

I. IDENTIFICATION DATA

Thesis name:	DSOS and SDSOS Optimization for Strategic Games
Author's name:	Tomáš Votroubek
Type of thesis :	master
Faculty/Institute:	Faculty of Electrical Engineering (FEE)
Department:	13136 Department of Computer Science
Thesis reviewer:	RNDr. Michal Červinka, Ph.D.
Reviewer's department:	Institute of Economic Studies, Faculty of Social Sciences, Charles University; Institute of Information Theory and Automation, Czech Academy of Sciences

II. EVALUATION OF INDIVIDUAL CRITERIA

Assignment	extraordinarily challenging
<i>Evaluation of thesis difficulty of assignment.</i>	
<p>The thesis involves an advanced topic of game theory, including semialgebraic games, and challenging classes of optimization problems such as semidefinite programming problems. The author of the thesis proposed a nontrivial modification of existing numerical methods and illuminates the performance a new solution technique on multiple academic examples. It is fair to say that I am not fully familiar myself with the regular standards on the extent of the original work in master's theses at FEE. However, as I could imagine that should the author have the desire to do so, part of his thesis involving the original material could be published in a suitable high quality international journal, I view this exceptionally challenging assignment completed in an outstanding fashion for a master's student.</p>	

Satisfaction of assignment	fulfilled
<i>Assess that handed thesis meets assignment. Present points of assignment that fell short or were extended. Try to assess importance, impact or cause of each shortcoming.</i>	
<p>I can safely confirm that all three items in the Guidelines section of the author's Master's Thesis Assignment are fully resolved in the final version of the thesis.</p>	

Method of conception	correct
<i>Assess that student has chosen correct approach or solution methods.</i>	
<p>The author first provides a rather too short introduction to two-player zero-sum games and polynomial games, and also, to my slight disappointment, provides only limited exposure to highly nontrivial concepts of the solution techniques involving sum-of-squares (SOS) decompositions, Putinar's Positivstellensatz, Lasserre hierarchies, moment an localization matrices, diagonally-dominant SOS (DSOS) and scaled diagonally-dominant SOS (SDSOS). As such, a reader with medium level of prior exposure to this topic would not be able to follow the author's line of thought and would not understand most of the development in the early parts of the thesis; and I believe that the first, roughly 20 pages of the manuscript could have been easily extended to provide much more smooth introduction to the current state of knowledge and results within this topic which would definitely benefit the reader (even the one familiar with those concepts). On the other hand, the main contribution of the author is presented in Chapters 4 and 5 which are more than satisfying.</p>	

Technical level	A - excellent.
<i>Assess level of thesis specialty, use of knowledge gained by study and by expert literature, use of sources and data gained by experience.</i>	
<p>The student demonstrates in the second part of the thesis (Chapters 4 and 5) the ability to apply the theoretical concepts, create computer codes for the proposed novel modification of the existing solution method, and provides illuminating comparison of the (successful) numerical performance of the new method with a few previously existing methods, most of which indeed fail to recover the solution of the considered examples of polynomial games.</p>	

Formal and language level, scope of thesis	D - satisfactory.
<i>Assess correctness of usage of formal notation. Assess typographical and language arrangement of thesis.</i>	

The more I am impressed with the technical level of the thesis and the scope of the contribution, the more I am disappointed by numerous notational, typographical and formal shortcomings of the thesis. I proceed with the list in the order of the text of the manuscript:

1/ the author confuses the reader with a fluent switch between the convention of vectors being rows and vectors being columns. This can be seen as early on page 3 (e.g. confront the expressions on page 3, line 9 from above with those on page 3, line 4 from below). In some cases, the reader can deduce the correct meaning but in some cases, this causes serious troubles to follow the author, e.g. in the case of the objective in "Semidefinite program 3" (Mom. Opt.) on page 11, where the result of the "product operation" could indeed be scalar as well as matrix.

2/ the author have many notational typos in the presented mathematical expressions, e.g. on page 3 line 7 from above, the characters for transpose should in fact have been stars. Similarly, on page 12, line 9 from below, the author minimizes with respect to variables with a dot superscript which are left unexplained.

3/ the author introduced an unnecessarily large list of notation, which could have been easily reduced, e.g. in Chapter 2, there is no reason to use functions p and f which both play the same role. The confusion caused by the author is magnified by occasional use of footnotes for which, I suppose, the author had only the good intentions, however, serve as a huge inconsistency of built material. That is, in some cases, the author repeats the meaning of a particular mathematical notation multiple times (e.g. footnote 13 on page 11 "introduces" notation for the coefficient vector, but its first appearance in the text can be found on page 9, line 11 from above, to which one can find footnote 10 on page 9; similarly with the notation for symmetric matrices – footnote 9 on page 9 and footnote 15 on page 14), while at the same time, occasionally provides a footnote which in fact does not explain the introduced term (e.g. footnote 1 on page 19 provides details to "Cholesky factor" but does not explain the introduced "Cholesky-like factor").

4/ in some cases, the author modifies the original statement to which he provides a proper reference. In particular, the formulation of Theorem 3 in the original source manuscript is formulated differently and the nontrivial difference in the formulation should have been explained and/or proved. On the other hand, to several of the claims in the text the author should have had provided a proper reference. In particular, I would prefer such a reference to statements e.g. on page 3, lines 1-3; on page 9, lines 11-9 from below and on page 12 lines 11-13 from above; and a reference e.g. to proof of Lemma 2 on page 19.

5/ one of the major formal troubles of the manuscript is the referencing system to previous results, expressions and sections of the text, which is an absolute mess. It took me nearly 20 pages of further text to realize that the reference to the "Gram-matrix method (3.1)" (cf. page 12, line 10 from above) actually refers to a specific (but precisely undefined) paragraph in the text of subsection 3.1 (hence (3.1)); and in the light of such unfortunate referencing system, the reference to "the previously discussed relaxations (3.1, 3.1)" is in fact not a typo (but just a confusing and useless note for the reader). These confusing references go on throughout the rest of the manuscript, e.g. "Minimax Theorem (1)" on page 12, line 7 from below; "algorithm 3.3" on page 14, line 3 from above; "optimal solution of 6" on page 15, line 9 from below; "algorithms (3.4, 3.3)" on page 15, line 2 from below; "hierarchy (3.4)" on page 16, line 9 from above; "polynomial (3.3)" on page 17, line 7 from above; "idea 3.3" on page 18, line 1 from above, to name a few. On the other hand, I believe "techniques from sections 3.3, 3.4, 3.5, 4.1 and 4.1" on page 22, lines 1-2 from above is indeed a typo.

6/ to present Definition 1 on page 18 after so many non-trivial mathematical concepts explained in the text or footnotes points out to further inconsistency of the overall manuscript.

7/ for some reason unknown to me, the author did not choose the presentation of the numerical results on selected academic examples in sections 5.1 and 5.2 in the same order, again, causing an unnecessary inconsistency of the overall manuscript.

8/ I do not understand the role of a subsection "Polynomial games with prescribed unique solutions" on pages 35 and 36, which is only after the main presentation of the authors numerical experiments and just prior a concluding mini-section 5.3. I would definitely present it either in some earlier part of the manuscript or consider it as an Appendix B.

Selection of sources, citation correctness**C - good.**

Present your opinion to student's activity when obtaining and using study materials for thesis creation. Characterize selection of sources. Assess that student used all relevant sources. Verify that all used elements are correctly distinguished from own results and thoughts. Assess that citation ethics has not been breached and that all bibliographic citations are complete and in accordance with citation convention and standards.

To best of my knowledge, the author used all relevant sources for a thesis on this particular topic and in most cases, uses the references on proper places in the text (see the previous comments for more details on my use of "in most cases"). On the other hand, the list of references on pages 45 and 46 contains serious flaws and I counted only 6 out of 17 provided references which contain full citation record and are presented in a proper citation standard. Items number 2, 3, 4, 7, 8, 10, 11, 12, 13, 14 and 15 are incomplete or not up-to-date. To illuminate on one of the remaining references, e.g. item number 2, paper by Ahmadi and Majumdar, was published in 2019 (not 2017) in SIAGA Vol 3, number 2, pages 193-230.

Additional commentary and evaluation

Present your opinion to achieved primary goals of thesis, e.g. level of theoretical results, level and functionality of technical or software conception, publication performance, experimental dexterity etc.

I would like to use this section of the evaluation form to present some further comments and objections which do not have a form of a question which could be addressed by the author during the defense:

1/ I would have welcomed the definition of Henkel and generalized Henkel matrices. Either way, I completely disagree with footnote 11 on page 10, and I do not see the similarity to a Hankel matrix. The ascending skew-diagonals from left to right are not constant at all, as can be seen easily on the third such skew-diagonal (1;2;1)! This part of the manuscript was exceptionally confusing to me.

2/ I was rather disappointed by an abrupt conclusion of the thesis in which the author could have indeed summarized for the reader the whole manuscript, highlighted the main achievements, stressed out to the reader the impact of the achieved results in further applications (i.e. to whom and how it is a useful investment of time to read this thesis) and possibly included some recommendations on further development in the field of polynomial games.

III. OVERALL EVALUATION, QUESTIONS FOR DEFENSE, CLASSIFICATION SUGGESTION

Summarize thesis aspects that swayed your final evaluation. Please present apt questions which student should answer during defense.

Questions which could be addressed during the defense

1/ In Definition 2 (page 19) you introduce scaled-diagonally-dominant matrices of the form DMD (cf. Definition 2 for details). Why is it not enough to use a "simpler" form MD? Would such a form make any difference for the proposed numerical experiments? Does, from the coding perspective, the form DMD have any advantage over the MD form of a scaled-diagonally-dominant matrix?

2/ The presentation of the pseudocode of Algorithm 1 (page 21) contains "initial guess" steps 2 and 3, where one selects a random starting points. To what extent does the particular initial guess influence convergence to the solution? Did you try to mitigate the risk of selecting "a particularly bad" starting point which would result in either convergence to an undesired local solution or saddle point, or in an uncharacteristically slow convergence rate?

3/ if the Double Oracle Algorithm is the main original contribution of the author, why does the section 5.2 contain less numerical results (fewer academic examples) than section 5.1?

4/ In example 1. (examples on simplices) on page 25 the reported solution does not seem to be correctly specified. In particular, my question is, how is it possible that for the μ -star component the weights add up to approximately 0.9 instead of 1? Similarly, in example 1. (examples on boxes) on page 26, where the weights of the μ -star component of the solution add up to 0.973 instead of 1; and in the first bullet (two semialgebraic examples) on page 34, where the weights of μ component of the solution add up to 0.8 instead of 1.

Overall evaluation

As an external reviewer without any experience with teaching at FEE, I rather refuse to comment on appropriateness of this topic with respect to the usual knowledge of a master student at FFE, but I suspect that the author must have self-study quite a large extent of the literature on this topic. The presentation of the pseudocode in section 4.3., further details to the original codes in Appendix A, and detailed comments on the outcomes of the author's own numerical experiments convince me of a contribution of this thesis being definitely on very high level.

However, as I have to take into account also the readability of the text and the author's ability to write his ideas and results in a reproducible way for a potential reader, in the light of my comments above, I evaluate handed thesis with classification grade **C - good**.

Date: **30.8.2020**

Signature: