Higher Order Neural Unit Adaptive Control and Stability Analysis for Industrial System Applications

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***Abstract*—**Higher-order neural units (HONUs) have proven to be comprehensible nonlinear polynomial models and computationally efficient for application as standalone process models or as a nonlinear control loop where one recurrent HONU is a plant model and another HONU is as a nonlinear state feedback (neuro)controller (via MRAC scheme). Alternative approaches as the widely used Lyapunov function, can be used for design of the control law or prove of stability for existing control laws in state space for a given equilibrium point and a given input. However, in practical engineering applications such methods although proving stability about an equilibrium point may still result in bad performance or damage if they are also not proven to be bounded-input-bounded-output/state (BIBO/BIBS) stable with respect to the control inputs.

The main contribution of this dissertation is the introduction of two novel real-time BIBO/BIBS based stability evaluation methods for HONUs and for their nonlinear closed control loops. The proposed methods being derived from the core polynomial architectures of HONUs themselves, provides a straightforward and comprehensible framework for stability monitoring that can be applied to other forms of recurrent polynomial neural networks. New results are presented from the rail automation field as well as several non-linear dynamics system examples. Further directions are also highlighted for sliding mode design via HONUs and multi-layered HONU feedback control presented as a framework for low to moderately nonlinear systems.

***Keywords*—model reference adaptive control; discrete-time nonlinear dynamic systems; polynomial neural networks; point-wise state-space representation; stability**

Abbreviations

BIBO … Bounded Input Bounded Output
BIBS … Bounded Input Bounded State
DHS … Discrete Time HONU Stability
DDHS… Discrete Time Decomposed HONU stability
GD … Gradient Descent
HONU… Higher-Order Neural Unit
HONU MRAC… Closed control loop with one HONU model and one HONU as a feedback controller
ISS … Input to State Stability
LM… Levenberg-Marquardt Algorithm
LNU, QNU… Linear, Quadratic Neural Unit
RLS … Recursive Least Squares Algorithm

HONU (QNU, CNU): $\tilde{y}=\mathbf{w}\cdot col^{r=2}(\mathbf{x}),\ \ \tilde{y}=\mathbf{w}\cdot col^{r=3}(\mathbf{x}),$

HONU-MRAC Control Law: $u(k)=r_q(k)\cdot\left(d(k)-\mathbf{v}(k)\cdot col^{\gamma}\left(\xi(k)\right)\right)$

parameter adaptation: $\mathbf{w}(k+1)=\mathbf{w}(k)+\Delta\mathbf{w}(k);\mathbf{v}(k+1)=\mathbf{v}(k)+\Delta\mathbf{v}(k)$

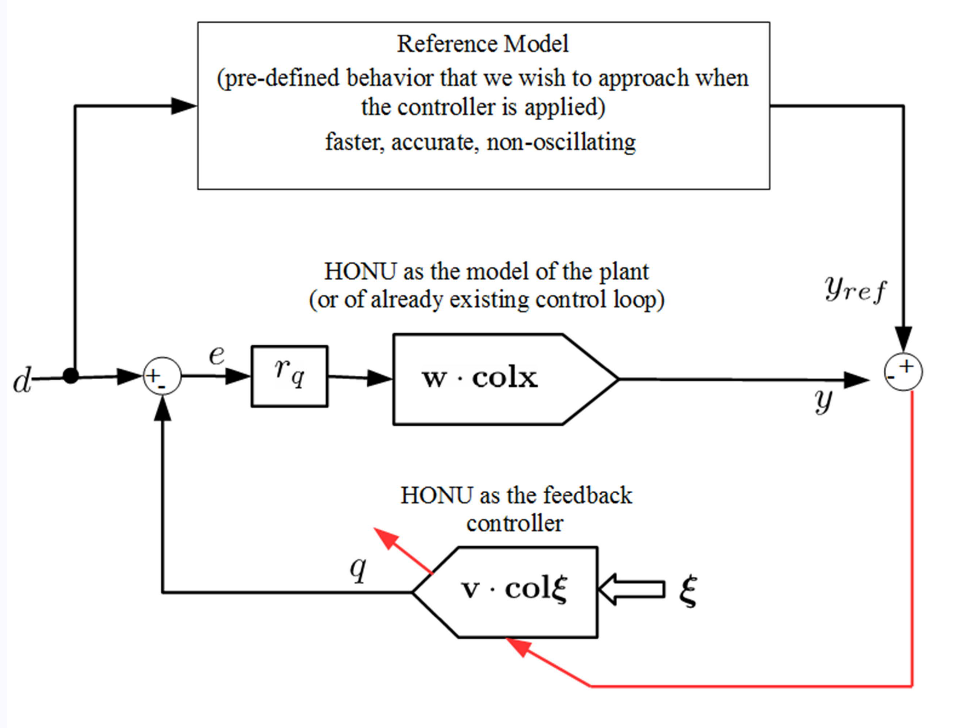
k … discrete index of time; r_q (or also denoted as p)… adaptive (opt. as a feedback) controller gain

$\mathbf{w};\mathbf{v}$ … vector of all adaptable parameters ($n_w \times 1$; $n_v \times 1$)

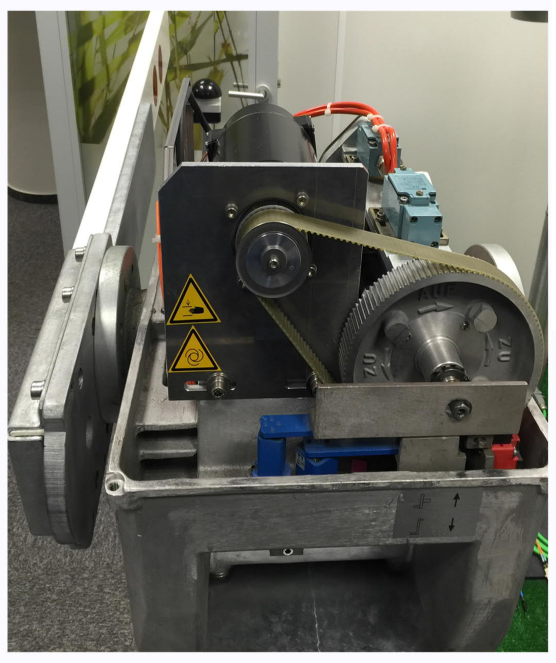
$\mathbf{x};\xi$ … vector of inputs (and feedback variables) (x_0; ξ_0=1)

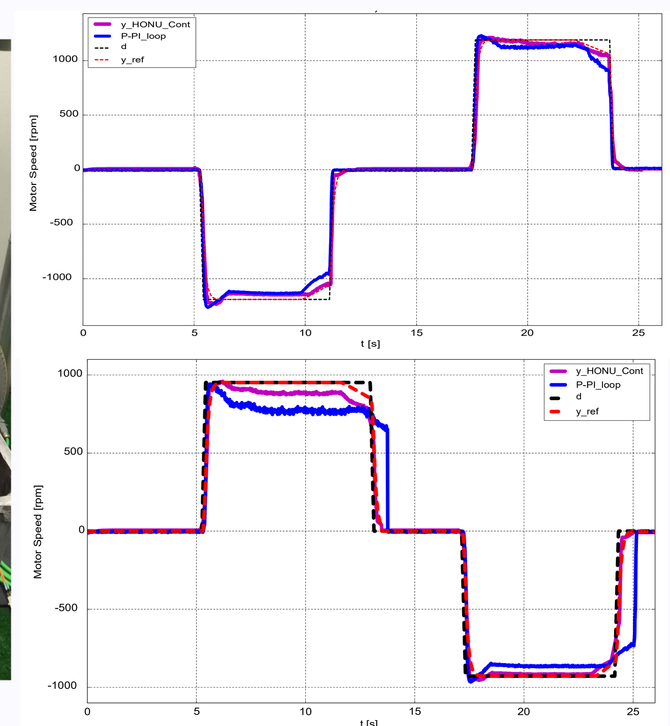
Fundamental Learning Algorithms

Gradient Descent(GD)	$\Delta\mathbf{w}=\mu\cdot e(k)\cdot\mathbf{colx}^T$
Levenberg-Marquardt (LM)	$\Delta\mathbf{w}=(\mathbf{J}^T\cdot\mathbf{J}+\frac{1}{\mu}\cdot\mathbf{I})^{-1}\cdot\mathbf{J}^T\cdot\mathbf{e}$
Recursive Least Squares (RLS)	$\Delta\mathbf{w}=e(k)\cdot\mathbf{colx}(k)^T\cdot\mathbf{R}^{-1}(k)$



HONU MRAC as a standalone control loop. One HONU=Plant model (optionally for identification of an existing control loop), the Second as a Feedback Controller. [3], [9]





Optimization of cascade control loop with const. parameter HONU-MRAC control loop (QNU-QNU) via RLS(plant), GD(cont.) on real barrier drive control board. Barrier 1 (above) standard boom. Barrier 2 (below) with loaded boom. [10]

Pointwise State-Space Representation of HONU

This method transforms the classic nonlinear polynomial representation of a HONU to a incremental linear approximation via the following pointwise state-space form

$$\Delta\bar{\mathbf{x}}(k+1)=\bar{\mathrm{A}}(k)\cdot\Delta\bar{\mathbf{x}}(k)+\Delta\bar{\mathbf{u}}(k);$$

where

$$\bar{\mathbf{A}}(k)=\frac{\partial\bar{\mathrm{f}}}{\partial\bar{\mathrm{x}}(k)}=\begin{bmatrix}0 & 1 & 0\ 0 & 0\\ \vdots & \dots & 1\ 0 & 0\\ \bar{a}_{n_y,1} & \bar{a}_{n_y,2} & \dots\dots & \bar{a}_{n_y,n_{\bar{x}}}\\ 0 & 0 & \dots\ 1 & 0\\ 0 & 0 & \dots\ 0 & 0\end{bmatrix}=\underset{\substack{i=1\dots n_{\bar{x}};\\ j=1\dots n_{\bar{x}}}}{\left[\{\bar{a}_{i,j}\}\right]},$$

Further, the coefficients maybe individually computed as

$$\bar{a}_{i,j}=\begin{cases}=1 & \text{for } i=2,3,\dots,n_x\wedge i\neq n_y;\ j=i+1\\ =\boldsymbol{\psi}_j col^r(\mathbf{x}(k+1))=w[p]+\frac{\partial}{\partial\bar{x}_j}\left(\bar{x}_{n_y}(k)\right) & \text{for } j=1,2,\dots,n_{\bar{x}}\wedge i=n_y\\ =0 & \text{else}\end{cases},$$

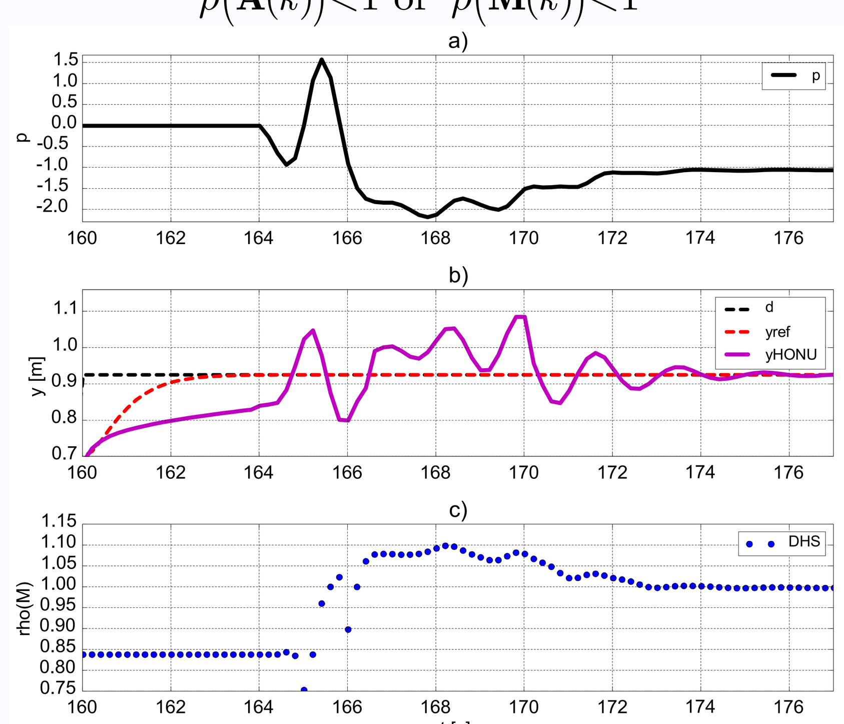
Analogically for a HONU-MRAC control loop, where desired behavior d and the extended matrix of dynamics is $\bar{\mathbf{M}}(k)$

$$\Delta\bar{\mathbf{x}}(k+1)=\bar{\mathbf{M}}(k)\cdot\Delta\bar{\mathbf{x}}(k)+\Delta\bar{\mathbf{d}}(k),$$

Discrete Time HONU Stability: DHS

Given the pointwise representation of a HONU, a HONU model and further whole HONU-MRAC control loop is BIBO if the following holds

$$\rho\left(\bar{\mathbf{A}}(k)\right)<1 \text{ or } \rho\left(\bar{\mathbf{M}}(k)\right)<1$$



DHS method under randomly changed controller gain from t>164[s] of adaptive QNU-QNU control loop on non-linear two-funnel tank system. [1]-[2], [3] .

Pointwise Decomposed State-Space Representation of HONU

The decomposed method re-expresses the classical HONU into a sub-polynomial representation as

$$\tilde{y}(k)=\sum_{i=0}^{n_x}\sum_{j=i}^{n_x}w_{i,j}\cdot x_i\cdot x_j$$
$$=w_{0,0}+\sum_{i=1}^{n_y}x_i\cdot\left(w_{0,i}+\sum_{j=i}^{n_x}w_{i,j}\cdot x_j\right)+\sum_{i=n_y+1}^{n_x}x_i\cdot\left(w_{0,i}+\sum_{j=i}^{n_x}w_{i,j}\cdot x_j\right),$$

Then re-expressing the above sub-polynomials, the following state-space representation yields, where the augmented input matrix is $\hat{\mathbf{B}}_{\mathbf{a}}$

$$\hat{\mathbf{x}}(k)=\hat{\mathbf{A}}(k\text{-}1)\cdot\hat{\mathbf{x}}(k-1)+\hat{\mathbf{B}}_{\mathbf{a}}\cdot\hat{\mathbf{u}}_{\mathbf{a}}(k-1);\ \tilde{y}(k)=\hat{\mathbf{C}}\cdot\hat{\mathbf{x}}(k-1),$$

Analogically, the concept extended to a HONU-MRAC loop yields where the input term is the desired behavior d and the extended matrix of dynamics is $\hat{\mathrm{M}}(k-1)$ and the augmented input matrix is $\hat{\mathrm{N}}_{\mathbf{a}}$

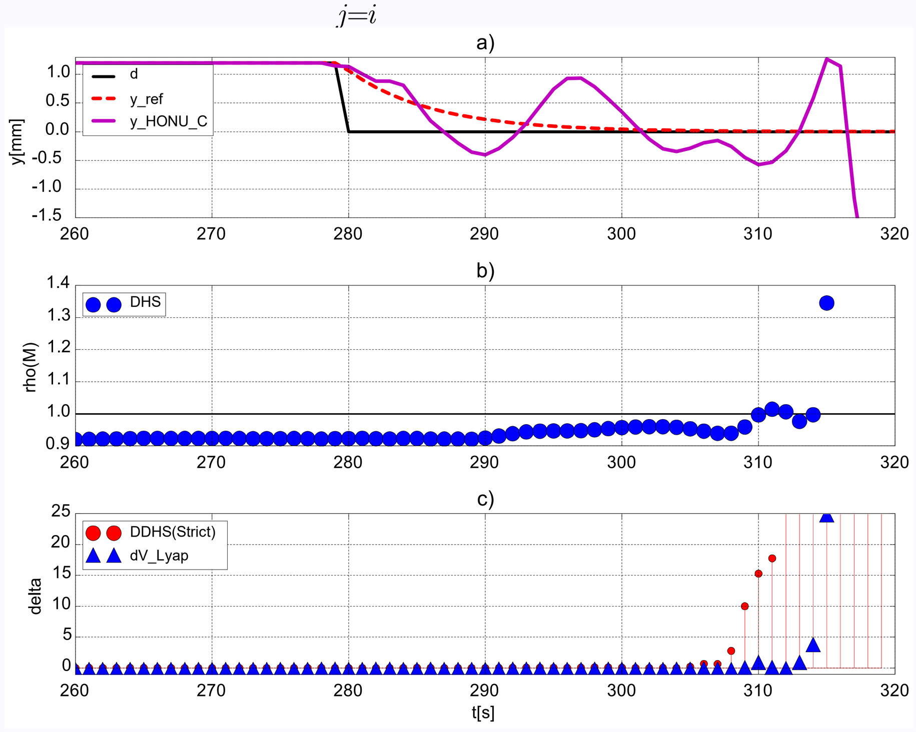
$$\hat{\mathbf{x}}(k)=\hat{\mathbf{M}}(k\text{-}1)\cdot\hat{\mathbf{x}}(k-1)+\hat{\mathbf{N}}_{\mathbf{a}}\cdot\hat{\mathbf{u}}_{\mathbf{a}}(k-1)\ ;\ \tilde{y}(k)=\hat{\mathbf{C}}\cdot\hat{\mathbf{x}}(k),$$

where

$$\hat{\mathbf{A}}=\begin{bmatrix}0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & \vdots\\ 0 & 0 & \ddots & \ddots & 0\\ 0 & 0 & \ddots & 0 & 1\\ \hat{a}_{n_y} & \hat{a}_{n_y-1} & \cdots & \hat{a}_2 & \hat{a}_1\end{bmatrix},\ \hat{\mathbf{B}}=\begin{bmatrix}0 & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 0\\ \hat{b}_{n_u} & \hat{b}_{n_u-1} & \cdots & \hat{b}_1\end{bmatrix}.$$

$$\hat{a}_i=\hat{a}_i\left(\hat{\mathbf{x}}(k-1),\hat{\mathbf{u}}(k-1),\mathbf{w}\right)=w_{0,i}+\sum_{j=i}^{n_x}w_{i,j}\cdot x_j(k-1),$$

$$\hat{b}_i=\hat{b}_i\left(\hat{\mathbf{u}}(k-1),\mathbf{w}\right)=w_{0,i}+\sum_{j=i}^{n_x}w_{i,j}\cdot x_j(k-1)\ ;\ i>n_y.$$



Comparative analysis of DDHS(Strict) with Lyapunov approach [4]-[5], [7]. Earlier detection via DDHS(Strict) of progressively unstable LNU-QNU control loop on conventional roller rig mathematical model.

Decomposed Discrete Time HONU Stability: DDHS

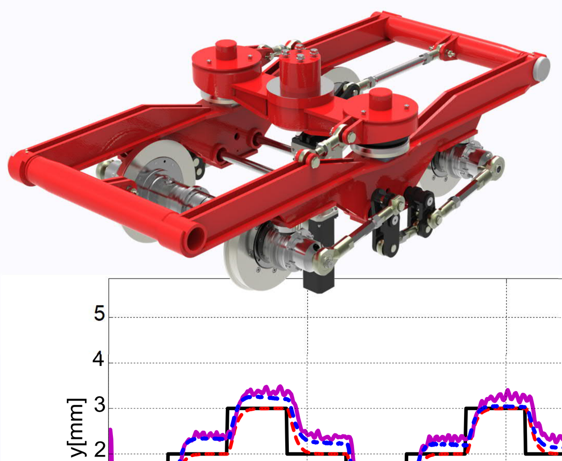
The decomposed HONU is BIBS if from an initial position in time k_0 the Input-to-State (ISS) stability relation is fulfilled

$$S(k)=\left\|\hat{\mathbf{x}}(k)\right\|-\left\|\prod_{\kappa=k_0}^{k-1}\hat{\mathbf{A}}(\kappa)\right\|\cdot\left\|\hat{\mathbf{x}}(k_0)\right\|-\sum_{\kappa=k_0}^{k-1}\left\|\prod_{i=\kappa}^{k-1}\hat{\mathbf{A}}(i)\cdot\hat{\mathbf{B}}_{\mathbf{a}}(\kappa)\right\|\cdot\left\|\hat{\mathbf{u}}_{\mathbf{a}}(\kappa)\right\|\leq 0\,.$$

Strict(DDHS)

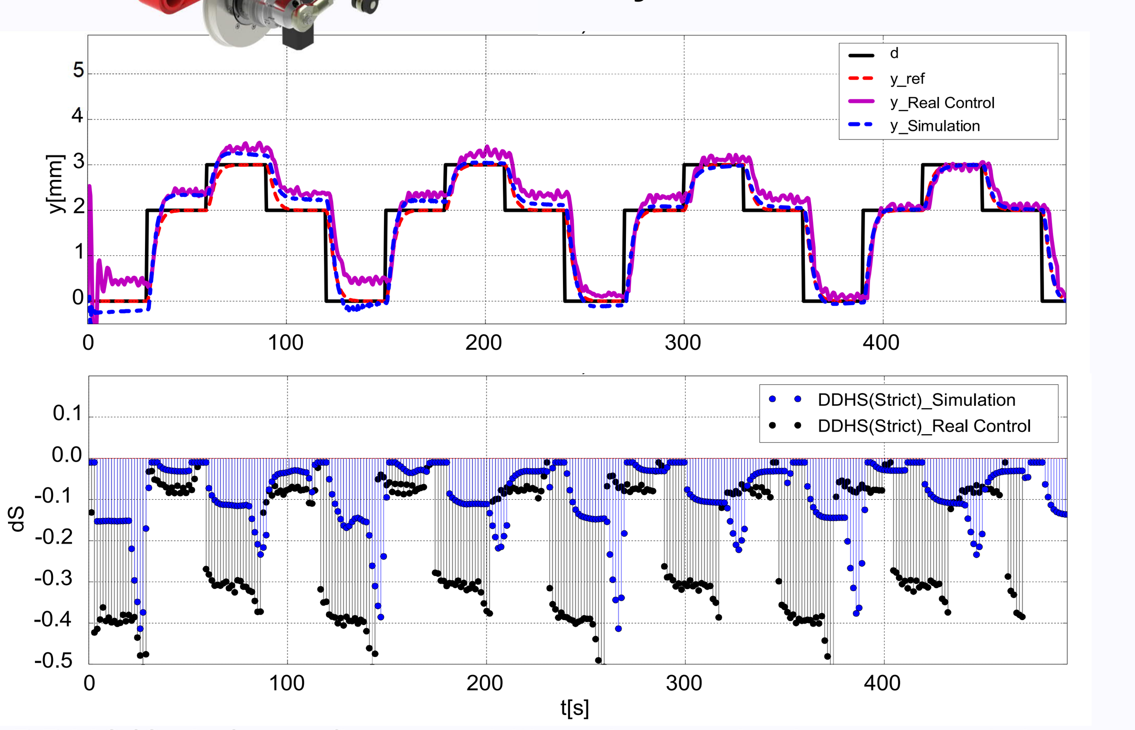
It may be further justified the BIBS of a HONU may be strictly satisfied if the difference of function S(k) in real-time is ≤ 0.

$$\Delta S(k)=S(k)-S(k-1)=\left\|\hat{\mathbf{x}}(k)\right\|-\left\|\hat{\mathbf{x}}(k-1)\right\|-\left\|\hat{\mathbf{B}}_{\mathbf{a}}(k-1)\cdot\hat{\mathbf{u}}_{\mathbf{a}}(k-1)\right\|$$
$$+\left(\left\|\hat{\mathbf{A}}(k-1)\right\|-1\right)\cdot\left(\left\|\hat{\mathbf{A}}(k-2)\cdot\hat{\mathbf{x}}(k-2)\right\|\right)+\left\|\hat{\mathbf{B}}_{\mathbf{a}}(k-2)\cdot\hat{\mathbf{u}}_{\mathbf{a}}(k-2)\right\|\leq 0\ \text{ for }\forall k>k_0.$$

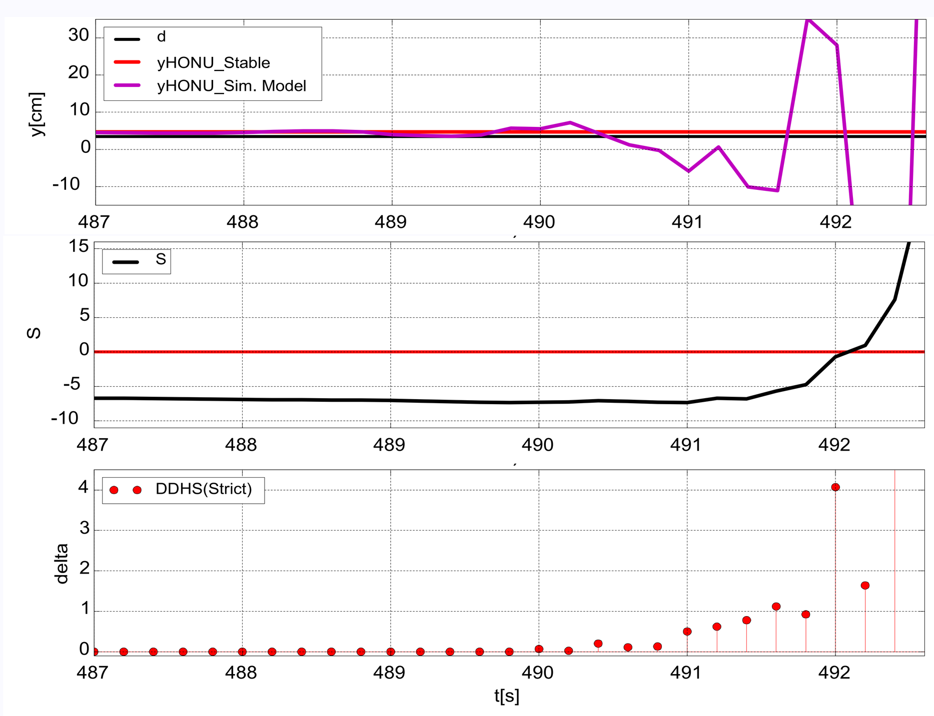


CTU Roller Rig: Fully adaptive QNU-LNU control loop

with real-time Strict(DDHS) analysis



DDHS(Strict) confirms stability via monitoring on real-time re-tuning of a fully adaptive QNU-LNU control loop for Real CTU Roller Rig with new dynamic behavior due to changed stiffness and damping properties. [4], [7]



DDHS and DDHS(Strict) comparison on nonlinear two-tank liquid level system proves LNU-QNU becoming unstable from t>488s. [2], [6], [8]

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Acknowledgements:

I would like to thank doc. Ivo Bukovský, Prof. RNDr. Sergey Čelikovský CSc. The grant SGS12/177/OHK2/3T/12 "Non-conventional and cognitive methods of dynamic system signal processing". The Technology Agency of Czech Republic Project No: TE01020038 "Competence Centre of Railway Vehicles" and the EU Operational Programme from the Center of Advanced Aerospace Technology CZ.02.1.01/0.0/0.0/16_019/0000826. In addition, to my workplace Siemens s.r.o.