Development of methodology for vehicle parameters identification

Prague 2020
I. Personal and study details

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<td>Cybernetics and Robotics</td>
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</table>

II. Bachelor’s thesis details

**Bachelor’s thesis title in English:** Development of methodology for vehicle parameters’ identification

**Bachelor’s thesis title in Czech:** Vývoj metodologie pro identifikaci jízdních parametrů vozu

**Guidelines:**

1. Familiarize with vehicle dynamics simulator “Live for Speed”.
2. Adopt low-fidelity and high-fidelity vehicle models.
3. Suggest suitable procedure for identification of parameters of:
   a. the rigid body;
   b. tires;
4. Provide validation of identification procedure on different car models implemented in the simulator.

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III. Assignment receipt

The student acknowledges that the bachelor’s thesis is an individual work. The student must produce his thesis without the assistance of others, with the exception of provided consultations. Within the bachelor’s thesis, the author must state the names of consultants and include a list of references.

Date of assignment receipt  Student’s signature
Acknowledgements
I would like to say a special thank you to my supervisor, Ing. Denis Efremov. His support, guidance, and overall insights in this field have made this an inspiring experience for me. And my biggest thanks to my family for all the support you have shown me through this research, the culmination of three years of distance learning. I am also grateful to the Department of Control Engineering, Faculty of Electrical Engineering, Czech Technical University in Prague. I am incredibly thankful and indebted for sharing expertise, and sincere and valuable guidance and encouragement extended to me.

I hereby declare that I have completed this thesis with the topic ”Development of methodology for vehicle parameters identification” independently and that I have included a full list of used references. I have no objection to the usage of this work in compliance with the act §60 Zákona č.121/2000 Sb. (copyright law).

In ................. date ................. signature of the author

Klíčová slova: "Life for Speed", single-track, twin-track, identifikace, identifikace parametrů pneumatik

Abstract: This thesis’s general goal is to create a possibility to test controllers in the "Life for Speed" vehicle simulator environment that opens the door to develop control algorithms for drive-by-wire vehicles. Completing this goal are adaptation vehicle models, creating tests for identification of the model’s parameters, and identification of model’s parameters. Single-Track and Twin-Track models were chosen as vehicle models. This work uses an adopted Single-Track model because it has sufficient fidelity to describe lateral vehicle dynamics and to test identification methods because it is simpler than the Twin-Track models (three versus ten states). Adopted Twin-Track was used because it allows testing identification on the more complex model (ten states) with more complicated physics. This thesis includes an adaptation of existing Single-Track and Twin-Track models for compatibility with signals from LFS, guide how to generate data for identification, creating and/or adapting formulas for calculating model parameters, and comparisons between each model and LFS. They were also created presets for generating needful data, automatic initialization files, and mat-files that calculate parameters for models.

Keywords: "Life for Speed", single-track, twin-track, identification, tyre identification
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1. Introduction

Technology has the power to do many things, and changing our world for a better place is one of them. We are privileged to be living in a time where science and technology can assist us, make our lives easier and safer, change the ways we think and solve daily problems. The technology we were already exposed to and accustomed to has paved the way for us to innovate further, and one of those technologies is a drive-by-wire technology.

At the moment when I heard that there is a chance to contribute to something that will bring the future closer, I realized that I could not lose this opportunity. This thesis is a part of a bigger project of adopting a vehicle dynamics simulator, ”Life for Speed,” for needing of automatic control department. The general goal of the team is to develop control strategies for the car of the future with fully drive-by-wire technology.

The general goal of this thesis is to create a possibility to test controllers in the ”Life for Speed” environment, that opens the door to develop control algorithms for drive-by-wire vehicles. Completing this goal are adaptation vehicle models, creating tests for identification of the model’s parameters, and identification of model’s parameters. Single-Track and Twin-Track models were chosen as vehicle models. It includes the Single-Track model because this type of model allows a physically plausible description of vehicles’ driving behavior without significant modeling and parameterization efforts. This fact means that a Single-Track is a perfect choice for the development of controllers. The Twin-Track model was chosen because of complexity. This complexity allows test developed controllers on the advanced model.

1.1 Outline

This work is divided into 8 parts.

In the first two parts, which are Introduction and Objectives, work description and goals are stated.

The 3th and 4th parts, which are Nonlinear Single-Track Model and Nonlinear Twin-Track Model introduces used vehicle’s and tire’s models.

In part, which is named Model identification, identification process for Single-Track and Twin-Track rigid bodies and tires is described.

The 6th chapter, which is Comparisons between LFS and Models, presents to reader’s attention a comparisons between various parameters from LFS and models.
The next part, **Results**, lists reached goals.

The last part, **Conclusion** summarises this thesis.
2. Objectives

The primary objectives of this thesis are:

- Familiarize with vehicle dynamics simulator ”Live for Speed” [1] (in text will be used abbreviation LFS).
- Adopt low-fidelity (Single-Track) and high-fidelity (Twin-Track) vehicle models
- Create experiments which generate data for lateral motion
- Create experiments which generate data for longitudinal motion
- Suggest suitable procedure for identification of parameters of:
  - the rigid body;
  - tires;
- Provide validation of identification procedure on different car models implemented in the simulator.
3. Nonlinear Single-Track Model

3.1 Main model

Nonlinear Single-Track model is adopted Single-Track model from [2]. Vehicle coordinate system used in this thesis is shown in Figure 3.1. It is the conventional right-hand Cartesian coordinate system. The $x$ axis follows from the center of gravity to the front of the vehicle. The $y$ axis goes towards the left side of the car, and $z$ axis lies from the center of gravity to the top of the vehicle. The vehicle’s yaw has a positive angle increment while turning to the left.

![Figure 3.1: The vehicle coordinate system [2]](image)

![Figure 3.2: The single-track model [2]](image)
The Single-Track model (Fig. 3.2) has 3 degrees of freedom and is used to represent the planar translation and yaw motion of a vehicle and consists of 3 states and 4 inputs, which are listed in Table 3.1. All parameters used in the model are presented in Table 3.2. The block diagram of the nonlinear single-track model is shown in Figure 3.3.

Table 3.1: States and inputs of the Single-Track

<table>
<thead>
<tr>
<th>State/Input</th>
<th>Symbol</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity of CG</td>
<td>$v$</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>Side-slip angle</td>
<td>$\beta$</td>
<td>rad</td>
</tr>
<tr>
<td>Yaw rate</td>
<td>$r$</td>
<td>rad s$^{-1}$</td>
</tr>
<tr>
<td>Steering angle of the front axle</td>
<td>$\delta_f$</td>
<td>rad</td>
</tr>
<tr>
<td>Steering angle of the rear axle</td>
<td>$\delta_r$</td>
<td>rad</td>
</tr>
<tr>
<td>Angular velocity of the front wheel</td>
<td>$\omega_f$</td>
<td>rad s$^{-1}$</td>
</tr>
<tr>
<td>Angular velocity of the rear wheel</td>
<td>$\omega_r$</td>
<td>rad s$^{-1}$</td>
</tr>
</tbody>
</table>

Table 3.2: Parameters of the Single-Track

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle mass</td>
<td>$m$</td>
<td>kg</td>
</tr>
<tr>
<td>Yaw moment of inertia</td>
<td>$I_{yaw}$</td>
<td>kg m$^{-2}$</td>
</tr>
<tr>
<td>Front axle-CG distance</td>
<td>$l_f$</td>
<td>m</td>
</tr>
<tr>
<td>Rear axle-CG distance</td>
<td>$l_r$</td>
<td>m</td>
</tr>
<tr>
<td>Radius of wheels</td>
<td>$p$</td>
<td>m</td>
</tr>
<tr>
<td>Aerodynamic drag coefficient</td>
<td>$c_{air}$</td>
<td>-</td>
</tr>
<tr>
<td>Air density</td>
<td>$\rho$</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>Frontal area of the car</td>
<td>$A$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Shaping coefficients for lateral dynamics</td>
<td>$c_{D,y}$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_{B,y}$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_{C,y}$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_{E,y}$</td>
<td>-</td>
</tr>
<tr>
<td>Shaping coefficients for longitudinal dynamics for throttling</td>
<td>$c_{D,x,t}$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_{B,x,t}$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_{C,x,t}$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_{E,x,t}$</td>
<td>-</td>
</tr>
<tr>
<td>Shaping coefficients for longitudinal dynamics for breaking</td>
<td>$c_{D,x,b}$</td>
<td>-</td>
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<tr>
<td></td>
<td>$c_{B,x,b}$</td>
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<tr>
<td></td>
<td>$c_{C,x,b}$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_{E,x,b}$</td>
<td>-</td>
</tr>
</tbody>
</table>
3.1.1 Used Assumptions and Simplifications

The nonlinear single-track model is used to describe planar vehicle motion consist next simplifications [3]:

- Vehicle motion include only lateral, longitudinal motions and yawing.
- Vehicle mass is concentrated at the center of gravity.
- Tires are represented as two tires, first tire on front and second tire on rear axle, with imaginary contact points between tires and surface on the center of axles.
- Neglecting of pneumatic trail and aligning torque resulting from a side-slip angle.
- Mass distribution on the axles is constant.
- Longitudinal forces on tires, resulting from a normalized tire slip angle, are neglected. All longitudinal forces acting on each axle are assumed to be strictly from the engine.

![Figure 3.3: The block diagram of the single-track model](image-url)
3.1.2 Block Representation

The block representation of the nonlinear single-track model can be found in Fig. 3.3. Each block is described more detailed in the next sub-sections. The model consists of:

- **rigid body dynamics** block is for a chassis representation;
- **steering angles projection** block respond for computing of forces and angular momentum acting on the rigid body;
- **tire models** block calculates longitudinal and lateral forces from slip variables and restrictions made by traction ellipse;
- **wheel kinematics**, calculation part, which compute side-slip angles of the tires in the particular coordinate frame;
- **slip ratios** block for calculating how each wheel spins.

3.1.3 Rigid Body Dynamics

The rigid body dynamics have three degrees of freedom represented by translation motion (as the velocity of CG \( v \)), side-slip angle \( \beta \), and rotation motion modeled by yaw rate \( r \).

The longitudinal motion is derived as

\[
F_x = F_{x,\text{tr}} + F_{x,\text{rot}} = ma_x - mr_v y = m(v \cos \beta)t - mvr \sin \beta = m(\dot{v} \cos \beta - v \sin \beta(\dot{\beta} + r)).
\]

The lateral motion is represented by equation

\[
F_y = F_{y,\text{tr}} + F_{y,\text{rot}} = ma_y + mr_v x = m(v \sin \beta)t + mvr \cos \beta = m(\dot{v} \sin \beta + v \cos \beta(\dot{\beta} + r)).
\]

The rotational motion is represented by equation

\[
M_z = I_{\text{yaw}} \dot{r}.
\]

Aerodynamics are represented as a dissipate force which reacts against the velocity vector in both, longitudinal and lateral directions as

\[
F_{\text{air},x} = \frac{1}{2} c_{\text{air}} \rho A (v \cos \beta)^2,
\]

\[
F_{\text{air},y} = \frac{1}{2} c_{\text{air}} \rho A (v \sin \beta)^2,
\]

In a matrix form rigid body dynamics have the next look:
\[
\begin{bmatrix}
\dot{\beta} \\
\dot{v} \\
\dot{r}
\end{bmatrix} = 
\begin{bmatrix}
\frac{1}{m v} & 0 & 0 \\
0 & \frac{1}{m} & 0 \\
0 & 0 & \frac{1}{I_{yaw}}
\end{bmatrix} 
\begin{bmatrix}
-\sin \beta & \cos \beta & 0 \\
\cos \beta & \sin \beta & 0 \\
0 & 0 & 1
\end{bmatrix} 
\begin{bmatrix}
F_x \\
F_y \\
M_z
\end{bmatrix} - 
\begin{bmatrix}
F_{air,x} \\
F_{air,y} \\
0
\end{bmatrix} - 
\begin{bmatrix}
r \\
0 \\
0
\end{bmatrix}.
\]

(3.6)

### 3.1.4 Steering Angles Projection

The steering angle projection translates forces acting on tires in wheel coordinate frames into forces and rotation momentum acting on a rigid body via transformation matrix as follows

\[
\begin{bmatrix}
F_x \\
F_y \\
M_z
\end{bmatrix} = 
\begin{bmatrix}
\cos \delta_f & -\sin \delta_f & \cos \delta_r & -\sin \delta_r \\
\sin \delta_f & \cos \delta_f & \sin \delta_r & \cos \delta_r \\
l_f \sin \delta_f & l_f \cos \delta_f & -l_r \sin \delta_r & -l_r \cos \delta_r
\end{bmatrix} 
\begin{bmatrix}
F_{xf} \\
F_{yf} \\
F_{xf} \\
F_{yf}
\end{bmatrix}.
\]

(3.7)

### 3.1.5 Tire Models

The block representation of the tire models can be seen in Figure 3.4. The block contains two tire models, which calculate via Simplified Pacejka Magic formula raw lateral and longitudinal forces for each wheel, then that forces are scaled in blocks named "Traction Ellipse".

![Figure 3.4: The block diagram of the tire models](image)

The tire model is the main part of any nonlinear car model because tires are primary system actuators of a vehicle dynamics. Simplified Pacejka Magic formula [4] is used for mapping a tire generated force on slip variable in both lateral and longitudinal directions defined as

\[
F_{x,f,raw}(\lambda_l) = c_{D,x} F_{x,f} \sin(c_{C,x} \arctan(c_{E,x} \lambda_l - c_{E,x}(c_{B,x} \lambda_l - \arctan(c_{B,x} \lambda_l)))),
\]

(3.8)
where load forces are constant and are calculated from the car parameters as follows

\[
F_{x,t,\text{raw}}(\lambda_t) = c_{D,x} F_{z,t} \sin(c_{C,x} \arctan(c_{B,x} \lambda_t - c_{E,x} \lambda_t - \arctan(c_{B,x} \lambda_t))), \quad (3.9)
\]
\[
F_{y,t,\text{raw}}(\alpha_t) = c_{D,y} F_{z,t} \sin(c_{C,y} \arctan(c_{B,y} \alpha_t - c_{E,y} \alpha_t - \arctan(c_{B,y} \alpha_t))), \quad (3.10)
\]
\[
F_{y,x,\text{raw}}(\alpha_t) = c_{D,y} F_{x,t} \sin(c_{C,x} \arctan(c_{B,x} \alpha_t - c_{E,x} \alpha_t - \arctan(c_{B,x} \alpha_t))), \quad (3.11)
\]

where \( g = 9.81 \text{ m s}^{-2} \) is a gravity coefficient of the Earth. For longitudinal force is used two sets of coefficients \( c_{B,x}, c_{C,x}, c_{D,x}, c_{E,x} \), first one for throttling and second one for braking.

\[
F_{z,f} = gm \frac{l_f}{l_f + l_r}, \quad F_{z,r} = gm \frac{l_r}{l_f + l_r}, \quad (3.12)
\]

\[F_{tot} = \sqrt{\frac{F_{x}^2}{c_{D,x}^2} + \frac{F_{y}^2}{c_{D,y}^2}} \leq \mu F_z, \quad (3.13)\]

where \( \mu \) is a friction coefficient of a road; \( c_{D,x} \) and \( c_{D,y} \) are parameters from Pacejka (in general, they are friction coefficients of the road in different directions).
The implementation of the traction ellipse can be adopted from [2]. The following algorithm (Eq. (3.14) - (3.18)) is applied to scale (if it is needed) the resulted force:

\[
\beta^* = \arccos\left(\frac{|\lambda|}{\sqrt{\lambda^2 + \sin^2(\alpha)}}\right),
\]

\[
\mu_{x,\text{act}} = \frac{F_{x,\text{raw}}}{F_z}, \quad \mu_{y,\text{act}} = \frac{F_{y,\text{raw}}}{F_z},
\]

\[
\mu_{x,\text{max}} = c_{D,x}, \quad \mu_{y,\text{max}} = c_{D,y},
\]

\[
\mu_x = \frac{1}{\sqrt{\left(\frac{1}{\mu_{x,\text{act}}}\right)^2 + \left(\frac{\tan(\beta^*)}{\mu_{y,\text{max}}}\right)^2}},
\]

\[
F_x = \left|\frac{\mu_x}{\mu_{x,\text{act}}}\right| F_{x,\text{raw}},
\]

\[
F_y = \left|\frac{\mu_y}{\mu_{y,\text{act}}}\right| F_{y,\text{raw}}.
\]

### 3.1.7 Wheel Kinematics

The wheel kinematics of the single-track model includes calculation of the velocity vector of each wheel and side-slip angle of each tire in the particular wheel coordinate frame. Velocity vectors for the front and rear wheels are calculated as

\[
\begin{pmatrix}
v_{xf} \\
v_{yf}
\end{pmatrix} = \begin{pmatrix}
\cos \delta_f & \sin \delta_f \\
-\sin \delta_f & \cos \delta_f
\end{pmatrix} \begin{pmatrix}
v_x \\
v_y + l_tr
\end{pmatrix} = \begin{pmatrix}
\cos \delta_t & \sin \delta_t \\
-\sin \delta_t & \cos \delta_t
\end{pmatrix} \begin{pmatrix}
v \cos \beta \\
v \sin \beta + l_tr
\end{pmatrix},
\]

\[
\begin{pmatrix}
v_{xr} \\
v_{yr}
\end{pmatrix} = \begin{pmatrix}
\cos \delta_r & \sin \delta_r \\
-\sin \delta_r & \cos \delta_r
\end{pmatrix} \begin{pmatrix}
v \cos \beta \\
v \sin \beta - l_tr
\end{pmatrix}.
\]

Thus, the side-slip angles of each wheel can be calculated using the definition as

\[
\alpha_f = -\arctan \frac{v_{yf}}{v_{xf}},
\]

\[
\alpha_r = -\arctan \frac{v_{yr}}{v_{xr}}.
\]
3.1.8 Slip Ratios

This block is used to calculate slip ratio per each wheel, which can be calculated using following definition:

\[
\lambda_f = \frac{\omega_{p} - v_{x,f}}{|v_{x,f}|}, \quad \lambda_r = \frac{\omega_{p} - v_{x,r}}{|v_{x,r}|}.
\] (3.23)
4. Nonlinear Twin-Track Model

4.1 Main Model

This chapter describes a Nonlinear Twin-Track vehicle model that was used for fitting car parameters. Actually, it is modified Twin-Track [5]. The notation used in this thesis is the same as in [3].

Vehicle coordinate system used for the description of the Twin-Track model is shown in Figure 4.1. It is the conventional right-hand Cartesian coordinate system. The x axis follows from the center of gravity to the front of the vehicle. The y axis goes towards the left side of the car, and z axis lies from the center of gravity to the top of the vehicle. The vehicle’s roll, pitch and yaw angles are measured conventionally (counter-clockwise to axis).

![Vehicle coordinate system](image)

Figure 4.1: Inertial (earth-fixed) and Vehicle (body-fixed) coordinate systems. [3]

The Twin-Track model (Fig. 4.2 and Fig. 4.3) is used to represent motion of a vehicle in 3 dimensional space and consists of 10 states and 8 inputs, which are listed in Table 4.1. All parameters used in the model are presented in Table 4.2. Note that in mathematical description is used symbol Ω, where is $\Omega = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$. The block diagram of the Nonlinear Twin-Track model is shown in Figure 4.4.

Tire forces $F_{Ri}$, forces from the tires acting on the chassis $F_i$, $r_{V}$, position vector of the center of gravity of the chassis $O_V$ from the pivot point $A_i$ are showed on Fig. 4.2.
Figure 4.2: Free body diagram of a twin track model - Top view.

Figure 4.3: Free body diagram of a twin track model - Side view.
### States/Inputs

<table>
<thead>
<tr>
<th>State/Input</th>
<th>Symbol</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position of the vehicle body in earth-fixed coordinate system on $OZ$ axis</td>
<td>$z$</td>
<td>m</td>
</tr>
<tr>
<td>Velocity of CG in each dimension</td>
<td>$v_{x,y,z}$</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>Roll rate, pitch rate, yaw rate</td>
<td>$\dot{\phi}, \dot{\theta}, \dot{\psi}$</td>
<td>rad s$^{-1}$</td>
</tr>
<tr>
<td>Euler's angles (earth-fixed coordinate system)</td>
<td>$\phi, \theta, \psi$</td>
<td>rad</td>
</tr>
<tr>
<td>Steering angle of each wheel</td>
<td>$\delta_i$</td>
<td>rad</td>
</tr>
<tr>
<td>Angular velocity of each wheel</td>
<td>$\omega_i$</td>
<td>rad s$^{-1}$</td>
</tr>
</tbody>
</table>

Table 4.1: States and inputs of the Twin-Track

### Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle mass</td>
<td>$m$</td>
<td>kg</td>
</tr>
<tr>
<td>Moment of inertia (matrix $3 \times 3$)</td>
<td>$I$</td>
<td>kg m$^{-2}$</td>
</tr>
<tr>
<td>Front/Rear axle-CG distance</td>
<td>$l_{v,h}$</td>
<td>m</td>
</tr>
<tr>
<td>Radius of wheels</td>
<td>$p$</td>
<td>m</td>
</tr>
<tr>
<td>Distance from CG to left/right wheels on $OY$ axis</td>
<td>$s_{l/r}$</td>
<td>m</td>
</tr>
<tr>
<td>Distance from CG to the point where springs are anchored on $OZ$ axis</td>
<td>$s_z$</td>
<td>m</td>
</tr>
<tr>
<td>Spring stiffness for front/rear wheels</td>
<td>$c_{a/r}$</td>
<td>N m$^{-1}$</td>
</tr>
<tr>
<td>Spring compression coefficient for front/rear wheels</td>
<td>$d_{a\text{compression},i/r}$</td>
<td>N m$^{-1}$</td>
</tr>
<tr>
<td>Spring rebound coefficient for front/rear wheels</td>
<td>$d_{a\text{rebound},i/r}$</td>
<td>N m$^{-1}$</td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>$c_{\text{air}}$</td>
<td>-</td>
</tr>
<tr>
<td>Air density</td>
<td>$\rho$</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>Frontal area of the car</td>
<td>$A$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Shaping coefficients for lateral dynamics</td>
<td>$c_D,y$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_B,y$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_C,y$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_E,y$</td>
<td>-</td>
</tr>
<tr>
<td>Shaping coefficients for longitudinal dynamics for acceleration</td>
<td>$c_D,x,t$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_B,x,t$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_C,x,t$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_E,x,t$</td>
<td>-</td>
</tr>
<tr>
<td>Shaping coefficients for longitudinal dynamics for breaking</td>
<td>$c_D,x,b$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_B,x,b$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_C,x,b$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$c_E,x,b$</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.2: Parameters of the Twin-Track
4.2 Scheme

The model is separated into 2 main parts: Chassis and Vehicle Body. Chassis represent tires and suspension, the suspension then supports the Vehicle Body (sprung mass).

4.3 Vehicle Body

The vehicle body can be described by next matrix form

\[
m \left( \begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix} + \Omega \times \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \right) = \sum_{i=1}^{4} \begin{bmatrix} F_{i,x} \\ F_{i,y} \\ F_{i,z} \end{bmatrix} - F_{\text{air}} \begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \end{bmatrix} - mg \right]. \quad (4.1)
\]

Forces \( F_i \) are in body-fixed coordinates, the matrix \( V T_E \) transforms the earth-fixed gravitational acceleration to vehicle-fixed coordinates. The rest of the variables are in vehicle-fixed coordinate frame. \( F_{\text{air}} = \frac{1}{2} c_{\text{air}} \rho A \sqrt{v_x^2 + v_y^2} \)

simulate resistance of air where \( c_{\text{air}} \) is drag coefficient, air density \( \rho \) and \( A \) is a frontal area of the car.

The rotational dynamics is described by:

\[
I \dot{\Omega} + \Omega \times (I \Omega) = \sum_{i=1}^{4} r_i \times F_i + r_w \times F_w, \quad (4.2)
\]

where \( I \) represent matrix of inertia, \( F_w \) are aerodynamic forces acting at the center of aerodynamics pressure \( r_w \), the vector is w.r.t. center of gravity in vehicle coordinates. The vector \( r_i \) is the point of application of the force \( F_i \) from wheels and suspension,
its values are determined from the dimensions of the body, e.g. for the first, front-left wheel, the value would be 
\[ r_i = \begin{bmatrix} l_f \\ s_l \\ -s_z \end{bmatrix}. \]

4.4 Chassis

4.4.1 Suspension

The suspension is modeled as spring-damper systems acting on each wheel individually.

Spring

Spring force acting on \( i^{th} \) wheel is defined as follows:

\[ V F_{Fi} = - (c_{a,i} \Delta l_{Fi}) V T_E \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \forall i \in \{1, 2, 3, 4\}, \quad (4.3) \]

where \( c_{a,i} \) is the stiffness coefficient of spring \( i \), \( \Delta l_{Fi} \) is the compression of spring \( i \), \( V T_E \) is a rotation matrix, transforming Inertial coordinates to Vehicle coordinates. The multiplication by \( \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \) means that the force acts only along the (inertial) \( z_E \)-axis (the spring is assumed to always point upwards with respect to the inertial coordinates).

Damper

Damping force acting on \( i^{th} \) wheel is defined as follows

\[ V F_{Di} = - (d_{a,i} \Delta \dot{l}_{Fi}) V T_E \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \forall i \in \{1, 2, 3, 4\}, \quad (4.4) \]

where \( d_{a,i} \) is the rebound coefficient of spring \( i \) for \( \Delta l_{Fi} \geq 0 \) and is the damping coefficient of spring \( i \) for \( \Delta l_{Fi} < 0 \).

4.4.2 Tire Interface

The model comes with the Simplified Pacejka tire model with constant coefficients and uses slip ratio \( \lambda \) and slip angle \( \alpha \) as their inputs.

Slip variables

Longitudinal (circumferential) slip:

\[ \lambda_i = \frac{\dot{x}_{R_i} - \rho \omega_{R_i}}{|\dot{x}_{R_i}|}. \quad (4.5) \]
Slip angle:
\[ \alpha_i = -\arctan \left( \frac{\dot{y}_{R_i}}{\dot{x}_{R_i}} \right), \quad (4.6) \]
where \( \dot{x}_{R_i} \) is velocity of the wheel center point along \( x \) in the wheel-fixed coordinate system, \( \dot{y}_{R_i} \) is velocity of the wheel center point along \( y \) in the wheel-fixed coordinate system, \( p \) is wheel radius, \( \omega_{R_i} \) Angular velocity of wheel \( i \).

**Wheel center point velocity**

![Diagram](image)

Figure 4.5: Clarification on the meaning of wheel center point vector \( r_{R_i} \) and the pivot point \( A_i \). [3]

The velocities of the wheel center point are obtained as follows:

\[ V_{r_{R_i}} = V_{r_{A_i}} + V_T E \begin{bmatrix} 0 \\ 0 \\ -l_{F_i} \end{bmatrix}, \forall i \in \{1, 2, 3, 4\}, \quad (4.7) \]

where \( V_{r_{R_i}} \) is a position of the wheel center point with respect to vehicle coordinates, \( V_{r_{A_i}} \) is a position of the spring anchor with respect to the vehicle coordinates, \( l_{F_i} \) is length of the spring.

The point \( V_{r_{A_i}} \) is where the spring is anchored to the vehicle chassis and where the tire forces are applied to the chassis.
\[ V_{V_R_i} = \begin{bmatrix} \dot{V}_{x_R_i} \\ \dot{y}_{R_i} \\ \dot{z}_{R_i} \end{bmatrix} = V_{V} + V_{\omega_V} \times V_{R_i} + V_{T_E} \begin{bmatrix} 0 \\ 0 \\ -\dot{\tau}_{F_i} \end{bmatrix}, \forall i \in \{1, 2, 3, 4\}, \quad (4.8) \]

where \( V_{V_R_i} \) is wheel velocity with respect to vehicle coordinates, \( V_{V} \) is vehicle velocity with respect to vehicle coordinates, \( V_{\omega_V} \) is vehicle angular velocity with respect to vehicle coordinates. Note: Vehicle = Chassis.

\[ R_i v_{V_R_i} = \begin{bmatrix} R_i \dot{x}_{R_i} \\ R_i \dot{y}_{R_i} \\ R_i \dot{z}_{R_i} \end{bmatrix} = R_i T_V V_{V_R_i}, \quad (4.9) \]

where \( R_i v_{V_R_i} \) wheel center point velocity with respect to wheel-fixed coordinates, \( R_i T_V \) rotational matrix from vehicle coordinates to wheel-fixed coordinates.

**Simplified Pacejka’s Magic Formula**

This model uses Pacejka’s magic formula for calculating generated forces on tire:

\[ F(k) = c_D F_x \sin(c_C \arctan(c_B k - c_E (c_B k - \arctan(c_B k)))) \], \quad (4.10) \]

where \( k \) is either sideslip angle \( \alpha \) or longitudinal slip \( \lambda \). \( F \) is either \( F_y \) or \( F_x \), depending on the input argument.

Coefficients \( c_B, c_C, c_D, c_E \), in this model, are constant for given \( F \). So for calculating \( F_y \) and \( F_x \), will be needed two sets of these parameters.

Dependency between forces \( F_x \) and \( F_y \) is expressed with traction ellipse \[ \text{Figure 4.6: Traction ellipse for constant } c_B, c_C, c_D, c_E. \]

And following algorithm (Eq. (4.11) - (4.18)) can be used to capture this depen-
dency. Let’s call the forces calculated from $F_{x,max}$ and $F_{y,max}$ as in Figure 4.6.

$$\alpha^* = s(\alpha),$$

$$\beta = \arccos\left(\frac{|\lambda|}{\sqrt{\lambda^2 + \alpha^*}}\right),$$

$$\mu_{x,act} = \frac{F_{x,max}}{F_z}, \quad \mu_{y,act} = \frac{F_{y,max}}{F_z},$$

$$\mu_{x,max} = \frac{D_x}{F_z}, \quad \mu_{y,max} = \frac{D_y}{F_z},$$

$$\mu_x = \frac{1}{\sqrt\left(\frac{1}{\mu_{x,act}}\right)^2 + \left(\tan(\beta)\right)^2},$$

$$\mu_y = \frac{\tan(\beta)}{\sqrt\left(\frac{1}{\mu_{y,act}}\right)^2 + \left(\tan(\beta)\right)^2},$$

$$F_x = \frac{\mu_x}{\mu_{x,act}}F_{x,max},$$

$$F_y = \frac{\mu_y}{\mu_{y,act}}F_{y,max}.$$  

Forces $F_x$ and $F_y$ are now respecting the traction ellipse from Figure 4.6. The force $F_z$ is load on the tire, the resultant force from the spring-damper system. Note that the shape of the ellipse is determined by $F_{x,max}$ and $F_{y,max}$.

### 4.5 Coordinate Transformations

In this section is described rotation matrices which are used for transformation between various coordinate systems ($c()$ and $s()$ is used instead of $\cos()$ and $\sin()$).

Wheel-fixed to body-fixed:

$$V_{T_{R_i}} = \begin{bmatrix} c(\delta_i)c(\theta) & -s(\delta_i)c(\theta) & -s(\theta) \\ s(\phi)s(\theta)c(\delta_i) + c(\phi)s(\delta_i) & \quad c(\phi)c(\delta_i) + c(\phi)s(\delta_i) \\ s(\phi)s(\theta)c(\delta_i) - c(\phi)s(\delta_i) & \quad -c(\phi)s(\theta)c(\delta_i) - s(\phi)c(\delta_i) \end{bmatrix},$$

where $\delta_i$ is the steering angle of wheel $i$, $\phi$ and $\theta$ are Euler angles (roll and pitch).

Body-fixed to inertial:

$$E_{T_V} = \begin{bmatrix} c(\theta)c(\psi) & s(\phi)s(\theta)c(\psi) - c(\phi)s(\psi) & c(\phi)s(\theta)c(\psi) + s(\phi)s(\psi) \\ c(\phi)s(\theta)c(\psi)c(\phi) & s(\phi)s(\theta)c(\psi) + c(\phi)c(\psi) & c(\phi)s(\theta)c(\psi) - s(\phi)c(\psi) \\ -s(\theta) & s(\phi)c(\theta) & c(\phi)c(\theta) \end{bmatrix},$$

where $\phi$, $\theta$ and $\psi$ are Euler angles. For more clarification on how these matrices were derived, see [3].
5. Model Identification

5.1 LFS Signals Preview

Thesis [7] describes methods for reading signals from "Life for Speed" simulator and information from files. This methods allow reading of the following signals, which are important for us, from the LFS in online mode for each wheel:

<table>
<thead>
<tr>
<th>Name of signal</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>XForce</td>
<td>tire generated force in lateral direction $F_{y,\text{raw}}$.</td>
</tr>
<tr>
<td>YForce</td>
<td>tire generated force in longitudinal direction $F_{x,\text{raw}}$.</td>
</tr>
<tr>
<td>VerticalLoad</td>
<td>load force $F_z$.</td>
</tr>
<tr>
<td>AngVel</td>
<td>angular velocity of a wheel $\omega$ in radians per second</td>
</tr>
<tr>
<td>Steer</td>
<td>steering of a wheel in radians</td>
</tr>
<tr>
<td>SlipRatio</td>
<td>slip ratio of a wheel $\lambda$.</td>
</tr>
<tr>
<td>TanSlipAngle</td>
<td>tangent of wheel’s sideslip angle $\alpha$.</td>
</tr>
</tbody>
</table>

Wheels are marked in the next way: W1 - rear left, W2 - rear right, W3 - front left, W4 - front right.

Car info.mat and car_info.txt files provide the following important information about chosen vehicle:

- **forceForward** represents a weight distribution on the front part of the vehicle;
- **Length of a wheelbase** $l$;
- **Total mass of the vehicle** $m$;
- **Moment of inertia of the rigid body** $I$ as matrix $3 \times 3$;
- **Rim’s radius** $r$ for each wheel in metres;
- **Width of a tyre** $w$ for each wheel in metres;
- **Sidewall height proportion** represents a ratio between tyre’s height and tyre’s width.
- **Aerodynamic drag coefficient** $c_{\text{air}}$

5.2 Single-Track Body Identification

The identification of the Single-Track rigid body parameters is determined as parsing of the values listed in the table. Car info.mat can provide the following information: total mass of the vehicle $m$, aerodynamics drag coefficient $c_{\text{air}}$ and yaw moment of inertia $I_{\text{yaw}}$ as third column and a third-row element of matrix $I$, air density is chosen as a conventional constant $\rho = 0.015 \text{ kg m}^{-3}$. Frontal area of the car $A$ cannot be
calculated within available information and empirically chosen as a constant \( A = 3 \, \text{m}^2 \).

Front axle-CG distance \( l_v \) can be calculated using the following formula:

\[
l_v = l \cdot \text{forceForward}.
\]  
(5.1)

Rear axle-CG distance \( l_h \) is calculated as:

\[
l_h = l \cdot (1 - \text{forceForward}).
\]  
(5.2)

Formula of wheel’s radius \( p \) have the following look:

\[
p = r + w \cdot \text{Sidewall height proportion}.
\]  
(5.3)

Actual radius of wheel’s can vary because of \( F_z \). It can be calculated using equation 5.4 in situation when the car is moving with constant velocity \( v_x \) and does not perform any lateral motion:

\[
p = \frac{\lambda \cdot |v_x| + v_x}{\omega}.
\]  
(5.4)

### 5.3 Twin-Track Body Identification

The identification of the Twin-Track rigid body parameters is determined as parsing of the parameters listed in the table 4.2. Car_info.mat can help with identification of Twin-Track body by providing the following important information: total mass of the vehicle \( m \), aerodynamic drag coefficient \( c_{\text{air}} \) and moment of inertia \( I \). Car_info.txt can provide the following parameters: distance from the CG to left/right wheels on \( OY \) axis \( s_{l/r} \), spring stiffness for front/rear wheels \( c_{a,l/r} \), spring compression coefficient for front/rear wheels \( d_{c,\text{compression},l/r} \), spring rebound coefficient for front/rear wheels \( d_{r,\text{rebound},l/r} \). Air density is chosen again as a constant \( \rho = 0.015 \, \text{kg/m}^3 \). Frontal area of the car \( A \) and distance from center of gravity to the point where springs are anchored on \( OZ \) axis \( s_z \), cannot be calculated within available information, so they are empirically chosen as a constant: \( A = 3 \, \text{m}^2 \) and \( s_z = 0.5 \, \text{m} \). Front axle-CG distance, rear axle-CG distance and wheel’s radius are calculated in the same way as for the Single-Track (equations 5.1, 5.2, and 5.4).

### 5.4 Tire’s Parameters Identification

The identification of shaping coefficients is done for general tire function, not for each tire. That and the fact that Single-Track and Twin-Track model use the same Pacejka Magic formula mean that shaping coefficients identification is the same for both Single-Track and Twin-Track.
Shaping coefficients can be calculated by using of nonlinear regression with Huber weight [\textsuperscript{8}] function

\[
\text{weight} = \frac{1}{\max(1, |\text{residual}|)} \tag{5.5}
\]

for robust fitting of unknowns parameters $c_D, c_C, c_B, c_E$ in equations 3.8, 3.9, and 4.10.

Lateral dynamics uses equation 3.10 for forward wheels and 3.11 for rear wheels with signals $X_{\text{Force}}$ as $F_y$, $\arctan(\text{TanSlipAngle})$ as $\alpha$, $W_4_{\text{VerticalLoad}} + W_3_{\text{VerticalLoad}}$ as $F_{z,f}$, $W_1_{\text{VerticalLoad}} + W_2_{\text{VerticalLoad}}$ as $F_{z,r}$ in a same time.

Longitudinal dynamics is described by a next equation 3.8 for forward wheels or 3.9 for rear for both situation: braking and throttling. Signals used in equations are $Y_{\text{Force}}$ as $F_x$, $\text{SlipRatio}$ as $\lambda$, $W_4_{\text{VerticalLoad}} + W_3_{\text{VerticalLoad}}$ as $F_{z,f}$, $W_1_{\text{VerticalLoad}} + W_2_{\text{VerticalLoad}}$ as $F_{z,r}$ in a same time.

The Nonlinear regression needs initial coefficient values which could be calculated within nonlinear curve-fitting. Initial values for nonlinear curve-fitting is represented in the table 5.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Limit</th>
<th>Initial guess</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaping coefficients</td>
<td>$c_D$</td>
<td>0.2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$c_B$</td>
<td>4..30</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>$c_C$</td>
<td>1..2</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>$c_E$</td>
<td>$-30..1$</td>
<td>$-4.5$</td>
</tr>
</tbody>
</table>
To collect the data for lateral dynamics parameter identification, the following maneuver is used: maximal turn of the steering wheel to the left and then to the right at maximum possible velocity for first gear. Fig. 5.1 shows inputs for this experiment.

Figure 5.1: Inputs for data generating for lateral coefficients identification

Described experiment allows data generation of all possible range for slip angles (Fig. 5.2)

Figure 5.2: Measured data for lateral dynamics tire parameter identification

Data acquisition experiment for longitudinal dynamics includes sharp start on the first gear and sharp braking at the maximal speed for the first gear, it’s done to
generate the data for all possible range for slip ratios. The inputs for that experiment are shown in Fig. 5.3

Figure 5.3: Inputs for data generating for longitudinal coefficients identification
Collected data are shown in Fig. 5.4. Notice that the slip ratio has a saturation limit in -1. That means that for calculation of coefficients for braking data where slip ratio > -1 is used. The fitting is used to estimate shaping coefficients for nonlinear curves, which describe the dependency of the generated lateral force on tire’s slip angle or the longitudinal force on tire’s slip ratio. The fitted curves are shown in figures 5.5, 5.6, and 5.7. Fitting scripts allow manual changes of Pacejka’s parameters and observing how it impact dependency.
Figure 5.4: Measured data for longitudinal dynamics tire parameter identification.
Figure 5.5: Curve fitting for the generated lateral force $F_y$
Figure 5.6: Dependence of the generated lateral force $F_x$ on tire slip ratio $\lambda$ for throttling
Figure 5.7: Dependence of the generated lateral force $F_{x,\text{brake}}$ on tire slip ratio $\lambda$ for braking
6. Validation Ride Tests

Resulting identified model were tested on 3 cars:

- **GTI**, car with front drive, this car was used to develop identification method.
- **UF1**, car with front drive.
- **XRG**, car with rear drive.

6.1 Single-Track

Figures 6.1, 6.2, 6.3 demonstrate that Single-Track well follows LFS in the following signals: velocity $v$, yaw rate $\dot{\psi}$. Slip ratios $\lambda$ and slip angles $\alpha$ are followed as well, but have some differences, because wheel radius in Single-Track model is constant, but in LFS tire are dynamically changed. This mean that parameters for longitudinal and lateral dynamics are calculated with sufficient accuracy.

![Graphs showing comparison between LFS and Single-Track models](image)

Figure 6.1: Comparison between LFS and the Single-Track model for GTI (blue line it is LFS, red line is the Single-Track)
Figure 6.2: Comparison between LFS and the Single-Track model for UF1 (blue line it is LFS, red line is the Single-Track)
6.2 Twin-Track

Figures 6.4, 6.5, and 6.6 demonstrate that Twin-Track well follow LFS in signals. Lateral and longitudinal dynamics are similar to Single-Track. Slip ratio $\lambda$ and slip angle $\alpha$ are well followed, but have some differences, because wheel radius in Twin-Track model are constant, but in LFS they are dynamically changed. Other parameters have more differences that described.
Figure 6.4: Comparison between LFS and the Twin-Track model for GTI (blue line it is LFS, red line is the Twin-Track)
Figure 6.5: Comparison between LFS and the Twin-Track model for UF1 (blue line is LFS, red line is the Twin-Track)
Figure 6.6: Comparison between LFS and the Twin-Track model for XRG (blue line is LFS, red line is the Twin-Track)
7. Results

All of the goals, listed in chapter [Objectives], are done. Namely:

- Familiarization with vehicle dynamics simulator "Live for Speed" was completed. Describing of information that can be taken from the simulator is placed in [LFS Signals Preview].

- Low-fidelity and high-fidelity vehicle models were adopted and described in [Nonlinear Single-Track Model] and [Nonlinear Twin-Track Model].

- Experiments for longitudinal motion were created and prepared as prescript inputs for MATLAB. Inputs are described in the chapter [Model identification].

- Experiments for lateral motion were created and prepared as prescript inputs for MATLAB. Inputs are described in the chapter [Model identification].

- Procedure for identification of parameters is described in [Model identification].

- Validation of identification procedure on different car models implemented in the simulator is provided in chapter [Comparisons].
8. Conclusion

This thesis completely describes the identification process for parameters of Single-Track and Twin-Track models. Two main modeling techniques exist for creating vehicle models. These techniques are mainly used to describe the vehicle dynamics: Single-Track [3, chapter 10], and Twin-Track models [3, chapter 11]. In this work, adopted Single-Track model, described in the article ”Nonlinear Single-Track Model” [2], was used because it has sufficient fidelity to describe a lateral vehicle dynamics, and to test identification methods because it is simpler than the Twin-Track models (three versus ten states). Twin-Track was used because it allows testing identification on more difficult model (ten states) to compare it in more ”complicated” design.

This thesis includes an adaptation of the existing Single-Track and Twin-Track models for compatibility with inputs from LFS, guide how to generate data for identification, creation and/or adaptation formulas for calculating of models parameters, and comparisons between Single-Track/Twin-Track model and LFS.

Result is presets for generating needful data, automatic initialization file, mat-files that calculate parameters for models.
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