

# Radial Radon Transform dedicated to Micro-object Localization from Radio Frequency Ultrasound Signal

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**Abstract**—In this paper we describe a method for automatic electrode localization in soft tissue from radio-frequency signal. The method is exploiting a property of the Radon Transform (RT) that allows to localize a line-segment in 3D data. The method directly processes the radio-frequency (RF) signal provided by the ultrasound (US) probe. Thus, there's no need for ultrasound image reconstruction. The method is able to detect line-segments that are discontinuous. The low computational cost allows to process data in real-time. The algorithm was tested on a radio-frequency signal acquired by scanning a phantom containing a thin metal electrode of 150 $\mu$ m in diameter. The experiments show that the developed technique is capable of reliably finding an arbitrarily positioned line-segment in a 3D image.

## INTRODUCTION

For reasons such as low-cost and real-time manipulation, the ultrasound imaging is frequently used to localize metallic inclusions in soft tissue. To detect the position of the electrode in brain, the interior of the brain is scanned by an three-dimensional ultrasound probe. In this way, we obtain a set of radio-frequency signals (Fig. 1), each of them representing one ultrasound beam. From the RF signals, a three-dimensional ultrasound image can reconstructed (Fig. 2).

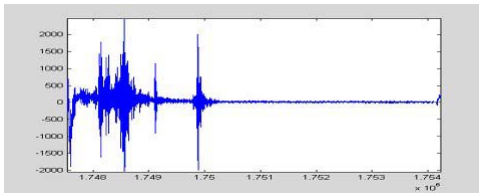


Fig. 1. Example of time behaviour of RF signal corresponding to one ultrasound beam.

The acoustical impedance of the tungsten electrode is very different from the one of biological tissue. Thus, the electrode will appear in the constructed US image as a region of high brightness (Fig. 2). Several techniques have already been developed to solve the problem of object localization from US images [1], [2], [3]. These methods however require reconstruction of ultrasound image from the radio-frequency signal. Our method is designed in such a way that it localize

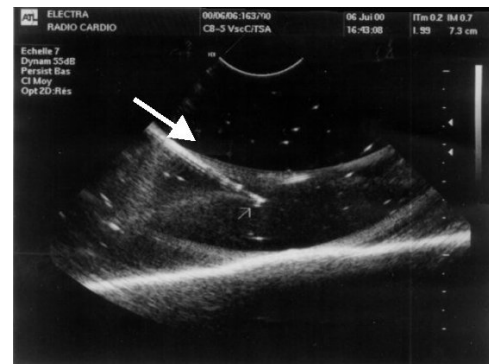


Fig. 2. Ultrasound image reconstructed from RF signals .

the position of the electrode directly from the RF signal. The electrode localization is more precise and works faster, because there is no need for the US image reconstruction. In this paper, we also present an interesting property of the Radon Transform that allows line-segment localization in 3D data.

## PROPOSED METHOD

The algorithm is based on the Radon Transform [4]. The RT is a linear transformation that is used mainly for reconstruction of tomographic reconstruction [5]. The RT has a property that allows to detect a line-segment in 3D data. For simplicity, we will demonstrate this property for the case of 2D data.

Let  $(x, y)$  be the cartesian coordinates of a point in a 2D space and  $f(x, y)$  a scalar function of two real arguments. The 2D RT denoted as  $R_f(\rho, \theta)$  is given by

$$R_f(\rho, \tau) = \iint_{-\infty}^{\infty} f(x, y) \delta(\rho - x \cos \theta - y \sin \theta) dx dy \quad (1)$$

Let the model of a line be

$$f(x, y) = \delta(\rho^* - x \cos \theta^* - y \sin \theta^*) \quad (2)$$

The Radon transformation of the line is:

$$R_f(\rho, \tau) = \iint_{-\infty}^{\infty} f(x, y) \delta(\rho - x \cos \theta - y \sin \theta) dx dy \quad (3)$$

$$R_f(\rho, \tau) = \delta(\rho^*, \theta^*) \quad (4)$$

The Radon transformation of a line with parameters  $\theta^*, \rho^*$  is the Dirac distribution at  $(\theta^*, \rho^*)$ . Therefore, it is possible to localize the line from the knowledge of the maximum position in the Radon space. The RT has a clear geometrical meaning in the sense that the  $R_f(\rho, \theta)$  is in fact a parallel projection of the function  $f(x, y)$  on a line passing through the origin of system of coordinates and at angle  $\theta$  with respect to the axis  $x$ .

The line localization by the RT in three-dimensional space is analogous to the RT in two-dimensional space.

Yet, in the case of the 3D RT, the Radon space has four dimensions  $\alpha, \beta, u, v$ , where  $\alpha, \beta$  are angles of rotation and  $u, v$  are the coordinates of projection plane. The procedure of line localization by the RT in 3D space can be summarized as follows:

- 1) The input image is transformed into the Radon space.
- 2) In the Radon space, the position of the maximum is detected.
- 3) Given the position of the maximum in the Radon space, the equation of the detected line is determined.

#### IMPLEMENTATION

The input to our algorithm are sampled RF signals from the ultrasound probe that operates in sectorial mode. Each RF signal represents one ultrasound beam. Its position in 3D space is defined by an angle  $\theta$  and one coordinate  $z$  (Fig. 3). Thus, the input data are given in cylindrical system of coordinates.

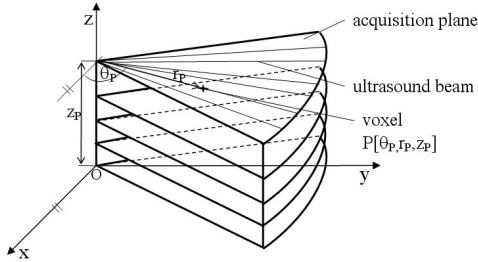


Fig. 3. Geometrie of received data (RF signal).

In the first step of the algorithm, the data are binarized to the value 0 or 1 by thresholding. The threshold was experimentally set to the value  $0.8 \cdot I_{max}$ , where  $I_{max}$  is the maximum voxel value.

The Radon transformation of the data is calculated by rotating the data and projecting them to a plane. As the center for rotation  $(m, n, o)$  we choose the center of a sphere comprising the data (Fig. 4). In the next step, the data are projected on the grid  $\sigma$  (Fig. 5).

The grid  $\sigma$  is parallel with the plane  $xy$  and passes through the center of the sphere.

We could gradually construct the complete Radon transformation by rotating the data by angles  $\alpha_i$ , resp.  $\beta_j$  around the axis  $i$ , resp.  $j$  followed by projection on the grid  $\sigma$ . However,

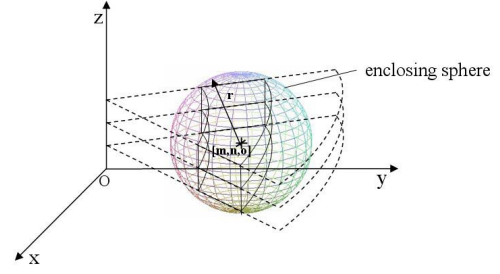


Fig. 4. Sphere enclosing the input image

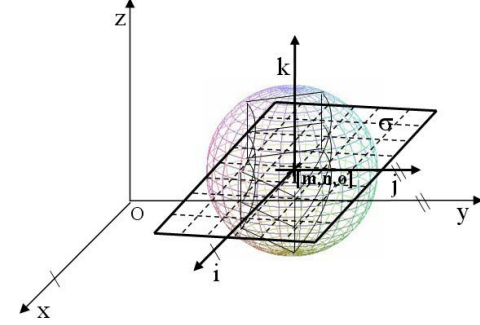


Fig. 5. Projection plane  $\sigma$ .

to determine the parameters of the line, we need to know only the position of the maximum in the Radon space.

Let  $(x, y, z)$  be the coordinates of a voxel  $X$  with value 1, and  $(x', y', z')$  coordinates of the same voxel turned by the angles  $\alpha_i, \beta_j$ , around the axis  $i, j$ . It can be shown that

$$\mathbf{X}' = \mathbf{M}_{\alpha, \beta} \cdot \mathbf{X} \quad (5)$$

where  $\mathbf{M}_{\alpha, \beta}$  is the rotation matrix by angles  $\alpha, \beta$  around the axis  $x, y$ .

Let's notice that the maximum in the Radon space occurs if the rotated line-segment is orthogonal to the plane  $\sigma$ . In this case all the voxels lying on the line-segment has the same coordinates  $x', y'$ . For each pair  $\alpha_i, \beta_j$ , we determine the histogram of coordinates  $x', y'$ . We denote them  $\text{hist}_{\alpha_i, \beta_j}(x')$ ,  $\text{hist}_{\alpha_i, \beta_j}(y')$  and their respective maximums  $\max \text{hist}_{\alpha_i, \beta_j}(x')$ ,  $\max \text{hist}_{\alpha_i, \beta_j}(y')$ . The maximums of the histograms occurs for some combination of angles  $(\alpha_{max}, \beta_{max})$  when the variables  $x', y'$  do not change (the case of the line-segment orthogonal to the projection plane  $\sigma$ ). With this knowledge, we construct a functional  $J(\alpha, \beta)$  defined by

$$J(\alpha, \beta) = \max_{x'}[\text{hist}_{\alpha_i, \beta_j}(x')] + \max_{y'}[\text{hist}_{\alpha_i, \beta_j}(y')] \quad (6)$$

The functional has the maximal value for angles  $\alpha_{max}, \beta_{max}$ . Therefore

$$\begin{aligned}
(\alpha_{max}, \beta_{max}) &= \arg \max_{\forall \alpha, \beta} J(\alpha, \beta) \quad (7) \\
&= \arg \max_{\forall \alpha, \beta} \{ \max_{\forall x'} [\text{hist}_{\alpha_i, \beta_j}(x')] + \\
&\quad + \max_{\forall y'} [\text{hist}_{\alpha_i, \beta_j}(y')] \}.
\end{aligned}$$

To find the values  $\alpha_{max}, \beta_{max}$ , we implemented the progressive mesh-size reduction, called log-D step [6]. We applied this algorithm on the functional  $J(\alpha, \beta)$  to obtain the values  $\alpha_{max}, \beta_{max}$ . The values  $u_{max}, v_{max}$  are the coordinates of the maximum in the plane  $\sigma$  rotated by angles  $\alpha, \beta$ .

In the last step, we determine the terminal points of the line-segment. Let  $z'_{min}$ , resp.  $z'_{max}$  be the minimal, resp. maximal value of the  $z'$  coordinates of the voxels turned by angles  $\alpha_{max}, \beta_{max}$ . The points  $\mathbf{P}'_1[u_{max}, v_{max}, z'_{min}]$ ,  $\mathbf{P}'_2[u_{max}, v_{max}, z'_{max}]$  are then the terminals of the detected line-segment rotated by angles  $\alpha_{max}, \beta_{max}$ . In order to determine the terminals in the original system of coordinates we multiply the coordinates by the inverse matrix  $M_{\alpha, \beta}$ . Formally:

$$\mathbf{P}_1 = \mathbf{M}_{\alpha_{max}, \beta_{max}}^{-1} \cdot \mathbf{P}'_1, \quad (8)$$

$$\mathbf{P}_2 = \mathbf{M}_{\alpha_{max}, \beta_{max}}^{-1} \cdot \mathbf{P}'_2. \quad (9)$$

The points  $\mathbf{P}_1, \mathbf{P}_2$  are the terminals of the detected line-segment.

#### OBJECT LOCALIZATION IN RF SIGNAL

The object localization by the Radon Transform can be done either in the radio-frequency data, or in the reconstructed ultrasound image. To justify the choice of object localization from the RF signal, we show the projection of the RF signal, resp. corresponding reconstructed image on the plane  $\sigma$  (Fig. 6 and Fig. 7).

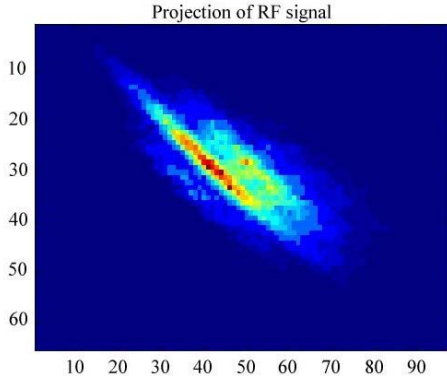


Fig. 6. Projection of the radio-frequency signal on the plane  $\sigma$ .

As we can be observed from these images, the size of the RF signal projection is smaller (more condensed) than the one the US image. From this, we conclude that the object localization is more accurate in the RF signal, since the maximum in the Radon space can be estimated more precisely.

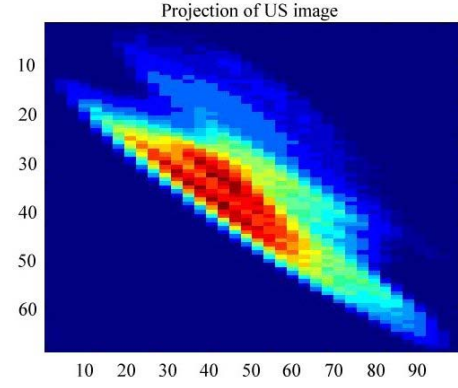


Fig. 7. Projection of the reconstructed ultrasound image on the plane  $\sigma$ .

#### EXPERIMENTS

The method was tested on RF signals from an ultrasound scanner. The acquisition was made on a cryogel phantom with a thin metallic electrode inserted. The acoustical properties of the phantom were closed to the acoustical properties of soft tissue.

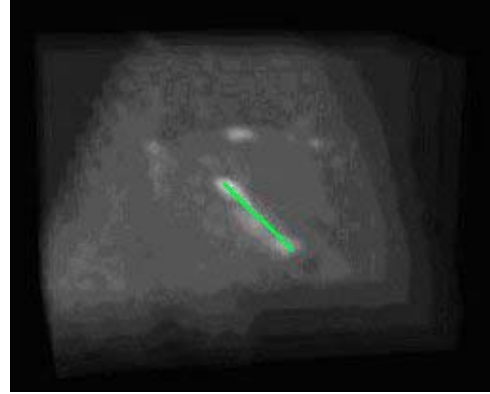


Fig. 8. Result of the algorithm. The detected electrode is marked in green.

The central frequency of the ultrasound probe was 7MHz, the RF signal was sampled at 27MHz. The size of the reconstructed image was 128x128x128 voxels. The time needed for processing one image was 15s on computer with 1GHz processor and 1GB of memory. Fig. 8 shows the reconstructed US image with the detected electrode.

#### CONCLUSION

This paper has shown a method that solves the problem of object localization from radio-frequency signal. The advantage of this approach reposes on the fact that this method does not require reconstructed ultrasound image. This saves the computational time and increases the precision of localization. The core of the method is based on the Radon Transform. It distinguish itself by a low computational cost allowing processing of data in real-time. Moreover, the algorithm is able to detect line-segments that are discontinuous, that is to say, when a part of the line-segment is missing.

The algorithm was tested on RF signal corresponding to a phantom with a metallic electrode inserted. The results show that the method is able to reliably find an electrode in three-dimensional data.

In the further research we would like to focus on the localization of more complex objects. Further, we would like to improve the computational cost of the algorithm.

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