



## ZADÁNÍ BAKALÁŘSKÉ PRÁCE

<b>Název:</b>	Lineární diskriminační analýza na proudu příznaků
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<b>Vedoucí:</b>	Ing. Jan Motl
<b>Studijní program:</b>	Informatika
<b>Studijní obor:</b>	Znalostní inženýrství
<b>Katedra:</b>	Katedra aplikované matematiky
<b>Platnost zadání:</b>	Do konce letního semestru 2020/21

### Pokyny pro vypracování

Implementujte lineární diskriminační analýzu (LDA) pracující na proudu příznaků.

Implementace musí:

- 1) být "inkrementální", tzn. schopna se přiučovat na nových příznacích bez nutnosti učit se celý model od začátku,
- 2) být schopna se vypořádat s kolineárními příznaky,
- 3) podporovat diskrétní (multi-class) klasifikaci,
- 4) vracet výsledky porovnatelné s "dávkovou" implementací LDA, kde všechny příznaky jsou od začátku k dispozici.

Proveďte:

- 1) rešerši,
- 2) popište implementaci,
- 3) porovnejte inkrementální a dávkovou implementaci LDA na syntetických datech a alespoň 15 reálných datových sadách,
- 4) vyhodnoťte výsledky.

### Seznam odborné literatury

Dodá vedoucí práce.

Ing. Karel Klouda, Ph.D.  
vedoucí katedry

doc. RNDr. Ing. Marcel Jiřina, Ph.D.  
děkan

V Praze dne 11. února 2020





**FAKULTA  
INFORMAČNÍCH  
TECHNOLÓGIÍ  
ČVUT V PRAZE**

Bakalářská práce

**XS<sup>^</sup>C-<sub>q</sub> @SWS S<sup>^</sup>-<sub>q</sub><sup>^</sup> -<sup>^</sup>-Y<-<sup>^</sup>- e<sub>q</sub>~@  
e <<sup>^</sup>-W**

*Ruslana Severa*

V-zC@q -eYwf-<sup>^</sup> \-zC\ -zW%o  
, C@b~< e<sub>q</sub><C=R'Li T-<sup>^</sup> | bzY

|\_i <sub>q</sub>CqfC<sup>^</sup><C | CFC



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**db@ Wbf-Δ**

p-@ 4%P eb@ Wbf-Y e- ^~ R'Li T- ^bfS[ bzYfS<- zqf Yf fC@C^ 4-WY-sW  
eq<C- <- f'<^ q @%Wq \ Seb\ bP%z-zb eq<S@W^çSi



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# dqPYC

dqPY ~S C U\ e C@B C^b~ eq<S f%eq <bf Y-g s \ bsz-z^ - C U\ ~fC@f-g fC Wq eb~ Sz S^Hq -ç^ <@pU f sb~Y@ s | Cz@S\W ebW%U b C&SWe eq f f%abW WWSW<P <-f çç^ <P eq <i

3Cq ^- f @\ > CsC^ - \ bUeq<S f<z P~Ueq f - ebfS^bszSf%Y f-U< <C<-W^ - ç i c|cw|CEEr 4i>-~zbçWPb<-W^> fC<^ ^ eb@U <P e C@eS i , sb~Y@s~zi Jvb@zi vzbPzb<-W^ - z \ zb~@ YIS^Cf Pq @^ beqf^ ^ fYS<SgW- Sz z zb \ bUeq<G- zb fçCz^ f C-P ebç z çbf <P eqLq \ >U U sb~U sb~ç-sz çSe YPb~>- fC Wq UUSP @bW\ C^z <Cf@-Ysb-Pq^ U^ „? Y“g- zb f C \ bsb4->Wq sSe CU? Y~ zi y%ab bsb4%U sb~ beqf^ ^%o ? Y~ z UW\ WWS<e sb4C >Wq ^Gs^S~UC Pb@^bz~ ? Y>- <- UW\ WWS çCC fçCz^ ~ Sz Wf @ Yç^ \ çCY \ g yzbz beqf^ ^ U ç-sbf >zCçQ zbç-Y S \ ^b szCf^ ^Cb\ G C^ i V- @-bsb4>Wq-f%o SC f C~fC@^b- YC<SsCf - W-f<~UC~@ YZ VCW @ \ ~ @Y>Wq f^ SWCf4%U< ç-szç ^- <-W@ ? Y> eq fb~ ? Y>sebUC \ ? Y s U \ @Y \ <- -<C^ \ ? Y @b@Y sb~4bq^ Pb çS<eq <bf^ \ ? Y fçCz^ e CW@g YC<S- Yseb fC f C~fC@^ \ çsP~ - <-çfC <e sz-e^ Sz<@pUf W@z-Wf Pb@Y - YQ seb scpf^ - zCY \ <e sb4C - fC scpf^ - zCY \ çsP~>UW UC<e sz-e^ ^ <@pUf W@? Yi

æCsW f%sbW ~çC^ zC^P^S^W f dq <C  
G WY: S^Hq\ -ç^ <P zC^P^b^bLS  
ö |ÇÇEp~sY^~ rCfCqi, C^P^~ eqf f%bq<C^~i  
y-zb eq<C f^SW UW WY @Y ^- æCsW f%sbW ~çC^ zC^P^S^W  
f dq <C G WY S^Hq\ -ç^ <P zC^P^b^bLSi dq <C Ç <Pq^ ^- eqf^ \ Se ÇÇQ  
eS%o \ G^S^~ç@ \ S \ Yf \ S b eqf ~ ~zbçW - eqfC^P sb~fSÇ <P  
s eqfÇ \ ~zbçW i V ÇÇ \ ~ ~ S > s f Ç W ~ 4G eYz^ <P <W^ ^ <P YQ  
<C^ < - ^- @q^ Ç beqf^ ^ ~fÇÇ^ <P f dq PY~C^ ^- e ÇÇPbx szq^ ^ > Ç  
^G^4%ç^ sb~PYS ~zbqi

**a @W< ^- z-zb eq<S**

rCfCq>p~sY^~i XS^G-q^ @SW S^~ç^ - ^- Y<- ^- eq~@ e <^~ Wi 3-WY-Q  
sW~eq<G dq P~æCsW f%sbW ~çC^ zC^P^S^W f dq <C G WY: S^Hq\ -ç^ <P  
zC^P^b^bLS>|ÇÇ



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## , 4szq W

y-zb eq<CsCf ^~LC'G-e^ @SWP S'-c^ -^-Y<Cpx C^ b fCzSWY S'Q  
WQ\C'z-<Ss W^sz ^z^ qL~Yqf-<i db@ebU C\ fCzSWY S'WQ\C'z-<CsC  
qx~\ e S@^ fCWbq e <^-W b4UW \ szb zq @S^ Pb @beY ^ \ ^b S%  
f<bwipL~Yqf-<CsY~ WCC^ eq4Y\ ~ \ ~YSW'G qf%fk-fSY<P e Q  
<^-Wg TCeb@ebpf^- @SWq z^ WsS W<G>WQq-ep4P-^- <-W@ 3-%Q  
sbf- qxPb@bf-<Pbeq fSYi dqb<q%PY^ f ebcz~ f^S^ ^ szqW-d%à b@CY  
LCf% S; PbYsWPb @CW\ ebx S<C>@be C@- <e z^-s-4szs-<G R eY\C'Q  
z<CUC^ -es^- fU<%Cd%Pb^ - UCzszbf^- ^- qC-Y <P @zbf <P s @<P  
- s%zCzS<W<P @ zC-Pi, sY@W%zCszbf^- ~W<~U> CWsS Wc^ \ b@CYs fCqQ  
zSWY S'WQ\C'z< \ C^4 @^b-z 10x q%PYU ~cC^ \ b@CY beqzSUPb  
<fWf \ ~ - ^- YL~ e SszU WsS Wc^ e G^bszS G^Y^ ç-sbf <q%PY^  
fCzSWY S'WQ\C'z-<C<-fS ^- \ ^b szf e <^-W - f<bwipL~Yqf-<CsY~

VYcbf-sYbf- Y'G-e^ @SWP S'-c^ -^-Y<->S'WQ\C'z-Y ~cC^ >b^Y'C  
~cC^ >qL~Yqf-<C>; PbYsWPb @CW\ ebx S<C>X?, >p?,

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## , 4szq <Z

yPCzPGSS @G Y ..SP Y'G q @S<f\ S'- ^z - ^- Y%SS C†zC^@C@ 4%fGqS- YS' <qQ  
 \ C^z...SP <b^sz ^z qL~Yqf - zb^i yPCzCq\ fGqS- YS' <q\ C^z \ G ^s - @S'L  
 @G<fzqbqf z~q fCzqbq bHb4U<zS S'szG @bHzq @SS^ - Y<b\ eY\ C^z bHsCz  
 bHs \ eYsi pL~Yqf - zb^ sCqfCs zbsbYfCzPCeq4Y\ bH ~YSbY'G qS%f@CQ  
 eC^@C^z @G<fzqbqg [ ~YSYss <YssS <- zb^ Ss s-eebqC@- ^@z- VCs eY<Cb^  
 zPC 4 sS bHzPC 3-%G @GSSb^ q-Yi yPC; PbYsW%@C<b\ ebsSSb^> Hq. : q@  
 - ^@ 4 <W. : q@s~4szsz-zb^ - qC ~sC@ zb - <<Cq zCzPC <b\ e~z- zb^ bHzPC S'Q  
 ^Cqszq<z~qCbHzPC\ b@CY R eY\ C^z zb^ Ss ..qszC^ S' d%Pb^ - ^@Ss zCsz@  
 b^ qG Y@z z sCz - ^@s%zPCz @z-i yPCqS- Ys bHzCsz'L sPb...zP-z zPC<YsQ  
 sS <- zb^ \ b@CY..SP fGqS- YS' <q\ C^z <b~Y@b' Cq10× HszCq\ b@CYzq S'SL  
 <b\ e- q@ zb Ss 4-zP - ^- YL ..PSCP- fS'L zPC s \ C<YssS <- zb^ eq<SSb^i  
 yPC " ^- YzS C- <<Cq zb^ bHfGqS- YS' <q\ C^z @GeC^@s b^ zPC - \ b~z bH  
 Hf z-qS - ^@s- \ eYsi

VC%obq@s Y'G q @S<f\ S'- ^z - ^- Y%SS>S' <q\ C^z YG q'SL>b^Y'CYG q'Q  
 S'L>qL~Yqf - zb^>; PbYsW%@C<b\ ebsSSb^>X?, >p?,

# a 4s P

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fb@ c
c ; Yeq<C {
| , ^-Y<- - ^-fQP I
|ic ?SWS S-ç^ -^-Y<- i i i i i i i i i i i i i i i i i i I
|i| 3-%Gbfsw çxPb@bf< WZ ç\ i i i i i i i i i i i i i i v
|i{ d @ebW@%o eb -@fW%oi i i i i i i i i i i i i i i u
|i{ic ?SWS S-ç^ -^-Y<- i i i i i i i i i i i i i i u
|i|i| Vf @q zSW@SWS S-ç^ -^-Y<- i i i i i i i i i i u
|i|i{i XS^C-q^ @SWS S-ç^ -^-Y<- i i i i i i i i i i D
|iJ a@P @e-q \ Czq i i i i i i i i i i i i i i i i i i _
|iI r<SVC q^ i i i i i i i i i i i i i i i i i i i i cCE
|iV pL~Yç<Ci i i i i i i i i i i i i i i i i i i i i c|
|iU RW\C C^z-Y e sz-e i i i i i i i i i i i i i i i i i i cJ
|iUic Vb^sz^z^ çL~Yç<C i i i i i i i i i i i i i i i i cv
|iU|i| a4^bfC^ <W@^ f^S^ szq-W~ç%oi i i i i i i i i i i cv
|iU|i{ ; PbYsWPb @W\ eb SCi i i i i i i i i i i i i i i cv

{ R eY\C C^z<C c_
{ic y @ X? , ; YssCq i i i i i i i i i i i i i i i i i i c_
|i| db~ S H^WC i i i i i i i i i i i i i i i i i i i i |c
|i|i| s<S%S'-Yi4Ysi@%qV i i i i i i i i i i i i i i i i |c
|i|i|i| s<S%S'-Yi4Ysi@zçf i i i i i i i i i i i i i i i i |c
|i|i|i{i| s<S%S'-Yi<PbYsW%o i i i i i i i i i i i i i i i i ||
|i|i|J s<S%S'-Yi4Ysi@LC f i i i i i i i i i i i i i i i i ||
|i|i|iI s<S%S'-Yi4Ysi@LC \ i i i i i i i i i i i i i i i i ||

J yGszbf^ ^ |{
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# rG ^- \ z 4~YW

|ic a4^bfC f^Sz ^ szq-W-q%S'Wq\ C^z-Y Pb\ b@CY i i i i i i i i cD

Jic dbeS f%4q ^ <P @zbf <P s-@ i i i i i i i i i i i i i i i i i i |J

Ji| [ SWpeq \ q^-W sS Wc^ e G^bsz ^- zq ^bf-<P @ zCP i i i i {|

Ji{ [ SWpeq \ q^-W sS Wc^ e G^bsz ^- zCzbf-<P @ zCP i i i i i i {{

JiJ [ SWpeq \ q^-f z ^bsz ^- zCzbf-<P @ zCP i i i i i i i i i i {J

JiI F<sub>1</sub> sWqC i {I

Jiv ; bPC^ f Wc^ SC^z Wee- ^- zCzbf-<P @ zCP i i i i i i i i i i {v

Jiu 3qCpfb sWqC ^- zCzbf-<P @ zCP i i i i i i i i i i i i i i i i {u

JiD , } ; Qa; sWqC ^- zCzbf-<P @ zCP i i i i i i i i i i i i i i i i {D

Ji\_ ? b4- zqf-^ ~cC^ \ b@CY ^- zq ^bf-<P @ zCP i i i i i i i i i i {\_

JicE? b4- zqf-^ f%zfb C^ eq@SWC\ b@CY ^- zq ^bf-<P @ zCP i i JCE





fb@

XŠC-ē @SWS S'-ç^ -^-Y<-^c UC WsS-W\ Czb@ sz-zSzSW%o szpUbf Pb  
~çC^ >Wq-sebç f-f ^-Y: C^ ^CUCe <P ŠC-ē <P W\ 4S'-< e <^-W eqp  
<- <C^ b4UW @b @fb- ^C4b f<C@SU^W^ <Pz @ | -Y: C^—W\ 4S'-<CsC  
eb- f-ēp q@-WS@S C^<Cedpszbq e C@^sY@-U< WsS' W< ^C4beqps-Q  
\bz^b- WsS' W<S, @^ eq<SsC<-4 f-\ <Wb-\ ^\ WsS' Wç^ PbebzC^Q  
<S-Y çx C^ \ Czb@%X?, i Š- ^ WbSW S~Y<P @GzSçZ f @C-WPb <Wb-Q  
\ ^\ Czb@ X?, < sWY Pb@ q<^ <P HqC\ Q:>9:iy%eSW\ e W@C\  
UC qL~Yçbf ^—@SWS S'-ç^ -^-Y<- |i | Czb@ p?, çx S~UC WsS' Wb-  
\ Czb@ X?, b qL~Yçf <S-Wq- C eq4Y\ ^-f<-UC <-fSY<P e <^-W  
- eb\ -P-e C@Uze Š^ \ ~e Se sb4C^ WsS' Wç^ Pb\ b@C\ @z \ fe CQ  
~çC^ g?-Y Hq\ b- \ Czb@%X?, UC S'Wç C^z-Y wb^ŠC@SWS S'-ç^ -^-Q  
Y<->Wq-sC@bW-Ce S-çZ -@W\ ^bf <P f<bqWi, ^ Wq <P e e-@C-P UC  
f-Wk sW^ ^bf <P f<bqW e Š^ -çç^ >- eqzb \ -s\ %Yf%fb Š\ b@C\  
Wq 4%çC@bW- Ye S-çZ sYb-e< \ ^bf <P fYsz^bsz US < sW^ <P b4UW i  
Š zbPzb @ fb@ ŠC\ sC çx Pb@Y S eY C^zbf-z \ Czb@ X?, >çx C^b-  
bf C^ \ ^ ^b- qL~Yçf <S- S'Wç C^z<S^bf <P e <^-Wi

Re Gzb CUC@G\ Czb@ X?, f^ \ ^- UWqçYzSf^ sz-q-sz-zSzSW\ CQ  
zb@>\ -e CWF-eSf @b4q f sY@W%o Cf <W\ ~< çW |CEEC~W- Y UW  
z \ ^CUCe <PY@SW<eq \ çf-^ WsS' Wç^ <P%Q:i, ~zb Sf <W\ ~  
@bse f-UW-f q> C\ Czb@ X?, ebsW%UCeq WsW%e Pb@ \ zWb eqp  
ebçf^ ^ s 4~@b<\ S-Ybçf\ %ç<PY@C\ Wzb\ ~> CUC ç%PY>s^ -@b S Q  
eY C^zbf-zY— @bsz-e^-fCsz-zSzSW<P WSPbf^<Pi, Cf <W\ ~< çW  
|CEEC\ Czb@ X?, ~f-@^- UW -zq Wsf^ -Yçf-zsf- WŠC-ē \ Czb@  
eb@e q^ <P fCWBq fr, | g WY ŠYe f ebçC^ sY ŠbszŠ UC@b@<P \ ~  
W^<Cez- - ç%PYU \ ~ ~çC^ 9:i

Š f C^ \ ^ ^ S' Hq -<C f%Yf> C \ Czb@ X?, \ -sz-Y ebzC^ <S-Y  
Wf Šsf UC@b@<PbsZ^ - e eq f~ - eb- f^ i ŠYe C^ - çx C^ zq @Š^ Pb

^Lyi XŠCq? ŠçŠ S'^z, ^-Yšš  
l-^Lyi pL~Yçf @? ŠçŠ S'^z, ^-Yšš

fb@

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e sz~e~ W\ Cz@ X?, ^-e W@eb\ b< qL~YqS~<C- S\ eY\ C^z~ç^ Pb  
ebsz~e~ \ C~@ Yz< @^ Pb \ b@CY fCY SsSY ^~szpUszpbUbf Pb ~çC^ i  
? b@G\ C\ Cz@ X?, Sp<C eb~ f^~ f q<^ <P b4YszC~P Sfbz~i [ GS  
e W@%e-z CW^b\ SW f <W\ %q:>qx ebx ^~^ Y@SWPbPYs~ qr:>ebP%4~  
qr - \ b< q>\ C@S^sW - 4SbYLSW f <W\ %q:>qr:>qCf sz~@C b ŠC\ S  
qc:>q|i

? - ^~eq<C UC szq~W~qf^~ @b WeSbY dqf^ ebeS~UC< Yeq<G @q~P~  
WeSbY sC f ^~UC~^~Y<C~ ^~fP~ C C^ Pb eqb4Y\ ~>ebsWz~UC zCbqZSW  
<~<C eqp eb<PbeC^ W^<Gez~ fCqSWY S Wq\ C^z~<G qL~YqS~<C- b4Q  
^bfC^ f^S^ szq~W~q%W sS Wç^ Pb \ b@CY i y Cz WeSbY ~f~@ S\ eYQ  
\ C^z~ç^ S Hq\ -<G dbsY@^ WeSbY sC~4 f~zCzbf^~ \ S\ eY\ C^z~<C^~  
qC~Y <P - s%zCZSW<P @ zC~Pi

V- e&by- **C**

; **Yeq**←C

dq\ -e' \ <Y\ @ ^ eq←CIC - ^-Y<- - S\ eY\ C^z <C WsS Wzbq ^- <-Q  
W@ \ Czb@%X?, >WcQ 4%@bW← Yeq <bf-z s f <C ^C @f \ - z @ \ S-zil  
eqf-@ z @SWjz ^ { WsS W-S ^- @ fsz-e ^ \ S@ z%oVq\ zbPb sC \ ~s WQ  
sS Wzbq~\ z f%ob -@ z s WbS^C-e' \ SfS^C-e' <-fSY \ Sge <^- Wøb\ b<  
qL-YqS - <G R eY\ C^z <C4%ã- W \ Y H^Lbf-z fC@fb~ \ @C-P=sz- ^@ q@Q  
^ \ >z @-fWj \ >W@%b@ <-ç-zW lsb~ <^→ %of C-P^%e <^- W%of<bq< >  
- S'Wq\ C^z-Y \ i R'Wq\ C^z-Y \ @ WsS Wzbq sC f%o^ -ç-IC s<Pbe^bsz  
\ b@CY S'Wq\ C^z-Y sCe S-çS ^- ^bf <P e <^- <<P>W@%ð b@CYe S\ C ^-  
fsz-e~ fCWbq ^bf Pb e <^- W - b4^bf f^S ^ szq-W-q- 4G ^-z^bszS~çS  
<CY \ b@CYb@<-ç-zWi

?-Y \ <→ q\ z zb eq←CIC zCzbf-Δ H^W^bszS f%afb C^ S\ eY\ C^Q  
z <C ^- <-W@ e Ceq <bf- ^ <P - e Seq fC^ <P @ z=- Yseb ^- cl qC-Y <P  
@ zbf <P s- @<P - s%zCzS<W<P @ zC-Pi yCzbf-Δ 4%ð Yb <-Pq^bf-z ebqfQ  
^→ zq @ç^ Pb @-fWj Pb \ b@CY s S'Wq\ C^z-Y S\ eY\ C^z < < PY@SW  
W sS Wç^ e G^bszS- ç-sbf <P ~W<- zCYi

{ \ ~YSYss

{



# , ^- Y<- - ^- fQ

|ic ? SWS S'-c^ - ^- Y<-

T-Wf%Yf-> ^-<f~>S^C-q^ @SWS S'-c^ - ^- Y<- UCUC@ \ < @q-P @SWSQ  
 \ S'-c^ - ^- Y<%o \ Cz@%d ^bPbqx \ q^ sz-zSzS-W-^-Y<%oWCq-> \ b ~UC  
 sz-@bf-z qx@Y%o G S@f \ - - f<C sWeS-\ S b4UCV eqp ^ WWSWeq\ ^Q  
 ^ <P ^- UC@b~ @{:i ? SWS S'-c^ - ^- Y<- UC b4C^ zCq\ ^> WCq sCz W-  
 ^ WWSW <Csb-fSCU<P sz-zSzS-W<P ebsz-e > WCq \ bPb- 4 z qx@Y^%o  
 eb@C \ Cz@S'zCpCz <C \ G SWeS'bf <P qx@Y ^- @SWS S'-S- WsS' Q  
 W<S ebx bpf^ i , z zb eq<S sC <- 4 f-> UC^b \ WsS' Wc^ \ S s<Pbe^bsz \ S  
 @SWS S'-c^ - ^- Y<%o

? SWS S'-c^ - ^- Y<- e-z @b sWeS%o Cz@~c^ s~c^SCC > zU UC~ SQ  
 zC^>V@%oe sY ^bsz b4UCV W sWeS' UCe CC\ <^>-i ] - z <Pz e CQ  
 @C \ <^> <P @zC-P eq4 P->c^ \ b@CYi 3 PC ~c^ ^- <W@ fsz-eQ  
 ^ <P Pb@bz @bP-< VCW^" L-q<S f^S ^ szq-Vz-q%o b@CY>WCq-e- WsY-  
 WCW sS' W<S , q<^b- fYsz^bsz @SWS S'-c^ - ^- Y<%oU W WsS' Wzbq UC  
 e CC\ <^> W^sz-^z^ W^C^ ebqCz @SU^W^ <P sWeS' ^C4b z @ VYQ  
 sS' W<Ce- Wwebq f-f~qC^ z @%o@ WCq e-z ebx bpf^ b4UCV i a 4UCV  
 ^C4b f: bqWUC e Csz fC^ qC-Y \ fC^bq \ x = (x\_1, ..., x\_d), x \in \mathbb{R}^d > UCQ  
 Pb W @-sY W bx^-c-UC Pb@bz-^- \ C^ fYsz^bszS>z e <^>-W zbPzb  
 b4UCV-i y \ zb Pb@bz-> sCz-W W-@SWS S'-c^ eqp \ ^^ i , Ybqz \ ~s  
 e S-<C^ b4UCV~ WUC@ < @C^bf-^ <P z @sC^< f-qx Pb@bf< eq fS@bi  
 TCzb<b4q<C^ < \ ^b S%e <^>-W X \subset \mathbb{R}^d @b \ ^b S%z @K=a(x) : X \Rightarrow Ki  
 ? SWS S'-^z%o C4b @SWS S'-c^ H^WC lsb~ H^WC>WCq sY~ Wqy SC^  
 \ G Sz @ \ S ? SWS S'-c^ H^WC @C^ ~U ebYp~ @SWS S'-c^ <P bs> WCq  
 qx@YU @qx \ q^ eqszbqe <^>-W b4UCV z-W-4%o@bP-<C^ W S'S -Q  
 ^ \ ~ e Cqj f^ z @ ? SWS S'-c^ H^WC sebY s @SWS S'-c^ \ Sbs \ S<-Q  
 IS ~U \ b ^bsz UC@^b^-c^ Pb e S-<C^ f: bqW 4 PC \ WsS' W<C f<-fS^bsz  
 ^- Pb@bz-<P e <^>-W zbPzb f: bqWi

Tsb- <^> %z S<-W@ WsS' Wc^ ebsz-e%o- Y C^ ^- @SWS S'-c^ <P

|i , ^- Y <- - ^' f q P

H^W <P QJ:i

á XS^C-é^ Ws^S Wzbq%Vc^sCWw @ z @ WW^szq~LC^S^C-é^ H^WG  
 a 4UW^Ue- Wé S-<C^ VCWQ z @ >ebW@Pb^bz - eSW<CWQ S^C-é^  
 H^WC^- fCWbqe <^- W zbPzbz b4UW^ LC \ - †S -Y i

á , <@-Y^bsz^ \ Cz@%Vc^b4UW^ Ue S-<C^ Wz @ >WQq-\ —^U C^  
 f:@-Y^bsz \ G Ssz C@C\ z zb z @% b4UW^C i ] Cq- sz U eb~ f^b~  
 f:@-Y^bsz^ \ CzPb~ ~ Ws^S Wzbq @ ^ Pb @q-P~ LC [ -P- Y^b4Sbf-  
 f:@-Y^bszi

á [ Cz@%o\ - †S -Y f qpPb^bsz^ ^C4b eq f@ eb@b4^bsz^ \ Cz@%Vc^  
 b4UW^ Ue S-<C^ VCz @ s ^Cj z ebszCfbq^ eq f@ eb@b4^bsz i

, q- <S @ ^ 4 WY-sW eq<C sC sb-sz C@ \ ^- z Cz ~fC@C^ Ws^S Wc^  
 ebsz-ei

**|i| 3- %GsbfsW qP Pb@bf- < Wfz qS \**

T-WUc\ U\$ <\ ^SY> eq f@ eb@b4^bsz^ Ws^S Wc^ \ Cz@%Jsb- <- Y C^%o  
 ^- ebszCfbq^ eq f@ eb@b4^bsz^ dbzCfbq^ eq f@ eb@b4^bsz Ueb@ \ ^ ^—  
 eq f@ eb@b4^bsz ^-Pb@^ Pb Ucf~>U f sY@CW-fS ^- ^ UW<P-ebszCfbqQ  
 ^ <P @ zC-P fCteCqS C^z-Y < sW^ S^Hq\ -<Gj TS^ \ SsYf%dc zb eq f@ ebQ  
 @b4^bsz Ucf~, <- e C@ebW@ eYz^bszSUCf~ 3i dbzCfbq^ eq f@ eb@b4^bsz  
 UCz s^ sebU^- s 3-%Gsbfb- f zb- beb@ \ ^ ^ eq f@ eb@b4^bszS@I:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}, \text{ ebW@ } P(B) \neq 0 \quad \text{f|i}cg$$

, W^zCz~ Ws^S W<Cebç z~ CebzCfbq^ eq f@ eb@b4^bsz e sY ^bsz^ Q  
 UWPb b4UW^ W-qS z @ P(k|x), x ∈ X, x = (x1, ..., xd), k ∈ Ki

$$P(k|x) = \frac{d}{i=1} \frac{P(x_i|k)P(k)}{P(x_i)} \quad \text{f|i}g$$

Vc^P(k) = πk UC-eqfbq^ eq f@ eb@b4^bsz e sY ^bsz^ ^-Pb@^ Pb f^bqW  
 Wz @ k>P(xi) UC-eqfbq^ eq f@ eb@b4^bsz> CSQ e <^- W^Pb@^ Pb b4Q  
 UW^ ^- 4 f~Pb@bz%xi> P(xi|k) UC eq f@ eb@b4^bsz> C ^-Pb@^ f^bqW  
 e sY ^ Wz @ k \ ^-Pb@bz~ SQbPbe <^- W qf^b~ xi Ob@bz%e <^- W  
 \ bPb~ ^- 4 f-z UW<PWYf Pb@bz> zb <^- \ C^> CUCb sebUz fCqS%dbQ  
 W@e <^- W%f q- <SU@ z @%@b@q ~UszU^ eq f@ eb@b4^bsz^ qP @ Y^ >  
 ebszCfbq^ eq f@ eb@b4^bsz YC f^J@ S fCzf-q

$$P(k|x) = \frac{\rho(x|k)\pi_k}{\rho(x)} \quad \text{f|i}g$$

$P(k|x) = \rho_k(x)$ 
 $P(x|k) = \rho_k(x)$ 
 $P(x) = \sum_{k \in K} P(k|x) \rho_k(x)$ 
 $P(k) = \sum_{x \in \mathcal{X}} P(x|k) \rho_k(x)$ 
 $P(x) = \sum_{k \in K} P(k) \rho_k(x)$ 
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 $P(x) = \sum_{k \in K} P(k) \rho_k(x)$ 
 $P(k) = \sum_{x \in \mathcal{X}} P(x) \rho_k(x)$

$$a(x) = \arg \max_k P(k|x) = \arg \max_k (\pi_k \rho_k(x)) \quad f|i jg$$

$P(x) = \sum_{k \in K} P(k) \rho_k(x)$ 
 $P(k) = \sum_{x \in \mathcal{X}} P(x) \rho_k(x)$ 
 $P(x) = \sum_{k \in K} P(k) \rho_k(x)$ 
 $P(k) = \sum_{x \in \mathcal{X}} P(x) \rho_k(x)$

**|i{ d C@ebW@%o eb - @ fW%o**

**|i{ic ? SWP S-ç^ - ^- Y<-**

- ? SWP S-ç^ - ^- Y<- W@Ce s^ eb - @ fW%o- fsz-e^ @z Quri
- á ?-z \ -s 4 z çx @ Y^ - Yseb @b @fb~ z @f4S-e^ WsS W<Gj
- á V- @-z @ \ -s 4 ze C@sz fC^ - Yseb @f \ - f<bqW%o
- á ; CWf ebcZ e <^-W f@SWP S-ç^ <P eq\ ^^ <Pg ^Gs\ e CWpçZ <CWf ebcZ b4CW i
- á d <^-W%à ~s 4 z Wf ^sz zsf^ - \ GSe <^-W%à ~s 4 z sY4-WçY<G
- á Vbf-ç^ç^ \ -zçC f G<P z @4%à %4 z <b ^Cf <CPb\ bLC^ ^ i
- á d C@ebW@-sC C-^- Y%bf-^ @SWP S-ç^ eq\ ^^ e C@sz f-U f Q 4 çf sb-4bq< f<Cç\ ç^ Pb ^bq\ -Y Pb çx @ Y^ i

**|i{i| Vf @q zSW@SWP S-ç^ - ^- Y<-**

$Vf @q zSW@SWP S-ç^ - ^- Y<-$ 
 $\chi^2$ 
 $N(\mu, \sigma^2)$ 
 $P(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$$p(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad f|i l g$$

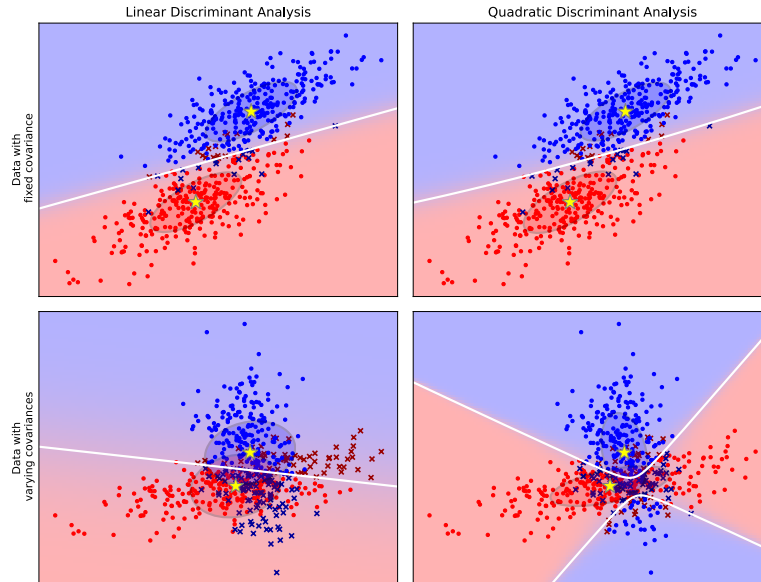
$N(\mu, \Sigma)$ 
 $P(x) = \frac{1}{(2\pi)^d \det \Sigma} e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)}$

$$p(x_1, \dots, x_p) = \frac{1}{(2\pi)^d \det \Sigma} e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)} \quad f|i v g$$





$$a 4q \leftarrow CW | ic = db \phi f^{\wedge} \Delta X?, - k?,$$



|iJ a @P- @e- q \ Czq

Š f' b q C @ S W S S - ç ^ P b s W q e p X?, f | i H g Y C f S @ z > C < - W @ ^ f ^ S z ^ sz q W - q W s S W ç ^ P b \ b @ C Y 4 % \ Y b 4 s P b f - z ^ - s Y @ - U k e q f W @ @

$$\acute{a} z @ ^ - e \phi b q ^ e q f @ e b @ b 4 ^ b s z S \pi_k, k \in K t$$

$$\acute{a} f C W b q % s z C @ ^ < P P b @ ^ b z - ^ C 4 b z @ ^ < P e q \setminus q \mu_k, k \in K t$$

$$\acute{a} s e b Y ç ^ - W b f - ç ^ ç ^ \setminus - z S C \Sigma i$$

, e q † S - Y c s W z C ç ^ P b @ ^ b z % z < P z b e q f W ^ C s b ~ < ^ \setminus > e q p z b s C U C s ^ - \setminus C b @ P - @ ^ b - z ^ - < - W @ @ b s z - e ^ < P z q ^ b f < < P @ z i | C P X \subset \mathbb{R}^d \cup W ^ C ç ^ - \setminus ^ b S - z q ^ b f < < P @ z > d \cup e b ç C z e < ^ - W > | X | \cup \setminus b P - z ^ b s z z q ^ b f < \setminus ^ b S % z i | < C W b f e b ç C z f' b q W > K \cup W ^ C ç ^ - \setminus ^ b S - z @ > | K | \cup < C W b f e b ç C z z @ > X\_k \subset X \cup e b @ ^ b S - e q f W e s Y ^ < P W z @ k \in K > | X\_k | \cup e b ç C z f' b q W e s Y ^ < P W z @ k > x\_{ki} \cup S Q f' b q C < f \setminus ^ b S X\_k > e - W f % P Y ^ J \setminus - † S - Y f q P b @ b @ P - @ % s Y C W f ^ S z ^ sz q W - q % Y C f % A d @ S z

J - ^ L Y i 4 S s C @

$\hat{\mu}_k = \frac{1}{|X_k|} \sum_{i=1}^{|X_k|} x_{ik}$

Proof

$$\hat{\mu}_k = \frac{1}{|X_k|} \sum_{i=1}^{|X_k|} x_{ik} \quad \text{f|icEg}$$

$$\hat{\mu}_k = \frac{1}{|X_k|} \sum_{i=1}^{|X_k|} x_{ik} \quad \text{f|icEg}$$

$$\hat{\Sigma} = \frac{1}{|X|} \sum_{i=1}^{|K|} \sum_{j=1}^{|X_i|} (x_{ij} - \hat{\mu}_i)(x_{ij} - \hat{\mu}_i)^T \quad \text{f|icEg}$$

$$= \frac{1}{|X|} \sum_{i=1}^{|K|} \sum_{j=1}^{|X_i|} (x_{ij} - \hat{\mu}_i)(x_{ij} - \hat{\mu}_i)^T \quad \text{f|icEg}$$

$$= \frac{1}{|X|} \sum_{i=1}^{|K|} |X_i| \frac{1}{|X_i|} \sum_{j=1}^{|X_i|} (x_{ij} - \hat{\mu}_i)(x_{ij} - \hat{\mu}_i)^T \quad \text{f|icEg}$$

$$= \frac{1}{|X|} \sum_{i=1}^{|K|} |X_i| \hat{\Sigma}_i \quad \text{f|icEg}$$

$$= \sum_{i=1}^{|K|} \hat{\pi}_i \hat{\Sigma}_i \quad \text{f|icEg}$$

Let  $\hat{\mu}_k$  be the sample mean of  $X_k$ . Then  $\hat{\mu}_k = \frac{1}{|X_k|} \sum_{i=1}^{|X_k|} x_{ik}$ . The sample covariance matrix  $\hat{\Sigma}$  is defined as  $\hat{\Sigma} = \frac{1}{|X|} \sum_{i=1}^{|K|} \sum_{j=1}^{|X_i|} (x_{ij} - \hat{\mu}_i)(x_{ij} - \hat{\mu}_i)^T$ . We can rewrite this as  $\hat{\Sigma} = \frac{1}{|X|} \sum_{i=1}^{|K|} |X_i| \frac{1}{|X_i|} \sum_{j=1}^{|X_i|} (x_{ij} - \hat{\mu}_i)(x_{ij} - \hat{\mu}_i)^T$ . Let  $\hat{\Sigma}_i = \frac{1}{|X_i|} \sum_{j=1}^{|X_i|} (x_{ij} - \hat{\mu}_i)(x_{ij} - \hat{\mu}_i)^T$  be the sample covariance matrix of  $X_i$ . Then  $\hat{\Sigma} = \sum_{i=1}^{|K|} \hat{\pi}_i \hat{\Sigma}_i$ , where  $\hat{\pi}_i = \frac{|X_i|}{|X|}$  is the relative frequency of  $X_i$ .

Proof

Let  $\hat{\mu}_k$  be the sample mean of  $X_k$ . Then  $\hat{\mu}_k = \frac{1}{|X_k|} \sum_{i=1}^{|X_k|} x_{ik}$ . The sample covariance matrix  $\hat{\Sigma}$  is defined as  $\hat{\Sigma} = \frac{1}{|X|} \sum_{i=1}^{|K|} \sum_{j=1}^{|X_i|} (x_{ij} - \hat{\mu}_i)(x_{ij} - \hat{\mu}_i)^T$ . We can rewrite this as  $\hat{\Sigma} = \frac{1}{|X|} \sum_{i=1}^{|K|} |X_i| \frac{1}{|X_i|} \sum_{j=1}^{|X_i|} (x_{ij} - \hat{\mu}_i)(x_{ij} - \hat{\mu}_i)^T$ . Let  $\hat{\Sigma}_i = \frac{1}{|X_i|} \sum_{j=1}^{|X_i|} (x_{ij} - \hat{\mu}_i)(x_{ij} - \hat{\mu}_i)^T$  be the sample covariance matrix of  $X_i$ . Then  $\hat{\Sigma} = \sum_{i=1}^{|K|} \hat{\pi}_i \hat{\Sigma}_i$ , where  $\hat{\pi}_i = \frac{|X_i|}{|X|}$  is the relative frequency of  $X_i$ .

<sup>1</sup> - ^LYi ebbY@

á s b y f C q = ^ - 4 < z S \ b ^ b s z S C C ^ W f s S W c ^ P b e q 4 Y \ ~ =

- „ s f @ ^ C 4 b s S L - Y e f ^ q x W f @
- „ X l q ^ ^ C 4 b \ C z b @ ^ U X C ^ < P c z f C q t
- „ C S L ^ ^ ^ C 4 b @ C W \ e b S c f Y s z ^ < P c s C Y

á s P q S W L C = e b @ e b q - U c q L - Y q S - < S W C q - U e b e s - ^ - f s C W S | i v > ^ - < - Q W @ P b @ ^ b z % g f s z - e ^ P b e - q \ C z q =

- „ / b ^ C ^ - \ C ^ - - @ b - q L - Y q S - < S
- „ - - z b ^ f C @ C W e b - S z f b q C X C @ b S z - „ b Y H | : e q b @ P - @ s P q S W L G
- ç s Y b z % • b - z f S z C q f - Y f C e g < - q - ç q L - Y q S - < S s b 4 @ q C ^ \ W C ^ < S ^ z C \ i

á e q b q S = e b f b Y - U C < - @ z z @ ^ - e q b q ^ e q f @ e b @ b 4 ^ b s z S W C q s C @ C Q H - Y ^ e b ç z U ^ - < - W @ z q ^ b f - < < P @ z t

á ^ € < b \ e b ^ C ^ z s = e b f b Y - U C < - @ z e b ç C z W \ e b ^ C ^ z e q q @ - W S @ S \ C ^ Q < S b ^ - Y z %

á s z b q C E < b f - q S ^ < C - ~ q - U C > @ U C e b z C 4 ~ Y S z W b f - q S ^ ç ^ \ - z S S

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[ G S @ f < - W @ ^ \ C z b @ % z @ % X S C q f S < q S ^ - ^ z ^ - Y % S ^ ~ z ^ W C W s S Q W < S e - z =

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- \ - z S C @ S W f S - ç ^ < P e q \ ^ ^ < P X > z i j \ - z S C P b @ ^ b z e < ^ - W b 4 U W t
- f C W b q f % s f z Y b f ^ e q \ ^ ^ y > z i j f C W b q e s Y ^ b s z S b 4 U W \ - Q z S C X W z @ - \ i

á e q C @ S z > W C q - ^ - < - W @ P b @ ^ b z e < ^ - W f s z - e ^ \ - z S C b 4 U W X f q - z e q C @ S W S ^ C 4 b W f s S W < S z < P z b 4 U W @ b \ ^ b S % z @ e C @ C @ C ^ b f - ^ 4 P C \ ~ ç C ^ i

C C ^ „ X l q ^ s C f % ^ - \ C ^ f - e b - S z \ \ C z b @ % ^ U X C ^ < P c z f C q 9 { : e q f e b ç C z z @ ^ < P W C ^ < S ^ z w\_k = \Sigma^{-1} \mu\_k > ç \ s C f % B 4 - C e Y S z ^ \ ~ s C Q s z p l C ^ S f C q ^ \ - z S C \Sigma^{-1} i y b z b < q % P Y C ^ U C f C X S @ Y C S z \ H W b q \ e q S ^ W C \ C ^ z - Y e s z - e > - e q z b U C \ e b - S Y e q f z b z b C C ^ < C z @ b s z - e Q ^ < P U W P Y f ^ e b @ W @ 4 P C \ S e Y \ C ^ z - C \ f Y s z ^ f C q C X ? , - z W e q ^ - s Y @ - U k z C z b f - < ç C % f W e S z b Y C J i C C ^ X ? , ^ - < - W @ @ C W \ e b Q < S C f Y s z ^ < P ç s C Y f , C S L ^ g s e b ç f - f b e z S \ - Y S - < S e b \ q - \ - z S C \ C S z @ f - q S S e % W \ - z S f ^ S q z @ f - q S S e % f G S P C q f b q x P b @ b f - < W f z q S \

|i , ^- Y <- - ^' fqP

9J:g rYb~ UWeq WsS W<S z- WS eq q@WS @S C^S^ Y%<b f - W  
^C^ z \ -zC z zb eq<G | - f < sCz zb C C^ b4q < Web \ - YU \ ~ CteYSSQ  
^ \ ~ ebsz-e~ f ebçz~ S'fCq CsebYc^ Wbf-ç ^ç^ \ -zSG C C^ <- Y C^  
^- sS'L~Yq' \ çx W@ f,,s/f@g^ çx@Yb@bsz-z^ <P @fb~ f 4C^ ^Ceb~ f~  
Wbf-ç ^ç^ \ -zSS- < zbPzb@ fb@ - ^S^Ceb@ebq-UCqL~Yçf- <S Wf YçC \ ~  
z-W ^Cf%Rbf~UC<Y \ @^ eq<G

### |iv pQL~Yçf- <C

TC@^ \ \ @ Y S <Peb - @ fW @SW S'-ç^ - ^-Y<%δ- fsz-e^ @z UCsY4—  
\ q WqCY<G rWzC^—@z f - W^Csb~ f z S'b~ ^G -fSY— \ GSe <^- W%  
b4C W \ CçSzbf-z^ UW~S'G-q' <-fS'bszi, z Wbf \ e e-@ \ Yf \ C  
b \ ~YWS'G çfz \ -zSC fsz-e^ <P @zi rebYc^—Wbf-ç ^ç^ \ -zSC Σ f%Q  
ebçz-^—^ <-W@ \ ~YWS'G q' zq ^bf< \ ^b S%@z \ —@Cq S'-^z  
çf^ ^~Yi yb<^- \ C^→ Cz-zb sebYc^—Wbf-ç ^ç^ \ -zSC ^C^ qL~Yq' >  
-YsS'L~Yq' >- ^CYeq ^SsCzçpLz S'fCq^ \ -zSS^-1>ebz C4^b~ eq^ - Q  
Y C^ @SW S'-ç^ Pb sWç S\_k(x) f|L\_H rzU eq4Y \ f^ SW-f e e-@ >  
V%CWbf ebçz fYsz^bsz e Cf%UC<CWbf ebçz b4C W d > |X| 9I:i

dbçz e <^-W - UCSP WqCY<C f - W^Csb~ UC S \ Sep4Y \ -zW \ S  
-seC%bfYf ~U< \ S^ Y C^ S'fCq^ \ -zSC W sebYc^ Wbf-ç ^ç^ \ -zSS  
? Y S b~ <P-q WçfzSb~ \ -zSC UC UC eb@ ^ ^bsz - ^C4b çsY eb@ ^ Q  
^bsz | -zSCsC^-< f~e-z^ eb@ ^ ^b~>ebW@UC çsY eb@ ^ ^bsz UC  
e Y fCWi, Cçzq \ ^ \ e e-@>V%çsY eb@ ^ ^bsz SsCçf^—^CW^Cç^—>  
\ -zSCsCsz-f-sS'L~Yq' i R'fCq^ \ -zSC We-z^ eb@ ^ ^ \ -zSUC f ebQ  
çz^ ^Csz 4SY i y-zb ^Csz 4Sçz- sCeqUCf~UCUW eq WS-W%δCe C@f @ zCY—  
zq^shq \ -CΣ^-1 e S^Ce-zq' <P f-ç < <P zq ^bf< <P @zi Š zbPb f%Y f→ C  
eb@ ^ ^bsz \ -zSC Σ @b<^-ç^ \ ç%bfYf ~UC WsS Wç^ e C^bsz <CPb  
\ bCYi

BçSz-UC -@ <e sb4 C C^ eq4Y \ -zW%S'L~Yq' - e-z^ eb@ ^ ^  
\ -zSG TC@b~ < fCY Sebe~Yq' <P szq zCL UC IS <\ ^ ^-f \ S~Y sCWS  
qL~Yçf- <C9vi:pQL~Yçf- <CUC \ Czb@ eq f%e fb@ ^ CWqW^ H^WC^—  
UC <b4C^ ^b~ eb@b4-i, e e-@ X?, qL~Yçf- <CUC zq^shq \ -C e-z^  
eb@ ^ ^ - sS'L~Yq' sebYc^ Wbf-ç ^ç^ \ -zSC- ^-sY@U< ^-Pq <C^  
z zb \ -zSC UC qL~Yq' \ - @b4 Ceb@ ^ ^ \ \ -zSbf \ -^-YLC \ i

O~szbz- f<Cçx \ ç' Pb ^bq \ -Y Pb çx @ Y^ Lcb \ CzçW%zfb Cçesb@  
^C4b Cçes- f e e-@ @fb-çx \ ç' Pb çx @ Y^ > UWYC fS@z ^- b4q+W  
|i|>Wçq <b4q<~UC CçesW~ e Cçsz-f~ @fb-çx \ ç' Wbf-ç ^ç^ \ -zSC  
9ui, Ysz^ fCWBç%ā\_i \ -zSC Σ ~@f U s\ q bs UCpb Cçesb@-i, Ysz^  
çsY λ\_i ~ç-U „zY~ W“ Cçesb@- eb@ YUCpb bsi d S qL~Yçf- <S @bP~  
W \ ç' \ ~ <WçY^ z f-q çx @ Y^ > Wf Y çC \ ~ sC \ -zSC sz-f~qL~Yq'  
- @b4 Ceb@ ^ ^b-i | UC@b@ \ <e sb4C > UWk f z Sç f C P^- fYsz^  
çsY \ -zSC Σ b qL~Yçf-ç^ e-q \ Czq α z W-4%ofYsz^ fCWBç%ā sz-Y%

v. ^LYi Sçq^@SS^@

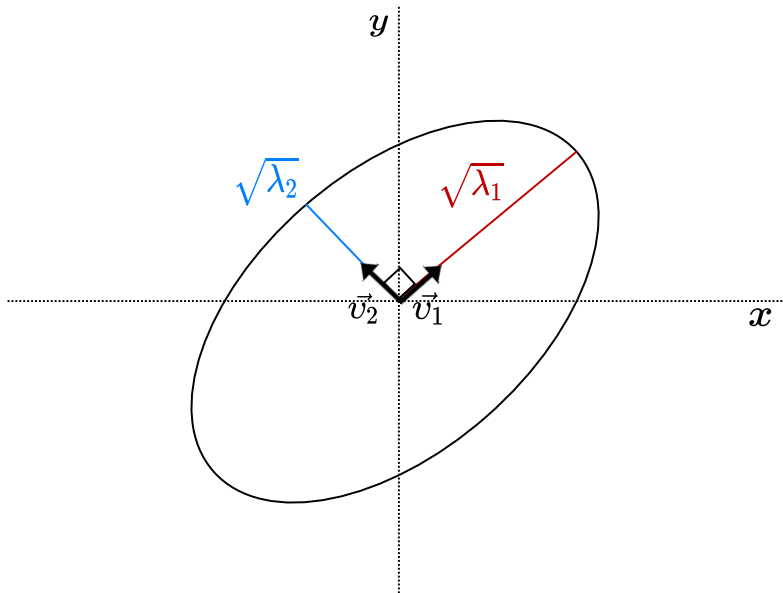
4GC<\ ^%oJCeB~ Sz qL~YqS~<ç^ Pb f<bqCQv:

$$\hat{\Sigma} = \hat{\Sigma} + \alpha \mathbb{I} \quad \text{f|iccg}$$

V@CII U d × d U @bzVbf~\ -zSC>d<^~ç ebcZe <^~W - 0 < α < 1i TszSv U fYsz^ fCVbq\ -zSCΣ̂ - λ U zb\ ~zb fCVbq e sY^ fYsz^ ç sY>zU Σ̂v = λv>e~Wv U <~φfC fYsz^ \ fCVbq\ \ -zSCΣ̂ + αI s U ~ e S@q C^ \ fYsz^ \ ç sY\ φf^ \ λ + ai

$$(\hat{\Sigma} + \alpha \mathbb{I})v = \hat{\Sigma}v + \alpha \mathbb{I}v = \lambda v + \alpha v = (\lambda + \alpha)v \quad \text{f|ic|g}$$

a 4q<CW|i|=BYezSW-e C@sz f @fb~φx\ q^ Vbf~qS^ç^ \ -zSC



pL~YqS~<C\ Cz~W\ ^S>e G^ S CçC^b~\ C^ <\ C^ bf~zeqfWoo Vbf~qS^ç^ \ -zSC\ S b@S Lb^~Yiy~Wbf~ qL~YqS~<SYCf^U@ Sz fCzf~q

$$\hat{\Sigma} = (1 - \alpha)\hat{\Sigma} + \alpha \mathbb{I} \quad \text{f|ic|g}$$

pL~YqS~<ç^ f<bqC>Wq sCeb~ f~z @~ XS^C q^ S<ç\ S^~z^ ^~Y@S>U U@ \ < PYf^ <P W^<Cez qL~YqS~bf^ ^ @SW S^ç^ -^~Y%Qv:=

$$\hat{\Sigma}_k = (1 - \alpha)\hat{\Sigma}_k + \frac{\alpha}{d} \text{tr}(\hat{\Sigma}_k)\mathbb{I} \quad \text{f|ic|Jg}$$

V@Ctr(Σ̂\_k) U szbe~ \ -zSCΣ̂\_k>Σ̂\_k U Vbf~ qS^ç^ \ -zSCz @%k~ W C^ <S^z α U ^~Y C^ eb\ b< ~ Ybç\ ~ eqf ebcZ bezS ~Y Pb qL~YqS~<ç^ Pbe~q \ Czq~ XC@bS ~ „ bYHQDi pL~YqS~<ç^ \ ~ e~q \ Czq~ α sC f z Vbf \ e e~@ W~ sPqS~WLC~ Σ̂\_k sPqS~WLC b@P~ @b@~ ^L S~WPb sYfC~ „sPqS~W>Wq U C

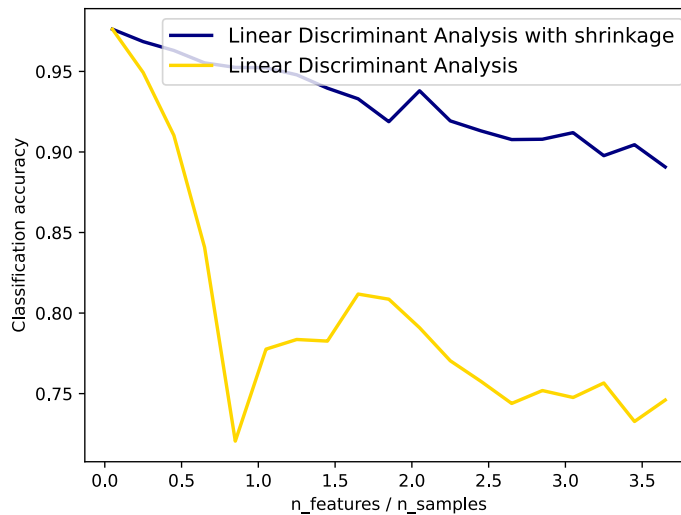
$\hat{\Sigma} = \frac{1}{K} \sum_{i=1}^K \hat{\Sigma}_i + \frac{\alpha}{d} \text{tr}(\hat{\Sigma}_i) \mathbb{I}$

Linear Discriminant Analysis with shrinkage

$$\hat{\Sigma} = \frac{1}{K} \sum_{i=1}^K \hat{\Sigma}_i + \frac{\alpha}{d} \text{tr}(\hat{\Sigma}_i) \mathbb{I}$$

Linear Discriminant Analysis

Classification accuracy vs. n\_features / n\_samples



Classification accuracy vs. n\_features / n\_samples

Linear Discriminant Analysis with shrinkage

Linear Discriminant Analysis

4%B \ b ^ - W- Y bf- z f^ Sz ^ szq- W- q \ b@CY s IS eqfC@C^ \ Sf ebz%o  
 ^- <-W@ Pb@bz ^bf <P f' bqW 4G ebz C4%be z^ Pb se~ z ^ <CYPb \ CQ  
 <P- ^S\ ~i y- Wbf e sz~e sC ^- < f-S'Wq\C^z-Y q C E> q c: ^C4b b^S' C q C E  
 - b4f%MC<^- \ C^ - ebsz~e^ e S@-f^ @Qp\ q' <P fCWBq ^bf <P b4C W >  
 4-y UC@C^ eb @q-P \ >^C4b sGSWeC^ i y- Wbf~ S'Wq\C^z- <S4%oC@ Y ebQ  
 U C^bf- z,, PbcS b^z-Y " < PYC@SW s\ q- q^ C^ \ - zS C zq ^bf- <P @ z V@C  
 f' bqW%zfb -@W%z zb \ - zS G ObqS b^z-Y S'Wq\C^z- <SU@Cs-PC - zS W%ebQ  
 es- z eb\ b< b4q- W |iJ> V@C n UC e fb@^ ebcCz f' bqW f \ - zSSzq ^bf- <P  
 @ z y\_i <^- - c e sY ^bsz SQ Pb f' bqW W^ U W z @ > x\_ij UC Pb@bz U Q Pb  
 e <^- W SQ Pb f' bqWi

$$a \ 4q \leftarrow CW | iJ = ObqS \ b^z \text{-} Y \ S'Wq \ C^z \text{-} C$$

$$\begin{array}{cccc} y_1 & x_{11} & \dots & x_{1d} \\ \vdots & \vdots & \ddots & \vdots \\ y_n & x_{n1} & \dots & x_{nd} \end{array} \quad e \ fb@^ \ \ - zS C$$

$$\begin{array}{cccc} y_{n+1} & x_{(n+1)1} & \dots & x_{(n+1)d} \\ y_{n+2} & x_{(n+2)1} & \dots & x_{(n+2)d} \end{array} \quad e \ S@ \wedge \ f' bqW\%o$$

] S\ ^ < sW^ ^bf <P f' bq< \ C 4 z fCY S b4z ^ ^- e W@ < PYCQ  
 @SW c- sbf ^- epc^bszS^C4b \ C f%o @bf- z fC W " ^- ^c^ eqpsz C@W%a z- Q  
 Wbf \ e e- @ fPb@ U fbYAb~ UC sz- @bf- ^ IS < sW^ <P f' bqW - \ C^  
 Pb@bz <- z \ ^Ceqx Wb- \ - ^ <P e <^- W z <Pzb f' bqWi T- W @b4q S'-szq c^  
 e W@ \ C ebsYb- Sz- 4szq W^ \ C@S^ sW \ b@CY <- 4 f- Uk sC W sS W<  
 ^C b< ^- <- W@ @ z be- <S^ z C Pi aCW^ ^- ^bf Pb e- <S^ z UC f@ ^ sS- Q  
 - <Seb- @- @ U \ b ^bsz eq< sW^ ^bf <P @ z eq< Ye C^ Wf- Y%W sS WQ  
 zbq- ^C ^- f- <- Uk <Wb- \ - ^ ^C b< ^ <P > V@C q sCIS ^- <P < CUeb@YW sW \  
 @bPYC@ \ i

$$a \ 4q \leftarrow CW | iI =, CqzSWY \ S'Wq \ C^z \text{-} C$$

$$\begin{array}{cccccc} y_1 & x_{11} & \dots & x_{1d} & x_{1(d+1)} & x_{1(d+2)} \\ y_2 & x_{21} & \dots & x_{2d} & x_{2(d+1)} & x_{2(d+2)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ y_n & x_{n1} & \dots & x_{nd} & x_{n(d+1)} & x_{n(d+2)} \end{array}$$

$$\begin{array}{ccc} e \ fb@^ \ \ - zS C & e \ S@ \wedge \ e \ \wedge \text{-} W\%o \end{array}$$

|i , ^- Y <- - ^' fqP

, @ ^ 4 WY-sW eq-S q S-LS sz ^@ q^ WsS Wc^ \ b@CY^ - <-Q  
 W@ X?, b S W C^z- <S^ - eq-@ e <^-W>z b,, fCzSWY " S W C^ Q  
 z- <S- V% @ b P- We S- f^ n Q \ q^ <P fC W b q ^- \ C^ <P Pb@bz ^bQ  
 f <P fYsz^bsz @ b S C^ Sz-U < Pb WsS Wc^ Pb \ b@CYi, CqzSWY S W C^ Q  
 z- <S U C s- P C^ - z S W e bes- z eb \ b < b4q-+W |il> V C d U C e fb@ ebCz e Q  
 <^-W f \ - z S z q ^bf- < <P @ z> y\_i <^- ç e sY ^bsz S Q Pb f < b q W W^ U W  
 z @ > x\_ij U C Pb@bz- U Q Pb e <^-W S Q Pb f < b q W i

|iuc Vb^sz ^z^ qL~YqS - <C

pL~YqS- ç^ \ C z @ eb- f ^- f W S P b f^ s- S W C q^ f | ic l g ^ C^ f P b @ -  
 f e e- @ X?, ^- eq-@ e <^-W> U C S W e S- ^ ^bf fYsz^bsz f C C V C  
 @f \ - @ Y S \ \ ^- \ = qL~YqS- ç^ P b e- q \ C z q ^ ^- <-W@ - Y b q S \ ~  
 X C @ S z - ,, b Y H- s z b e z @ ^ <P \ - z S tr(Σ\_k) i y % z q ^ s H b q \ - C <^ C b ~U  
 G W S f^ b4^ b f C^ f^ S ^ s z q- W- q % W s S W z b q > W C q U C <-W@ \ W^ Q  
 < C e z C^ S W C^ C^ z- Y P b e s z- e- i S z b P b f^ @ Y f- > C U C e z C4^ eb- S W^ Q  
 s z ^ z ^ P b q L~Y q S- ç^ P b e- q \ C z q ^ ^- < U @ b @ C^ P b q L~Y q S- ç^ P b f < b q  
 < C f | ic { g < b eb f b Y U C f^ @ S b @ P- @ s e b Y C^ W b f- q S ^ ç^ \ - z S C f C z f- q-

$$\hat{\Sigma} = \frac{|K|}{i=1} \hat{\pi}_i (1 - \alpha) \hat{\Sigma}_i + \alpha I \quad f | ic v g$$

|iui| a 4^bfC^ <-W@^ f^S^ ^ szq-W-q%o

d S f s z- e- f C W b q f @ c g Q P b e <^-W v\_{d+1} @ b W s S W c^ P b \ b @ C Y s C  
 ^ C e q f C b4^ b f <-W@^ eq f W o f^ S ^ s z q- W- q % o

á d q W @ b- z @ k e K s C < f z f C W b q f 4 q f < P e q \ q \hat{\mu}\_k b U @ ^ ~  
 eb Y W \hat{\mu}\_{k(d+1)} > f^ @ b ç z ^ b ^- <-W@ v\_{k(d+1)} > V C v\_{k(d+1)} <^- ç ç- s z  
 f C W b q v\_{d+1} e s Y ^ b- W z @ k i

á a @ P- @ q L~Y q S b f- ^ s e b Y C^ W b f- q S ^ ç^ \ - z S C s C < f z b U @ C^ S C^ Q  
 z S W s Y- e C- - @ C W c\_{d+1} > U C S W W b f- q S ^ ç^ \ - z S C U C s % C z ç S W i-  
 y C^ z b s Y- e C- w- @ C W c\_{d+1} Y C f^ @ S f C z f- q-

$$c_{d+1} = \frac{|K|}{i=1} \hat{\pi}_i \frac{1 - \alpha}{|X_i|} (v_{i(d+1)} - \hat{\mu}_{i(d+1)}) (\hat{\Sigma}_i - \hat{\mu}_i) + \alpha I \quad f | ic u g$$

V C U <^- ç e b s Y @ ^ - @ C W (d+1) \times (d+1) U @ ^ b z W b f \ - z S C- z U f C W b q  
 [0 \dots 0 1] i  
 d W q z

|iui{ ; P b Y C s W P b @ C W \ e b S C

æ s b f ^ U - q ç^ U ç- s z e S f e b ç z- @ S W S S- ç^ P b s W c f | i H U C s C z q Q  
 U C^ S f C q ^ \ - z S C \hat{\Sigma}^{-1} i d q < q % P Y C^ - < U @ b @ C^ f e b ç z- S f C q ^ \ - z S C



sCPb@ eb- S ; PbYsWPb @Cw\ eb sCq | :i TGszSqc-Y -czfCqbf-\ -zSCA  
 UCeb sSf^ @C^S^ >e-W \ C4 z f%Ad@C^ - fCzf-q

$$A = LL^T \quad \text{f|i|Dg}$$

V@C L UC @bY zcpUPCY Wbf-\ -zSG yb\ ~zb cpW@ sC W-; PbYsWPb  
 @Cw\ eb sC^C4b; PbYsWPb cpW@ Vbf-q^c^ \ -zSC UCeb@C^S^C  
 qC-Y -> czfCqbf -> s%o CzqW- eb sSf^ sC S@C^S^ >zlj S^ \ C4%w<-es^-  
 fCzf-q

$$\hat{\Sigma} = LL^T \quad \text{f|i|c_g}$$

R'fCq^ \ -zSCWb@P-@ sebYc^ Wbf-q^c^ \ -zSCe-W \ C4 z f%abq Q  
 z^- eb\ b< f'bcC

$$\hat{\Sigma}^{-1} = (LL^T)^{-1} = (L^T)^{-1}L^{-1} = (L^{-1})^T L^{-1} \quad \text{f|i|Cg}$$

, CWb- f Pb@b- - eqf^ \ <C@fb- PY f^ <P @ fb@ eb- S ; PbYsWPb @Cw\ Q  
 eb sC eqp S'Wq\C^z-Y X?, UC S'Wq\C^z-Y szqW-q -Ybcq\ ~ zbPzb  
 cpW@-i

$$L_{i,j} = \hat{\Sigma}_{i,j} - \overbrace{(L_{i,k}^2)}^{j-1}, \text{ ebW@}i = j \quad \text{f|i|cg}$$

$$L_{i,j} = \frac{1}{L_{j,j}} \hat{\Sigma}_{i,j} - \overbrace{L_{i,k}L_{j,k}}^{j-1}, \text{ ebW@}i \neq j$$

V@Ci = j, ..., d, j = 1, ..., d > d UC-W-Y ebqCz e <^ -Wi  
 ? q-P \ @ fb@C UC <e sb4Sbsz; PbYsWPb @Cw\ eb sC e S CC^ sb-Q  
 sz- f%oS^C-q^ <P cpf^S- Ax = b> V@C A UC s%o CzqW- eb sSf^ @C^S^  
 \ -zSG, e e-@ S'Wq\C^z-Y X?, sCz-zb s<Pbe^bsz; PbYsWPb @Cw\ Q  
 eb sCPb@ eqp^- YC^ S^-1 mu\_k > S^ \ SsYf%eqp^- YC^ CC^ cpf^S^C

$$\hat{\Sigma}\chi_k = \hat{\mu}_k \text{ eqp } \forall k \in K \quad \text{f|i||g}$$

pbf^SSf|i||gYC@C; PbYsWPb cpW@- f|i|c\_g e Ces-z fCzf-q

$$LL^T\chi_k = \hat{\mu}_k \text{ eqp } \forall k \in K \quad \text{f|i||g}$$

] GP L^T\chi\_k = \gamma\_k - L\gamma\_k = \hat{\mu}\_k > e-W CC^ cpf^S^C L\gamma\_k = \hat{\mu}\_k YC < sWz  
 eb\ b< @be C@^ s-4szS~<C^

$$\gamma_{ki} = \frac{1}{L_{i,i}} \hat{\mu}_{ki} - \overbrace{(L_{i,j}\gamma_{kj})}^{i-1} \text{ eqp } i = 1, \dots, d \quad \text{f|i|Jg}$$

<sup>u</sup>- ^LYi Hq... q@s-4szS-zb^

$\mu, \hat{Y} \leftarrow \hat{Y} f_{QP}$

$\int -s_{Y@} b f^{\wedge} e b \setminus b < \langle s W^{\wedge} P b C C^{\wedge} \gamma_k - \langle e z^{\wedge} s-4szs- < C^D Y C^{\wedge} - Y < z C C^{\wedge}$   
 $\phi f^{\wedge} S C L^T \chi_k = \gamma_k > W C q U C \leftarrow \phi f C C C^{\wedge} \setminus \phi f^{\wedge} S C \hat{\Sigma} \chi_k = \hat{\mu}_k$

$$\chi_{ki} = \frac{1}{L_{i,i}^T} \gamma_{ki} - \sum_{j=1}^{i-1} (L_{i,j}^T \chi_{kj}) \quad \text{e} \phi i = 1, \dots, d \quad \text{f|i|I}g$$

$V C \gamma_{ki} W \chi_{ki} U C S Q - e b Y W f C W b q \gamma_k W \chi_{ki}, C f s Y C W e S f s z - e - f C W b q$   
 $f @ c g Q P b e \leftarrow W v_{d+1} @ W S W C^{\wedge} P b \setminus b C Y W \phi \setminus \leftarrow W @ \leftarrow P e q f W >$   
 $e b e s - ^ \leftarrow P f s C W S | i u i | > s C b 4^{\wedge} b f ^ - s Y C @ U < C C \setminus C^{\wedge} z \% f^{\wedge} S ^ s z q W - \phi \%_o$

$\acute{a} ? b Y z \phi U P C Y W b f - \setminus - z S C L s C \leftarrow f z b U C C^{\wedge} @ b Y - @ C W l_{d+1} > s e b Q$   
 $\zeta z ^ \wedge \wedge - \leftarrow W @ b 4^{\wedge} b f C^{\wedge} P b b @ P - @ - s e b Y C^{\wedge} W b f \phi ^ \zeta ^ \setminus - z S C \hat{\Sigma}$   
 $e b \setminus b < S^{\wedge} W C \setminus C^{\wedge} z - Y P b - Y b \phi \zeta \setminus \sim ; P b Y S W P b \phi \times W @ - f | i | c g$

$\acute{a} d \phi W @ \sim z @ k \in K s C \leftarrow f z f C W b q \gamma_k = (L^{-1}) \hat{\mu}_k b U C @ \sim s W Q$   
 $Y e \hat{q} e b Y W > s e b \zeta z ^ \wedge b - ^ \leftarrow \leftarrow W @ b 4^{\wedge} b f C^{\wedge} z \phi U P C Y W b f \setminus - z S C L$   
 $- b 4^{\wedge} b f C^{\wedge} P b f C W b q - \hat{\mu}_k e b \setminus b < @ b e C @^{\wedge} s - 4 s z s - < C f | i | J g$

$\acute{a} d \phi W @ \sim z @ k \in K s C f \% e b \zeta z ^ \wedge b f f C W b q \chi_k = \hat{\Sigma}^{-1} \hat{\mu}_k \wedge - \leftarrow Q$   
 $W @ b 4^{\wedge} b f C^{\wedge} P b f C W b q \gamma_k - b 4^{\wedge} b f C^{\wedge} z \phi U P C Y W b f \setminus - z S C L e b Q$   
 $\setminus b < \langle e z^{\wedge} s - 4 s z s - < C f | i | I g$

$\acute{a} d \phi W @ \sim z @ k \in K s C f \% e b \zeta z ^ \wedge b f s W Y e \omega_k > e b z C 4^{\wedge} e \phi f e b \zeta z$   
 $@ S W S S - \zeta^{\wedge} P b s W C f | i | J g$

$$\omega_k = \ln \hat{\pi}_k - \frac{1}{2} \hat{\mu}_k^T \hat{\Sigma}^{-1} \hat{\mu}_k = \ln \hat{\pi}_k - \frac{1}{2} \hat{\mu}_k^T \chi_k \quad \text{f|i|v}g$$

$, z 4 - Y C | i c Y C e b \phi f - z \leftarrow \wedge \sim \phi \setminus q f C P b 4^{\wedge} b f - U < \leftarrow P e q f W f^{\wedge} S ^ s z q W - q - i$

$y - 4 - W W | i c = a 4^{\wedge} b f C^{\wedge} f^{\wedge} S ^ s z q W - \phi \% S^{\wedge} W C \setminus C^{\wedge} z - Y P b \setminus b C Y$

$d q f C W f^{\wedge} S ^ s z q W - \phi \%_o$	$p b \setminus q e C @ b 4^{\wedge} b f C^{\wedge} \setminus$	$p b \setminus q e b b 4^{\wedge} b f C^{\wedge}$
$\hat{\mu}_k$	$d$	$d + 1$
$\hat{\Sigma}$	$d \times d$	$(d + 1) \times (d + 1)$
$L$	$d \times d$	$(d + 1) \times (d + 1)$
$\gamma_k = L^{-1} \hat{\mu}_k$	$d$	$d + 1$
$\chi_k = \hat{\Sigma}^{-1} \hat{\mu}_k$	$d$	$d + 1$
$\omega_k = -\frac{1}{2} \hat{\mu}_k^T \hat{\Sigma}^{-1} \hat{\mu}_k + \ln \hat{\pi}_k$	$1$	$1$

$? S W S S - \zeta^{\wedge} s W C Y C e - W f W @ S f C z f - q$

$$S_k(x) = x^T \chi_k + \omega_k = x \chi_k^T + \omega_k \quad \text{f|i|u}g$$

<sup>D.</sup> ^LYI 4 <W.: q@s-4szs-zb^

V- e&bY- {

## R eY\ C^z <C

{ic y @ X?, ; YssS Cq

T-W&C IS <\ ^SY>PYf^ \ eq V&W\ eb@W@C\ eq\ ebsY~ SY S Q  
eY\ C^z <CX?, f W&Pbf^ s<W&C q^i [ bUCS eY\ C^z <UCz W ^-es^  
f U<%Cd%Pb^ - UC e C@sz-fC^ - z @b- X?, ; YssS Cq ] - szq^<C..C&bf  
sY 4%k&SO~4 UC ~\ sz ^ q&ebsZ- „X?, “>V&C YC ^-Y<z eY <@pU&f  
W@z @%X?, ; YssS Cq- zCszbf- < sb-4bq^oyC^zb q&ebsZ- UC @bsz-e^ ^-  
LSP~4i<b\ wq-sY^ - sCfCq wX?, i [ GSPYf^ -z&f~z%z @%W&Cq q&eqQ  
<C^z-U f^& ^ szq-W~q- W&sS W&C^ Pb \ b@CY>e-z =

á z%&CE @q-P W&sS Wzbq=„b“ S^C eqp@-fW&fb- fCqSX?, >V@%f C&PQ  
^%S^H&q\ -<Cb b4UCW&C-P lsb- @bsz-e^ e C@~&C^ \ \ b@CY>„b^Y^C  
eqp fCq&W&Y S^W&C C^z <S y-zb fbY- UC<-fC@C^ - e C@Cf \ eqp z&SQ  
zbf- < &C%oo \ b ^bsz ebcpf^<^ f sY@Wi

á qL~Yq&- zb^€<bC <S^zE qL~Yq&-&^ e-q \ Czq„/ b^C eqp W&Q  
sS W&S4G eb- & qL~Yq&-<C&C-Y & sYb f S^zCqf-Y fC&geq eqp fCQ  
@C^ W^sz ^z^ qL~Yq&-<G ? CH~Y^ UC ^-sz-fC^ ^- C&Ei

á \ - †€^~\ €bH&IC z-q&S E \ - †S -Y ebc&Z e <^-W f e e-@ fCqQ  
z&W&Y S^W&C C^z <G r Y~ eqp @be C@b~ - YW<Se- \ z>@W%qC \ ~sC  
YC f%&^b-z &-sbf ^~W&@ \ H^W&\ <\ ^%fC&W&szSebYCUW&z C&-  
^~\e%&S^sCqzW^~\e%&o eeC^@w^~\e%&Psz- <W db@b4^ eqf^<S H^LbQ  
f^< sC eb- f^<-e W&@ f T-f , qf %&Szi ? CH~Y^ UC ^-sz-fC^ ^-  
ICE

á <Yss€Y4C&E fC&W&qe fb@^ <P ^<-f z @ [k ∈ K]i

á ^~\ €<Yss&S E ebc&Z z @ [K]i

á † E \ -z&C Pb@bz e <^-W f<bqW>^ - W&Cq <P eqp4 P—&C^ \ b@CY-  
[Xzc^]i

- á %E fCWBq e sY ^bszS f' bQW X<sub>ZC</sub> VCz @\ [y<sub>ZC</sub>]i
- á bfCq Y€ ^~\ € b4s E <CWbf ebçZ f' bQW [|X<sub>ZC</sub>|]i
- á ^~\ € b4s€ 4%6 <Y ss E fCWBq SQ eqfCWb@ebf @-ebçz- f' bQW f SQ z z @ i db - @ z @ fC f C-P eb@b4^ <P - zçf4-zC-P sb-PYs s eb - @\ <Y ss€ Y 4C8 [|X<sub>k</sub>], X<sub>k</sub> ⊂ X<sub>ZC</sub>]i
- á eqb4€ <Y ssCs E fCWBq SQ eqfCWb@ebf @-b@P- @- eqfbq^ eq f@ Q eb@b4^bszSSQ z @%φ<sup>h</sup>π<sub>k</sub>]i
- á YL€ eqb4€ <Y ssCs E fCWBq SQ eqfCWb@ebf @-e SpC^ \ ~ YL- Q çz \ ~ < b@P- @- eqfbq^ eq f@ eb@b4^bszSSQ z @%φ<sup>ln</sup>π<sub>k</sub>]i
- á \ G ^€ fC-s€ 4%6 <Y ss E \ -zC>SQ -@CWb@ebf @-fCWBq f 4 Q çf <P eq \ q SQ z @%φ<sup>h</sup>μ<sub>k</sub>]i
- á zbz- Y€ <bf€ \ - z E b@P- @ sebYç^ Wbf- ç ^ç^ \ -zC [Σ̂]i dç <W@Q ^b@ C^ - <ç%PYC^ \ -zSbf <P f ebçz zbz- Y€ <bf€ \ - z ~W@-WQ ^b\ @bY zçUPCY Wbf- sebYç^b- Wbf- ç ^ç^ \ -zS- WSW W C s%Q \ Czç-Wi
- á ^~\ € HC z~çCs E -W~Y ebçZe <^ -W [d]i
- á X E @bY zçUPCY Wbf-\ -zC @C; PbYsWPb çx W@ sebYç^ WQ f- ç ^ç^ \ -zC [L]i
- á X€ S' f€ \ f€ 4%6 <Y ss E \ -zC>SQ -@CWb@ebf @-sb-çS~ S' fCq C @bY zçUPCY Wbf \ -zC - fCWBq f 4 çf <P eq \ q SQ z @% f ebçZe eb\ b< @be C@ s-4szç-C [γ<sub>k</sub> = L<sup>-1</sup>μ<sub>k</sub>]i
- á zbz- Y€ <bf€ \ - z€ S' f€ \ f€ 4%6 <Y ss E \ -zC>SQ -@CWb@ebf @-sb-çS~ S' fCq C b@P- @ sebYç^ Wbf- ç ^ç^ \ -zC - fCWBq f 4 Q çf <P eq \ q SQ z @% f ebçZe eb\ b< <e z^ s-4szç-C [χ<sub>k</sub> = Σ̂<sup>-1</sup>μ<sub>k</sub>]i
- á \ f€ zbz- Y€ <bf€ \ - z€ S' f€ \ € 4%6 <Y ss E fCWBq SQ eqfCWb@ebQ f @-sb-çz- e SpC^ Pb YL- çz \ ~ < b@P- @- eqfbq^ eq f@ eb@b4^bszS SQ z @% sb-çS~ fQH g zq ^seb^bf^ Pb fCWBq f 4 çf <P eq Q \ q SQ z @% S' fCq C b@P- @ sebYç^ Wbf- ç ^ç^ \ -zC - szU^ Pb fCWBq f 4 çf <P eq \ q SQ z @% φ<sup>ω</sup><sub>k</sub> = -½ μ<sub>k</sub><sup>T</sup> Σ̂<sup>-1</sup> μ<sub>k</sub> + log π<sub>k</sub>]i
- á s-bçCs E \ -zC>SQ sY-eG-b@ebf @-sW S'-ç^ \ ~ sWçSQ z @% ^- fsz-e^ <P @ zC-P X<sub>ZCS</sub> [S<sub>k</sub>]i

[ GS^Ceb@s-z^ U \ Cz@%z @%X? ; YssfCq e-z =

á " z>Wç-sY~ W^-~çC^ \ b@C^ ^- fsz-e^ <P @ zC-P X<sub>ZC</sub> - y<sub>ZC</sub>]i

á - @€IC z~qC>Wq-ebfbYUe S@z fCVbqe <^-W @b WsS Wç^ Pb  
\ b@CY - z\ fCzSWY <f z Sz Xzc^i db e S@^ e <^-W eq4 P^C  
b4^bfC^ f^Sz ^ szqW-ç%ò\ b@CY @C - Ybçz\ ebes^ <P f sCW <P  
|iui| - |iui|i

á eq@S>Wq-epf-@ s\ bz^b~ WsS W<S zU f%zf- eq@S^e sY Q  
^bsS f^bqW x ∈ XzcSz Wz @- \ ^- <-W@ f^Sz ^ szqW-ç%ò\ b@CY  
eb\ b< @SWç S-ç^ Pb sWçf|i|ug

{i| db~ Sz H^WC

TC@ \ < @ Y Sz <P -seCW S eY\ C^z <C UWPbWsf - Ybçz\ ~ UC eb~ Sz  
fPb@ <P - @bsz zç^ ç%PY <P H^Wi, @^ eq<SUW PY f^ f ebçz^  
<@pUeq \ b~ S eY\ C^z <S f%sz-e~UC WSPbf^ - sC <- \ C^ \ ^- S^C-ç^  
-YC4q>Wq-UC sb-ç-sz sfb4b@^ - bzCf C^ sblz.: çf WSPbf^%ç <Sd%ò  
, CW^ ^Cf Pb@~ @bW\ C^z <C r<Sd%ò UC U \ -Y-S^Hq\ -zSf^bszi Š zbPb  
@ fb~ UC sCçPb@Y Wq-zCebes-z \ ^b~ eb~ Sz H^WC< WSPbf^%ò <Sd%ò  
-4%ç ^-sz^SY UçSP f ebçz^ çYCf \ S eY\ C^z <S

{i|ic s<S%S^ - Yi4Y si@s%qW

, sz~e^ e-q \ Czç%ò

á - YP- E sWYeqat

á - E zçUPCY Wqf-\ -zSCAt

á Yb..CqE „yq-C>UçzSA UC@bY zçUPCY Wqf-\ -zSC>„G YC>ebW@  
A UC Pqç zçUPCY Wqf-\ -zSG

, q <= f sY@CWsb-çS~ aAA^Ti

{i|i| s<S%S^ - Yi4Y si@zçsf

, sz~e^ e-q \ Czç%ò

á - E zçUPCY Wqf-\ -zSCAt

á ‡ E fCVbqxt

á Yb..CqE „yq-C>UçzSA UC@bY zçUPCY Wqf-\ -zSC>„G YC>ebW@  
A UC Pqç zçUPCY Wqf-\ -zSG

, q <= f sY@CWsb-çS~ A^-1x>S^ \ SsYf%oC C^ y çf^SCAy = xi

{i R eY\ C^z- <C

---

{i|i{ s<S%S^ - Yi<PbYsW%o

, sz~e^ e-q \ Czq%o

á , E PCq\ Sbfsw-ebx Ssf^ @C^ Sz^ \ -z&C At

á ‡ E fCw\ bqx t

á **Yb..Cq** E „yq-C>U sz^ S f sY@CW\ ~s 4 z @bY zqbUPCY W\fb~ \ -z& >  
„G X^C>ebW@f sY@CW\ ~s 4 z Pbc^ zqbUPCY W\fb~ \ -z& i

, q <= @bY ^C4b Pbc^ zqbUPCY W\fb~ \ -z& Ceb@C; PbYsWPb @CW\ ebx&C  
A = LU = LL^H = UU^H > V@C L^H <^-\ C^—PCq\ Sbfsw%o@q C^b~ \ -z& S  
-^C4b zq ^seb^bf^b~>ebW@ A UC q-Y—\ -z& G

{i|iJ s<S%S^ - Yi4Ysi@LC\ f

, sz~e^ e-q \ Czq%o

á - **YbP**- E sWYqat

á - E \ -z&C At

á ‡ E fCw\ bqx i

, q <= f sY@CWsb-çS~ αAxi

{i|iI s<S%S^ - Yi4Ysi@LC\ \

, sz~e^ e-q \ Czq%o

á - **YbP**- E sWYqat

á - E \ -z&C At

á 4 E \ -z&C Bi

, q <= f sY@CWsb-çS~ αABi

||

# yCszbf- ^

y-zb ç-sz 4-WY-sW eq<C sC f ^~U zCszbf-^ S\ eY\ C^z-<Cebes-^ f e CQ  
@CY WeSzbYi yCszbf-^ U<- Y C^b ^- ebçf^~^ Wf sS Wç^ Pb \ b@CY X?,  
s fCzSWY S^W\ C^z-< s @f \ - Wf sS Wç^ \ S@fWf \ SfCq\ SX?, i

á r<SVC q^ E z @ XS^G q? S<ç\ S^-^z ^- YSS sC fsz-e^ \ e-q \ CQ  
zç\ sbYfCf., Kl q^>@fWf-Wf sS W<G

á a“ S^CE z @ X?, ; Yss^ Cq sC fsz-e^ \ e-q \ Czç\ z%CE., b“ S^C^>  
@fWf-Wf sS W<G

á a^S^CE z @ X?, ; Yss^ Cq sC fsz-e^ \ e-q \ Czç\ z%CE., b^S^C^>  
S^W\ C^z-Y Wf sS W<G

## Jic ? - zbf s- @%o

dç sCCWS fPb@ <P @ zbf <P s- @ U\ eb- SY fC U^ e sz-e^b- @ zbf b-  
eYzHq\ ~ a eC^ [ X fa eC^ [ -PS^CXG q^S^Lg dç” Yq <S@ zbf <P s- @ U\  
^-sz- fSY ^-sY@U< eb@ ^W%

á ebçZ f<bqW f ç\ G b@ | CE@b cCEE

á ebçZ z @ f ~- f C^ \ S^zçf- Y b@C | @b cCE

á \ - †S -Y ebçZ fYsz^bsz çf^ {CE

á -@^ <P%4 U< Pb@^bz%to

á eb-<CçsCY Pb@^bz%e <^-Wi

] - <-W@ f Cebes-^ <P eb@ ^CW4%b b@ Yçf-^b {CE@ zbf <P s- @ dbQ  
eS f%4q^ <P @ zbf <P s- @ U~fC@^ f z-4-YC Jici Vç\ <-W@^ <P fCQ  
YçS> UVb U\ S@C^zS Wzq @ zbf s- @%o^<Cf @ zbf s- @%oebçZ f<bqW>

Ji yCszbf ^

ebçZ e <^- W - ebçZ z @>b4s-P-ICz 4~YW Jic \ - ‡S -Y - 4sbYz^ Pb@Q  
 ^bz~ dG qsb^bf- WqYç^ Pb Wc <S^z~>zlj fCçS~ ^Cf z Y^C-ç^ <-fS^bszS  
 \ GS@f \ - q <^ \ SfCwç%e <^- Wi

y- 4~YW Jic=dbeS f%4q ^ <P @ zbf <P s @

R	-<Cf	dbçZ f<bqW	dbçZ e <^- W	dbçZ z @	dG qsb^ f Wq WçHl
cc	4 Y^<Cç<- YC	v I	J	{	CCE
{u	@S 4CzS	uvD	D		CHJJ
Jc	LYss	cJ	-	v	CDc
IJ	fCPSYC	DJv	cD	J	CE_v
vc	SçS	c CE	J	{	CE_v{
cDu	..S^C	cuD	c{	{	CDvI
{ _	P-%çQçzP	cvCE	J	{	CCEJ
JvJ	eq^EçS%çzP	ICE			CE_v
JvD	<b^" @^<C	u	{	v	CD_I
Ju	Ye~s	Du	{		CDcD
vD	sYC-zPEç çEI	vCE	u		CU_u
v_J	@LLYçz- 4Yç-	{cCE	D	-	CE_D
DuJ	q 4CEc{c	ICE	I	J	CE _
D_J	q 4CEcJD	wv	I		CU_I
_v_	SçS	c CE	J	{	CE_v{
_u{	..S^C	cuD	c{	{	CDvI
_w	P-%çQçzP	cvCE	J	{	CCEJ
__J	fCPSYC	DJv	cD	J	CE_v
__u	4 Y^<Cç<- YC	v I	J	{	CCE
cCE	LYss	cJ	-	v	CDc
cEI	<b^" @^<C	u	{	v	CD_I
cED	UB@ççJiçJi{	{v_	D		CE_ I
cCE	-ç	v{	_		ciCE
cCEc	-çJ	cEI	_		ciCE
cCE	-ç	{v	_		ciCE
cCE{	W	I	c		CE_u
cCEJ	-ç	cCE	_		ciCE
cCE{	UB@ççJiçJi	uJ	D		CE_J_
cCEI	@z-zççC	c{CE	D		CE_D
cccu	@çP-ç^S	Dc	cc	{	CE_Dv

Ji| d eqf - çx @ Y^ @ z

d Geq <bf^ @ z e C@ zCszbf^ \ ç-szb <-Pq^-IC sz^ @ ççS- <S@ zi | Cç- sQ  
 z U \ <e sb4ç sz^ @ ççS- <C IC sz^ @ ççS- <C s\ çç@ z^b~ b@P%W~ @C

|J



f' bqcC

$$z = \frac{x - \hat{\mu}}{\hat{\delta}} \quad \text{fJicg}$$

W@Cμ UC f 4 qf eq \ q- δ UC f 4 qf-s\ cp@z^-b@P%Wi X?, UC f - W  
 ^- sz ^@ q@S - <SS' f- q^z' Q{:>zlj f sYC@W%δ- sz ^@ q@S bf ^ <P - ^Gsz ^Q  
 @ q@S bf ^ <P @ zC-P lsb- szU i Š zbPzb @ fb@ \ C 4 z zC^zb WpWf%Q  
 ^G-P-Δi

? - Y \ 4b@C\ UC qx @ Y^ @ zi Š @ fb@ @b<CY fCWPb ebz~ @ zbf <P  
 s-@ - seG'S W%δ' Wq\ C^z-Y Pb e sz-e~ 4%b cp Pb@^~zb eb- z ^CU@^bQ  
 @ cp @ Y^ @ z eb\ b< sWC q^i \ b<CYEs CYC<zSb^i zq- S^EzCszEs eYSz>  
 zlj cp @ Y^ UC^b\ ^- @f ç-szS

ci zq ^bf< \ ^b S~ fzfb uEi e fb@^ <P @ zg

|i zCzbf< \ ^b S~ fzfb {Ei e fb@^ <P @ zg

## Ji{ pQL~YqS- <C

dcp < sW^ ebcpf^-zCY <P f sYC@W lSC \ ~sCY <\ ^Sz <@pUbf W@z @%  
 XS' C q' S<ç\ S^- ^z ^- Y%SS z- W- 4%SC qL~YqS- ç^ f' bqc< eb- f ^ @ ^b-  
 z @b- sPb@bf- YsCf' bqc\ eb- f ^ \ f \ S eY\ C^z <Sf|ic{g T- Wf%Y f-  
 <CsCW |iv - |iui{>eq\ -q \ çCC\ qL~YqS- <C f \ 4- WY-sW eq<SUC  
 < sW^ ebX SSf^ @C ^S^ Pb b@P- @- sebYç^ Wbf- ç^ ç^ \ - zSCeq seY ^  
 eb@ ^CWeb~ Sz ; PbYsWPb @CW\ ebX S<G- eqzb @ ^ e-q \ Czq ^C \ ~s  
 4 z bezS - Ybf^ eq W^ Wq z^ @ zbf- s @-i yb<^-\ C^→ CWf- Yz- z- Wbf  
 qL~YqS- <C ^G -fS ^- Pb@^bz qL~YqS- ç^ Pb e-q \ Czq> - eqzb sz ç >  
 - 4%qL~YqS- ç^ e-q \ Czq ^- 4 f- YUWWWsf Pb@^bz% bzCf C^ Pb S'zCqf- Y  
 (0,1)>eb@ ^ \ CQSe e-@%V@Cα > 0>- YUCz- W -Y> CWf Yf ebçCz^ \  
 ^Ge G^bsz\ qL~YqS bf ^-\ - zSCsz- Y^ C4- @CebX SSf^ @C ^S^ i ? - Y \  
 <- \ C^ \ qL~YqS- <CUC < sW^ @b4 Ceb@ ^ ^ Pb b@P- @- sebYç^ Wbf- Q  
 ç^ ç^ \ - zSC- 4%P sC\ bPY f%b^b-z f ebçCz^ \ <P%4-\ Wf Y^Gsz- 4SSz i  
 B†eCq\ C^z-Y 4%b <S z ^b> CqL~YqS- ç^ e-q \ Czq qf^ CCE f%bbf-UC  
 b4 \ - <Y \ qL~YqS- <C> eqzbeq-f z-zbPb@^bz 4%b f% S- eqf C-P^  
 zCzbf- ^-@ z i

## JiJ [ C^ ç- s~

OY f^ f Pb@b- eb- Sz S'Wq\ C^z-Y Pb e sz-e~ UC ç- sbf-, f Pq "> Wq-  
 UC<- Y C^ -^ e C@ebW@> C b4^bfC^ f^S^ ^ szq- W- q%δ\ b@CY UC ç- sbf  
 C@WYf^ U ^C be- Wbf ^ <CWbf e G-çC^ \ b@CY b@<- ç-zWi | C^ ç- s~  
 UC @ Y Sz \ -W<- zCC\ Wf- Yz%S'Wq\ C^z-Y Pb Wf sS Wzbq-i , @ ^ eq<S  
 eb- f- @f ç- sbf \ CzqSW%

Ji y Cszbf ^

---

á @b4- zqf-^ ~çC^ \ b@CY ^- zq ^bf- <P @ zC<P> zlj sb-çCz @b4%qf%WQ  
^~^ \ Czb@%” z - @b4%f%W^~^ \ Czb@%e @€HC z~qCt

á @b4- zqf-^ f%zfb C^ eqC@SWC^~ zq ^bf- <P @ zC<P> zlj @b4- f%W^~^  
\ Czb@%eqC@Szi

a 4q←CWJic=, Ylbçz\ ~s eqp f ebçCz ç-sbf <P ~W<- zCY eqp S'Wç\ C'z-Y  
W'sS W<S

HszçzS\C E ÇE  
eqC@S<zçzS\C E ÇE  
z\è E zS\G eqb<CszçzS\Cf g  
\b@CY E \b@CYi Hszf†çzq- S^9=> ÇE> %çzq- S^g  
HszçzS\C j E zS\G eqb<CszçzS\Cf g Q z\è  
z\è E zS\G eqb<CszçzS\Cf g  
\b@CYi eqC@S<z f†çzq- S^9=> ÇEg  
eqC@S<zçzS\C j E zS\G eqb<CszçzS\Cf g Q z\è  
for S in q ^LCf c> †çzq- S^i sP- eC9c: g=  
z\è E zS\G eqb<CszçzS\Cf g  
\b@CY E \b@CYi - @€HC- z~qCf†çzq- S^9=> S: g  
HszçzS\C j E zS\G eqb<CszçzS\Cf g Q z\è  
z\è E zS\G eqb<CszçzS\Cf g  
\b@CYi eqC@S<z f†çzq- S^9=> =S j c: g  
eqC@S<zçzS\C j E zS\G eqb<CszçzS\Cf g Q z\è

a 4q←CWJi|=, Ylbçz\ ~s eqp f ebçCz ç-sbf <P ~W<- zCY eqp @-fWçfb- W'sSQ  
"W<S

HszçzS\C E ÇE  
eqC@S<zçzS\C E ÇE  
for S in q ^LCf ÇE †çzq- S^i sP- eC9c: g=  
z\è E zS\G eqb<CszçzS\Cf g  
\b@CY E \b@CYi Hszf†çzq- S^9=>=Sj c: > %çzq- S^g  
HszçzS\C j E zS\G eqb<CszçzS\Cf g Q z\è  
z\è E zS\G eqb<CszçzS\Cf g  
eqC@S<zC@çzq- S^S^L E \b@CYi eqC@S<z f†çzq- S^9=>=Sj c: g  
eqC@S<zçzS\C j E zS\G eqb<CszçzS\Cf g Q z\è

r- \ bz^ \ C^ ç-s~ sC çç YS ~UC eqpsz C@^Szf \ -Ylbçz\ ^- b4q←<P  
Jic - Jil i db <be- Wçf-^ @ ^ <P - Ylbçz\ 10× W^çç^ Pb@^bz%çç- sbf <P

~W<-zCY>zlj <CWF-@b4 ~çC^ - <CWF-@b4 f%afB C^ eq@SWC>lsb- f%Q  
 ebçzC^%UW eq \ ç%o \ G Sf sY@W eq @ ^ W sS Wç^ \ b@CY

## Jil [ CzqSW%o

Vç\ ç-sbf <P ~W<-zCY lç\ eb- S^ ^-sY@-U< \ CzqSW%o eq ebsb-<C^  
 Wf- Y%o\ eY\ C^zbf- ^ Pb W sS Wzbq- QJ:i

á VYsS Wç^ e G^bsz- eq 4S^-q^ W sS W<S~f-@ eb\ q \ G S f< bQW%o  
 WCq 4%o%eq@SWf-^%UW W@ ^ >- sWzCç^ \ Seb S S f^ \ S f< bQW%o  
 , e e-@ @SWj z^ W sS W<Cse C@ebW@C\ çf^bf-P%z @UCeb- Sz-  
 \ SWpeq \ q^ -e G^bsz  $\frac{TP}{TP+FP}$  > V@C TP UC <CWF ebçz sWzCç^ <P  
 eb S S f^ <P Pb^bz eq f C^P^%z @%o FP UC <CWF ebçz Pb^bz  
 HYC ^ eq@SWf- ^ <P UW eb S S f^ eq f C^P^%z @%o

á , z ^bsz<sup>CE</sup>UC\ q eb@b4^-e G^bsz>WCq- eq 4S^-q^ W sS W<SebQ  
 çz-eb@YsWzCç^ <P eb S S f^ <P f< bQW> WCq 4%o%seq-f^ W sS WQ  
 f-^%o, e e-@ @SWj z^ W sS W<Cse C@ebW@C\ çf^bf-P%z @UC  
 - ^- YLSW%oe SWf- f< bQ< eq \ SWpeq \ q^ b- f z ^bsz  $\frac{TP}{TP+FN}$  > V@C  
 TP UC <CWF ebçz sWzCç^ <P eb S S f^ <P Pb^bz eq f C^P^%z @%o  
 - FP UC <CWF ebçz Pb^bz HYC ^ eq@SWf- ^ <P UW ^L- zsf^ eq  
 f C^P^%z @%o

á F<sub>1</sub> sWç<-fS ^- Pb^bz-<P W sS Wç^ e G^bszS- f z ^bszS y-zb<-Q  
 fS YbS YC f%U@ Sz eb\ b< f< bQCF<sub>1</sub> = 2 \*  $\frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}$  i [ SWpeq \ q^  
 F<sub>1</sub> sWç<S Cebçz-szU - ~W<-~LC\ q çf^bf-P%o\ G S \ SWpeq \ çQ  
 ^b~ e G^bsz - \ SWpeq \ q^ f z ^bsz i

á ; bPC^ f WC' <S^z Wee- κ f eb@sz- z f%U@ ~LC> WSWj z Pb^b<C^  
 W sS Wzbqeb@f- YCe f W^ ^C ^-Pb^ W sS Wzbq WCq çx Pb@-UC  
 eb~<C^ - ~W@ çz^bszS f sWz- f< bQç f W @ z @ i

$$\kappa = \frac{p_o - p_e}{1 - p_e} = 1 - \frac{1 - p_o}{1 - p_e} \quad \text{fJi|g}$$

V@C p<sub>o</sub> UC< sW^ -Pb^bz- - p<sub>e</sub> UC bçCWf ^ -Pb^bz- i VYsS Wzbqs κ ≤ 0  
 UC ^Ceb- S CY i

á , } ; Qa; sWç\ eY<P~ eb@ W S fW~> WCq-<b4q<-~LC f< z-P \ G S  
 eq f@ eb@b4^bsz\ S seq-f^ fbs- yg- <P%4^ fbs- xg W sS W<C sWQ  
 zCç^ Pb eb S S f^ Pb f< bQW f f z ^bsz- HYC ^ ebeY<Pg TS^ \ S sYf%o  
 , } ; Qa; sWç~ç~LC s<Pbe^bsz \ b@CY çx Ybf-z \ G Sz @ \ S dç  
 @SWj z^ W sS W<S, } ; Qa; sWç UC ebçz-^ UW eq \ q<C f C^P  
 \ b ^ <P W\ 4S^-< z @ , } ; Qa; sWç\ ~s ^-4 f-z - Yseb CE>  
 -4%Wf- Yz- W sS Wç^ Pb \ b@CY 4%o eb f- bf-^ - e S U z CY b-i

-- ^LYi eq<SS^  
 c<sup>CE</sup> ^LYi ç<- Y

Ji yCszbf ^

á 3qCqfb sWqC UC WYq ç^ \ CzqW> WqC-ebç z-sz C@ Wf-@q zS W-  
 <P%4- \ GSeqC@Wf ^ \ SebszCfbq^ \ Seq f@ eb@b4^bsz \ S- bçCWQ  
 f- ^ \ SPb@bz- \ S ] CXCe Pb@bz- 3qCqf- sWqC sC qf^ -E3qCq f  
 sWqC ecp @S Wj z^ W sS Wç^ \ b@CYC f%j @ S fCzf- q

$$BS = \frac{1}{|X_{zCsz}|} \sum_{i=1}^{|X_{zCsz}|} \sum_{j=1}^{|K|} (p_{ij} - o_{ij})^2 \quad fJi\{g$$

W@C|X\_{zCsz}| < ^- \ C^ -ebç Cz f- bç W f zCszbf- < \ ^b S^ @ z- p\_{ij} < ^- \ C^ -  
 ebszCfbq^ eq f@ eb@b4^bsz > C f- bç W x\_i e- z @ b z @ %j i

$$o_{ij} = \begin{cases} 1 & \text{UCszSj} = k \\ 0 & \text{S}^ - W \end{cases}$$

dbszCfbq^ eq f@ eb@b4^bsz p\_{ij} ^ C4b p\_k(x) YC f%ebç z- z ^- < -W @  
 f q < C^ Pb @S W S^ - ç^ Pb sWqC S\_k(x) ecp UC @bz S f z @ %k \in K ebQ  
 \ b < - e S W < C ç eb^ C^ < S^ Y H^ W G

$$p_k(x) = \frac{e^{S_k(x)}}{\sum_{i=1}^{|K|} e^{S_i(x)}} \quad fJiJg$$

dçp f C P %qf Cebes- ^ \ CzqW% s f US Wb- 3qCqf- sWqC eYz > C f z  
 Pb@bz- < ^- \ C^ -Ye Wf- Yz- W sS Wzbq- i

Jiv , sYC@W% zCszbf- ^ ^- @ zbf < P s- @C<P

] CUsb- < b4q < C^ %z- 4- YW% UC @bz S f < P \ CzqW ebes- ^ < P f sCWS Ji i V- @-  
 z- 4- YW b4s- P- UC ^- \ C^ Pb@bz %b @ebf @ Uk e sY ^ \ ~ W sS Wzbq-  
 - S@C^ zS Wç^ @ UC b@ Yçp f- ^ < P @ zbf < P s- @ [ S W p e q \ q^ - W sS Wç^  
 e G^bsz ^- zq ^bf- < < P @ zC<P> \ S W p e q \ q^ - W sS Wç^ e G^bsz ^- zCq  
 zbf- < < P @ zC<P> \ S W p e q \ q^ - f z ^bsz ^- zCszbf- < < P @ zC<P> \ S W p e q Q  
 \ q^ F\_1 sWqC ^- zCszbf- < < P @ zC<P>; bPC^ f W C^ < S^ z Wee- ^- zCszbf- < < P  
 @ zC<P> 3qCqfb sWqC ^- zCszbf- < < P @ zC<P> - , } ; Q a ; sWqC ^- zCszbf- Q  
 < < P @ zC<P> sCsPb@- U ecp UC @bz S f zCszbf- ^ S e Y \ C^ z < G y b < ^- \ C^ -> C  
 S^ W q \ C^ z- Y^ W sS Wzbq ^ C- sz- e- UC @- f W b f \ ~ W sS Wzbq- < P Y C @ S W Pb @ Q  
 ^b < C^ eq C @ S W ^ < P s- P be ^bsz i TC @ Y S eb ^- \ C^ -z C W ^ W j z^ f sYC@W%  
 Usb- fCX S- fSY ^- Wf- Yz @ zbf s- @ %z j ^- W S W @ ^- @ zbf- s @ seY- UC  
 eb- @ fW% X?, ^- fsz- e^ @ z- - ^- W S W C fPb@- ecp W sS W < S \ b @ C C  
 X?, i ? b4q \ e W @ C \ UC z- 4- YW Jiv > < C W C q YC f% sz > C @ zbf- s- @  
 s S@C^ zS Wzbq \ c C | ^ C^ @ b4q- f b Y b- ecp W sS Wzbq ^- < - W @ Y C- q^  
 @ S W S S^ - ç^ - ^- Y < %o

? - Y C Usb- ~ fC@C^ % @ f z- 4- YW% Pb@bz ç- sbf < P ~ W < - zCY > ^- \ C^ < P  
 @ Y - Y b ç \ ebes- ^ < P f sCWS Ji Ji TCS W b4^bf C^ f^ S ^ szq- W- ç % ecp Q  
 4 P- UC b \ 4 P C \ ~ ç C^ \ b @ C^ > ^ C f z ç- sbf b- „ f P q “ S^ W C \ C^ z- Y Pb

WfS Wzbq ^-@-@fWfb- fCq YC<-<^- \ C^-z f z 4-YC Ji\_i ? YC ç- sbf <P f sY@W - ebeS~ @ zbf <P s @YCe C@ebW@ z> C fCSWsz ç- sbf „f Pq%oUC seblC^-s <Cwbf \ ebçz\ e <^- Wi yC^zbe C@ebW@ UC ^-sY@bf^ bf C^- s%azCzS-W@P @ zC-P f sCWSJiui

Jiu r%azCzS-W@z-

r%azCzS-W@z- lsb- ~\ Y f%aC^Cpf- ^-@z- eqp seC-S <W çC%oWCq-^CQ 4%b < sW^A- CteCq\ C^z-Y \ <\ C^- \ i, W^zC^z- X?, YC \ Y fS z b f%aCQ ^Cpf- ^ <P @ zC-P> fPb^ <P eqp WfS W-S@^ \ WfS Wç^ \ \ b@CC i, Pb@b- s%azCzS-W@P @ z lsb- ULS-P f eb@sz-z ^Cb\ G C^ çx \ ç%oCf PbQ @b- ULS-P ~\ Y-szq-W-q - ^Cs-Pbe^bsz b@q <S sY Sb- bç- ^S- <Sc-Y Pb sf z-i V^Pbf^- s<SVC ç^ ^-4 < H^WSsWC q^i @z- sCzsi \-V@<Y-ssSQ HS<-zSb^ Wf%afb C^- \-zSC@z^- <-W@ ^bç- Y Pb çx @ Y^ eqp zCszbQ f- WfS Wzbq i | GS<-W@ fsz-e^ e-q \ Czç%e-z =

á ^€s- \ eYCs E ebçz f bçWt

á ^€HC z-qCs E ebçz e <^- Wt

á ^€S^Hbç\ -zSfCE ebçz S^Hbç\ -zSf^ <P e <^- Wt

á ^€çC@-^@^z E ebçz e C4%ç^ <P e <^- W> WCq lsb- YC-ç \ S W\ 4S^- <C S^ -Pb@ f%4q^ <P S^Hbç\ -zSf^ <P e <^- Wt

á ^€çCeG zC@ E ebçz @-eYSS^ <P e <^- W> WCq lsb- ^-Pb@ f%Q 4q-^% S^Hbç\ -zSf^ <P - e C4%ç^ <P e <^- Wt

á ^€<YssCs E ebçz z @

] - <-W@ s%azCzS-W%f%aC^Cpf- ^ <P @ zbf <P s @ lC\ f%afb SY @f- Lq H%o WCq b@q- <-fSYsz ç- sbf <P ~W<-zCY ^- ebçz- e <^- W e S” †^ \ ebçz- f bç fb4q+W%Ji\ - JiJç-@-Y lC\ f%afb SY @f- Lq H%o WCq b@q- <-fSQ Ysz ç- sbf <P ~W<-zCY ^- ebçz- f bçW e S” †^ \ ebçz- e <^- W fb4q+W%o JiI - Jivg y%zb Lq H%ebeS-Uç- sbfb- „f Pq-“ S^Wç\ C^z-Y Pb WfS Wç^ Pb \ b@CY ^-@-@fWfb- fCq i db@C^- \ C^- <P Pb@bz @bW-C fCzSWY S^Q Wç\ C^z- <Ceç \ ç^ 10x <ç%PYZ ~ç^ WfS Wç^ Pb \ b@CY-i] - e W@-e S |CEE f bç <P - ICE e <^- < <P „b^Y^C X?, \ b@CYeqf-@ 5x ç%PYU zq Q ^bf-^ beçzS„b“ S^C X?, \ b@CY - 15x ç%PYU zq ^bf-^ beçzS„sVC ç^“ X?, \ b@CY-i d S{CEE f bç <P - |CEE e <^- < <P „b^Y^C X?, \ b@CYeqf-@ 7x ç%PYU zq ^bf-^ beçzS„b“ S^C X?, \ b@CY - 17x ç%PYU zq ^bf-^ beçzS„sVC ç^“ X?, \ b@CY-i ? - zbf s-@%^- WCq <P 4%b eqpfC@^b @ ^ \ C^- >4%bç%afb C^%eb\ b< e W< =

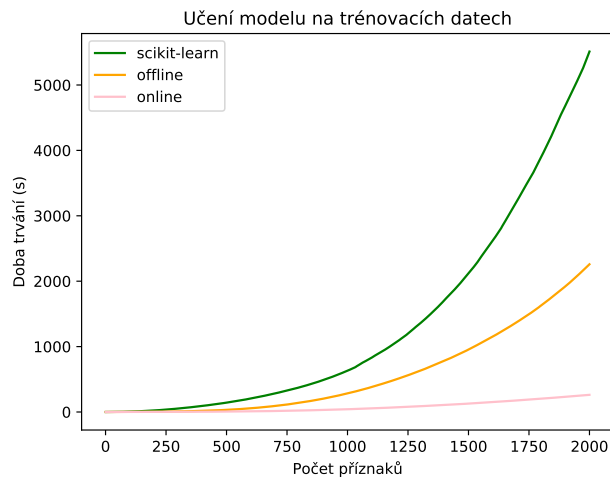
@z- sCz€HC z-qCs E \-V@<Y-ssSHS<-zSb^f^€s-\eYCsE{CEE ^€HC z-qCsE|CEE ^€S^Hbç\ -zSfCEcICE ^€çC@-^@^zE{CE

## Ji y Cszbf ^

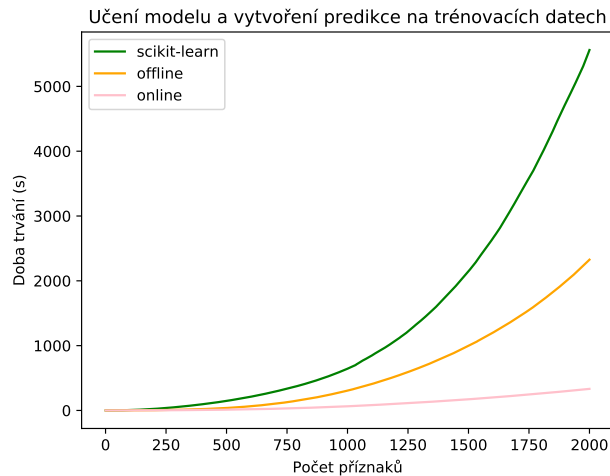
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^€qCeC- zC@EcCEE ^€<Y- ssCsEcI g  
@ z- sCz€s- \eYCs E \- VC€<Y- ssSH€<- zSb^f ^€s- \eYCsE| CCEE  
^€HC- z~qCsEI CEE ^€S^Hbq\ - zSfCEJCEE ^€qC@~^@ ^zE|I >  
^€qCeC- zC@E|I > ^€<Y- ssCsEcI g

a 4q<CWJi{=dbqpf^Δ ç-sbf <P f sY@W f~çC^ g X?, \ b@CY f <-fS^bszS  
^- \ ^b szf e <^-W

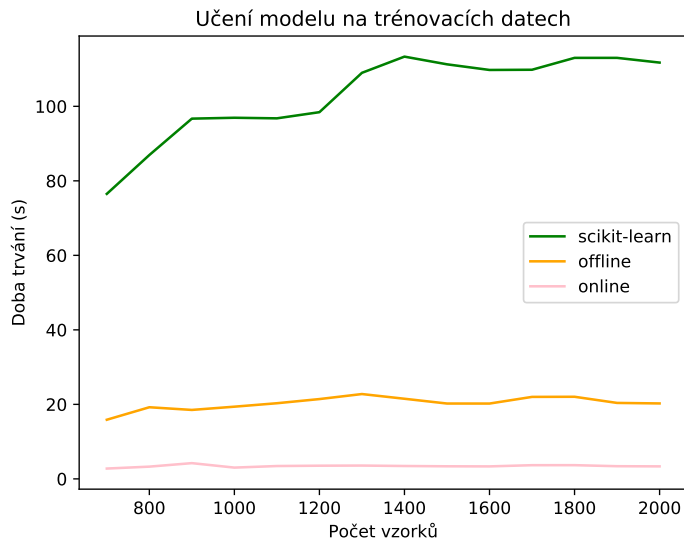


a 4q<CWJiJ=dbqpf^Δ ç-sbf <P f sY@W f~çC^ - f%afb C^ eq@SWG X?,  
\ b@CY f <-fS^bszS^- \ ^b szf e <^-W

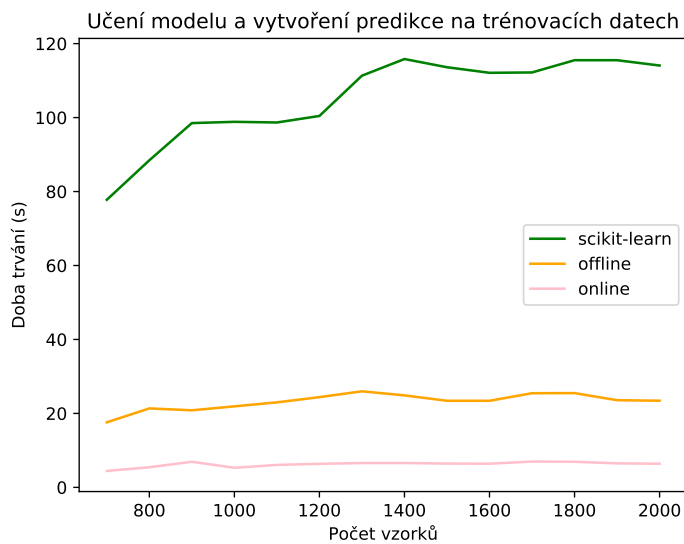


{E

a)  $f: \mathbb{R}^n \rightarrow \mathbb{R}$  je funkce  $f(x) = x^2$ ,  $x \in \mathbb{R}$ .  
 ^- \ ^b szf f< bqW



a)  $f: \mathbb{R}^n \rightarrow \mathbb{R}$  je funkce  $f(x) = x^2$ ,  $x \in \mathbb{R}$ .  
 ^- \ ^b szf f< bqW



y-4~YW Ji|= [ SWpeq \ q^—W sS Wç^ e G^bsz ^- zq ^bf < <P @ zG-P

R	] →Cf	r<SWçq^	a“ S’C	a ^S’C
cc	4-Y^<Cq<-YC	HD_Cev	HD_Cev	HD_Cev
{u	@S 4GzS	Eu_cJ{	Eu_cJ{	Eu_cJ{
Jc	LYss	Ev{Eu	Ev{Eu	Ev{Eu
IJ	fPS>Y	HDc I	HDc I	HDc I
vc	SfS	Eu_ucJ{	Eu_ucJ{	Eu_ucJ{
cDu	..S’C	ciCE	ciCE	ciCE
{ _	P-%GqzP	EvcvEi	EvcvEi	EvcvEi
JvJ	eq^ ^Es%zP	HD_cJ{	HD_cJ{	HD_cJ{
JvD	<b^” @C^<C	Eu_J	Eu_J	Eu_J
Ju	Ye~s	Eu_vvu	Eu_vvu	Eu_vvu
vD	sY-zPE C  CEI	EuD_uc	EuD_uc	EuD_uc
v_J	@SLY€z-4Y€-	ciCE	ciCE	ciCE
DuJ	q 4€c{c	Eu_J Dv	Eu_J Dv	Eu_J Dv
D_J	q 4€cJD	Eu_uD v	Eu_uD v	Eu_uD v
_v_	SfS	Eu_ucJ{	Eu_ucJ{	Eu_ucJ{
_u	..S’C	ciCE	ciCE	ciCE
_uJ	P-%GqzP	EvcvEi	EvcvEi	EvcvEi
__J	fPS>Y	HDc I	HDc I	HDc I
__u	4-Y^<Cq<-YC	HD_Cev	HD_Cev	HD_Cev
cEE	LYss	Ev{Eu	Ev{Eu	Ev{Eu
cEI	<b^” @C^<C	Eu_J	Eu_J	Eu_J
cED	UB@z€Ji €Ji{	EvJ{Jc	EvJ{Jc	EvJ{Jc
cEVE	-q	Eu_IJII	Eu_IJII	Eu_IJII
cEvc	-qJ	EuD D	EuD D	EuD D
cE	-q	ciCE	ciCE	ciCE
cE{	W	HDvDJ_	HDvDJ_	HDvDJ_
cEJ	-qv	Eu_cJ _	Eu_cJ _	Eu_cJ _
cE{	UB@z€Ji €Ji	EuCEDc	EuCEDc	EuCEDc
cEI	@z-zfC	Eu_ {CE	Eu_ {CE	Eu_ {CE
cccu	@Gp-q^ -S	HD_ _	HD_ _	HD_ _



y-4~YW Ji{=[ SWpeq \ q^—W sS Wç^ e G^bsz ^- zCzbf-<P @zCzP

R	] -<Cf	r<SWCQ q^	a " S'C	a ^'S'C
cc	4 Y^<CQ< YC	CDuuv	CDuuv	CDuuv
{u	@S 4CzS	CUJI_	CUJI_	CUJI_
Jc	LYss	CVclJ	CVclJ	CVclJ
IJ	fCPS>YC	CUlI_c	CUlI_c	CUlI_c
vc	SfS	ciCE	ciCE	ciCE
cDu	..S'C	CE_DcJD	CE_DcJD	CE_DcJD
{ _	P-%GqzP	CHJcvu	CHJcvu	CHJcvu
JvJ	eq^<€s%zP	CD vvu	CD vvu	CD vvu
JvD	<b^" @C^<C	CU u u	CU u u	CU u u
Ju	Ye~s	CUuuD	CUuuD	CUuuD
vD	sYC-zPEC  CEI	CD {{{	CD {{{	CD {{{
v_J	@SLY€z-4Y€-	ciCE	ciCE	ciCE
DuJ	q 4CEc c	CE {{{	CE {{{	CE {{{
D_J	q 4CEcJD	CD	CD	CD
_v_	SfS	ciCE	ciCE	ciCE
_u	..S'C	CE_DcJD	CE_DcJD	CE_DcJD
_u	P-%GqzP	CHJcvu	CHJcvu	CHJcvu
_J	fCPS>YC	CUlI_c	CUlI_c	CUlI_c
_u	4 Y^<CQ< YC	CDuuv	CDuuv	CDuuv
cCE	LYss	CVclJ	CVclJ	CVclJ
cCEI	<b^" @C^<C	CU u u	CU u u	CU u u
cCED	UB@S€Ji €Ji{	CH_JI_	CH_JI_	CH_JI_
cCECE	-q	CD cc	CD cc	CD cc
cCEc	-q	CE_CCE	CE_CCE	CE_CCE
cCE	-q	CV{v{v	CV{v{v	CV{v{v
cCE{	W	CU_vcD	CU_vcD	CU_vcD
cCEJ	-q	CDCEJI	CDCEJI	CDCEJI
cCE	UB@S€Ji €Ji	CUcCEJ	CUcCEJ	CUcCEJ
cCEI	@z-zc CfC	CE_ {CE	CE_ {CE	CE_ {CE
cccu	@GSP-q^~S	CEv	CEv	CEv

y-4-YW JiJ=[ SWpeq \ q^-f z ^bsz ^- zCszbf- <P @ zC<P

R	] ->Cf	r<SVC q^	a " S'C	a ^S'C
cc	4-Y^<CQ<-YC	FDuuvv	FDuuvv	FDuuvv
{u	@S4GzGs	FuJI_	FuJI_	FuJI_
Jc	LYss	FvclJ	FvclJ	FvclJ
IJ	fPS<YC	FuI_c	FuI_c	FuI_c
vc	Ss	ciCE	ciCE	ciCE
cDu	..S'C	CE_DcJD	CE_DcJD	CE_DcJD
{ _	P-%GQzP	FJcvu	FJcvu	FJcvu
JvJ	eq^<E s%zP	FD vvu	FD vvu	FD vvu
JvD	<b^" @C<C	Fu u u	Fu u u	Fu u u
Ju	Ye~s	FuuuD	FuuuD	FuuuD
vD	sYC-zPE C  CEI	FD{}}	FD{}}	FD{}}
v_J	@SLY€z-4Y€-	ciCE	ciCE	ciCE
DuJ	q 4€c{c	CE_}}}}	CE_}}}}	CE_}}}}
D_J	q 4€cJD	FD	FD	FD
_v_	Ss	ciCE	ciCE	ciCE
_u	..S'C	CE_DcJD	CE_DcJD	CE_DcJD
_uJ	P-%GQzP	FJcvu	FJcvu	FJcvu
__J	fPS<YC	FuI_c	FuI_c	FuI_c
__u	4-Y^<CQ<-YC	FDuuvv	FDuuvv	FDuuvv
cCE	LYss	FvclJ	FvclJ	FvclJ
cCEI	<b^" @C<C	Fu u u	Fu u u	Fu u u
cCED	UB@z€Ji €Ji{	FJ_JI_	FJ_JI_	FJ_JI_
cCCE	-q	FDJ cc	FDJ cc	FDJ cc
cCvc	-q	CE_CCE	CE_CCE	CE_CCE
cCv	-q	CV{v{v	CV{v{v	CV{v{v
cCv{	W	Fu_vcD	Fu_vcD	Fu_vcD
cCvJ	-qv	FDCJI	FDCJI	FDCJI
cCv{	UB@z€Ji €Ji	FrcDJ	FrcDJ	FrcDJ
cCEI	@z-zCfC	CE_ {CE	CE_ {CE	CE_ {CE
cccu	@GSP-q^-S	CEv	CEv	CEv

y- 4-YW Jil = F<sub>1</sub> sWqC

R	]-Cf	r<SVC q^	a " SC	a ^ SC
cc	4 Y^<Cq< Y	CDuvv	CDuvv	CDuvv
{u	@S 4GzS	CUJI_	CUJI_	CUJI_
Jc	LYss	CVclJ	CVclJ	CVclJ
IJ	fPS-Y	CUlI_c	CUlI_c	CUlI_c
vc	Ss	ciCE	ciCE	ciCE
cDu	..SC	CE_DcJD	CE_DcJD	CE_DcJD
{ _	P-%GqzP	CHJcvu	CHJcvu	CHJcvu
JvJ	eq^E s%zP	CD vvu	CD vvu	CD vvu
JvD	<b^" @C^<C	CU u u	CU u u	CU u u
Ju	Ye~s	CUuuD	CUuuD	CUuuD
vD	sY-zPE C  CEI	CD {{{	CD {{{	CD {{{
v_J	@SLY€z- 4Y€-	ciCE	ciCE	ciCE
DuJ	q 4CEc c	CE {{{	CE {{{	CE {{{
D_J	q 4CEcJD	CD	CD	CD
_v_	Ss	ciCE	ciCE	ciCE
_u	..SC	CE_DcJD	CE_DcJD	CE_DcJD
_uJ	P-%GqzP	CHJcvu	CHJcvu	CHJcvu
__J	fPS-Y	CUlI_c	CUlI_c	CUlI_c
__u	4 Y^<Cq< Y	CDuvv	CDuvv	CDuvv
cCE	LYss	CVclJ	CVclJ	CVclJ
cCEI	<b^" @C^<C	CU u u	CU u u	CU u u
cCED	UB@S€ Ji  € Ji{	CH_JI_	CH_JI_	CH_JI_
cCEE	- q	CD cc	CD cc	CD cc
cCEc	- q	CE_CCE	CE_CCE	CE_CCE
cCE	- q	CV{v{v	CV{v{v	CV{v{v
cCE{	W	CU_vcD	CU_vcD	CU_vcD
cCEJ	- q	CDCEJI	CDCEJI	CDCEJI
cCE	UB@S€ Ji  CE Ji	CUcCEJ	CUcCEJ	CUcCEJ
cCEI	@ z- zCfC	CE_ {CE	CE_ {CE	CE_ {CE
cccu	@Gp- q^ - S	CEv	CEv	CEv

y-4~YW Jiv=; bPC^ f WC^ <SC^z Wee- ^- zCszbf- <P @ zCP

R	] → Cf	r < S^C q^	a " S^C	a ^ S^C
cc	4 Y^ <CQ< YC	CUu vc	CUu vc	CUu vc
{u	@S 4CzS	CECE{c	CECE{c	CECE{c
Jc	LYss	CEEu	CEEu	CEEu
IJ	fCPSYC	CEuJ{u	CEuJ{u	CEuJ{u
vc	SfS	ciCE	ciCE	ciCE
cDu	..S^C	CE_u cJ	CE_u cJ	CE_u cJ
{ _	P-%GQzP	CE DvJ_	CE DvJ_	CE DvJ_
JvJ	eq^ ^€s%zP	CEI{D_	CEI{D_	CEI{D_
JvD	<b^" @C^<C	CEuJDD	CEuJDD	CEuJDD
Ju	Ye~s	CE	CE	CE
vD	sYC-zPE C  CEI	CEI D {	CEI D {	CEI D {
v_J	@SLYCEz 4YCE-	ciCE	ciCE	ciCE
DuJ	q 4CE c{c	CE CECE	CE CECE	CE CECE
D_J	q 4CE cJD	CEvcI{D	CEvcI{D	CEvcI{D
_v_	SfS	ciCE	ciCE	ciCE
_u{	..S^C	CE_u cJ	CE_u cJ	CE_u cJ
_uJ	P-%GQzP	CE DvJ_	CE DvJ_	CE DvJ_
_J	fCPSYC	CEuJ{u	CEuJ{u	CEuJ{u
_u	4 Y^ <CQ< YC	CUu vc	CUu vc	CUu vc
cCE	LYss	CEEu	CEEu	CEEu
cCEI	<b^" @C^<C	CEuJDD	CEuJDD	CEuJDD
cCED	UB@S€Ji €Ji{	CE__	CE__	CE__
cCEE	-q	CE c{  I	CE c{  I	CE c{  I
cCEc	-qJ	CEu{  u	CEu{  u	CEu{  u
cCE	-q	CE	CE	CE
cCE{	W	CE u cv	CE u cv	CE u cv
cCEJ	-qv	CE{DD_	CE{DD_	CE{DD_
cCE{	UB@S€JiCEJi	CE Jvc	CE Jvc	CE Jvc
cCEI	@z-zqCfC	CE	CE	CE
cccu	@GSP- q^ -S	CE  vcI	CE  vcI	CE  vcI

y- 4~YW Jiu=3qCpfb sWqC ^- zCzbf < <P @ zC<P

R	]-<Cf	r<SvzC q^	a " S' C	a ^' S' C
cc	4- Y^<Cq<- YC	CcIJ{v	CcIJ{v	CcIJ{v
{u	@S 4CzGs	Cc{C_	Cc{C_	Cc{C_
Jc	LYss	Cccl_	Cccl_	Cccl_
IJ	fCPS<Y	CcclCJ	CcclCJ	CcclCJ
vc	Ss	CcC u	CcC u	CcC u
cDu	..S' C	CcC_ {	CcC_ {	CcC_ {
{ _	P-%CqzP	Cc_u{	Cc_u{	Cc_u{
JvJ	eq^<€s%zP	Cc{C_	Cc{C_	Cc{C_
JvD	<b^" @C^<C	CcVJ_	CcVJ_	CcVJ_
Ju	Ye~s	Cc u u	Cc u u	Cc u u
vD	sY<-zP€C  CcI	CcJ{I_	CcJ{I_	CcJ{I_
v_J	@SLY€z- 4Y€-	CcC	CcC	CcC
DuJ	q 4C€c  c	CcC u	CcC u	CcC u
D_J	q 4C€cJD	Cc u_ID	Cc u_ID	Cc u_ID
_v_	Ss	CcC u	CcC u	CcC u
_u{	..S' C	CcC_ {	CcC_ {	CcC_ {
_u	P-%CqzP	Cc_u{	Cc_u{	Cc_u{
__J	fCPS<Y	CcclCJ	CcclCJ	CcclCJ
__u	4- Y^<Cq<- YC	CcIJ{v	CcIJ{v	CcIJ{v
cCCE	LYss	Cccl_	Cccl_	Cccl_
cCEI	<b^" @C^<C	CcVJ_	CcVJ_	CcVJ_
cCED	UB@S€Ji €Ji{	CcVl_	CcVl_	CcVl_
cCCE	- q	Cc _vcI	Cc _vcI	Cc _vcI
cCec	- qJ	Cc v_	Cc v_	Cc v_
cC	- q	Cc u u	Cc u u	Cc u u
cC{	W	Cc IIv	Cc IIv	Cc IIv
cCEJ	- qv	Cc DvcI	Cc DvcI	Cc DvcI
cC{	UB@S€JiC€Ji	Cc{{{vu	Cc{{{vu	Cc{{{vu
cCEI	@z-zCfC	Cc{JC	Cc{JC	Cc{JC
cccu	@Cp- q^ - S	Ccvc_D	Ccvc_D	Ccvc_D

y-4-YW JiD=, } ; @a; sWqC^- zGzbf < <P @ zCP

R	] -<Cf	r<SVC q^	a " SC	a ^SC
cc	4-Y^<Cq<- YC	CE_{uD	CE_{uD	CE_{uD
{u	@S 4GzG	CE D v	CE D v	CE D v
Jc	LYss	CEu D	CEu D	CEu D
IJ	fPSYC	CE_JCEJ	CE_JCEJ	CE_JCEJ
vc	Ss	ciCE	ciCE	ciCE
cDu	..SC	ciCE	ciCE	ciCE
{ _	P-%GqzP	CECEcc	CECEcc	CECEcc
JvJ	eq^ ^Es%zP	CEJDv _	CEJDv _	CEJDv _
JvD	<b^" @C^<C	CE_vvuu	CE_vvuu	CE_vvuu
Ju	Ye~s	CE{ c	CE{ c	CE{ c
vD	sYC-zPE C  CEI	CE	CE	CE
v_J	@SLYCE-z 4YCE-	ciCE	ciCE	ciCE
DuJ	q 4CEc{c	CE_Dvcc	CE_Dvcc	CE_Dvcc
D_J	q 4CEcJD	CE_Jcvu	CE_Jcvu	CE_Jcvu
_v_	Ss	ciCE	ciCE	ciCE
_u{	..SC	ciCE	ciCE	ciCE
_w	P-%GqzP	CECEcc	CECEcc	CECEcc
__J	fPSYC	CE_JCEJ	CE_JCEJ	CE_JCEJ
__u	4-Y^<Cq<- YC	CE_{uD	CE_{uD	CE_{uD
cCE	LYss	CEu D	CEu D	CEu D
cCEI	<b^" @C^<C	CE_vvuu	CE_vvuu	CE_vvuu
cCED	UB@S€Ji €Ji{	CEvcuuv	CEvcuuv	CEvcuuv
cCECE	- q	CEuCE{	CEuCE{	CEuCE{
cCEc	- qJ	CEucJ{	CEucJ{	CEucJ{
cCE	- q	CEvvuu	CEvvuu	CEvvuu
cCE{	W	CE_ I	CE_ I	CE_ I
cCEJ	- qv	CE_J I_	CE_J I_	CE_J I_
cCE{	UB@S€JiCEJi	CE_wJ	CE_wJ	CE_wJ
cCEI	@ z-zCfC	CEcul_{	CEcul_{	CEcul_{
cccu	@GSP-q^ -S	CEu{cI	CEu{cI	CEu{cI

y-4-YW Ji\_=? b4- zqf-Δ ~çC^ \ b@CY ^- zq ^bf- < P @ zC&P

R	]-<Cf	r<S&ZC q^	a " S' C	a ^'S' C
cc	4- Y ^<CQ<- YC	CRCEI {	CRCEI I	CRCEID {
{u	@S 4CzGs	CRCEI_D	CRCEuC	CRCEc{ {
Jc	LYss	CRCEcul	CRCE_C	CRCEvD
IJ	fCPS-YC	CRCEuuD	CRCEcC	CRCE{u_
vc	SfS	CRCE_	CRCE_C	CRCEI v
cDu	..S' C	CRCEu{ {	CRCEcC	CRCE{   v
{   _	P- %GqzP	CRCEID	CRCEc  J	CRCE  c
JvJ	eq ^€s%zP	CRCE_C	CRCEu_	CRCEuv
JvD	<b^" @C^<C	CRCE_C	CRCEJ { I	CRCE_C
Ju	Ye~s	CRCE_C	CRCE{ v	CRCEuu
vD	sY<-zP€C  CEI	CRCEvu	CRCE{ _c	CRCEI J
v_J	@LLY€z- 4Y€-	CRCE{   _	CRCEccc	CRCEI_J
DuJ	q 4C€c{ c	CRCEI D	CRCE_D_v	CRCE{ { _
D_J	q 4C€cJD	CRCE_C	CRCEI u	CRCEvI
_v_	SfS	CRCEI c	CRCE_C	CRCE{ _
_u	..S' C	CRCE_c{	CRCEc{ I	CRCE{   {
_uJ	P- %GqzP	CRCEI D	CRCE_C	CRCEvD
__J	fCPS-YC	CRCEu{	CRCE_C_D	CRCEJv_
__u	4- Y ^<CQ<- YC	CRCE	CRCEuc	CRCEDu
cCE	LYss	CRCE_Cv	CRCEI uc	CRCEI_c
cCEI	<b^" @C^<C	CRCE  C	CRCEI C	CRCE{ C
cCED	UB@S€Ji  €Ji{	CRCEc{	CRCEJvc	CRCEJvJ
cCE	- q	CRCEc  v	CRCEuuJ	CRCEJDu
cCEc	- qJ	CRCE_cD	CRCEI u_	CRCE{ cI
cC	- q	CRCEI Jc	CRCEI   u	CRCE{ Jc
cC{	W	CRCEI v	CRCE__D	CRCE  cD
cCEJ	- qv	CRCE_C_	CRCE_  _	CRCE_Cv
cCE{	UB@S€Ji€Ji	CRCE_C	CRCEJ   v	CRCEcu_
cCEI	@ z- zCfC	CRCEc{ c	CRCEJ_v	CRCEcu
cccu	@Gp- q^ - S	CRCEI C	CRCE{ Ju	CRCE  v

y-4~YW JicE? b4- zqf-^ f%af b C^ eq@SWC\ b@CC\ ^- zq ^bf < <P @ zC-P

R	] → Cf	r<SWC q^	a " S'C	a ^S'C
cc	4- Y ^<CQ<- YC	CRCEE	CRCEEJ	CRCEED
{u	@S 4GzS	CRCEEu{	CRCEEI	CRCEED
Jc	LYss	CRCEEc_	CRCEE	CRCEE_
IJ	fCPS>YC	CRCEE_I	CRCEE_c	CRCEE__
vc	SfS	CRCEED	CRCEED	CRCEEc
cDu	..S'C	CRCEE {	CRCEE	CRCEEJ
{ _	P-%GzP	CRCEEv	CRCEE	CRCEE
JvJ	eq^ ^Es%zP	CRCEEI	CRCEE	_CQE
JvD	<b^" @C^<C	CRCEEI	CRCEE	CRCEEc
Ju	Ye~s	CRCEEc	CRCEE{	CRCEEc
vD	sYC-zP€ C  CEI	CRCEEv	CRCEEJ	CRCEEv
v_J	@SLY€z- 4Y€-	CRCEE	CRCEED	CRCEEDu
DuJ	q 4C€c{c	CRCEEv	CRCEEv	CRCEEv
D_J	q 4C€cJD	CRCEEJ	CRCEE_	CRCEE
_v_	SfS	CRCEEI	CRCEE_	CRCEEc
_u{	..S'C	CRCEEI c	CRCEE_	CRCEEDc
_uJ	P-%GzP	CRCEED	CRCEE_	CRCEEc
__J	fCPS>YC	CRCEE_	CRCEEvJ	CRCEEvc
__u	4- Y ^<CQ<- YC	CRCEEv	CRCEEJ	CRCEEJ
cCEE	LYss	CRCEE D	CRCEEDJ	CRCEEv
cCEI	<b^" @C^<C	CRCEE_	CRCEEI	CRCEEu
cCED	UB@S€ Ji  € Ji{	CRCEEcc	CRCEEI	CRCEEI
cCvE	- q	CRCEEI v	CRCEE_D	CRCEEJ
cCvc	- qJ	CRCEEc	CRCEED	CRCEEJ
cCv	- q	CRCEEu{	CRCEEcu	CRCEEc
cCv{	W	CRCEED	CRCEE_{	CRCEEu_
cCvJ	- qv	CRCEEQ	CRCEE	CRCEEvJ
cCv{	UB@S€ Ji€ Ji	CRCEE	CRCEEc	CRCEEu
cCEI	@ z- zCfC	CRCEE	CRCEED	CRCEEv
cccu	@GzP- q^ - S	CRCEE{	CRCEE	CRCEEc



# Š-f q

OY f^ \ < Y\ z zb eq < C 4%b ^- f q^ b-z - S e Y\ C^ zbf-z @S Wj z^ S Wj CQ  
 \ C^ z-Y W sS Wzbq ^- < -W @ Y C-q^ @S Wj S^ -c^ - ^- Y < % Wj Cq 4% sC  
 @b W- Ye S- cS ^- ^bf < P e < ^- < < Pi ? - Y \ eb - @ f W\ ^- W sS Wzbq 4% b  
 W P b s- P b e^ b s z e b q @S s S s e p 4 Y \ C \ ~ Y S W Y C q s % d c p ^- e Y ^ z b P b z b  
 < Y 4% b e p f C @ C^ - ^- Y < - C t S z - U < P b @ - f W j f P b C C^ i ] - < -W @ z z b  
 - ^- Y < % 4% b ^- f q C^ - Y b q s \ ~ s C W s f^ P b b 4^ b f C^ f^ S ^ s z q - W - q % W Q  
 s S W c^ P b \ b @ C Y e b \ b < ; P b Y s W P b @ C W \ e b S C s e b - S z \ W ^ s z - ^ z ^  
 q L - Y q s - < G , ^- Y < - - ^- f q C^ S W j C^ z - Y e b s z - e U b - ^ C @ Y S U \ S  
 c - s z \ S z z b e q < C > e q z b C e b f b Y U f % j b S z f C q S W Y S W j C^ z - Y W s S Q  
 W z b q X ? , e p s z C @ S z f \ U W W Y s f f P b @^ z C P ^ b Y L S G  
 ] - s Y C @^ 4% b f % j b C^ - S e Y\ C^ z - C f U < % C d % z P b ^ - 4% b e p f C @ C^ b  
 z C s z b f - ^ z z b S e Y\ C^ z - C ^- q C - Y < P @ z b f < P s @ - P - s % z C z S - W - P @ Q  
 z C - P i , s Y C @ W % z C s z b f - ^ ~ W < ~ U > C f C q S W Y S W j C^ z - C f W ^ z C i z - X ? ,  
 \ - e b z C^ - S - Y U C S W \ C ^ - 4 @ b - z 10 x q % P Y U ~ c^ W s S W c^ P b \ b Q  
 @ C Y b e q z S @ - f W j f \ ~ - ^- Y L - e S s z C U W - Y s e q C @ S W ^ < P s - P b e^ b s z i | C Q  
 f P b @ b - f C q S W Y S W j C^ z - Y X ? , U b - e s^ e b - @ f W j C^ - f s z - e ^  
 @ z - > < @ @ ^ f C P ^ C @ b s z - z W @ - f W j f f C q C - b \ G C^ f % S W ^ < C e z -  
 q L - Y q s - < G ; Y C @ C ^ b f - ^ f W e S z b Y C c 4% b % s e Y ^ %



# 3SfYbLq " C

- ci Oap-Va, ->BYSWi pb4-sz^ \ Cz@%f @SWp\ S^-c^ -^- Y<C9^YS^Ci 3q^b> |CE\_ 9Si |CECEQE:i ? bsz~e^ <=Pzzes=wwSsi \~^Si << wzPw <\_-uw?SeYb\Wi e@H ? SeY\ bf~eq<G [ -s q^Mbf } ^SfCq Sz >d qQ @bf @C@W-HWY-i, C@b-< eq<C [ -qC Ga p3BXrV-i
- li ] , , p-yRxa, ->rS\ b^-i / b@Cq^ \ Cz@%f @SWp\ S^-c^ -^- Y<%9^Q Y^Ci 3q^b> |CE\_ 9Si |CECEQE:i ? bsz~e^ <=Pzzes=wwSsi \~^Si << w zPwPIIqH?dE^- fq-zSYbf-€sS\b^-ie@H ? SeY\ bf~eq<G [ -s q^Mbf } ^SfCq Sz >d q@bf @C@W-HWY-> sz-f \ -zC\ -zW%o sz-zS%o, CQ @b-< eq<Cp-@S ] , , p-yRXi
- li yTB] QRB] >XS t ,, BRQ R] >XbPt rORO> ^~QP-^i , ; b\ e-qs^ bH dq@S-zs^ , <<-q <%o; b\ eYtS%o-^@ yqS^S^L yS C bHyPSp^QPqC a Y@-^@] C..; YssS <-z^, YbqzP\ si / -<PS^CXG q^S^L 9^Y^Ci |CECE qci {>ci JCEsi |CE ||D 9Si |CECEQE:i Rr] cIu{QEVli ? bsz~e^ < ? a R=cCEcCE{w=cCEvCE} |J| |\_i
- li [ R, VR- [ it VR] >^ it 3, ] ? ByyR R] di , it VpBKBrVapyB>] i ; b\ e-qs^ bH\ ~Ysf- q zC <YssS Cq -^@ qCseb^sC ^bq\ -Ys-zb^s Hq e-zzCq^S^Hq\ -z^ H] pR / C-qR -LC 9^Y^Ci |CECE qci I{>ci c>si cCE ccD 9Si |CECEQE:i Rr] cCE{Dcc\_i ? bsz~e^ < ? a R=cCEcCEvwü ^C-qbS\ -LG |CECECEi CEci
- li ] r, X> [ CP\ CzK q%o] , Š [ , ] >B<LS R^fCzL-z^L sb-SQ-b^b\ S- q^V^L bH<SS S^ y~qV%o-s^L @z- C^fCbe\ C^z-^- Y%ss f? B, g-^@ Y^G q@S<q S^-z-^- Y%ss fX?, g , ^^ -Y bHa eCq zb^s pCsG q-P 9^Q Y^Ci |CED-si c cl 9Si |CECEQE:i Rr] cIu|\_{D ? bsz~e^ < ? a R=cCEcCEvscHu\_QCEuQ} wJDCE
- vi O} , ] K> [ CSXS^Lt; OB] >OsS^ ^ S O} , ] K>, GS; PC^Ltyr, R^ S^ ~i XS^G q@S<q S^-z-^- Y%ss -^@-qS <S Y^G-q Y^Cz.bqWHq LY~<b\ - @SL^bsS ~s^Ls<-^s^L YsCqebY q Czq^q-q 4Y<bq^G <b\ eC^s-zb^ \ G s-qC C^zS y-S.-^ ; PS^Gcbe~Yzb^i Kq CCs , qPS^C Hq; YQ

^S- Y- ^@ BteCq\ C^z Ya ePz- Y bVL%b^S^Ci |CECE qci |JD> ci {> si  
J{I JJc 9Si |CECECECE Rr} CE|cD|†i? bsz~e^ < ? a R=cCE cCEw  
sCEHcuQCEQc|I\_Qi

ui r} >~z^Lt XR^ - ^Lt XR} >, ^-^i a eC^QSC..P-\ -^ -<Sb^ qCbl^SSb^  
4-s@b^ S^G q@S<q\ S^-^z-^- Y%ssi / ~Y\ C@S ybbY^-^@, eeY<-zb^s  
9b^S^Ci |CE> qci uD> ci c>si uvu uD| 9Si |CECECECE Rr} c|u{uu|ci  
? bsz~e^ < ? a R=cCEcCEwscCE|QCEQc|v|uQvi

Di rR? Rk R[ ~P-\ \ -@O-\ CC@, XR-p-P\ -^t VO, |>, @SY| CP\ bb@  
d, pV>^ b-^L y-<WXBB>r~^L%b-^Li O-\ -^ G-<S YBteqCsb^ pCbQ  
L^SSb^ } s^LrzCe..SXC^G q? S<q\ S^-^z, ^- Y%SS -^@OSe@C; b^@SQ  
zb^- Yp-^@b\ GSC@si BBB yq ^s- <zb^s b^ R -LC dqb-Css^L 9b^S^Ci  
|CE|> qci |J> ci J>si c{Dv c{D 9Si |CECECECE Rr} cCEucJ\_i ? bQ  
sz~e^ < ? a R=cCEccCEwRdi |CEi |JCE{Jvi

\_i „ } >| SP-CY; itŠO, |K>XS^Lsb^Lt,, , |K>ŠP-biŠ; OpRyR |R?-Q  
fS@; itXR| >† P^Li re-qC^S^G q@S<q\ S^-^z-^- Y%SS Hqqs\ ~Y-^Cb-s  
zCz^SL HqzPCsL^S< ^<CbH LC^CsCzve-zP..: % ^@LC^CsCC-zS^i 3SQ  
bS^Hbq\ -zSs 9b^S^Ci |CE> qci |I> ci \_>si ccJI cc|c 9Si |CECECECE  
Rr} c{v|JCEi ? bsz~e^ < ? a R=cCECE{w|SbS^Hbq\ -zS<sw4zeCE\_i

cCE ^, |K>T-^LKS^VR| >T-CV..b^tV, |K>} ^K~tXBB>^ b-^LObi; bqbQ  
^- q%PG qz @SG sCbezS S-zb^ s%sz\ b^ -@ ezSfCQ Cz.bqVQ- sC@H<<%  
S^HC^<Cs%sz\ -^@ S^G q@S<q\ S^-^z-^- Y%SS f, |GR^Q?, g dCqsbQ  
^- Y-^@} 4I~Sb-s; b\ e~z^L 9b^S^Ci |CEJ> qci cD> ci v>si c{Ic c{v|  
9Si |CECECECE Rr} cvcuJ\_Cei ? bsz~e^ < ? a R=cCE cCEwscCEu\_Q  
CE{QE{uQE

cci y} |K>V- VSt; , [ d>; P-qC? i rbYq<%YC...q\ S^L-z zPCB-qzP s  
s-qH<CS^]; Bd -^@Bp, QCE@z=, S^G q@S<q\ S^-^z-^- Y%SSi Tb-qQ  
^- YbHK CbeP%SS- YpCsG q-P, z\ bsePCqCs 9b^S^Ci |CED> qci cc{> ci I>  
si c cc 9Si |CECECECE Rr} CEJDCE|ui ? bsz~e^ < ? a R=cCE cCE\_w  
|CEET?CEcvJi

c|i , yVa, ->K-4qCYt |a, ay] >V-qCY dpa VB >X-4b\ q Op? Q  
XR^V, >, Yt V, R Bp>Tb CH] a, ay] >T-^t |, XR| , >p-@b\ q  
dpa; O, ŠV, >? -fSai G-sz SaC^zS<zb^ bH4S\ S^Cq Y 4% G ^s bH  
sz-^@Q YsCqS@-<C@4q Veb..^ seCzqps<be%oS^L S^G q@S<q\ S^-^z  
-^- Y%SS -^@-qS<S Y^C-q Y^Cz.bqVi reCzpb<PS S<, < Qd-qz 3  
, zb\ S-reCzpb<be%oS^S^Ci |CE|> qci u{>si c v 9Si |CECECECE Rr}  
CEDJDI Jui ? bsz~e^ < ? a R=cCEcCEw|s-4i |CE|i CEi CECE

c{i O^ p? XB>, bYL-^L V-qYrR| , p>X bebYai ? S<q\ S^-^z, ^- Y%SSi R=  
, eeY@/ ~Ysf-qzCrz zszS- Y, ^- Y%SS 9b^S^Ci Ji f%ai 3CqS>OCSCQ  
4CqL=rec^LQc|CE|>si JCE J|I 9Si |CECECECE Rr} |\_uDQ Qv|QI cucQ  
ui ? bsz~e^ < ? a R=cCEcCEw\_uDQ Qv|QI cucQui

cJi p, ?O, VpR^O], p, a>; i ? S<q\ S'-zbc%a ^- Y%ss fR^C^zS <- zS^g  
R'=XS^G q r z zS zS- YR^Hq^<C- ^@Ss, eeY<- zS^si |i f%a } r, =TbP^  
, SC%o rb^s>|CEE>si JCI J|Ii., SC%a CqCs S' dcb4- 4SS%o ^@r z zS Q  
zS si R 3] \_uDQEQucQcDul Qi

cIi pRlXB^ >3i ? i 3-%G q Ys Hbq Wb..^ @SzqS4-zS^si R'=d-zzCq^ pCbQ  
L^Sb^ - ^@] C-q Y] Cz.bqM; - \ 4q@LC} ^S@VS^L@b\ =; - \ 4q@LC  
} ^SfCpS%dcqSs>c\_v>si cD |vi R 3] CE|cQvCEvQi

cvi GpRB? [ , ] >TCp\ C Oi pCL~YqC@ @S<q\ S'-^z ^- Y%ssi Tb-q^- YbH  
zPC, \ Cq<- ^ rz zS zS- Y, sss<S zS^ 9^Y^Ci c\_D\_> qci DJ> ci JCE> si  
cvi cul 9Si |CECEQE:i Rr] cI{u|u†i ? bsz~e^ < ? a R=cCE cCEw  
CEv|cJI\_ic\_D\_icCEuDuI |i

cui ?} ?, >pSP-q@a it O, py>dCzCqBit rya pV>? - fS@Ki [ ~YsYC? S Q  
<q\ S'-^z, ^- Y%ssi R'=d-zzCq^ ; YssS' <zSi |i f%a ] C..^ bqW., SC%Q  
R'zCq<S^<C>|CEEsi Ju Ici R 3] \_uDQEQucQEvv\_QE

cDi pRlXB^ >3i ? i XS^G q ? S<q\ S'-^z, ^- Y%ssi R'=d-zzCq^ pCbL^Sb^  
- ^@] C-q Y] Cz.bqM; - \ 4q@LC} ^S@VS^L@b\ =; - \ 4q@LC} ^SfCQ  
sS%dcqSs>c\_v>si \_c c|ci R 3] CE|cQvCEvQi

c\_i XS^G q - ^@k ~- @q zS ? S<q\ S'-^z, ^- Y%ss ...SP <bf- qS ^<C CYesh@  
9^Y^Ci |CEE Q|CE\_>s<SVZQC q^ @CfCbeCq f3r? XS^C^sG 9Si |CECEQE  
c|i ? bsz~e^ <=Pzzes=wss<SVZQYC- q^i bqlwsz- 4YCW ~zbEC†- \eYCSw  
<Y- ssSH<- zSb^weYbzEY@ €I @ i Pz\Yi

|CE O, ryRB>yCfbq yRrORp, ] R pb4Cqt GpRB? [ , ] >TCp\ G XS^G q  
? S<q\ S'-^z, ^- Y%ssi R'=yPCBY^ C^z bHrz zS zS- YXC q^S^L=? - z  
/ S^S^L>R^Hq^<C- ^@dq@S zS^ 9^Y^Ci |i f%a ] C..^ bqW reqS^LCq  
|CE>si cCE cc{ 9Si |CECEQE:ci reqS^LCq rCqCs S' rz zS zS si R 3] \_uDQEQ  
DuQJDI Dqi ? bsz~e^ < ? a R=cCE cCEw\_uDQEQ DuQJDI Dqi

|ci sVC q^i@S<q\ S'-^z€- ^- Y%ssi XS^G q? S<q\ S'-^z ^- Y%ss 9^Y^Ci |CEE  
Q|CE\_>s<SVZQC q^ @CfCbeCq f3r? XS^C^sG 9Si |CECEQE|:i ? bsz~e^  
<=Pzzes=wss<SVZQYC- q^i bqlwsz- 4YCW\b@- YCSwLC^Cq zC@vsVWC- q^i  
@Ss<q\ S'-^z€- ^- Y%ssi XS^G q? Ss<q\ S'-^z, ^- Y%ssi Pz\Yi

|i XB? a Ry>a YfCq „ a XG> [ SP-CY Ob^C%oRrPq^WzPC r-\ eY; bQ  
f- qS ^<C [ -zq†i yPC Tb-q^- YbHdbq†Hb / - ^- LC^ C^z 9^Y^Ci |CEE>  
qci J>ci {CEsi ccEcc\_ 9Si |CECEQE:i ? bsz~e^ < ? a R=cCE |c\_{w  
ssq^i J}{DJCE

{i ?} ?, >pSP-q@a it O, py>dCzCqBit rya pV>? - fS@Ki [ S^S ~\ rl~Q  
q@Bqpdq<C@-qsi R'=d-zzCq^ ; YssS' <zSi |i f%a ] C..^ bqW., SC%Q  
R'zCq<S^<C>|CEEsi |D {ui R 3] \_uDQEQucQEvv\_QE

|Ji GR^OBp>pb^@, %X Cq yPC} sCbH| ~YsYC [ G s-q\ C^zS S' y- †bQ  
^b\ S dcb4Y^ si, ^^ Y bHB~LC^Ss 9^Y^Ci c\_{v>ci u>si cu\_ cDD 9Si  
|CECEQE:i ? bsz~e^ < ? a R=cCEccccwU cJv\_QcDEi c\_{vi z4CEc{ui †i

|Ii V, XR , >T ^t , , XB] y, >Š@C^ WyB33B] r>T-qC^ ? ~S^zCq ; b\ Q  
e~z zB^ bHpCL~YqC@XS^G q? S<q\ S^- ^z, ^- Y%Si ; b\ e~z rz zi R^zi  
; b^H 9^ Y^Ci |CEI>si c|D c{| 9Si |CECEQE:i ? bsz-e^ <=Pzze=ww  
.....i <si <- si << w@S^zUCqzC44C^sve~4sw b\esz- z|CEJi e@H

|vi ^, kR ] K} atypB, ap O, ryRt pa 3Bpy yRrOrp, ] R pCL~YqCQ  
<@? S<q\ S^- ^z, ^- Y%SS - ^@Rs, eeY<- zB^ S [ S<qb- qf %ai 3Ssz- zS^Q  
zS^S 9^ Y^Ci |CEE>qci c>ci c>si c cD9Si |CECEQE:i ? bsz-e^ < ? a R<-  
cCEccI w|CEI wu\_u|DE

|ui Ob... zb ? q... BYesC bH; bf qS ^<C | -zqf 9^ Y^C 9Si |CECEQE:i  
? bsz-e^ <=Pzzes=ww<bbVScqb4bzS<si <b\wCEIwi

|Di XB? a Ry>a YfScq ,, a XG>| SP- OY R eqfC@ GzS\ - zB^ bHzPC <bf qCQ  
- ^<C \ - zqf bHszb<Wqz~q^s ..SZP - ^ - eeY<- zB^ zb ebqzHYb sCC-zB^i  
Tb~q^ - YbHB\ eSf<- YGS^ - ^<C 9^ Y^Ci |CEE> qci I>ci cE si vCE v|c  
9Si |CECEQE:i Rr] CE|ul{ \_D ? bsz-e^ < ? a R<- cCE cCEvwrCE|uQ  
I{ \_Df CEgCEEEQE

|\_i / bq\ - Y- ^@rPqS^ WLC XS^G q ? S<q\ S^- ^z, ^- Y%SS Hbq <YssS^ <- zB^  
9^ Y^Ci |CEI Q|CE\_>s<SVZQC q^ @CfCbeCq f3r? XS^C^sG 9Si |CECEQE  
cl:i ? bsz-e^ <=Pzzes=ww<SVZQYC q^i bqLwsz- 4YCW ~zb€Cf- \eYCsw  
<Y- ssSH<- zSb^weYbz€Y@ i Pz\Yi

{CE d, ] K>rP-b^S^Lt aŠ, ,, , >rCS^PS V, r, 3a, >] SWYi R^<q\ C^z YSQ  
^G q@S<q\ S^- ^z- ^- Y%SS Hbq <YssS^ <- zB^ bH@z- szqC \ si BBB yq ^si  
r%zi / - ^i ; %Cq^i 9^ Y^Ci |CEE> qci {I>ci I>si \_CE \_cJ 9Si |CECEQE  
CEi Rr] cCEJJc\_i ? bsz-e^ < ? a R<- cCEccEwyr[ ; 3i |CEI DJuuJi

{ci ; O) >? CS^tXR a>XS^PS ] K>| SP- CYV..bWdbt ,, , ] K>† S b%o^i R^Q  
<q\ C^z YXS^G q? S<q\ S^- ^z, ^- Y%SS=, G-sz, YbqSP\ - ^@; b\ e- qCQ  
sb^si BBB yq ^s- zB^s b^ ] C-q Y] Cz.bqW^ - ^@XC q^S^Lr%z\ s 9^Q  
Y^Ci |CEI> qci |v>ci cc>si |ucv |u{I 9Si |CECEQE:i Rr] |cv|{|DI  
? bsz-e^ < ? a R<- cCEccEwy|] Xri |CEI |{ \_c|CEi

{|i VpR^O] , [ a apyO^ >, q fS^@Pt [ B] a ] >? CE- W [ - zqf S^fCpSb^  
~s^L ; PbYGSW%@C^b\ ebsSb^i rL^ - Ydq<Css^L Q, YbqSP\ s>, qPSQ  
zCz-qS>, qf ^LC C^zs> - ^@, eeY<- zB^s ; b^Hq^<C dq<C@S^Ls>r d,  
9^ Y^Ci |CE{>ci {>si uCEu] 9Si |CECEQE:i Rr] |{|vCEv|i ? bsz-e^  
<=Pzzes=ww q†Sfi bqLwe@Hcccci JcJi e@H

{|i p, r; OV, >rC4- szS ^i XS^G q? S<q\ S^- ^z, ^- Y%SS=3S4%3S 9^ Y^Ci  
|CE{ QCEf C4- szS ^ p- s<PW>|CEJ 9Si |CECEQE:i ? bsz-e^ <=Pzzes=  
wwsC4- szS- ^q- s<PW i <b\w qzS<YCs w|CEJ €e%zPb^€Y@ i Pz\Y^ Y@ Q  
S^Q QszCesi

{Ji / CqfS - ^@s<bqS^L=l~ ^zHSL zPCl~ Y%bHeqC@S zB^s 9^ Y^Ci |CEI  
Q|CE\_>s<SVZQC q^ @CfCbeCq f3r? XS^C^sG 9Si |CECEQE:i ? bsz-e^  
<=Pzzes=ww<SVZQYC q^i bqLwsz- 4YCW b@- YCs w b@CY€Cf- Y-- zSb^i  
Pz\Yi

d YbP- ,

---

**rG ^- \ eb~ Sz <P <Wq zCW**

?, ?SWP S-ç^ -^-Y<-

X?, XSG-ç^ @SWP S-ç^ -^-Y<-

k?, Vf-@q zSW @SWP S-ç^ -^-Y<-

p?, pL~Yçbf-^-@SWP S-ç^ -^-Y<-

d; , , ^-Y<- PYf^ <P W eb^Cz





