

Review report on Doctoral Thesis  
“Advanced Moment Invariants for Pattern Matching”  
by Jitka Kostková

This report is based on a manuscript received for review on June 16, 2020. It is an article thesis that consists of a separate introduction part of 35 pages followed by four research papers that have been published in international peer-reviewed journals.

### **1. Research problem**

Ms. Kostková’s thesis deals with moment invariants, which is a topic that has been widely investigated already for many decades since the seminal work by Ming-Kuei Hu published in 1962. Despite of the long history it is still an active field of research. As pointed out in the thesis, more than 12,000 new articles on moment invariants were published during 2014-19, and the trend has been growing, which is an interesting observation, because the deep learning boom, that has taken place at the same time, has not made the researchers abandon this classical topic. Moment invariants have been mainly used for finding image features that are insensitive to geometric deformations such as affine transformation, and more recently also to certain image degradations. Conventionally, invariants are computed for grayscale or color images, whereas this thesis explores the problem of extending moment invariants to vector fields. Images of vector fields are essential representations in many areas of application including computer vision, mechanical engineering, fluid dynamics, and meteorology. Another problem investigated is invariance to convolution with anisotropic Gaussian function that may appear, for example, as image blur under the influence of atmospheric turbulence, but also as distortion of probability multivariate distributions when estimated based on observations corrupted by additive Gaussian noise. Both research problems addressed in the thesis are somewhat new, and because of their application potential, they are also highly relevant to the research community.

### **2. General organization and comments**

The thesis consists of two parts. The first part has been divided into four chapters so that Chapter 1 introduces several fundamental concepts of image analysis and describes the basics of image moments and moment invariants. Chapter 2 puts the work into perspective by presenting a taxonomy and a summary of different research topics on moment invariants. Chapter 3 defines the goals and contributions of the thesis. They are divided into two topics: vector fields and convolution with the Gaussian function. The contributions include moment invariants to both these areas. Chapter 4 gives a short overview of the main results without going to details. Apparently, the purpose of the first part is to give a general idea of the work to readers who are not familiar with the topic or the whole area of image analysis, which is, in

my opinion, well achieved thanks to many illustrations and pictures that are obviously worth a thousand words.

The second part consists of four research papers that provide a detailed description of the invariants and the methods developed in the thesis work. These papers have several authors, and it is not explicitly mentioned anywhere, what role different authors had, but Ms. Kostková is the first author in three papers and the second author in one paper, which gives an impression that her role has been substantial. Besides these main papers, the thesis lists six other related papers authored or co-authored by Ms. Kostková. All these 10 papers have appeared in 2017-2020, which is a very short interval, when taking into account that they are full peer-reviewed papers in high-quality journals and conferences. This report covers only the four main papers that are included in the thesis.

Paper 1 has been published in Pattern Recognition that is among the leading journal in image analysis (IF 7.196). It proposes an invariant-based method for analysis of vector field patterns under an unknown rotation. Unlike scalar images, vector fields require invariance to total rotation that includes both inner and outer components. The invariants presented are constructed from Gaussian-Hermite or Zernike moments. Both are orthogonal moments which makes them perform favorably compared to geometric moments. While the vector field extension of Zernike moments seems to be relatively straightforward, the paper mainly focuses on Gaussian-Hermite moments that require more extensive treatment to make them invariant to total rotation. The performance of the method has been evaluated only on synthetic data that is a small weakness of the otherwise nice theoretical paper.

Paper 2 has been published in IEEE Transactions on Pattern Analysis and Machine Intelligence (IEEE TPAMI) which is the flagship journal in image analysis and computer vision (IF 17.861). This paper proposes moment invariants for vector fields with respect to total affine transformation that is a more general problem than rotation invariance addressed in Paper 1, which makes it also more challenging. The theoretical treatment of the problem is very comprehensive. Besides the equations for constructing the moment invariants the paper also presents an approach for selecting an irreducible and independent set of the invariants. Unlike in Paper 1, the experimental evaluation has been carried out by using both synthetic and real data while the emphasis is still on synthetically generated data.

Paper 3 is another paper published in Pattern Recognition. It deals with analysis of blurry images where the degradation is caused by convolution with an anisotropic Gaussian point spread function. The paper proposes moment invariants that do not require any prior knowledge of the Gaussian parameters. This is achieved by using a non-linear projection operator, which extracts a primordial image that is insensitive to Gaussian blur. The paper also shows that the corresponding moment invariants can be computed in the image domain without explicit construction of the projections. Furthermore, the paper proposes combined invariants to blur and affine transformation. The method is again theoretically sound and elegant, but it may suffer from the problem that exact Gaussian blur rarely exists in real images, and if the blur is non-Gaussian the performance is likely to degrade.

Paper 4 has been published in Multidimensional Systems and Signal Processing (IF 1.810). It proposes a method to quantify multivariate probability density functions when they have been estimated from observations subject to additive Gaussian noise. A representation invariant to noise with arbitrary covariance matrix is achieved by using moment invariants that are essentially the same as described in Paper 3. The method is an extension of convolution invariants originally proposed by Höschl and Flusser

to deal with multivariate distributions. The main use case in image analysis is characterization of color histograms of noisy images. The paper also envisages an application of noise-robust content-based image retrieval using color histograms but does not provide any concrete results to support this idea.

### 3. Conclusions

The thesis presents new theory that goes beyond the state-of-the-art in the area of moment invariant image analysis. Experimental evaluation of the methods is mainly based on synthetic data that is not as convincing as benchmarking with real image data. Due to the strong theoretical contributions this is, however, still a minor issue, and the work in general is likely to have a substantial impact on the future research carried out in the field. To conclude, it was my great pleasure to read and review Jitka Kostková's doctoral thesis, and now I am more than happy to recommend its acceptance to the Faculty of Nuclear Sciences and Physical Engineering.

Sincerely,

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