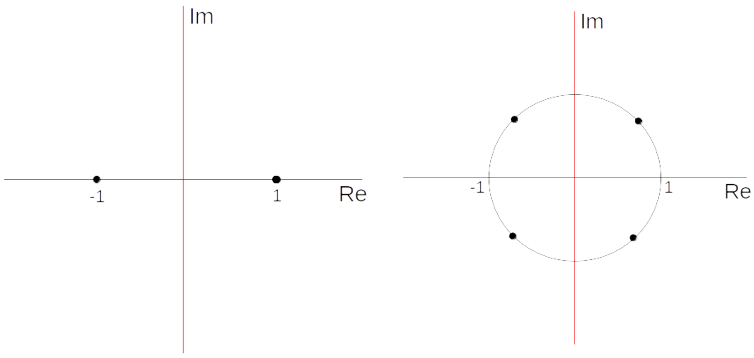
where M_q is size of alphabet.

Binary Phase Shift Keying (BPSK) is modulation with $M_q = 2$ and its constellation points are

$$q_n \in \{-1, 1\}. \tag{2.14}$$

Quaternary Phase Shift Keying (QPSK) has $M_q = 4$ and constellation points

$$q_n \in \{\frac{1+j}{\sqrt{2}}, \frac{-1+j}{\sqrt{2}}, \frac{-1-j}{\sqrt{2}}, \frac{1-j}{\sqrt{2}}\}. \tag{2.15}$$



(a) : BPSK **(b) :** QPSK

Figure 2.1: PSK decision regions

2.2 AWGN channel

In reality all signals include distortion and thus we have to use mathematical models. One of the most used models is Additive White Gaussian Noise (AWGN) channel. This noise has zero mean, his spectral power density $S_w(f)$ is constant for all frequencies

$$S_w(f) = \frac{N_0}{2}, \tag{2.16}$$

and distrubution of this noise is normal with zero mean. Input-output relation of AWGN channel is

$$y(t) = x(t) + w(t), \tag{2.17}$$

where $y(t)$ is received signal, $x(t)$ is sent signal and $w(t)$ is Additive White Gaussian Noise.

Probability density function (PDF) of w is

$$p_w(w) = \frac{1}{2\pi\sigma_w^2} e^{-\frac{w^2}{2\sigma_w^2}}, \tag{2.18}$$

where σ_w^2 is variance of w.

Chapter 3

Statistical Signal Processing

One of the main topics of this thesis will be Statistical Signal Processing (SSP). The main goal is to reconstruct the signal initially sent from the source and find the channel's properties.

3.1 Nuisance parameters

Nuisance parameter ω from

$$x = x(\theta, \omega), \tag{3.1}$$

is parameter that is unnecessary for us, but has impact on observation.

Nuisance parameters can be eliminated from the observation

$$p(\boldsymbol{x}|\boldsymbol{\theta}) = \int p(\boldsymbol{x}|\boldsymbol{\theta}, \boldsymbol{\omega}) p(\boldsymbol{\omega})\, d\boldsymbol{\omega} \tag{3.2}$$

$$p(\boldsymbol{x}|\boldsymbol{\theta}) = \int \prod_k p(x_k|\boldsymbol{\theta}, \boldsymbol{\omega}) p(\boldsymbol{\omega})\, d\boldsymbol{\omega} \tag{3.3}$$

$$p(\boldsymbol{x}|\boldsymbol{\theta}) = \prod_i \int_{\{\omega_i\}} \prod_{\mathcal{A}_T(\omega_i)} p(x_k|\boldsymbol{\theta}, \boldsymbol{\omega}_i) p(\boldsymbol{\omega}_i)\, d\boldsymbol{\omega}_i, \tag{3.4}$$

where $\mathcal{A}_T(\boldsymbol{\omega}_i)$ is is set of indices k for which $\boldsymbol{\omega}_i$ affects x_k, it is called Trace set. Cardinality of Trace set is Trace length

$$L_t(\boldsymbol{\omega}_i) = \text{card}\mathcal{A}_T(\boldsymbol{\omega}_i), \tag{3.5}$$

$$L_t(\boldsymbol{\omega}) = \max_i L_t(\boldsymbol{\omega}_i). \tag{3.6}$$

ow we can use this to elimininate noise from WPNC channel from previous chapter. We have channel in form

$$\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{w}, \tag{3.7}$$

hence we can write

$$p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{w}) = \delta(\boldsymbol{y} - \boldsymbol{x} - \boldsymbol{w}). \tag{3.8}$$

Now we can eliminate noise

$$p(\boldsymbol{y}|\boldsymbol{x}) = \int_{\{\boldsymbol{w}\}} p(\boldsymbol{y}|\boldsymbol{x},\boldsymbol{w}) p_w(\boldsymbol{w})\, d\boldsymbol{w} \tag{3.9}$$

$$p(\boldsymbol{y}|\boldsymbol{x}) = \int_{\{\boldsymbol{w}\}} \delta(\boldsymbol{y}-\boldsymbol{x}-\boldsymbol{w}) p_w(\boldsymbol{w})\, d\boldsymbol{w}. \tag{3.10}$$

Thanks to the sampling property of Dirac delta we get

$$p(\boldsymbol{y}|\boldsymbol{x}) = p_w(\boldsymbol{y}-\boldsymbol{x}) \tag{3.11}$$

$$p(\boldsymbol{y}|\boldsymbol{x}) = \prod_k \frac{1}{2\pi\sigma_w^2}\, e^{-\frac{\|y-x\|^2}{2\sigma_w^2}} = c\, e^{-\frac{\|y-x\|^2}{2\sigma_w^2}}. \tag{3.12}$$

3.2 Cramer-Rao Lower Bound

Cramer-Rao Lower Bound is criterion that tells us, how good can be unbiased estimator at the best in terms of stochastical parameters.

Theorem 3.2.1 (CRLB for scalar $\theta \in \mathbb{R}$)**.** The regularity condition for the observation is

$$\mathrm{E}\left[\frac{\partial \ln p(\mathbf{x}|\theta)}{\partial \theta}\right] = 0, \tag{3.13}$$

if it holds, then variance of any unbiased estimator is

$$\mathrm{var}[\hat{\theta}] \geq \left(-\mathrm{E}\left[\frac{\partial^2 \ln p(\mathbf{x}|\theta)}{\partial^2 \theta}\right]\right)^{-1}, \tag{3.14}$$

or equivalently

$$\mathrm{var}[\hat{\theta}] \geq \left(-\mathrm{E}\left[\frac{\partial \ln p(\mathbf{x}|\theta)}{\partial \theta}\right]^2\right)^{-1}.[5] \tag{3.15}$$

Theorem 3.2.2 (CRLB for $\boldsymbol{\theta} \in \mathbb{C}^{N\times 1}$)**.** The regularity condition for is

$$\mathrm{E}\left[\frac{\tilde{\partial} \ln p(\mathbf{x}|\boldsymbol{\theta})}{\tilde{\partial} \boldsymbol{\theta}^*}\right] = 0, \tag{3.16}$$

if it holds, then the error covariance matrix $\mathbf{C} = \mathrm{E}\left[(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta})(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta})^{\mathrm{H}}\right]$, where $^{\mathrm{H}}$ denotes the conjugate transpose, is

$$\mathbf{C} \geq \mathbf{J}^{-1}(\boldsymbol{\theta}), \tag{3.17}$$

where $\mathbf{J}^{-1}(\boldsymbol{\theta})$ is Fisher information matrix

$$\mathbf{J}(\boldsymbol{\theta}) = \mathrm{E}\left[\left(\frac{\tilde{\partial} \ln p(\mathbf{x}|\boldsymbol{\theta})}{\tilde{\partial} \boldsymbol{\theta}^*}\right)\left(\frac{\tilde{\partial} \ln p(\mathbf{x}|\boldsymbol{\theta})}{\tilde{\partial} \boldsymbol{\theta}}\right)^{\mathrm{T}}\right].[5] \tag{3.18}$$

3.3 Sufficient statistic

Definition 3.3.1 (Statistic)**.** Statistic is function $\mathbf{y} = \boldsymbol{T}(\mathbf{x})$, that does not depend on $\boldsymbol{\theta}$.[5]

Definition 3.3.2 (Sufficient Statistic)**.** Sufficient statistic is statistic $\mathbf{y} = T(\mathbf{x})$, that contains all available information necessary for estimator $\hat{\boldsymbol{\theta}}$ such that

$$\hat{\boldsymbol{\theta}}(\mathbf{x}) = \hat{\boldsymbol{\theta}}(\boldsymbol{T}(\mathbf{x})).[5] \tag{3.19}$$

3.4 Types of estimators

There are a couple of different estimators. We will provide some of the most popular types of them.

3.4.1 Maximum likelihood

The goal of Maximum likelihood (ML) estimator is to find the most probable input in input-output relation

$$\hat{\boldsymbol{\theta}} = \arg\max_{\check{\boldsymbol{\theta}}} p(\mathbf{x}|\check{\boldsymbol{\theta}}). \tag{3.20}$$

ML estimator in linear AWGN model. Assume linear AWGN model $\mathbf{x} = \mathbf{H}\boldsymbol{\theta} + \mathbf{w}$, where $\mathbf{w}$ is gaussian noise with covariance matrix $\mathbf{C}$. Then the ML estimation is

$$\hat{\boldsymbol{\theta}} = (\mathbf{H}^{\mathrm{H}}\mathbf{C}^{-1}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{H}}\mathbf{C}^{-1}\mathbf{x} \tag{3.21}$$

and Fisher information matrix is

$$\mathbf{J}(\boldsymbol{\theta}) = \mathbf{H}^{\mathrm{H}}\mathbf{C}^{-1}\mathbf{H}. \tag{3.22}$$

ML estimators are relatively straightforward and usually provide good performance, but on the other hand we have to know stochastical properties of observation.

3.4.2 Bayesian estimators

In Bayesian estimators we define Loss function $\boldsymbol{L}(\boldsymbol{\theta}, \check{\boldsymbol{\theta}})$ representing price for making error. Mean value of Loss function is called Bayesian risk

$$\boldsymbol{R}(\check{\boldsymbol{\theta}}) = \mathbf{E}_{\mathbf{x},\boldsymbol{\theta}}\left[\boldsymbol{L}(\boldsymbol{\theta}, \check{\boldsymbol{\theta}})\right] \tag{3.23}$$

$$\boldsymbol{R}(\check{\boldsymbol{\theta}}) = \int_{\{\mathbf{x}\}}\int_{\{\boldsymbol{\theta}\}} \boldsymbol{L}(\boldsymbol{\theta}, \check{\boldsymbol{\theta}}) p(\boldsymbol{\theta}|\mathbf{x}) p(\mathbf{x})\, \mathrm{d}\boldsymbol{\theta}\, \mathrm{d}\mathbf{x}. \tag{3.24}$$

Goal of Bayesian estimators is to minimize Bayesian risk. We can also define Conditional Bayesian risk

$$\boldsymbol{R}(\check{\boldsymbol{\theta}}|\mathbf{x}) = \int_{\{\boldsymbol{\theta}\}} \boldsymbol{L}(\boldsymbol{\theta}, \check{\boldsymbol{\theta}}) p(\boldsymbol{\theta}|\mathbf{x})\, \mathrm{d}\boldsymbol{\theta}. \tag{3.25}$$

$p(\boldsymbol{\theta}|\mathbf{x}) \geq 0$, so minimization of $\boldsymbol{R}(\check{\boldsymbol{\theta}}|\mathbf{x})$ minimizes $\boldsymbol{R}(\check{\boldsymbol{\theta}})$ as well.

MAP

There are couple of different Bayesian estimators, first of them is Maximum A posteriori (MAP) estimator. Loss function of MAP estimator is

$$\boldsymbol{L}(\boldsymbol{\theta},\check{\boldsymbol{\theta}}) = \begin{cases} 0, & \|\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}\| < \Delta_{\boldsymbol{\theta}} \\ 1, & \|\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}\| \geq \Delta_{\boldsymbol{\theta}} \end{cases}, \; \Delta_{\boldsymbol{\theta}} \to 0^+. \tag{3.26}$$

Bayesian risk $\boldsymbol{R}(\check{\boldsymbol{\theta}})$ in MAP is

$$\boldsymbol{R}(\check{\boldsymbol{\theta}}) = \int_{\{\mathbf{x}\}}\int_{\{\boldsymbol{\theta}\}} \boldsymbol{L}(\boldsymbol{\theta},\check{\boldsymbol{\theta}})p(\boldsymbol{\theta}|\mathbf{x})p(\mathbf{x})\,\mathrm{d}\boldsymbol{\theta}\,\mathrm{d}\mathbf{x} \tag{3.27}$$

$$\boldsymbol{R}(\check{\boldsymbol{\theta}}) = \int_{\{\mathbf{x}\}}\left(1-\int_{\{\Delta_{\boldsymbol{\theta}}(\check{\boldsymbol{\theta}})\}} \boldsymbol{L}(\boldsymbol{\theta},\check{\boldsymbol{\theta}})p(\boldsymbol{\theta}|\mathbf{x})\,\mathrm{d}\boldsymbol{\theta}\right)p(\mathbf{x})\,\mathrm{d}\mathbf{x}. \tag{3.28}$$

Now we can discuss inner integral

$$\lim_{\Delta_{\boldsymbol{\theta}}\to 0^+}\int_{\{\Delta_{\boldsymbol{\theta}}(\check{\boldsymbol{\theta}})\}} \boldsymbol{L}(\boldsymbol{\theta},\check{\boldsymbol{\theta}})p(\boldsymbol{\theta}|\mathbf{x})\,\mathrm{d}\boldsymbol{\theta} = \lim_{\Delta_{\boldsymbol{\theta}}\to 0^+}\int_{\{\Delta_{\boldsymbol{\theta}}(\check{\boldsymbol{\theta}})\}} \boldsymbol{L}(\boldsymbol{\theta},\check{\boldsymbol{\theta}})\,\mathrm{d}\boldsymbol{\theta}p(\check{\boldsymbol{\theta}}|\mathbf{x}), \tag{3.29}$$

the integral of Loss function is non-negative function, so we can write this expression as $ap(\check{\boldsymbol{\theta}}|\mathbf{x})$, where $a \geq 0$. Bayesian risk $\boldsymbol{R}(\check{\boldsymbol{\theta}})$ is then minimized by maximization of $p(\check{\boldsymbol{\theta}}|\mathbf{x})$

$$\hat{\boldsymbol{\theta}} = \arg\min_{\check{\boldsymbol{\theta}}} \boldsymbol{R}(\check{\boldsymbol{\theta}}) = \arg\max_{\check{\boldsymbol{\theta}}} p(\check{\boldsymbol{\theta}}|\mathbf{x}). \tag{3.30}$$

MMSE

Another option is Minimal Mean Square Error (MMSE) estimator. Bayesian loss function is square error

$$\boldsymbol{L}(\boldsymbol{\theta},\check{\boldsymbol{\theta}}) = \|\boldsymbol{\theta}-\check{\boldsymbol{\theta}}\|^2. \tag{3.31}$$

As the name of this estimator suggests, Bayesian risk is minimal mean square error

$$\boldsymbol{R}(\check{\boldsymbol{\theta}}) = \int_{\{\mathbf{x}\}}\int_{\{\boldsymbol{\theta}\}} \|\boldsymbol{\theta}-\check{\boldsymbol{\theta}}\|^2 p(\boldsymbol{\theta}|\mathbf{x})p(\mathbf{x})\,\mathrm{d}\boldsymbol{\theta}\,\mathrm{d}\mathbf{x}. \tag{3.32}$$

Criterion of optimality does not depend on $\mathbf{x}$, so

$$\arg\min_{\check{\boldsymbol{\theta}}} \boldsymbol{R}(\check{\boldsymbol{\theta}}) = \arg\min_{\check{\boldsymbol{\theta}}} \int_{\{\boldsymbol{\theta}\}} \|\boldsymbol{\theta}-\check{\boldsymbol{\theta}}\|^2 p(\boldsymbol{\theta}|\mathbf{x})\,\mathrm{d}\boldsymbol{\theta}. \tag{3.33}$$

The minimum of this function is at a stationary point $\frac{\partial}{\partial\check{\boldsymbol{\theta}}}\mathbf{R}(\check{\boldsymbol{\theta}})|_{\check{\boldsymbol{\theta}}=\hat{\boldsymbol{\theta}}} = 0$, so at first we take the derivative of $\mathbf{R}(\check{\boldsymbol{\theta}})$ with respect to $\check{\boldsymbol{\theta}}$

$$\frac{\partial}{\partial\check{\boldsymbol{\theta}}}\int_{\{\boldsymbol{\theta}\}} \|\boldsymbol{\theta}-\check{\boldsymbol{\theta}}\|^2 p(\boldsymbol{\theta}|\mathbf{x})\,\mathrm{d}\boldsymbol{\theta} = \int_{\{\boldsymbol{\theta}\}} -2(\boldsymbol{\theta}-\check{\boldsymbol{\theta}})p(\boldsymbol{\theta}|\mathbf{x})\,\mathrm{d}\boldsymbol{\theta}, \tag{3.34}$$

and for $\check{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}$ we have to find the solution of this equation at zero

$$\int_{\{\boldsymbol{\theta}\}} -2(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})p(\boldsymbol{\theta}|\mathbf{x})\,\mathrm{d}\boldsymbol{\theta} = 0 \tag{3.35}$$

$$\hat{\boldsymbol{\theta}} = \int_{\{\boldsymbol{\theta}\}} \boldsymbol{\theta}p(\boldsymbol{\theta}|\mathbf{x})\,\mathrm{d}\boldsymbol{\theta} = \mathbf{E}[\boldsymbol{\theta}|\mathbf{x}]. \tag{3.36}$$

3.4.3 LS

A goal of Least Squares (LS) estimator is to minimize difference between received signal and signal model. Unlike ML knowing of stochastical parameters is not needed. Metric of LS estimator is

$$\hat{\boldsymbol{\theta}} = \arg\min_{\check{\boldsymbol{\theta}}} \|\mathbf{x} - \mathbf{s}(\check{\boldsymbol{\theta}})\|^2, \tag{3.37}$$

where $\mathbf{s}(\boldsymbol{\theta})$ is signal model and $\mathbf{x}(\mathbf{s}(\boldsymbol{\theta}))$ is measured signal.

Assume linear signal model $\mathbf{s} = \mathbf{H}\boldsymbol{\theta}$, LS estimation is then

$$\hat{\boldsymbol{\theta}} = \arg\min_{\check{\boldsymbol{\theta}}} \|\mathbf{x} - \mathbf{H}\check{\boldsymbol{\theta}}\|^2. \tag{3.38}$$

The solution of this equation is

$$\hat{\boldsymbol{\theta}} = (\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{H}}\mathbf{x}. \tag{3.39}$$

Expression $(\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{H}}$ is called Pseudoiverse of $\mathbf{H}$ and its notation is $\mathbf{H}^{\dagger}$. Linear LS hence can be written in form

$$\hat{\boldsymbol{\theta}} = \mathbf{H}^{\dagger}. \tag{3.40}$$

Notice similarity with ML estimation of linear AWGN signal. LS is basically ML with ignoring covariance $\mathbf{C}$. If noise $\mathbf{w}$ in is independent and identically distributed, then $\mathbf{C} = \mathbf{I}$, and hence ML=LS.

Chapter 4
WPNC

4.1 The 2-way relay channel

We can show advantage of using WPNC in 2-way relay channel (2-WRC), which is the simplest topology, in which WPNC can be used, consisting of 2 sources A and B and 1 relay R.

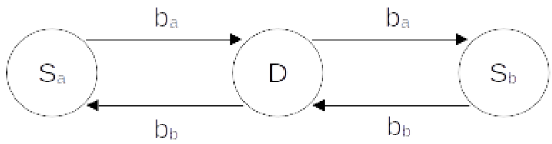


Figure 4.1: 2WPNC in traditional radio network

In traditional way, we need 4 phases to transmit data from A to B and data from B to A. In first phase A transmits data b_a to R, then R retransmits it to B, in next 2 steps data b_b from B are transmit from B to R and finally from R to A.

Network-Layer Network Coding on the other hand needs only three phases, first 2 phases are reserved for transmit data b_a from A to R and data b_b from B to R, now we need to describe data b_r on R, which are function of both b_a and b_b, such that b_r is exclusive OR (XOR) function of b_a and b_b,

$$b_r = b_a \oplus b_b. \tag{4.1}$$

In third phase R transmits b_r to both nodes A and B, where another XOR function is needed, for node A we get

$$b_b = b_r \oplus b_a = (b_a \oplus b_b) \oplus b_a = b_b \oplus (b_a \oplus b_a) = b_a \oplus 0, \tag{4.2}$$

and similarly for node B

$$b_a = b_r \oplus b_b = b_a \oplus 0. \tag{4.3}$$

In WPNC first 2 phases are joint into 1, so in the first phase data b_a and b_b are sent to R, and then R transmits b_r simultaneously to both A and B, and overall just 2 phases are needed.

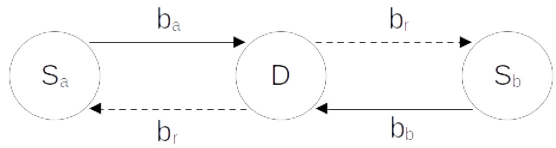


Figure 4.2: 2WPNC

4.2 Relay strategies

The simplest strategy is to directly store received signal and then retransmit it, this strategy is called *Amplify and Forward* (AF). The disandvantage of AF is fact, that noise at the receiver is also amplified.

The other option is to etimate received signal before retransmiting it to another node, this strategy is known as *Decode and Forward* (DF) and has 2 basic ways, how to do it.

The first of them consists of decoding each source separately, computing the network code and forwarding it, the commonly used name of this strategy is *Joint Decode and Forward* (JDF).

The second one is the strategy described above, in which relay decodes network code function of received signal and retransmit it without knowledge of each source. We call this strategy *Hierarchical Decode and Forward* (HDF).

We need 2 stages to successfully transmit data from sources to destination, at the first step, called *Multiple Access Channel* (MAC), sources transmit information to neighboring relays, where are data from multiple sources are processing and furthermore send to destination, at the second stage denoted *Broadcast Channel* (BC).

Network coded symbol, transmited in BC phase, we denote *Hierarchical Information* (HI). For succesful decoding destination needs also direct information about some source, known as *Hierarchical Side Information* (HSI). Note that HSI can me transmit to destination in MAC phase.

We can describe this on example of butterfly network, consists of 2 sources S_a and S_b, 1 relay R and 2 destinations D_x and D_y. In MAC phase S_a transmit data b_a to destination D_x and relay R, simultaneously b_b is send from S_a to R, and D_y. Then in R function $b_r = f(b_a, b_b)$ is computed and furthermore, in BC phase, retransmit to destinations D_x and D_y.

4.3 BPSK hierarchical MAC

We assume two-source BPSK MAC channel with messages b_a and b_b. Codesymbols $c_{A,n}, c_{B,n} \in \{0, 1\}$ use BPSK alphabet $s_{A,n}, s_{B,n} \in \{\pm 1\}$. Observation model is memoryless AWGN channel of length N

$$x_n = s_{A,n}(c_{A,n}) + s_{B,n}(c_{B,n}) + w_n, \tag{4.4}$$

where noise has variance σ_w^2 and probability density function

$$p_w(w) = \frac{1}{2\pi\sigma_w^2}\exp(-\frac{w^2}{2\sigma_w^2}). \tag{4.5}$$

The hierarchical mapping function is XOR

$$c_n = c_{A,n} \oplus c_{B,n}. \tag{4.6}$$

However, in real systems, messages contains fading, denoted h_A, respectively h_B, observation function is then in the shape

$$x_n = h_{A,n}s_{A,n} + h_{B,n}s_{B,n} + w_n, \tag{4.7}$$

note, that fading is complex function. We can define function $u_n = f(s_A, s_B, h_a, h_B)$, this function

$$u_{A,B,n} = h_{A,n}s_{A,n} + h_{B,n}s_{B,n}. \tag{4.8}$$

In the two-source channel we can also use relative fading h

$$h_A s_A + h_B s_B = h_A(s_A + \frac{h_B}{h_A}s_B),\ h = \frac{h_B}{h_A}. \tag{4.9}$$

For M-source MAC the function u is

$$u_n = \sum_{m=1}^{M} h_{m,n}s_{m,n}, \tag{4.10}$$

where M denotes number of messages and n is length of codewords. Input-output system for this channel is

$$x_n = \sum_{m=1}^{M} h_{m,n}s_{m,n} + w_n = u_n(c_n) + w_n. \tag{4.11}$$

4.3.1 Hierarchical demodulator in Gaussian channel

As in chapter 2, we need some metric to decide, which symbol was send. The goal is to find most probable code from set $c = f(\check{c})$. Probabilities for each codesymbols in memoryless AWGN channel are

$$p(x_n|\check{c}_n) = p_w(x_n - u_n(\check{c}_n)) = \frac{1}{2\pi\sigma_w^2}\exp(-\frac{1}{\sigma^2}\|x_n - u_n(\tilde{c}_n)\|^2), \tag{4.12}$$

This function is Gaussian, and hence it is maximized by minimization of exponent, so we find the point from c_n closest to x, this point is denoted minimum hierarchical distance point. The metric used in this scenario is Euclidian distance

$$\rho^2_{Hmin}(x, c_n) = \min_{\check{c}_n : f(\check{c}_n) = c_n} \|x_n - u_n(\check{c}_n)\|^2, \tag{4.13}$$

note, that this metric is usable only for medium to high SNR. This metric is called *Hierarchical distance (H-distance) metric.*

Chapter 5

Two source network MAC phase estimation with BPSK

This chapter aims to construct an estimator of MAC phase in the butterfly network with eliminating noise, fading consisting of phase and amplitude changes, and time delay. We will use the BPSK alphabet and a set of two orthogonal signals.

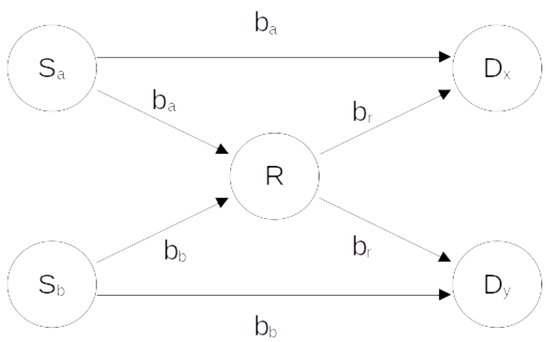


Figure 5.1: Butterfly network

The signal received in relay is combination of messages from both sources burdened by fading and time delay. The channel is AWGN with complex noise

$$x(t) = h_a s_a(c_a, t - \tau_a) + h_b s_b(c_b, t - \tau_b) + w(t), \tag{5.1}$$

where $h_i = \eta_i e^{j\varphi_i}$, $\eta_i, \varphi_i \in \mathbb{R}$ and $s_a(t)$, $s_b(t)$ are orthogonal signals originated as combinations of modulation pulses g

$$s_i(t) = \sum_n q_i g(t - nT_s). \tag{5.2}$$

However, if time delays $\tau_a, \tau_b = 0$, we can work on the level of constelation space, where $s_i = 1$ for $c_i = 1$ and $s_i = -1$ for $c_1 = 0$, as we use BPSK modulation. Hence we can rewrite equation 5.1 into the form

$$x = h_a s_a(c_a) + h_b s_b(c_b) + w. \tag{5.3}$$

b_a, b_b	s_a, s_b	$b_r = b_a \oplus b_b$	s_r
0,0	-1,-1	0	-1
0,1	-1,1	1	1
1,0	1,-1	1	1
1,1	1,1	0	-1

Table 5.1: Decoding table for two source BPSK channel

5.1 Hierarchical demodulator in two source channel

We will start with the simplest case, the AWGN channel without any fading and delay.

$$x_r = s_a + s_b + w. \tag{5.4}$$

Hierarchical demodulator for AWGN channel is as follows

$$p(x|s_A, s_b) = \frac{1}{2\pi\sigma_w^2} \exp\left(-\frac{1}{2\sigma_w^2}(x_R - (s_a + s_b))^2\right). \tag{5.5}$$

Useful function is

$$u_r(c_r) = s_a + s_b, \tag{5.6}$$

where c_r is sequence $b_a\, b_b$, it means that for example if the data $c_r = 10$, so $b_a = 1$ and $b_b = 0$. Minimal hierarchical distance of this channel is

$$\rho^2_{Hmin}(x_r, u_r(c_r)) = \min_{\check{c}_r} \|x_r - u_r(\check{c}_r)\|^2. \tag{5.7}$$

As $b_r = 0$ for $u_r(c_r) = \pm 2$, and $b_r = 1$ for $u_r(c_r) = 0$, we can write

$$\hat{b}_r = 1 \text{ for } |\Re[x_r]| \leq 1 \tag{5.8}$$

$$\hat{b}_r = 0 \text{ for } |\Re[x_r]| > 1. \tag{5.9}$$

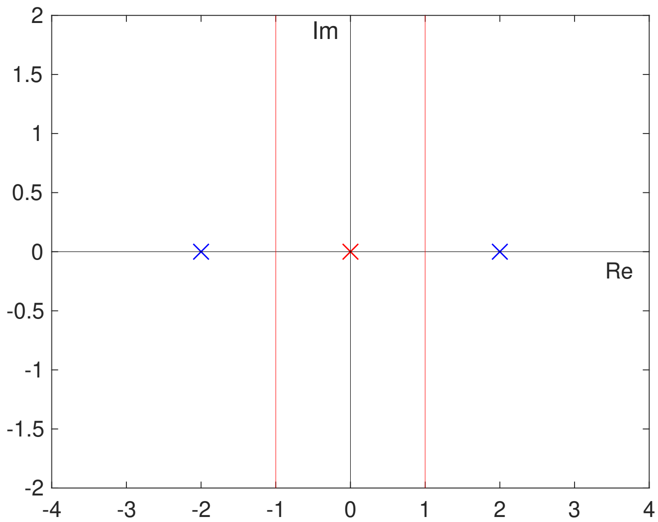


Figure 5.2: Decision regions for $h_a = h_b = 1$

And for message retransmit to the receiver holds

$$s_r = 2(\hat{b}_r - \frac{1}{2}). \tag{5.10}$$

5.2 AWGN channel with fading

The channel model for AWGN channel with fading, but without any delay, is

$$\mathbf{x} = h_a \mathbf{s_a} + h_b \mathbf{s_b} + \mathbf{w}. \tag{5.11}$$

Function $\mathbf{u_r}(\mathbf{c_r}) = h_a \mathbf{s_a} + h_b \mathbf{s_b}$, that means, that hierarchical PDF is

$$p(x_n | h_a s_{a,n}, h_b s_{b,n}) = \frac{1}{2\pi\sigma_w^2} \exp\left(-\frac{1}{2\sigma_w^2} \| x_n - (h_a s_{a,n} + h_b s_{b,n}) \|^2\right), \tag{5.12}$$

$$p(\mathbf{x} | h_a \mathbf{s_a}, h_b \mathbf{s_b}) = \sum_{n=1}^{N} \frac{1}{2\pi\sigma_w^2} \exp\left(-\frac{1}{2\sigma_w^2} \| x_n - (h_a s_{a,n} + h_b s_{b,n}) \|^2\right), \tag{5.13}$$

where N is dimension of messages. Hierarchical demodulator is then

$$p(\mathbf{x} | h_a \hat{\mathbf{s}}_\mathbf{a}, h_b \hat{\mathbf{s}}_\mathbf{b}) = \arg\min_{\check{\mathbf{s}}_\mathbf{a}, \check{\mathbf{s}}_\mathbf{b}} \sum_n \frac{1}{2\pi\sigma_w^2} \exp\left(-\frac{1}{2\sigma_w^2} \| x_n - (h_a s_{a,n} + h_b s_{b,n}) \|^2\right), \tag{5.14}$$

and appropriate hierarchical distance

$$\rho^2_{Hmin}(x_n, [h_a s_{a,n}, h_b s_{b,n}]) = \sum_n \min_{\check{s}_{a,n}, \check{s}_{b,n}} \| x_n - (h_a \check{s}_{a,n} + h_b \check{s}_{b,n}) \|^2. \tag{5.15}$$

Now please let us look at this function forming Euclidian metric in equation 5.15. We assume the properties of noise are unknown. We will hence use that metric for the rest of this section. Note that this is the Least Squares estimator mentioned above

$$\rho^2(x_n,[h_a,h_b,s_{a,n},s_{b,n}]) = \|x_n\|^2 + \|h_a s_{a,n}\|^2 + \|h_b s_{b,n}\|^2 + 2\Re[< h_a s_{a,n}; h_b s_{b,n} >] \\ -2\Re[< x_n; h_a s_{a,n} >] - 2\Re[< x_n; h_b s_{b,n} >]. \tag{5.16}$$

$e^{j\varphi_a}, e^{j\varphi_b}$ lies on unity circle and $s_{a,n}, s_{b,n} = \pm 1$, $> \|e^{j\varphi_a} s_a, n\|^2$, respectively $\|e^{j\varphi_b} s_b, n\|^2$ is then equals to 1 and hence can be discarded

$$\rho^2(x_n,[h_a,h_b,s_{a,n},s_{b,n}]) = \|x_n\|^2 + \|\eta_a\|^2 + \|\eta_b\|^2 + 2\Re[\eta_a\eta_b s_{a,n} s_b^* e^{j(\varphi_a-\varphi_b)}] \\ -2\Re[|x_n|e^{j\varphi_{x,n}}\eta_a s_{a,n}^* e^{-j\varphi_a}] - 2\Re[|x_n|e^{j\varphi_{x,n}}\eta_b s_{b,n}^* e^{-j\varphi_b}]. \tag{5.17}$$

Assuming s_a, s_b are real, we can write this equation in the shape

$$\rho^2(x_n,[h_a,h_b,s_{a,n},s_{b,n}]) = \|x_n\|^2 + \|\eta_a\|^2 + \|\eta_b\|^2 + 2\eta_a\eta_b s_{a,n} s_{b,n}^* \cos(\varphi_a-\varphi_b) \\ -2|x_n|\eta_a s_{a,n}^* \cos(\varphi_{x,n}-\varphi_a) - 2|x_n|\eta_b s_{b,n}^* \cos(\varphi_{x,n}-\varphi_b), \tag{5.18}$$

and if in addition s_a, s_b are orthogonal, we can furthermore simplify the expression by discarding the term of both s_a and s_b

$$\rho^2(x_n,[h_a,h_b,s_{a,n},s_{b,n}])^2 = \|x_n\|^2 + \|\eta_a\|^2 + \|\eta_b\|^2 \\ -2|x_n|\eta_a s_{a,n}^* \cos(\varphi_{x,n}-\varphi_a) - 2|x_n|\eta_b s_{b,n}^* \cos(\varphi_{x,n}-\varphi_b). \tag{5.19}$$

Hierarchical distance for the whole sequence is sum of all components

$$\rho^2(\mathbf{x},[h_a,h_b,\mathbf{s_a},\mathbf{s_b}]) = \sum_n \|\mathbf{x}\|^2 + \|\eta_a\|^2 + \|\eta_b\|^2 + 2\eta_a\eta_b \mathbf{s_a},\mathbf{s_b}^* \cos(\varphi_a-\varphi_b) \\ -2|\mathbf{x}|\eta_a \mathbf{s_a}^* \cos(\varphi_{\mathbf{x}}-\varphi_a) - 2|\mathbf{x}|\eta_b s_{b,n}^* \cos(\varphi_{\mathbf{x}}-\varphi_b), \tag{5.20}$$

respectively

$$\rho^2(\mathbf{x},[h_a,h_b,\mathbf{s_a},\mathbf{s_b}]) = \sum_n \|\mathbf{x}\|^2 + \|\eta_a\|^2 + \|\eta_b\|^2 + 2\eta_a\eta_b \mathbf{s_a},\mathbf{s_b}^* \cos(\varphi_a-\varphi_b) \\ -2|\mathbf{x}|\eta_a \mathbf{s_a}^* \cos(\varphi_{\mathbf{x}}-\varphi_a) - 2|\mathbf{x}|\eta_b s_{b,n}^* \cos(\varphi_{\mathbf{x}}-\varphi_b), \tag{5.21}$$

if the signals are orthogonal.

5.2.1 Estimation of real fading

We will at first discuss the case of the known phase shift. Without loss of generality, we assume that phase shift of both signals are equal to zero, $\varphi_a = \varphi_b = 0$, $0 < \eta_a, \eta_b \leq 1$. The signal received in relay is

$$\mathbf{x} = \eta_a \mathbf{s_a} + \eta_b \mathbf{s_b} + \mathbf{w}. \tag{5.22}$$

For PDF in channel with real fading holds

$$p(\mathbf{x}|\eta_a \mathbf{s_a}, \eta_b \mathbf{s_b}) = \sum_n \frac{1}{2\pi\sigma_w^2} \exp\left(-\frac{1}{2\sigma_w^2}\|x_n - (\eta_a s_{a,n} + \eta_b s_{b,n})\|^2\right), \quad (5.23)$$

Hierarchical distance metric is modification of equation 5.20 for $\varphi_a = \varphi_b = 0$

$$\rho^2(\mathbf{x}, [\eta_a \mathbf{s_a}, \eta_b \mathbf{s_b}]) = \sum_n (\|x_n\|^2 + \|\eta_a\|^2 + \|\eta_b\|^2 + 2\eta_a\eta_b s_{a,n} s_{b,n}^* - 2\Re[x_n]\eta_a s_{a,n}^* - 2\Re[x_n]\eta_b s_{b,n}^*). \quad (5.24)$$

Now our goal is to find the minimum of this function $[\hat{\eta}_a, \hat{\eta}_b] = \arg\min_{\check{\eta}_a, \check{\eta}_b} \rho^2$. We will do it be derivating the function ρ^2 with respect to η_a and η_b and set it equal to zero. These derivates are

$$\frac{\partial \rho^2(\mathbf{x}, [\eta_a \mathbf{s_a}, \eta_b \mathbf{s_b}])}{\partial \eta_a} = \sum_n (2\eta_a + 2\eta_b s_{a,n} s_{b,n}^* - 2\Re[x_n] s_{a,n}^*) \quad (5.25)$$

$$\frac{\partial \rho^2(\mathbf{x}, [\eta_a \mathbf{s_a}, \eta_b \mathbf{s_b}])}{\partial \eta_b} = \sum_n (2\eta_b + 2\eta_a s_{a,n} s_{b,n}^* - 2\Re[x_n] s_{b,n}^*), \quad (5.26)$$

and for $\eta_i = \hat{\eta}_i$ holds

$$\hat{\eta}_a = \frac{1}{n} \sum_n (\Re[x_n] s_{a,n}^* - \hat{\eta}_b s_{a,n} s_{b,n}^*) \quad (5.27)$$

$$\hat{\eta}_b = \frac{1}{n} \sum_n (\Re[x_n] s_{b,n}^* - \hat{\eta}_a s_{a,n} s_{b,n}^*). \quad (5.28)$$

CRLB

For minimal variance of fading parameter η_a, η_b in this channel holds

$$\mathrm{var}(\hat{\eta}_a) \geq \left(-\mathrm{E}\left[\frac{\partial^2 \ln p(\mathbf{x}|\eta_a, \eta_b)}{\partial \hat{\eta}_a^2}\right]\right)^{-1} \quad (5.29)$$

$$\mathrm{var}(\hat{\eta}_b) \geq \left(-\mathrm{E}\left[\frac{\partial^2 \ln p(\mathbf{x}|\eta_a, \eta_b)}{\partial \hat{\eta}_b^2}\right]\right)^{-1}, \quad (5.30)$$

where $-\ln p(\mathbf{x}|\eta_a, \eta_b) = \frac{1}{\sigma_w^2}\rho^2$. ρ^2 is already known and was mentioned in equation 5.24. First derivatives are

$$\frac{\partial \ln p(\mathbf{x}|\eta_a, \eta_b)}{\partial \hat{\eta}_a} = -\frac{1}{\sigma_w^2} \sum_n (2\eta_a + 2\eta_b s_{a,n} s_{b,n}^* - 2\Re[x_n] s_{a,n}^*) \quad (5.31)$$

$$\frac{\partial \ln p(\mathbf{x}|\eta_a, \eta_b)}{\partial \hat{\eta}_b} = -\frac{1}{\sigma_w^2} \sum_n (2\eta_b + 2\eta_b s_{a,n} s_{a,n}^* - 2\Re[x_n] s_{b,n}^*). \quad (5.32)$$

Second derivatives are straightforward

$$\frac{\partial^2 \ln p(\mathbf{x}|\eta_a, \eta_b)}{\partial \hat{\eta}_a^2} = -\frac{1}{\sigma_w^2} \sum_n 2 = -\frac{2N}{\sigma_w^2} \quad (5.33)$$

$$\frac{\partial^2 \ln p(\mathbf{x}|\eta_a,\eta_b)}{\partial \hat{\eta}_b^2} = -\frac{1}{\sigma_w^2}\sum_n 2 = -\frac{2N}{\sigma_w^2}. \tag{5.34}$$

Fisher information matrices are

$$\mathrm{J}_{\eta_a,\eta_a} = \left(-\mathrm{E}\left[\frac{\partial^2 \ln p(\mathbf{x}|\eta_a,\eta_b)}{\partial \hat{\eta}_a^2}\right]\right) = \frac{2N}{\sigma_w^2} \tag{5.35}$$

$$\mathrm{J}_{\eta_b,\eta_b} = \left(-\mathrm{E}\left[\frac{\partial^2 \ln p(\mathbf{x}|\eta_a,\eta_b)}{\partial \hat{\eta}_b^2}\right]\right) = \frac{2N}{\sigma_w^2}. \tag{5.36}$$

And according to CRLB for variances holds

$$\mathrm{var}(\hat{\eta}_a) \geq \frac{\sigma_w^2}{2N} \tag{5.37}$$

$$\mathrm{var}(\hat{\eta}_b) \geq \frac{\sigma_w^2}{2N}. \tag{5.38}$$

Example

Assume set of two orthogonal signals given by matrix $\mathbf{G}$

$$\mathbf{G} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \tag{5.39}$$

where the first column represents signal s_a and the second one signal s_b. It means, that $s_{a,1}$, $s_{a,2}$ are equal to 1 for $c_a = 1$, and to -1 for $c_a = 0$. For $c_b = 1$ we get $s_{b,1} = 1$, $s_{b,2} = -1$ and for $c_b = 0$ $s_{b,1} = -1$, respectively $s_{b,2} = 1$. We can write the matrix $\mathbf{S}$ of signals $\mathbf{s_a}$, $\mathbf{s_b}$ as

$$\mathbf{S} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}, \tag{5.40}$$

where $d_i = 1$ for $c_i = 1$ and $d_1 = -1$ for $c_i = 0$. The signal $\mathbf{x}$ can be written in the matrix form at the shape

$$\mathbf{X} = \begin{bmatrix} h_a & h_b \\ h_a & -h_b \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{5.41}$$

respectively

$$\mathbf{X} = \begin{bmatrix} \eta_a & \eta_b \\ \eta_a & -\eta_b \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}. \tag{5.42}$$

And in the equation form

$$\mathbf{x} = h_a s_{a,1} + h_b s_{b,1} + w_1 + h_a s_{a,2} + h_b s_{b,2} + w_2 \tag{5.43}$$

$$\mathbf{x} = \eta_a s_{a,1} + \eta_b s_{b,1} + w_1 + \eta_a s_{a,2} + \eta_b s_{b,2} + w_2. \tag{5.44}$$

First step is to decode signals s_a and s_b. We will use hierarchical distance metric approximation, so $|\Re[x_n]| < 1$ is decoded as a symbol 0, $\Re[x_n] < -1$

x_1	x_2	$\hat{c}_a$	$\hat{d}_a$	$\hat{c}_b$	$\hat{d}_b$
0	2	1	1	0	-1
0	-2	0	-1	1	1
2	0	1	1	1	1
-2	0	0	-1	0	-1

Table 5.2: Decoding table for channel with real fading

as a symbol -2 and $\Re[x_n] > 1$ as a 2. It is not an optimal demodulator and does not work well for low SNR and significant fading, but it is the best option if both fading and SNR are unknown. The following table shows, how c_a, c_b, respectively d_a, d_b are decoded.

From $\hat{d}_a$ and $\hat{d}_b$ we can now easily get $\hat{s}_a$ and $\hat{s}_b$

$$\hat{\mathbf{S}} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \hat{d}_1 \\ \hat{d}_2 \end{bmatrix}. \tag{5.45}$$

Hierarchical distance metric is in this case

$$\begin{aligned} \rho^2(\mathbf{x}, [\eta_a \hat{\mathbf{s}}_\mathbf{a}, \eta_b \hat{\mathbf{s}}_\mathbf{b}]) = \sum_{n=1}^{2} (&\|x_n\|^2 + \|\eta_a\|^2 + \|\eta_b\|^2 + 2\eta_a\eta_b\hat{s}_{a,n}\hat{s}_{b,n} \\ &-2\Re[x_n]\eta_a\hat{s}_{a,n} - 2\Re[x_n]\eta_b\hat{s}_{b,n}), \end{aligned} \tag{5.46}$$

and its derivations are

$$\begin{aligned} \frac{\partial \rho^2(\mathbf{x}, [\eta_a \hat{\mathbf{s}}_\mathbf{a}, \eta_b \hat{\mathbf{s}}_\mathbf{b}])}{\partial \eta_a} = &4\eta_a + 2\eta_b\hat{s}_{a,1}\hat{s}_{b,1} - 2\Re[x_1]\hat{s}_{a,1} \\ &+2\eta_b\hat{s}_{a,2}\hat{s}_{b,2} - 2\Re[x_2]\hat{s}_{a,2} \end{aligned} \tag{5.47}$$

$$\begin{aligned} \frac{\partial \rho^2(\mathbf{x}, [\eta_a \hat{\mathbf{s}}_\mathbf{a}, \eta_b \hat{\mathbf{s}}_\mathbf{b}])}{\partial \eta_b} = &4\eta_b + 2\eta_a\hat{s}_{a,1}\hat{s}_{b,1} - 2\Re[x_1]\hat{s}_{b,1} \\ &+2\eta_a\hat{s}_{a,2}\hat{s}_{b,2} - 2\Re[x_2]\hat{s}_{b,2}. \end{aligned} \tag{5.48}$$

From the properties of s_a s_b holds $\hat{s}_{a,1} = \hat{s}_{a,2}$, $\hat{s}_{b,1} = -\hat{s}_{b,2}$, hence terms with both $\hat{s}_a$ and $\hat{s}_b$ will be deducted, and for $\hat{\eta}_a$, $\hat{\eta}_b$ holds

$$\hat{\eta}_a = \frac{1}{2}(\Re[x_1]\hat{s}_{a,1} + \Re[x_2]\hat{s}_{a,2}) \tag{5.49}$$

$$\hat{\eta}_b = \frac{1}{2}(\Re[x_1]\hat{s}_{b,1} + \Re[x_2]\hat{s}_{b,2}). \tag{5.50}$$

CRLB in this two sources example give us

$$\text{var}(\hat{\eta}_a) \geq \frac{\sigma_w^2}{4} \tag{5.51}$$

$$\text{var}(\hat{\eta}_a) \geq \frac{\sigma_w^2}{4}. \tag{5.52}$$

5.2.2 Estimation of complex fading

Now let us consider channel with complex fading $h_i = \eta_i e^{j\varphi_i}$. Model of this channel with two sources is

$$\mathbf{x} = \eta_a \mathbf{s_a} e^{j\varphi_a} + \eta_b \mathbf{s_b} e^{j\varphi_b} + \mathbf{w}. \tag{5.53}$$

The hierarchical distance metric for this case was already mentioned above

$$\begin{aligned} \rho^2(x_n, [h_a s_{a,n}, h_b s_{b,n}]) = \|x_n\|^2 + \|\eta_a\|^2 + \|\eta_b\|^2 + 2\eta_a\eta_b s_{a,n} s_{b,n}^* \cos(\varphi_a - \varphi_b) \\ -2|x_n|\eta_a s_{a,n}^* \cos(\varphi_{x,n} - \varphi_a) - 2|x_n|\eta_b s_{b,n}^* \cos(\varphi_{x,n} - \varphi_b) \end{aligned} \tag{5.54}$$

$$\begin{aligned} \rho^2(\mathbf{x}, [h_a \mathbf{s_a}, h_b \mathbf{s_b}]) = \sum_n (\|x_n\|^2 + \|\eta_a\|^2 + \|\eta_b\|^2 + 2\eta_a\eta_b s_{a,n} s_{b,n}^* \cos(\varphi_a - \varphi_b) \\ -2|x_n|\eta_a s_{a,n}^* \cos(\varphi_{x,n} - \varphi_a) - 2|x_n|\eta_b s_{b,n}^* \cos(\varphi_{x,n} - \varphi_b)) \end{aligned} \tag{5.55}$$

Minimization of amplitude and phase fading is given by the following equations

$$\begin{aligned} \frac{\partial \rho^2(x_n, [\hat{h}_a \mathbf{s_{a,n}}, \hat{h}_b \mathbf{s_{b,n}}])}{\partial \hat{\eta}_a} = \sum_n (2\hat{\eta}_a + 2\hat{\eta}_b s_{a,n} s_{b,n}^* \cos(\hat{\varphi}_a - \hat{\varphi}_b) \\ -2|R|[x_n] s_{a,n}^* \cos(\varphi_{x,n} - \hat{\varphi}_a)) = 0 \end{aligned} \tag{5.56}$$

$$\begin{aligned} \frac{\partial \rho^2(x_n, [\hat{h}_a \mathbf{s_{a,n}}, \hat{h}_b \mathbf{s_{b,n}}])}{\partial \hat{\eta}_b} = \sum_n (2\hat{\eta}_b + 2\hat{\eta}_a s_{a,n} s_{b,n}^* \cos(\hat{\varphi}_a - \hat{\varphi}_b) \\ -2|R|[x_n] s_{b,n}^* \cos(\varphi_{x,n} - \hat{\varphi}_b)) = 0 \end{aligned} \tag{5.57}$$

$$\begin{aligned} \frac{\partial \rho^2(x_n, [\hat{h}_a \mathbf{s_{a,n}}, \hat{h}_b \mathbf{s_{b,n}}])}{\partial \hat{\varphi}_a} = \sum_n (- 2\hat{\eta}_a\hat{\eta}_b s_{a,1} s_{b,1}^* \sin(\hat{\varphi}_a - \hat{\varphi}_b) \\ -2|x_n|\eta_a s_{a,2}^* \sin(\varphi_{x,n} - \hat{\varphi}_a)) = 0 \end{aligned} \tag{5.58}$$

$$\begin{aligned} \frac{\partial \rho^2(x_n, [\hat{h}_a \mathbf{s_{a,n}}, \hat{h}_b \mathbf{s_{b,n}}])}{\partial \hat{\varphi}_b} = \sum_n (2\hat{\eta}_a\hat{\eta}_b s_{a,1} s_{b,1}^* \sin(\hat{\varphi}_a - \hat{\varphi}_b) \\ -2|x_n|\hat{\eta}_b s_{b,2}^* \sin(\varphi_{x,n} - \hat{\varphi}_b)) = 0. \end{aligned} \tag{5.59}$$

CRLB

The results for CRLB of $\hat{\eta}_a$, $\hat{\eta}_b$ will be the same as in the previous section. We can then write

$$\mathrm{var}(\hat{\eta}_a) \geq \frac{\sigma_w^2}{2N} \tag{5.60}$$

$$\mathrm{var}(\hat{\eta}_b) \geq \frac{\sigma_w^2}{2N}. \tag{5.61}$$

First derivatives with respect to φ_a, respectively φ_b are

$$\frac{\partial \ln p(\mathbf{x}|\eta_a,\eta_b)}{\partial \hat{\eta}_a} = -\frac{1}{\sigma_w^2}\sum_n \big(-2\hat{\eta}_a\hat{\eta}_b s_{a,1}s_{b,1}^* \sin(\hat{\varphi}_a - \hat{\varphi}_b) \\ -2|x_n|\eta_a s_{a,2}^* \sin(\varphi_{x,n} - \hat{\varphi}_a)\big) \tag{5.62}$$

$$\frac{\partial \ln p(\mathbf{x}|\eta_a,\eta_b)}{\partial \hat{\eta}_b} = -\frac{1}{\sigma_w^2}\sum_n \big(2\hat{\eta}_a\hat{\eta}_b s_{a,1}s_{b,1}^* \sin(\hat{\varphi}_a - \hat{\varphi}_b) \\ -2|x_n|\hat{\eta}_b s_{b,2}^* \sin(\varphi_{x,n} - \hat{\varphi}_b)\big). \tag{5.63}$$

And for second derivatives holds

$$\frac{\partial^2 \ln p(\mathbf{x}|\eta_a,\eta_b)}{\partial \hat{\eta}_a^2} = -\frac{1}{\sigma_w^2}\sum_n \big(-2\hat{\eta}_a\hat{\eta}_b s_{a,1}s_{b,1}^* \cos(\hat{\varphi}_a - \hat{\varphi}_b) \\ +2|x_n|\eta_a s_{a,2}^* \cos(\varphi_{x,n} - \hat{\varphi}_a)\big) \tag{5.64}$$

$$\frac{\partial^2 \ln p(\mathbf{x}|\eta_a,\eta_b)}{\partial \hat{\eta}_b^2} = -\frac{1}{\sigma_w^2}\sum_n \big(-2\hat{\eta}_a\hat{\eta}_b s_{a,1}s_{b,1}^* \cos(\hat{\varphi}_a - \hat{\varphi}_b) \\ +2|x_n|\hat{\eta}_b s_{b,2}^* \cos(\varphi_{x,n} - \hat{\varphi}_b)\big). \tag{5.65}$$

If signals s_a, s_b are orthogonal, this equations are reduced to forms

$$\frac{\partial^2 \ln p(\mathbf{x}|\eta_a,\eta_b)}{\partial \hat{\eta}_a^2} = -\frac{1}{\sigma_w^2}\sum_n 2|x_n|\eta_a s_{a,2}^* \cos(\varphi_{x,n} - \hat{\varphi}_a) \tag{5.66}$$

$$\frac{\partial^2 \ln p(\mathbf{x}|\eta_a,\eta_b)}{\partial \hat{\eta}_b^2} = -\frac{1}{\sigma_w^2}\sum_n 2|x_n|\hat{\eta}_b s_{b,2}^* \cos(\varphi_{x,n} - \hat{\varphi}_b). \tag{5.67}$$

And lover bounds of variance of φ_a and φ_b are

$$\mathrm{var}(\hat{\varphi}_a) \geq \frac{\sigma_w^2}{\sum_n \big(-2\hat{\eta}_a\hat{\eta}_b s_{a,1}s_{b,1}^* \cos(\hat{\varphi}_a - \hat{\varphi}_b) + 2|x_n|\eta_a s_{a,2}^* \cos(\varphi_{x,n} - \hat{\varphi}_a)\big)} \tag{5.68}$$

$$\mathrm{var}(\hat{\varphi}_b) \geq \frac{\sigma_w^2}{\sum_n \big(-2\hat{\eta}_a\hat{\eta}_b s_{a,1}s_{b,1}^* \cos(\hat{\varphi}_a - \hat{\varphi}_b) + 2|x_n|\hat{\eta}_b s_{b,2}^* \cos(\varphi_{x,n} - \hat{\varphi}_b)\big)}, \tag{5.69}$$

and for orthogonal signals

$$\mathrm{var}(\hat{\varphi}_a) \geq \frac{\sigma_w^2}{\sum_n 2|x_n|\eta_a s_{a,2}^* \cos(\varphi_{x,n} - \hat{\varphi}_a)} \tag{5.70}$$

$$\mathrm{var}(\hat{\varphi}_b) \geq \frac{\sigma_w^2}{\sum_n 2|x_n|\hat{\eta}_b s_{b,2}^* \cos(\varphi_{x,n} - \hat{\varphi}_b)}. \tag{5.71}$$

Example

Consider the same set of orthogonal signals as in previous section, given by matrices

$$\mathbf{S} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \tag{5.72}$$

$$\mathbf{X} = \begin{bmatrix} h_a & h_b \\ h_a & -h_b \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}. \tag{5.73}$$

The equation form of the channel model is then

$$\mathbf{x} = h_a s_{a,1} + h_b s_{b,1} + w_1 + h_a s_{a,2} + h_b s_{b,2} + w_2 \tag{5.74}$$

$$\mathbf{x} = \eta_a e^{j\varphi_a} s_{a,1} + \eta_b e^{j\varphi_b} s_{b,1} + w_1 + \eta_a e^{j\varphi_a} s_{a,2} + \eta_b e^{j\varphi_b} s_{b,2} + w_2. \tag{5.75}$$

The demodulator again uses hierarchical distance metric, where decision regions are bounden by lines $\Re = \pm 1$. The following table shows, how c_a and c_b are decoded. And thence we can easily get $\hat{s}_{a,n}$, $\hat{s}_{b,n}$, which will be used

x_1	x_2	$\hat{c}_a$	$\hat{d}_a$	$\hat{c}_b$	$\hat{d}_b$
0	2	1	1	0	-1
0	-2	0	-1	1	1
2	0	1	1	1	1
-2	0	0	-1	0	-1

Table 5.3: Decoding table for channel with complex fading

in minimization of parameters of fading. The hierarchical distance in this channel is

$$\rho^2(\mathbf{x}, [h_a\hat{\mathbf{s}}_\mathbf{a}, h_b\hat{\mathbf{s}}_\mathbf{b}]) = \sum_{n=1}^{2} \big(\|x\|^2 + \|\eta_a\|^2 + \|\eta_b\|^2 + 2\eta_a\eta_b\hat{s}_{a,n}\hat{s}_{b,n}\cos(\varphi_a - \varphi_b) \\ -2|x_n|\eta_a\hat{s}_{a,n}\cos(\varphi_x - \varphi_a) - 2|x_n|\eta_b\hat{s}_{b,n}\cos(\varphi_x - \varphi_b)\big). \tag{5.76}$$

For derivations holds

$$\frac{\partial\rho^2(x_n, [\hat{h}_a\hat{\mathbf{s}}_{\mathbf{a,n}}, \hat{h}_b\hat{\mathbf{s}}_{\mathbf{b,n}}])}{\partial\hat{\eta}_a} = \sum_n \big(2\hat{\eta}_a + 2\hat{\eta}_b\hat{s}_{a,n}\hat{s}_{b,n}\cos(\hat{\varphi}_a - \hat{\varphi}_b) \\ -2|x_n|\hat{s}_{a,n}\cos(\varphi_{x,n} - \hat{\varphi}_a)\big) = 0 \tag{5.77}$$

$$\frac{\partial\rho^2(x_n, [\hat{h}_a\hat{\mathbf{s}}_{\mathbf{a,n}}, \hat{h}_b\hat{\mathbf{s}}_{\mathbf{b,n}}])}{\partial\hat{\eta}_b} = \sum_n \big(2\hat{\eta}_b + 2\hat{\eta}_a\hat{s}_{a,n}\hat{s}_{b,n}\cos(\hat{\varphi}_a - \hat{\varphi}_b) \\ -2|x_n|\hat{s}_{b,n}\cos(\varphi_{x,n} - \hat{\varphi}_b)\big) = 0 \tag{5.78}$$

$$\frac{\partial\rho^2(x_n, [\hat{h}_a\hat{\mathbf{s}}_{\mathbf{a,n}}, \hat{h}_b\hat{\mathbf{s}}_{\mathbf{b,n}}])}{\partial\hat{\varphi}_a} = \sum_n \big(- 2\hat{\eta}_a\hat{\eta}_b\hat{s}_{a,1}\hat{s}_{b,1}\sin(\hat{\varphi}_a - \hat{\varphi}_b) \\ -2|x_n|\eta_a\hat{s}_{a,2}\sin(\varphi_{x,n} - \hat{\varphi}_a)\big) = 0 \tag{5.79}$$

$$\frac{\partial \rho^2(x_n, [\hat{h}_a \hat{\mathbf{s}}_{\mathbf{a},\mathbf{n}}, \hat{h}_b \hat{\mathbf{s}}_{\mathbf{b},\mathbf{n}}])}{\partial \hat{\varphi}_b} = \sum_n \big(2\hat{\eta}_a \hat{\eta}_b \hat{s}_{a,1} \hat{s}_{b,1} \sin(\hat{\varphi}_a - \hat{\varphi}_b) \\ -2|x_n| \hat{\eta}_b \hat{s}_{b,2} \sin(\varphi_{x,n} - \hat{\varphi}_b)\big) = 0. \tag{5.80}$$

Thanks to the orthogonality of $\hat{s}_a$ and $\hat{s}_b$ (regardless of $\mathbf{c}$), terms with both $\hat{s}_a$ and $\hat{s}_b$ are zero, and these equations are a function of the only h_a, respectively h_b, and this system is easily solvable. From first two equations we found out, that $\hat{\eta}_a$, $\hat{\eta}_b$ holds

$$\hat{\eta}_a = \frac{1}{2}(|x_1| \hat{s}_{a,1} \cos(\varphi_{x,1} - \hat{\varphi}_a) + |x_2| \hat{s}_{a,2} \cos(\varphi_{x,2} - \hat{\varphi}_a)) \tag{5.81}$$

$$\hat{\eta}_b = \frac{1}{2}(|x_1| \hat{s}_{b,1} \cos(\varphi_{x,1} - \hat{\varphi}_b) + |x_2| \hat{s}_{b,2} \cos(\varphi_{x,2} - \hat{\varphi}_b)) \tag{5.82}$$

We will get the third and fourth equation, by substituting η_i and multiplication by -1, to the form

$$\sin(\varphi_{x,1} - \hat{\varphi}_a) \cos(\varphi_{x,1} - \hat{\varphi}_a) |x_1|^2 \hat{s}_{a,1}^2 + \sin(\varphi_{x,2} - \varphi_a) \cos(\varphi_{x,2} - \hat{\varphi}_a) |x_2|^2 \hat{s}_{a,2}^2 \\ +2|x_1||x_2| \hat{s}_{a,1} \hat{s}_{a,2} \big(\sin(\varphi_{x,1} - \hat{\varphi}_a) \cos(\varphi_{x,2} - \hat{\varphi}_a) + \sin(\varphi_{x,2} - \hat{\varphi}_a) \cos(\varphi_{x,1} - \hat{\varphi}_a)\big) = 0 \tag{5.83}$$

$$\sin(\varphi_{x,1} - \hat{\varphi}_b) \cos(\varphi_{x,1} - \hat{\varphi}_b) |x_1|^2 \hat{s}_{b,1}^2 + \sin(\varphi_{x,2} - \hat{\varphi}_b) \cos(\varphi_{x,2} - \hat{\varphi}_b) |x_2|^2 \hat{s}_{b,2}^2 \\ +2|x_1||x_2| \hat{s}_{b,1} \hat{s}_{b,2} \big(\sin(\varphi_{x,1} - \hat{\varphi}_b) \cos(\varphi_{x,2} - \hat{\varphi}_b) + \sin(\varphi_{x,2} - \hat{\varphi}_b) \cos(\varphi_{x,1} - \hat{\varphi}_b) = 0. \tag{5.84}$$

We know, that $|\hat{s}_{i,n}| = 1$, $i = a, b$, $n = 1, 2$ and also $\hat{s}_{a,1} = \hat{s}_{a,2}$, $\hat{s}_{b,2} = -\hat{s}_{b,2}$ hence these equations can be simplified. After some manipulations, including using of trigonometric identity $\cos(a)\sin(a) = \frac{1}{2}\sin(2a)$, we get the final expressions

$$\frac{1}{2}\sin(2\varphi_{x,1} - 2\hat{\varphi}_a)|x_1|^2 + \frac{1}{2}\sin(2\varphi_{x,2} - 2\hat{\varphi}_a)|x_2|^2 + \sin(\varphi_{x,1} + \varphi_{x,2} - 2\hat{\varphi}_a)|x_1||x_2| = 0 \tag{5.85}$$

$$\frac{1}{2}\sin(2\varphi_{x,1} - 2\hat{\varphi}_b)|x_1|^2 + \frac{1}{2}\sin(2\varphi_{x,2} - 2\hat{\varphi}_b)|x_2|^2 - \sin(\varphi_{x,1} + \varphi_{x,2} - 2\hat{\varphi}_b)|x_1||x_2| = 0. \tag{5.86}$$

And lover bounds of parameters provided by CRLB are

$$\mathrm{var}(\hat{\eta}_a) \geq \frac{\sigma_w^2}{4} \tag{5.87}$$

$$\mathrm{var}(\hat{\eta}_a) \geq \frac{\sigma_w^2}{4}. \tag{5.88}$$

$$\mathrm{var}(\hat{\varphi}_a) \geq \frac{\sigma_w^2}{\sum_{n=1}^{2} 2|x_n| \eta_a s_{a,2}^* \cos(\varphi_{x,n} - \hat{\varphi}_a)} \tag{5.89}$$

$$\mathrm{var}(\hat{\varphi}_b) \geq \frac{\sigma_w^2}{\sum_{n=1}^{2} 2|x_n| \hat{\eta}_b s_{b,2}^* \cos(\varphi_{x,n} - \hat{\varphi}_b)}. \tag{5.90}$$

5.2.3 Estimation of delay

However, the situation in a channel with time delays is more complex because signals are not orthogonal anymore. In addition, we cannot use constellation space representation, as function x is a function of time. Also, finding the maximum by differentiating with respect to delay can be problematic, as these derivatives are for some pulses discontinuous. Model of this channel is

$$x(t) = h_a s_b(c_a, t - \tau_a) + h_a s_b(c_b, t - \tau_b) + w(t). \tag{5.91}$$

PDF for $x(t)$ in this channel is given by the equation 5.92

$$p(x(t)|u(t)) = \frac{1}{2\pi\sigma_w^2} \exp\left(-\frac{1}{2\sigma_w^2}\|x - (h_a s_a(c_a, t - \tau_a) + h_b s_b(c_b, t - \tau_b))\|^2\right), \tag{5.92}$$

and by dropping the properties of noise, we will get the hierarchical metric distance

$$\rho^2(x(t), u(t)) = \|x - (h_a s_a(c_a, t - \tau_a) + h_b s_b(c_b, t - \tau_b))\|^2. \tag{5.93}$$

Probability for whole signal x is given by integral with respect to time throughout all time

$$p(x|u) = \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma_w^2} \exp\left(-\frac{1}{2\sigma_w^2}\|x - (h_a s_a(c_a, t - \tau_a) + h_b s_b(c_b, t - \tau_b))\|^2\right) \mathrm{d}t, \tag{5.94}$$

and similarly for hierarchical distance

$$\rho^2(x, u) = \int_{-\infty}^{\infty} \|x - (h_a s_a(c_a, t - \tau_a) + h_b s_b(c_b, t - \tau_b))\|^2 \, \mathrm{d}t. \tag{5.95}$$

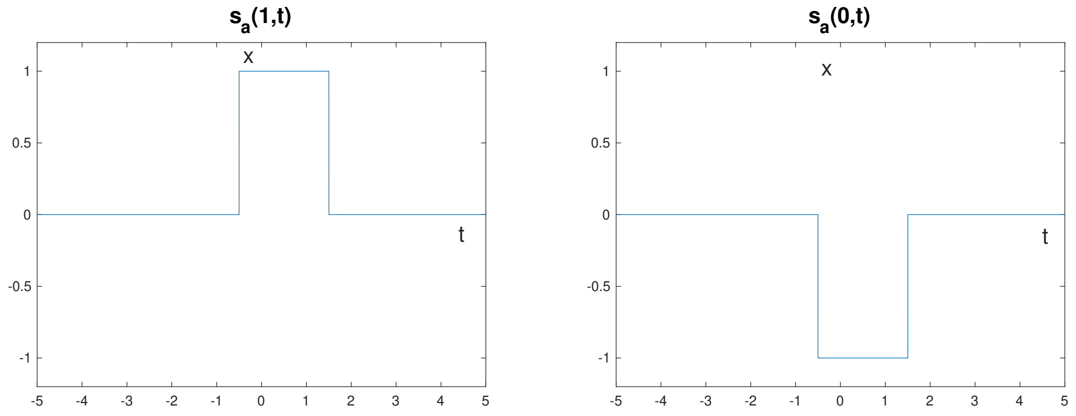
Assume signals $s_a(c_a, t)$, $s_b(c_b, t)$ are combinations of rectangular pulses $\Pi(t) = \begin{cases} 1, & \text{if} -\frac{1}{2} \le t \le \frac{1}{2} \\ 0, & \text{else,} \end{cases}$ and $\Pi(t-1) = \begin{cases} 1, & \text{if} \frac{1}{2} \le t \le \frac{3}{2} \\ 0, & \text{else,} \end{cases}$ such that

$$s_a(1, t) = \begin{cases} 1, & \text{if} -\frac{1}{2} \le t \le \frac{3}{2} \\ 0, & \text{else} \end{cases} \tag{5.96}$$

$$s_a(0, t) = \begin{cases} -1, & \text{if} -\frac{1}{2} \le t \le \frac{3}{2} \\ 0, & \text{else} \end{cases} \tag{5.97}$$

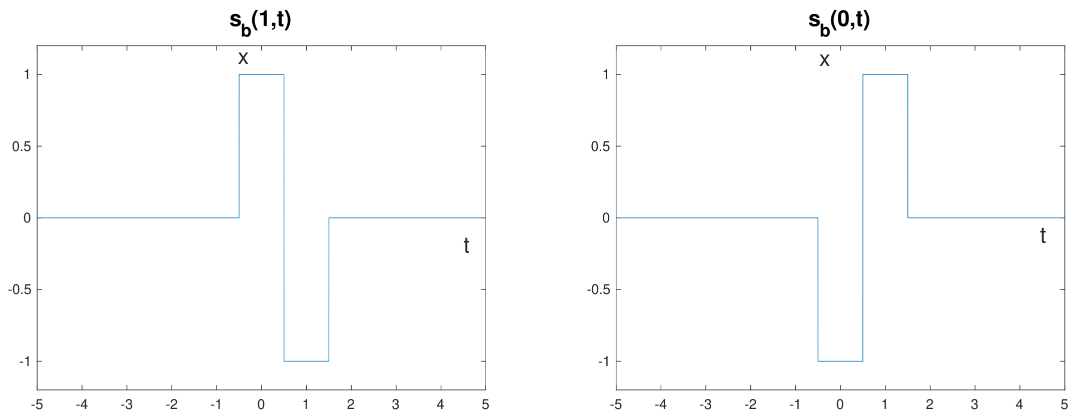
$$s_b(1, t) = \begin{cases} 1, & \text{if} -\frac{1}{2} \le t \le \frac{1}{2} \\ -1, & \text{if} \frac{1}{2} < t \le \frac{3}{2} \\ 0, & \text{else} \end{cases} \tag{5.98}$$

$$s_b(0, t) = \begin{cases} -1, & \text{if} -\frac{1}{2} \le t \le \frac{1}{2} \\ 1, & \text{if} \frac{1}{2} < t \le \frac{3}{2} \\ 0, & \text{else.} \end{cases} \tag{5.99}$$



(a) : $c_a = 1$ **(b) :** $c_a = 0$

Figure 5.3: Signals possibly sent from source A



(a) : $c_a = 1$ **(b) :** $c_a = 0$

Figure 5.4: Signals possibly sent from source B

However, in our simulation, the relay does not know the exact continuous course, but only its samples. The model of the channel hence can be rewritten to the shape

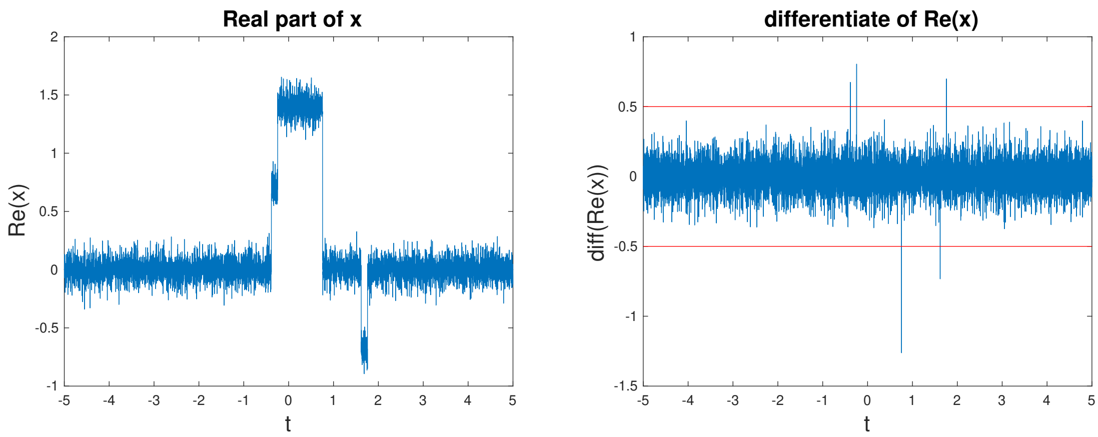
$$x(kT_n) = h_1 s_1(c, (k - \tau_a)T_n) + h_2 s_2((k - \tau_b)T_n) + w(t), \tag{5.100}$$

where T_n is step between two neighboring samples. Probabilities from equations 5.93 and 5.94 then will not be given by integrals but by the sums

$$p(x|u) = \sum_{k=-\infty}^{\infty} \frac{1}{2\pi\sigma_w^2} \exp\left(-\frac{1}{2\sigma_w^2} \|x - (h_a s_b(c_a, (k - \tau_a)T_n) + h_b s_b(c_b, (k - \tau_b)T_n))\|^2\right) \tag{5.101}$$

$$\rho^2(x, u) = \sum_{k=-\infty}^{\infty} \|x - (h_a s(c_a, (k - \tau_a)T_n) + h_b s(c_b, (k - \tau_b)T_n))\|^2. \tag{5.102}$$

As mentioned above, finding analytic expressions is tricky for this problem, so our goal is to find the numerical solver for this particular example. The sum from equation 5.102 is meaningful only for samples, for which at least one of the pair $s_a(c_a, t-\tau_a)$, $s_b(c_b, t-\tau_b)$ is non-zero, however it is not useful, if we do not know the time delays, hence our first step will be estimation of delays. For properties used in simulation ($\eta_a, \eta_b \in (0.8, 1)$, $\phi_a, \phi_b \in (-\frac{\pi}{2}, \frac{\pi}{2})$, $\tau_a, \tau_b \in (0, \frac{1}{2})$) the best way seems to be examine real part of $x(t)$, respectively its differentiate. If absolute value of differentiate is greater than 0.5, this sample is investigate as possible point of leading edge of signal $s_a(t-\tau_a)$, respectively $s_b(t-\tau_b)$. And as lengths of signals, distance between its changes and also time, in which signals without fading start, are known, we can estimate delays τ_a and τ_b.



(a) : $\Re(x)$ **(b) :** differentiate of $\Re(x)$

Figure 5.5: Example of real part of x and its differentiate

As we have estimations $\hat{\tau}_a$ and $\hat{\tau}_b$, we can use it in demodulation of $s_a(c_a)$, respectively $s_b(c_b)$ according to expression

$$\rho^2(x, [s_a(c_a), s_b(c_b)]) = \arg\min_{\check{c}_a, \check{c}_b} \sum_{k_{min}}^{k_{max}} \|x - (s_a(\check{c}_a, (k-\hat{\tau}_a)T_n) + s_b(\check{c}_b, (k-\hat{\tau}_b)T_n))\|^2, \tag{5.103}$$

where k_{min} lays in $-\frac{1}{2} + \min(\hat{\tau}_a, \hat{\tau}_b)$, and k_{max} in $\frac{3}{2} + \max(\hat{\tau}_a, \hat{\tau}_b)$. And the last step is estimating the fading h_a, h_b

$$\rho^2(x, u) = \arg\min_{\check{\eta}_a, \check{\varphi}_a, \check{\eta}_b, \check{\varphi}_b} \sum_{k_{min}}^{k_{max}} \|x - (\check{\eta}_a \check{\varphi}_a s_a(\hat{c}_a, (k-\hat{\tau}_a)T_n) + \check{\eta}_b, \check{\varphi}_b s_b(\hat{c}_b, (k-\hat{\tau}_b)T_n))\|^2. \tag{5.104}$$

However, due the noise is impossible to find exact solution of this equation, so it is solved numerically by comparing each options within regions (0.8,1), respectively $(-\frac{\pi}{4}, \frac{\pi}{4})$, with given step between them, simulation uses step 0.01 for $\hat{\eta}_a$, $\hat{\eta}_b$ and 0.05 for $\hat{\varphi}_a$, $\hat{\varphi}_b$.

Note. This algorithm is not robust and does not work for low SNR, in which estimation of delays fails.

Chapter 6

Conclusion

This thesis provides algorithms for estimating channel parameters (amplitude, phase and delay) in H-MAC of WPNC. In the beginning, fundamentals of digital communication and statistical signal processing are introduced. The next chapter describes the principles of AWGN on the example of simple topologies. The main part is focused on BPSK modulated two source H-MAC channel, for which we created Least squares estimator for channel parameters using data decoded from Hierarchical demodulator. For estimation of amplitude and phase, analytical expressions were founded, and for the estimation of delay, numerical algorithms were used. For cases without delay, limits were established by CRLB. These results were accompanied by Matlab simulations for simple examples.

Further work could focus on more complicated topologies and modulations. Also, algorithms for non-orthogonal pilot signals could be provided.

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Matlab files

decision_regions.m
example_realfading.m
example_complexfading.m
signals.m
example_delay.m

BACHELOR'S THESIS ASSIGNMENT

I. Personal and study details

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Department / Institute: **Department of Radioelectronics**

Study program: **Open Electronic Systems**

II. Bachelor's thesis details

Bachelor's thesis title in English:

Network Channel State Estimation for WPNC Radio Networks

Bachelor's thesis title in Czech:

Network Channel State Estimation for WPNC Radio Networks

Guidelines:

Student will first get acquainted with fundamentals of estimation and detection theory and with fundamentals of Wireless Physical Layer Network Coding (WPNC). The goal of the work is to develop estimation and detection algorithms for radio Network Channel State (NCS) estimation suited for WPNC based networks. The NCS involves channel parameters (e.g. SNR, channel coefficients, delays) and also the network topology. The ultimate target is to obtain a distributed NCS estimate at each node without the need of explicit assignment of orthogonal signal resources (e.g. for pilots). Student should however include also simple orthogonal pilot based algorithms (at least as a reference scenario). Then the work should focus on suitably selected non-orthogonal sharing scenarios (e.g. NCS with limited set of unknown parameters, etc.). The performance of the algorithms should be evaluated theoretically (e.g. MSE, detection probability) and should also be verified by a computer simulation and, if possible, in selected cases with a simple over-the-air experiment using laboratory transceivers.

Bibliography / sources:

[1] S. M. Kay: Fundamentals of statistical signal processing-estimation theory, Prentice-Hall 1993
[2] S. M. Kay: Fundamentals of statistical signal processing-detection theory, Prentice-Hall 1998
[3] J. Sykora, A. Burr: Wireless Physical Layer Network Coding, Cambridge University Press 2018

Name and workplace of bachelor's thesis supervisor:

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Name and workplace of second bachelor's thesis supervisor or consultant:

Date of bachelor's thesis assignment: **03.02.2020** Deadline for bachelor thesis submission: **21.05.2021**

Assignment valid until: **30.09.2021**

prof. Ing. Jan Sýkora, CSc.	doc. Ing. Josef Dobeš, CSc.	prof. Mgr. Petr Páta, Ph.D.
Supervisor's signature	Head of department's signature	Dean's signature

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The student acknowledges that the bachelor's thesis is an individual work. The student must produce his thesis without the assistance of others, with the exception of provided consultations. Within the bachelor's thesis, the author must state the names of consultants and include a list of references.

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