

CZECH TECHNICAL UNIVERSITY IN PRAGUE  
FACULTY OF MECHANICAL ENGINEERING  
DEPARTMENT OF ENVIRONMENTAL ENGINEERING

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**SOLAR IRRADIANCE DECOMPOSITION USING THE ERBS  
MODEL**

BACHELOR THESIS



# BACHELOR'S THESIS ASSIGNMENT

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## II. Bachelor's thesis details

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**Solar Irradiance Decomposition Using the Erbs Model**

Bachelor's thesis title in Czech:

**Rozklad slunečního ozáření s užitím Erbsova modelu**

Guidelines:

Apply the model developed by Erbs for the decomposition of solar irradiance into direct and diffuse components. Compare the results based on the Erbs model with the solar data for a specific location (i.e. data available from an external source). Solar irradiance data may be processed in a spreadsheet tool (e.g. Excel) or in a general mathematical tool (e.g. Matlab).

Bibliography / sources:

Duffie J. A., Beckman W. A. Solar Engineering of Thermal Processes. 3rd ed. Hoboken : Wiley, 2006.  
Erbs D. G., Klein S. A., Duffie J. A. Estimation of the Diffuse Radiation Fraction for Hourly, Daily and Monthly-Average Global Radiation. Solar Energy 28 (4), 1982.  
Reindl D. T., Beckman W. A., Duffie J. A. Diffuse Fraction Correlations. Solar Energy 45 (1), 1990.  
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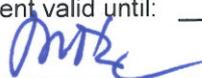
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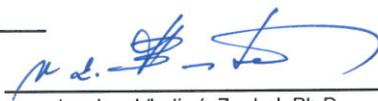
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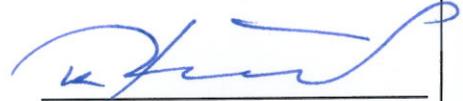
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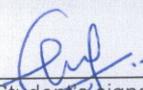
  
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## Summary

With the latest rise in solar energy initiatives around the world there is a rising need of right estimation of solar radiation. Sun radiation consists of two components: direct and diffuse. Diffuse and direct radiation data plays a crucial role in energy estimation from solar photovoltaics and are also needed in building performance stimulations. The diffuse and direct components can be measured using an instrument. However, such measurements might be complicated and expensive. This is why there has been a lot of research conducted to compute these components by mathematical models. In the current study, Erbs model is applied for the decomposition of global solar irradiance into direct and diffuse components. The solar radiation data from Třeboň were used to compare the calculated and measured values. The solar irradiance data were processed in Excel spreadsheet. The graphs of different months are presented to understand the characteristics of the model in different atmospheric conditions. It is noted that Erbs model performs relatively well when the clearness index is on the medium range, whereas when the clearness index is above 0.8 the correlation between the calculated and measured data is not so good.

## Keywords

solar radiation, diffuse irradiance, direct irradiance, clearness index, Erbs model

## Declaration

I declare that this bachelor thesis entitled “Solar Irradiance Decomposition Using the Erbs Model” is my own work performed under the supervision of Ing. Martin Barták, Ph.D. with the use of the literature presented at the end of my thesis in the list of references.

In Prague 31.07.2020

Nihal Muhammed Kannanari

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## Nomenclature

$F_D$	Diffuse fraction [-]
$G$	Irradiance on a horizontal plane [ $W/m^2$ ]
$G_{SC}$	Solar constant [ $W/m^2$ ]
$H$	Daily radiation on a horizontal plane [ $J/m^2$ ]
$I$	Hourly radiation on a horizontal plane [ $J/m^2$ ]
$K_T$	Clearness index [-]
$\alpha_s$	Solar altitude angle [°]
$\gamma$	Azimuth angle [°]
$\delta$	Solar declination angle [°]
$\theta_z$	Zenith angle [°]
$\phi$	Latitude [°]
$\omega$	Hour angle [°]

### Indexes:

B	Beam
BN	Direct normal (Beam)
D	Diffuse
H	Global
ON	Extraterrestrial

## 1 Introduction

The Sun is the primary source of energy that determines life on Earth. Solar radiation is electromagnetic energy from the Sun. This radiation provides light and heat needed for all living organisms on the Earth. The radiation entering the Earth's atmosphere is partly scattered or reflected. Sun radiation consist of two components, direct and diffuse components.

There is an accurate estimation of the incoming radiation from the Sun which can be essential for climate monitoring. In recent years the usage of solar energy has been increased a lot which helps to decrease global warming. It is important to determine how much diffuse irradiance and direct irradiance is being received as it provides knowledge to make crucial decisions on future energy efficiency, overall performance, required in building simulations as an input and the maintenance required for the operations. One of the easiest ways to measure the irradiance intensity is to measure the global irradiance on a horizontal surface by using the instrument called pyranometer. As we know that the large part of this radiation is scattered in the atmosphere which results in diffuse irradiance.

By using a pyranometer it is possible to estimate both direct and diffuse irradiance on a horizontal plane but it would be more expensive to measure it both rather than measuring global irradiance. So this why there has been a lot of studies conducted to find the easiest way to determine the diffuse irradiance and beam irradiance from the global irradiance on a horizontal surface. Software like (e.g. TRNSYS, HOMER) can be used to find the diffuse irradiance but they might be a bit expensive for the process.

So the calculation of diffuse components seems to be more effective. Various studies are done to explore different mathematical models in several world locations and many comparisons have been done to find the most accurate model that can be used irrespective of the location to improve the diffuse irradiance data.

Present mathematical computation helps to find the most convenient method to calculate the diffuse irradiance which will result in improving the solar energy resource data hence the bankability of the solar photovoltaic system and solar thermal. This will make renewable energy more attractive to governments around the globe and will lead to an increase in the participation of the renewables in the electric field. Most of the calculations are done on an hourly, daily, or monthly- average basis to find the accuracy of the model during a certain period.

The main aim of this study is to find the efficiency of Erbs model to calculate the diffuse irradiance and direct irradiance on a horizontal plane. Comparison between calculated data using Erbs model and measured values graphs was shown. Performed for the Třeboň in South Bohemia. All the calculations were processed in Excel spreadsheet.

## 2 Solar geometry

The angles are really important as it explains the position of Earth concerning the Sun. So, the angles have a lot to do with the amount of heat we receive from the Sun given each location.

The geometric relationships between a surface of any specific orientation relative to the earth at any time (whether that surface is fixed or changing relative to the earth) and the incoming direct solar radiation, that is, the position of the sun relative to that surface, can be defined in terms of several angles. The solar geometry helps to understand some important parameter to find the diffuse and direct radiation on that surface.

### 2.1 Solar declination

The Earth axis of rotation is tilted to  $23.45^\circ$  angle from the orbit of the Earth around the Sun. So, the declination is the angular displacement from the Sun's north or south to the Earth's equator. The North declination is designated as positive whereas the south is negative. So, when the southern hemisphere is tilted towards the Sun, the north will be experiencing winter and it is summer in the south with this angle.

The maximal and minimal declination values in this Earth's orbit result in having different seasons throughout the year. Declination range varies from  $+ 23.45^\circ$  and  $- 23.45^\circ$ . [2]

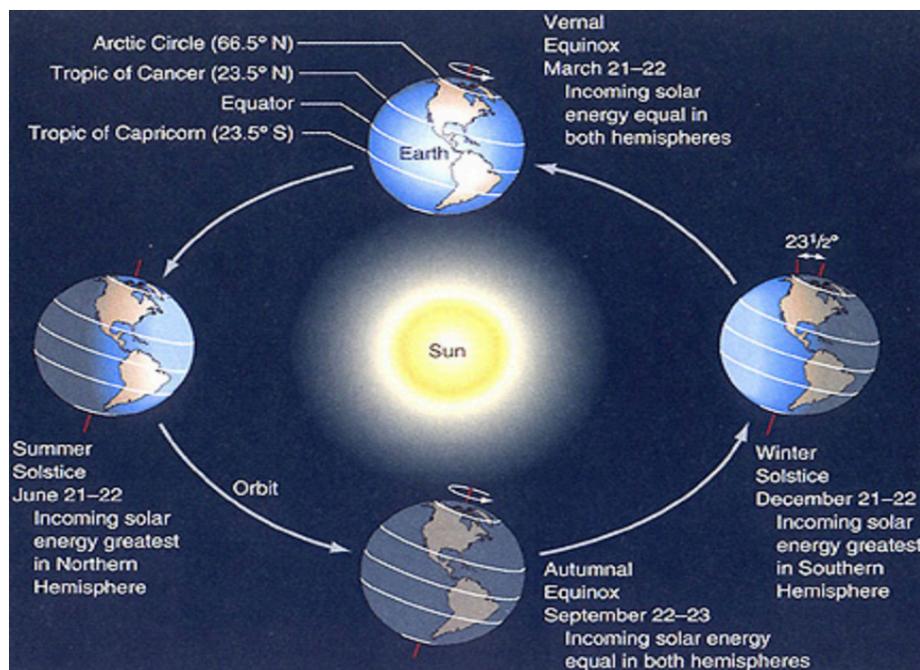


Figure 1: When the northern hemisphere is tilted toward the Sun, the northern region will experience summer whereas the Southern will experience winter. [3]

The declination angle ( $\delta$ ) can be calculated from the equation of Cooper (1969), [1]

$$\delta = 23.45 \sin \left( 360 \frac{284 + n}{365} \right) \quad (2.1)$$

where n is the day number in the year, with January 1 equal to 1.

## 2.2 Hour angle

The main idea of the hour angle is to describe the rotation of the Earth around its orbit axis which has an approximate value from  $+15^\circ$  per hour during the day time (morning) and  $-15^\circ$  in the evening. At noon the value of the hour angle will be 0. It is actually the angle between the observer's zenith and the meridian whose plane contains the Sun.[2] The hour angle also changes the local solar time into the number of degrees according to the movement of the Sun across the sky.

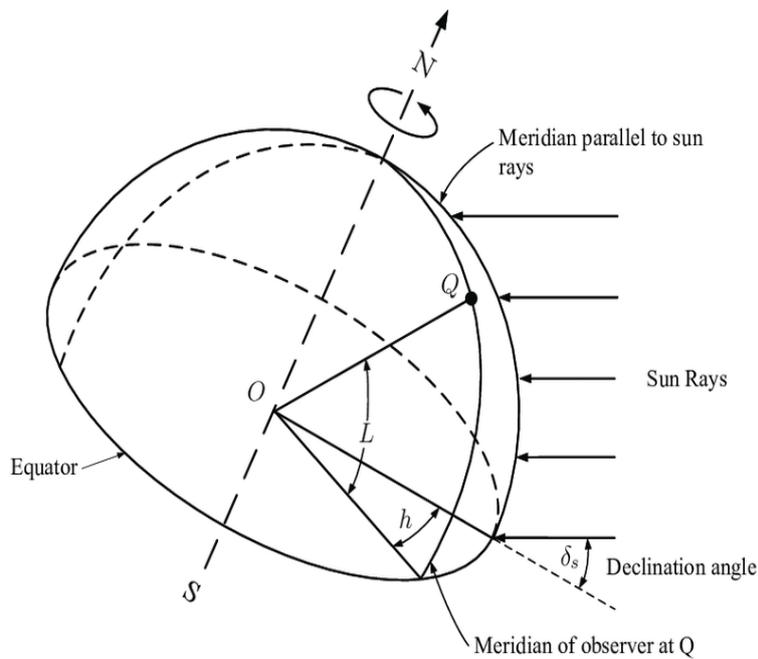


Figure 2: Hour angle, solar declination angle and latitude[4]

The hour angle can be calculated from the equation below:

$$\omega = 15(12 - ST) \quad (2.2.A)$$

$$ST = LT + \frac{ET}{60} + \frac{4}{60} [L_S - L_L] \quad (2.2.B)$$

where  $ST$  is the local solar time and unit is in hours,  $L_L$  is the longitude of the location,  $L_S$  is the standard meridian,  $LT$  is the local standard time and  $ET$  is the equation of time (in minutes) from Spencer (1971).

$$ET = 229.2 ( 0.000075 + 0.001868 \cos B - 0.032077 \sin B - 0.014615 \cos 2B - 0.04089 \sin 2B ) \quad (2.2.C)$$

where,  $B = (n - 1) \frac{360}{365}$ , ( $n$  is the number of day in the year, where January 1 equal to 1).

### 2.3 Latitude

It is the angular position with relevance to north or south of the equator. The variation of the latitude is from  $0^\circ$  to  $90^\circ$  it is positive in the northern hemisphere and negative in the southern hemisphere. And it is  $0^\circ$  at the equator and  $\pm 90^\circ$  at the poles.

### 2.4 Longitude

Longitude, measures how far east or west a point is from a specific meridian at the Earth's surface. It is an angular measurement and expressed in degrees.

### 2.5 Azimuth angle

The azimuth angle gives the compass direction from which the Sunlight is coming. It is the angle of the Sun's rays measured in a horizontal plane from the south for the northern hemisphere and vice versa. Displacements in azimuth angle east of west are -ve and west of south are noted as +ve.[5]

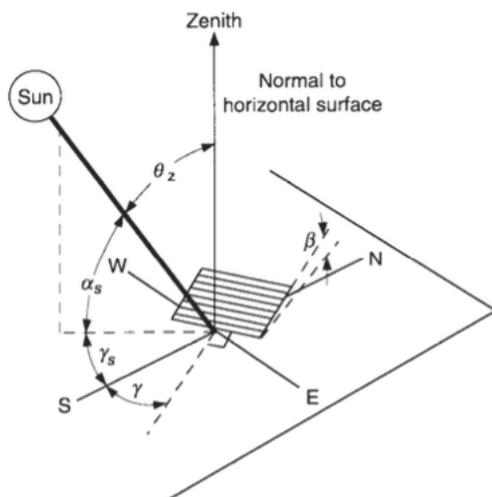


Figure 3: Azimuth angle , zenith angle and solar altitude angle[6]

## 2.6 Zenith angle and solar altitude angle

The zenith angle is the angle between the Sun rays and vertical plane which will result in the angle of incidence of the beam on a horizontal surface. Whereas the solar altitude is the angle between the Sun rays and the horizontal plane in which the angle of incidence of the beam will be on the vertical surface.[5]

Zenith angle mainly depends on the latitude, hour angle, and solar declination angle.

$$\cos \theta_z = \cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta \quad (2.6)$$

where,

$\phi$  = latitude of the location [ ° ]

$\delta$  = solar declination angle [ ° ]

$\omega$  = hour angle [ ° ]

$\theta_z$  = zenith angle [ ° ]

### 3 Solar radiation quantities

Solar radiation is the radiant energy released from the Sun. It can be measured in various forms like solar irradiance and radiant energy density. Irradiance or energy flux is the rate of solar radiation transfer obtained per unit area by a given surface, it can be expressed in watts per meter square [ $W/m^2$ ]. Whereas radiant energy density is expressed in joules per square centimetre [ $J/cm^2$ ] which includes integration in over the time as part of the measurement. The data used for the current study is done in radiant energy density, i.e. incident on a horizontal plane and integrated intervals of 15 minutes between them.

There is a reduction in the strength of solar radiation due to reflection within the atmosphere, scattering dust particles, and also due to the absorption in the atmosphere. based on the interaction of radiations with the atmosphere it can be classified into these main components like global radiation, diffuse radiation and direct radiation.[7]

#### 3.1 Solar constant

The term solar constant  $G_{SC}$  means the total energy reaching the topmost or extraterrestrial surface of the Earth where the irradiance will be around  $1367[W/m^2]$  (Iqbal, 1983). As the radiation cannot be fully utilized because of the above-mentioned atmospheric impacts. This constant is being measured mostly at noon. That's when the radiation intensity is higher than any other time.[5][1]

#### 3.2 Variation of extraterrestrial irradiance

The distance between the Sun and Earth is quite large and the quantity of the irradiance power is hard to determine from the surface of the Earth because of the atmospheric impacts. A formula of calculation was discovered by Garg (1982) which has helped to find the most accurate value.

The Sun is closest to the Earth around January; for this reason the solar heat is quite large on the edge of the Earth's atmosphere and the measured values are around  $1400 [W/m^2]$ . When it is during June, the Sun is farthest away from the surface of the Earth as the irradiance value will be  $1330 [W/m^2]$ . [2]

$$G_{on} = G_{SC} \left[ 1 + 0.033 \cos \left( \frac{360n}{365} \right) \right] \quad (3.2.A)$$

where,

$G_{on}$  = extraterrestrial irradiance measured on a horizontal plane to the radiation on nth day of year [ $W/m^2$ ]

$G_{SC}$  = solar constant [ $1367W/m^2$ ] from Iqbal in (1983).

The calculation of hourly extraterrestrial radiation;

$$I_{on} = \frac{12 \times 3600}{\pi} G_{SC} \left( 1 + 0.033 \cos \frac{360n}{365} \right) \times \left[ \cos \phi \cos \delta (\sin \omega_2 - \sin \omega_1) + \frac{\pi(\omega_2 - \omega_1)}{180} \sin \phi \sin \delta \right] \quad (3.2.B)$$

where,

$\omega_1$  and  $\omega_2$  are the hour angle [°]

$I_{on}$  = hourly extraterrestrial radiation [ $J/m^2$ ]

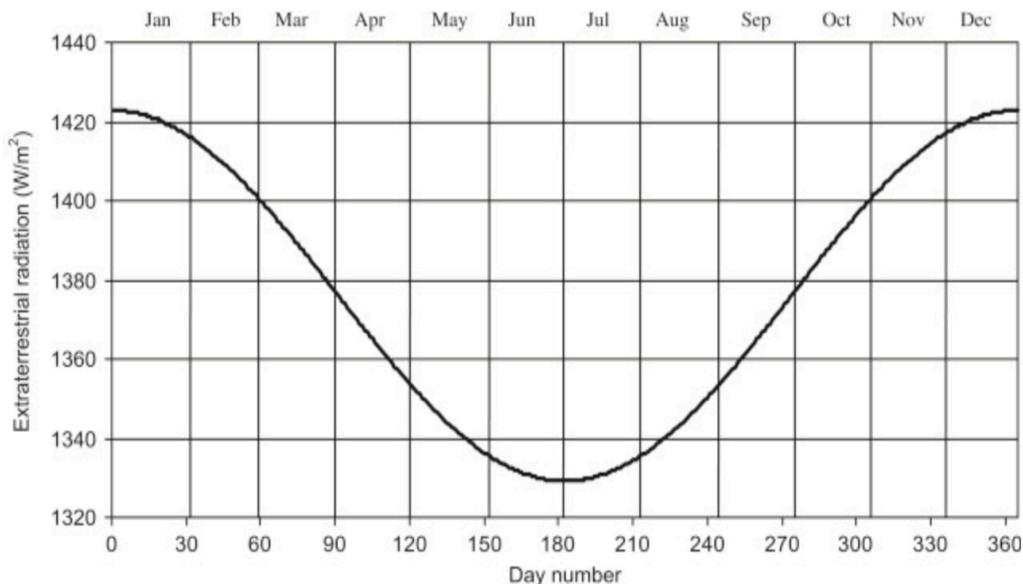


Figure 4: Variation of extraterrestrial solar irradiance within a year.[1]

### 3.3 Beam irradiance

The undisturbed Sun irradiance incidents on a horizontal surface of the Earth is called beam irradiance or direct irradiance. The beam irradiance is usually high during the noon as there won't be much cloudy in the sky as the irradiance won't scatter much in the atmosphere which is due to the air mass. The direct normal irradiance is the amount of solar radiation received per unit area by a surface which is perpendicular or normal to the sun rays that radiates in straight line from the direction of the Sun. This direct normal irradiance can be calculated using the zenith angle  $\theta_z$  which is the angle between the Sun rays and the normal plane. So the intensity of the irradiance can be calculated by both equation mentioned below[2][1]

$$G_B = G_H - G_D \quad (3.3.A)$$

where,

$G_H$  = global solar irradiance on a horizontal surface [ $W/m^2$ ]

$G_D$  = diffuse solar irradiance on a horizontal surface [ $W/m^2$ ]

$G_B$  = direct irradiance on a horizontal surface [ $W/m^2$ ]

$$G_{BN} = \frac{G_B}{\cos \theta_z} \quad (3.3.B)$$

where,

$G_{BN}$  = direct normal irradiance [ $W/m^2$ ]

$\cos \theta_z$  = zenith angle [ ° ]

The direct irradiance on a horizontal surface can be measured by an instrument called pyrheliometer. Sunlight enters through its window and directed onto a thermopile which converts sunlight to an electrical signal that can be recorded. The acceptance angle is only  $5^\circ$  for this instrument.[8][2]



Figure 5: Pyrheliometer for measuring the beam irradiance[6]

### 3.4 Diffuse irradiance

Diffuse irradiance is the irradiance from the Sun that is being scattered due to the atmospheric impacts such as dust, water, etc. When the sky is covered with full of clouds, the radiation enter on the horizontal surface will be diffuse radiation mainly.[3]

The diffuse irradiance on a horizontal surface can be calculated by the instrument called the pyranometer. A pyranometer is operated primarily based on the dimension of temperate difference between clear surface and dark surface. The black coating on the thermopile sensor absorbs the solar irradiance and the clear surface reflects it back.

This device can be used to calculate the beam irradiance as well but needs an extra shaded ring or ball to measure the diffuse irradiance in this instrument.[7][9]

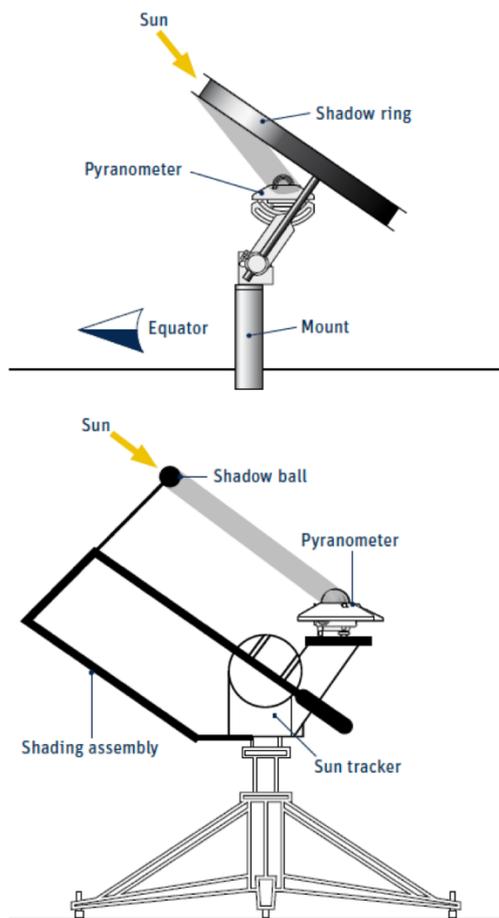


Figure 6: Pyranometer with a shading ring or ball[6]

There are many calculations formed over the years to find the most accurate diffuse irradiance value based on diffuse fraction. The below mentioned is the equation can be used to find the diffuse irradiance on a horizontal plane.[9]

$$G_D = F_D G_H \quad (3.4)$$

Where,

$G_D$  = Diffuse irradiance on a horizontal plane [ $W/m^2$ ]

$F_D$  = Diffuse fraction (value depends on the clearness index) [-]

$G_H$  = Global solar irradiance on a horizontal surface [ $W/m^2$ ]

### 3.5 Total solar irradiance

It is the sum of beam and diffuse irradiance on a horizontal plane, most termed as the global irradiance on the surface.[10]

$$G_H = G_D + G_B \quad (3.5)$$

where,

$G_D$  = diffuse irradiance on a horizontal surface [ $W/m^2$ ]

$G_B$  = beam irradiance on a horizontal surface [ $W/m^2$ ]

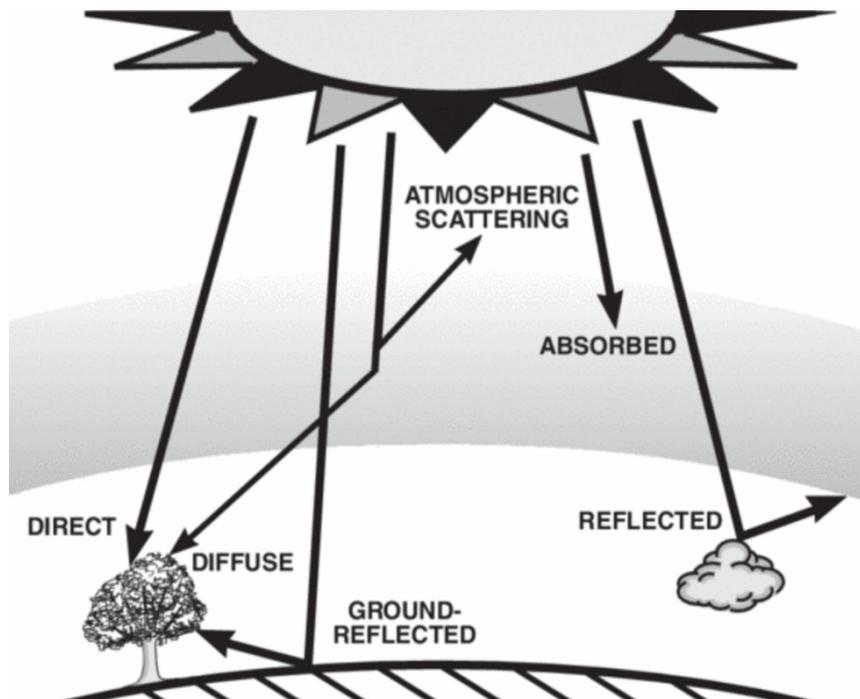


Figure 7: Types of irradiance on a horizontal plane[11]

## 4 Diffuse irradiance calculations

We know that the diffuse irradiance is really important as most of our radiation is being scattered in the atmosphere due to dust, water, gases, etc.

There has been a lot of improvements in finding the accuracy in diffuse irradiance value. Liu and Jordan in 1960 have laid the beginning in the calculation of modelling for assessment of diffuse irradiance. The equation by them couldn't provide any hourly estimations. This equation is mentioned below:

$$\frac{H_D}{H_{on}} = 0.2710 - 0.2939 \frac{H_B}{H_{on}} \quad (4)$$

where,

$H_D$  = daily diffuse radiation on a horizontal surface [ $J/m^2$ ]

$H_{on}$  = daily extraterrestrial radiation on a horizontal surface [ $J/m^2$ ]

$H_B$  = daily direct radiation on a horizontal surface [ $J/m^2$ ]

Orgill and Hollands (1976) have estimated a new model to calculate the hourly diffuse irradiance, which they experimented with the latitude between 40°N and 60°N.

They have accumulated the clearness index which represents the condition of the sky if it is the clear sky or cloudy conditions. The main aim of this is to find the average clearness index over the years.[13]

Erbs et al. (1982) have estimated a new relationship between the clearness index and diffuse fraction by measuring various cities in the United States. The values of the clearness index over 2 to 4 years were included in the measurement. They also added the clearness index with a constant diffuse fraction for  $0.8 < K_T$  that has been used by Orgill and Hollands. The main part of his calculation was based on the clearness index and sunset hour angle which depends on the seasons. They had made sure to implement the mean bias error and standard deviation to his calculations to know how accurate the values are concerning the measured values. The most basic idea of the Erbs model is the relation between the  $K_T$  and  $F_D$  into fitted polynomials. This has been first investigated by Liu and Jordan (1960) who discovered the relation between global and diffuse irradiance at a daily interval.[1], [6]

These were some of the early discoveries made in the estimation of diffuse irradiance which then followed or improved by various scientists like Reindl in (1992), Janjai in (1996), Li et al. (2011), etc. Several analytical studies have been conducted to improve the data across the world. It is often compared to the most similar terrain or environmental factors for getting better results and accuracy in values.[11]

#### 4.1 Clearness index

The clearness index is the measured global horizontal irradiance divided by the extraterrestrial irradiance. If it is a rainy day or sky fully covered by clouds, perhaps one can assume that all that irradiance will be diffuse.[1][2]

It is a dimensionless number between zero and 1. The clearness index is useful because it serves, somehow, as a measure of "how cloudy" the day is. The clearer the day, the less cloudy. That is why it makes sense to use the clearness index as an indicator of the fractions of diffuse/direct irradiance. The higher the clearness index, the less cloudy is the day, and thus the larger the fraction of beam irradiance. The clearness index value is used in the Erbs model correlation to calculate the diffuse fraction of solar irradiance, depended to this value range then the diffuse irradiance is being calculated. [3]

The  $K_T$  can be defined on an hourly basis

**Hourly clearness index:**

$$K_T = \frac{I_H}{I_{on}} \quad (4.1)$$

where,

$I_H$  = hourly global radiation on a horizontal surface [ $J/m^2$ ]

$I_{on}$  = hourly extraterrestrial radiation on a horizontal surface [ $J/m^2$ ]

#### 4.2 Various models for the calculation of diffuse irradiance on a horizontal surface

Measuring diffuse irradiance is a complicated process. There are two ways we can measure the diffuse components. The first method is to measure it with a pyranometer as this technique is expensive and requires a lot of maintenances. The second method is to use a shadow ring that should be kept parallel to the Sun path and this will result in blocking the beam irradiance. The problem with this method is that it not only blocks beam irradiance but also some part of diffuse irradiance reaching the surface hence it will provide a poor estimation.[12]

For this there is a need for the mathematical calculation to determine the diffuse irradiance from the global irradiance, which is proposed by various scientists. Some of this mathematical model is being used in simulation software like Trnsys, Homer, etc. In the next chapter, Erbs model is used to decompose the solar irradiance to find the diffuse components.

## 5 Erbs model

### 5.1 Estimating diffuse fraction of hourly global irradiance

The value of the diffuse fraction of the hourly total irradiance is highly dependent on the clearness index. From a general perspective, if the atmosphere is clearer then a small fraction of irradiance is scattered in the atmosphere.

The diffuse irradiance fraction for an hour is only dependent upon the type and formation of clouds during the measured time, which is the reason why it largely dependent to the different seasons. The diffuse irradiance formed by this equation mainly based on the relationship between clearness index and the diffuse fraction. The relationship between diffuse fraction and clearness index is combined in order to find out the total irradiance for the hour and average value are being calculated for each interval in clearness index is 0.025.[6]

#### Erbs model (ER)

Interval:  $K_T \leq 0.22$ ;  $F_D = 1 - 0.09 K_T$

Interval:  $0.22 \leq K_T \leq 0.8$ ;  $F_D = 0.9511 - 0.1604K_T + 4.388K_T^2 - 16.638K_T^3 + 12.336K_T^4$

Interval:  $K_T > 0.8$ ;  $F_D = 0.165$

(5.1)

The advantage of Erbs model is that the calculation procedure is simple and can be used to any geographical terrain, but the performers of the model may vary. In the Erbs model, the diffuse fraction is used as the ratio of diffuse irradiance to the horizontal irradiance is a function of clearness index. Whereas the clearness index is defined as the ratio of extraterrestrial irradiance to global irradiance.

The diffuse fraction for an hour is highly dependent on the condition of the sky which is the distribution of clouds. The clearness index and the diffuse fraction cannot be used as a function of Sunshine percentage in the sky. For this why the hourly diffuse fraction is separated into three bins using the hourly possible percent of Sunshine. The first has 0-20 percent, which is the most clouded condition like when it is rainy, 21-80 percent which is said to be partly clouded and 81-100 is the clearer sky region like daytime.[6][1]

## 5.2 Correlation between measured and calculated data

The measured value of the diffuse irradiance is compared against the calculated value of the diffuse irradiances using Erbs models. The calculation is completed using coefficient of determination.[1]

$$R^2 = 1 - \frac{\sum_{i=1}^N (y_i - f_i)^2}{\sum_{i=1}^N (y_i - \bar{y})^2} \quad (5.2)$$

where,

$R^2$  = coefficient of determination

$y_i$  = measured diffuse irradiance on a horizontal surface [ $W/m^2$ ]

$f_i$  = calculated diffuse irradiance on a horizontal surface [ $W/m^2$ ]

$\bar{y} = \frac{1}{N} * (\text{maximum measured diffuse irradiance for given month})$  or known as mean value

The ideal correlation will have ( $R^2 = 1$ ), i.e. all the calculated data will match the measured values. It also helps to show the dispersion of calculated points from the measured values. The lower value indicates a weaker correlation between the calculated and measured irradiance.

## 6 Methodology

### 6.1 Solar data used

The data of hourly global irradiance and diffuse irradiance on a horizontal plane were provided by dr. Vladimír Jirka from ENKI, o.p.s. (non-profit organisation) in Třeboň, South Bohemia. All data set (global irradiance and diffuse irradiance) is of 15 minutes intervals which  $4 \times 24 \times 365$  values, is then converted to the duration of one hour which gives us the result as hourly global irradiance and diffuse irradiance on a horizontal plane. The values were given in irradiance [ $W/m^2$ ] and radiant energy density [ $kJ/m^2$ ], which was then converted into [ $J/cm^2$ ] to do the analysis.

Table 1: Sample input solar data

date	time	global flux	global energy	diffuse flux	diffuse energy
		W/m <sup>2</sup>	kJ/m <sup>2</sup>	W/m <sup>2</sup>	kJ/m <sup>2</sup>
19/11/07	10:15	130	114	124	111
19/11/07	10:30	164	131	157	126
19/11/07	10:45	267	161	218	146
19/11/07	11:00	231	200	217	180
19/11/07	11:15	215	171	201	158
19/11/07	11:30	433	287	233	212
19/11/07	11:45	138	155	132	137
19/11/07	12:00	145	110	142	106
19/11/07	12:15	114	104	106	101
19/11/07	12:30	113	107	106	103
19/11/07	12:45	249	114	138	99

The latitude of the location is  $49.005^\circ$  north and longitude is  $14.774^\circ$  east and the time zone is GMT +1 hour. This data was measured for the year 2007. All the calculations were done in an Excel file. The experiment covers the behaviour of diffuse irradiance on a horizontal plane during different months in Třeboň and determining how accurate is the calculated diffuse irradiance and beam irradiance from the measured data by using the Erbs model.

### 6.2 Data processing

The global irradiance on a horizontal plane and the hourly extraterrestrial irradiance equation (3.2.B) was calculated to find the clearness index of the hour. The Erbs model is used to calculate the diffuse fraction which depends on the clearness index value. Graphs of 12 months were created, depending upon each month there is a difference between the measured value and the calculated value. Both values are again compared to find the  $R^2$  value to find the correlation between them.

## 7 Results

This section shows the results of Erbs model and correlation of calculated diffuse irradiance and direct irradiance on a horizontal plane. Graphs with various irradiance components are included in this section to support the result. The Erbs model always estimates the diffuse irradiance first. The direct irradiance on a horizontal plane can be calculated from total solar irradiance and estimated diffuse irradiance on a horizontal plane.

### 7.1 Correlation of clearness index and diffuse fraction

The diffuse fraction describes the relationship between the global radiation and diffuse radiation on a horizontal plane. The clearness index is the ratio between the hourly global radiation and the calculated hourly extraterrestrial radiation.[5][11]

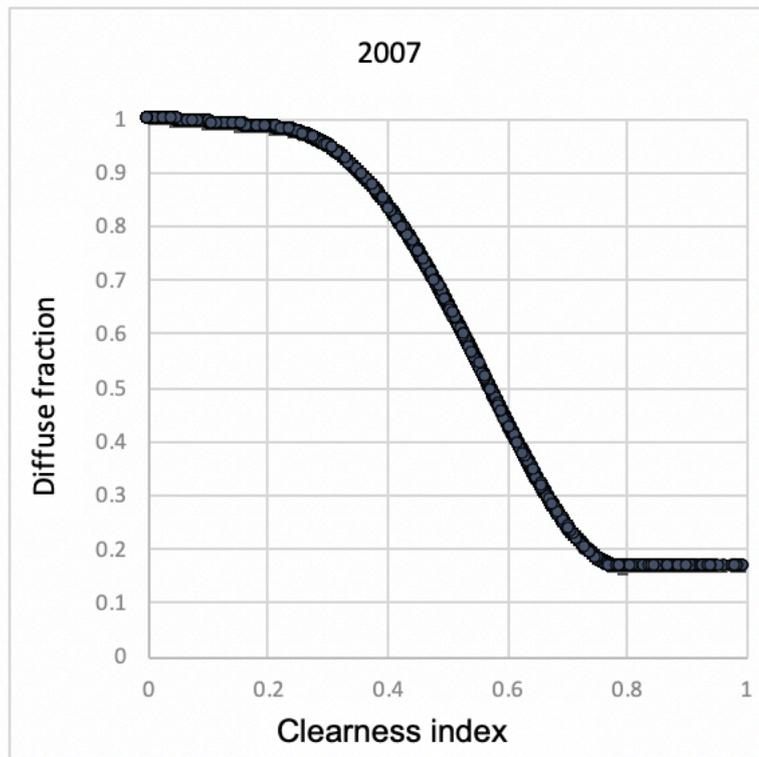


Figure 8: The average hourly weighted diffuse correlation using Erbs model

Now the next step is to make a graph of calculated diffuse irradiance value against measured diffuse irradiance to find the accuracy of the calculation. The graphs are shown below for different months. As previously discussed the Erbs model tends to calculate diffuse irradiance or diffuse radiation energy. The graphs of 12 different months are shown above. The trendline shows the ideal line for the data, helps to represent the dispersion of values in the chart. When the value is below the trendline, i.e. the calculated diffuse radiation is underestimated compared to the measured diffuse radiation energy and also when the value is above the trendline that means the calculated value is overestimated.

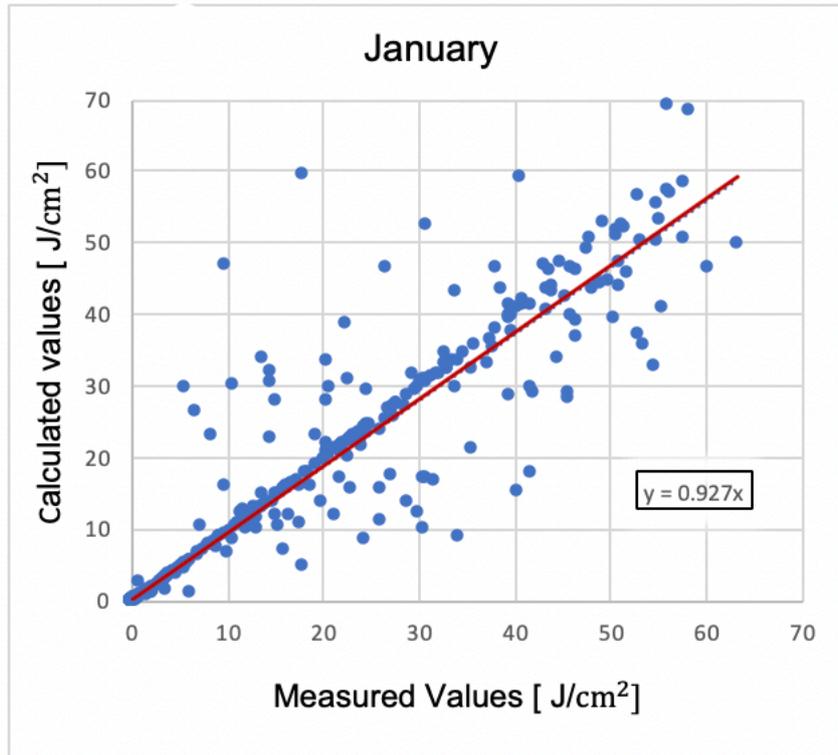


Figure 9: Diffuse radiation on a horizontal plane; measured data vs calculated data for January

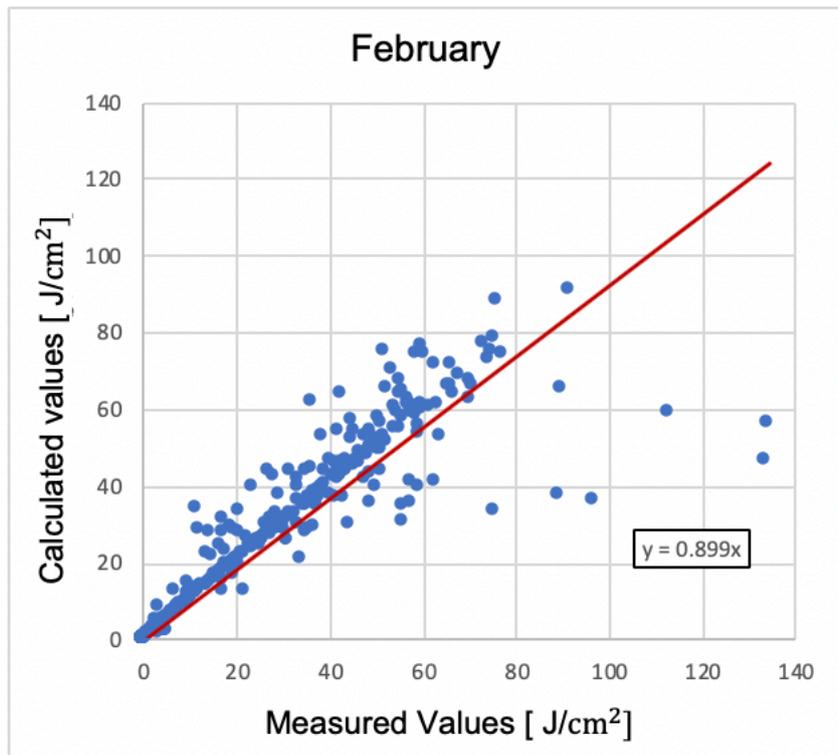


Figure 10: Diffuse radiation on a horizontal plane; measured data vs calculated data for February

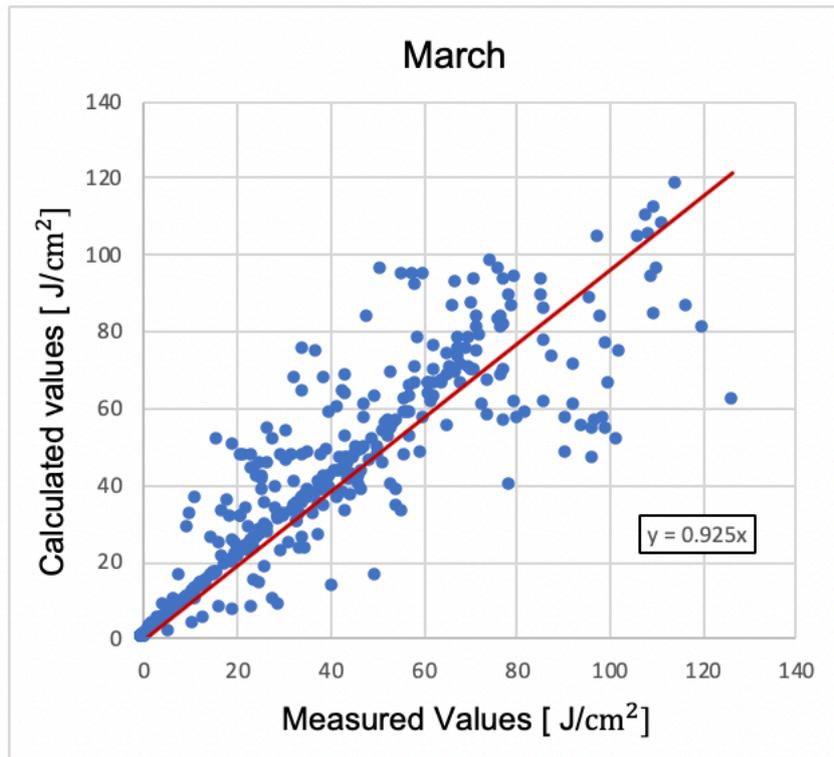


Figure 11: Diffuse radiation on a horizontal plane; measured data vs calculated data for March

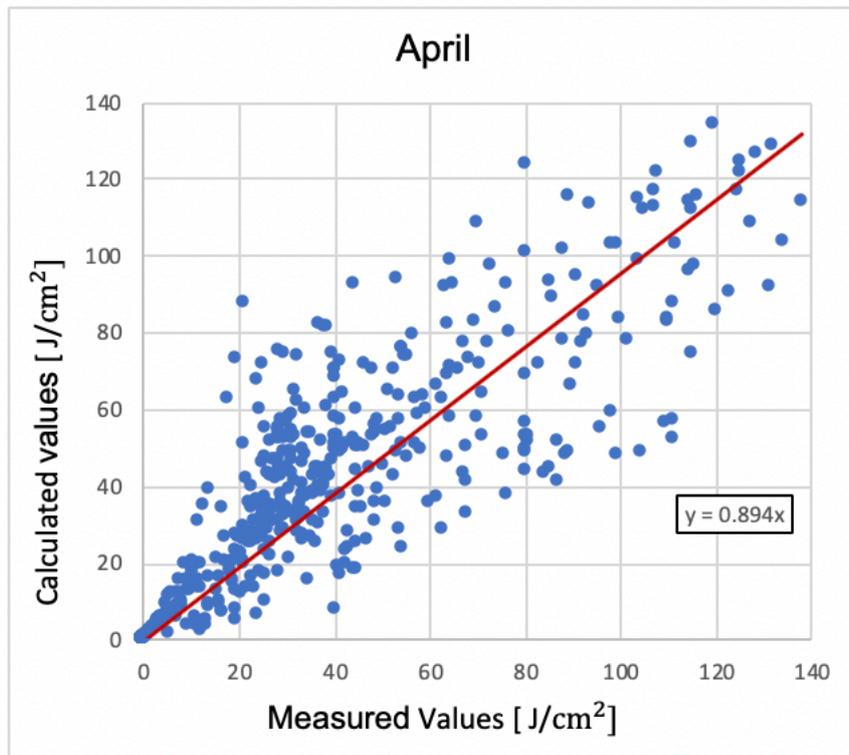


Figure 12: Diffuse radiation on a horizontal plane; measured data vs calculated data for April

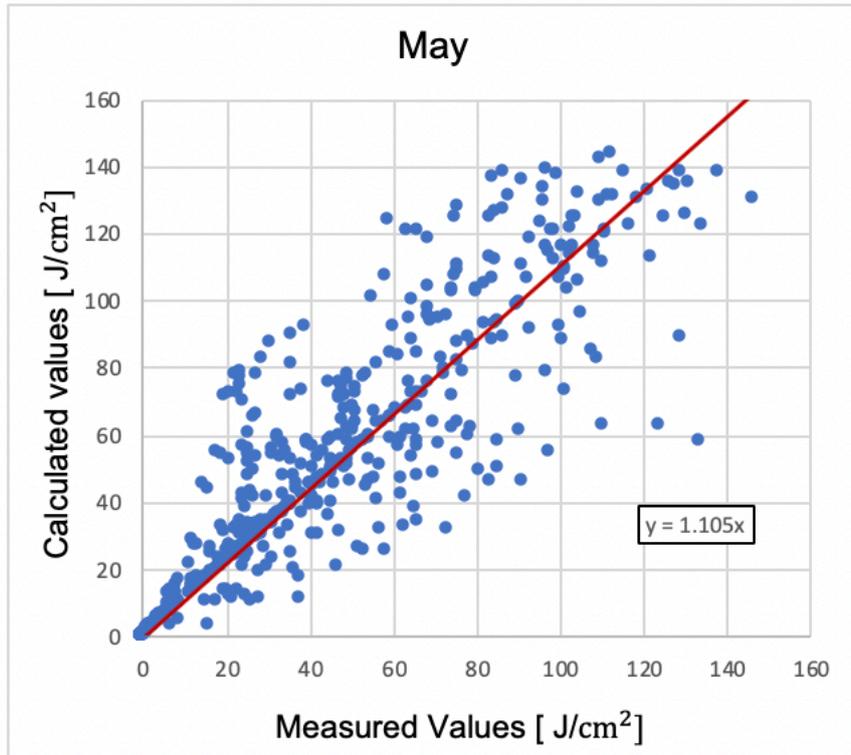


Figure 13: Diffuse radiation on a horizontal plane; measured data vs calculated data for May

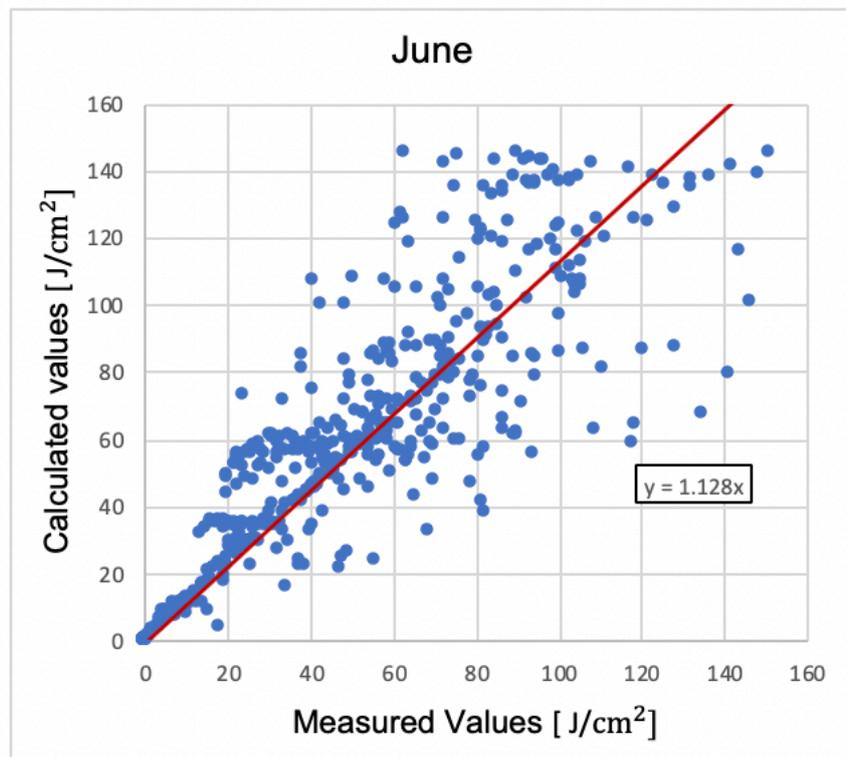


Figure 14: Diffuse radiation on a horizontal plane; measured data vs calculated data for June

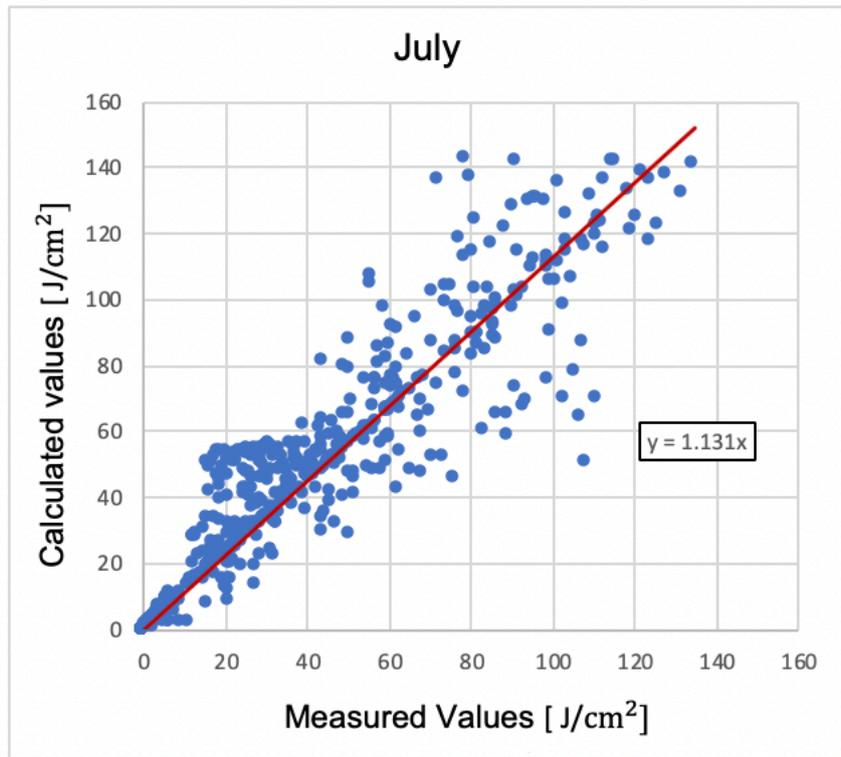


Figure 15: Diffuse radiation on a horizontal plane; measured data vs calculated data for July

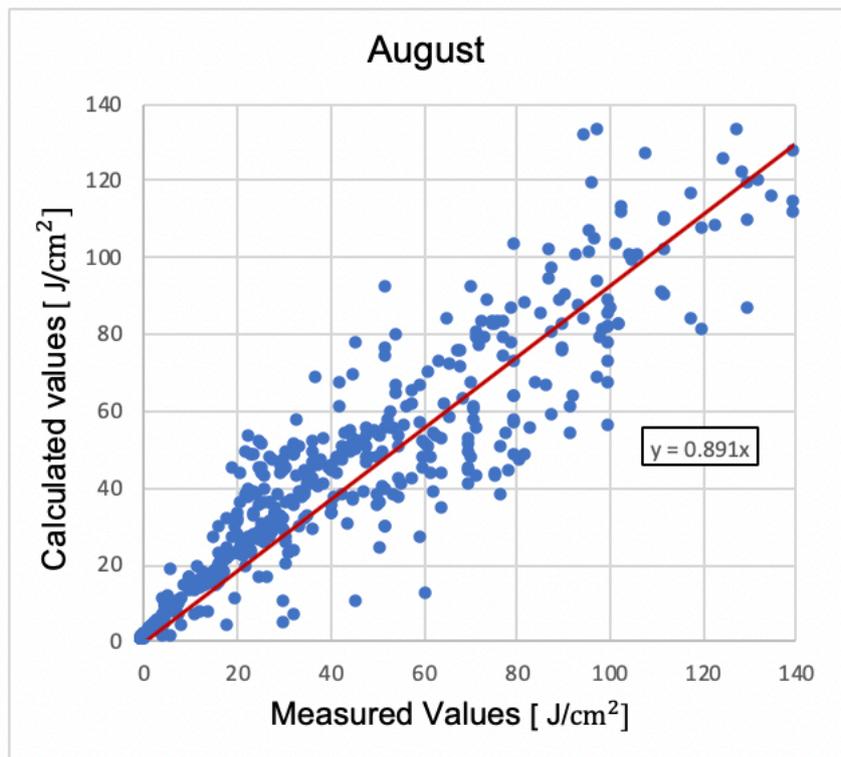


Figure 16: Diffuse radiation on a horizontal plane; measured data vs calculated data for August

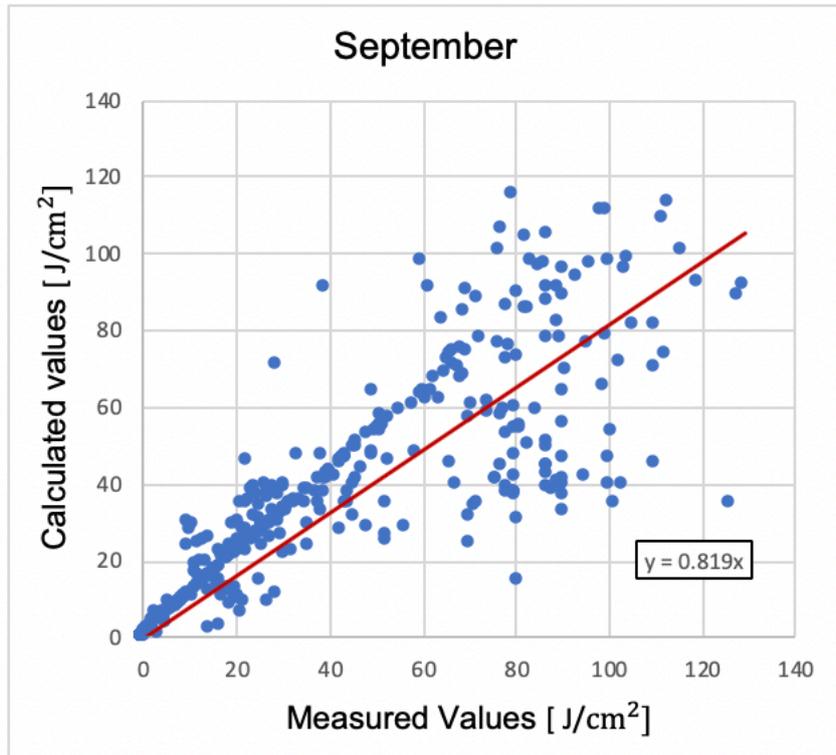


Figure 17: Diffuse radiation on a horizontal plane; measured data vs calculated data for September

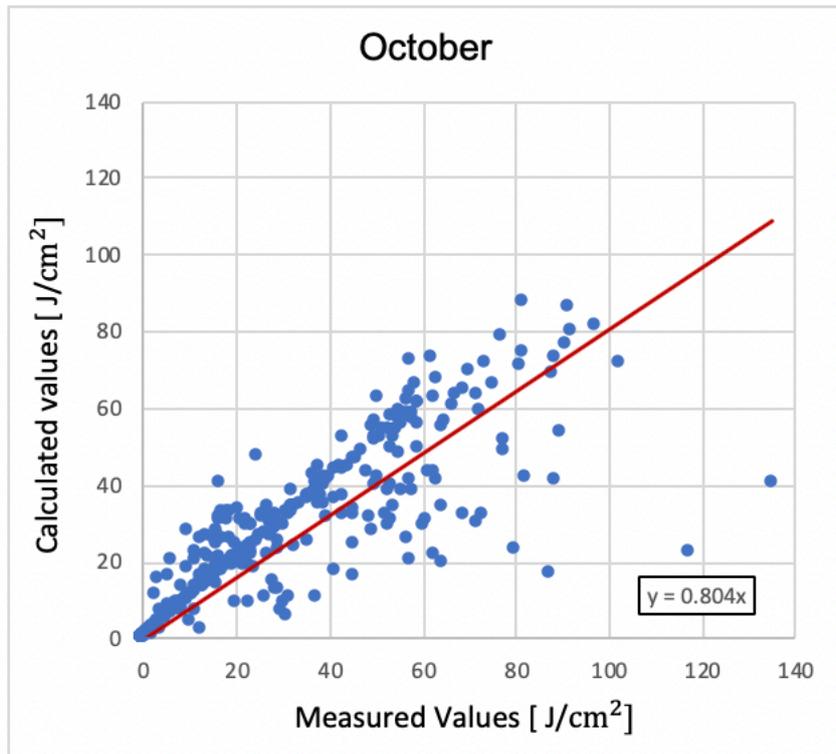


Figure 18: Diffuse radiation on a horizontal plane; measured data vs calculated data for October

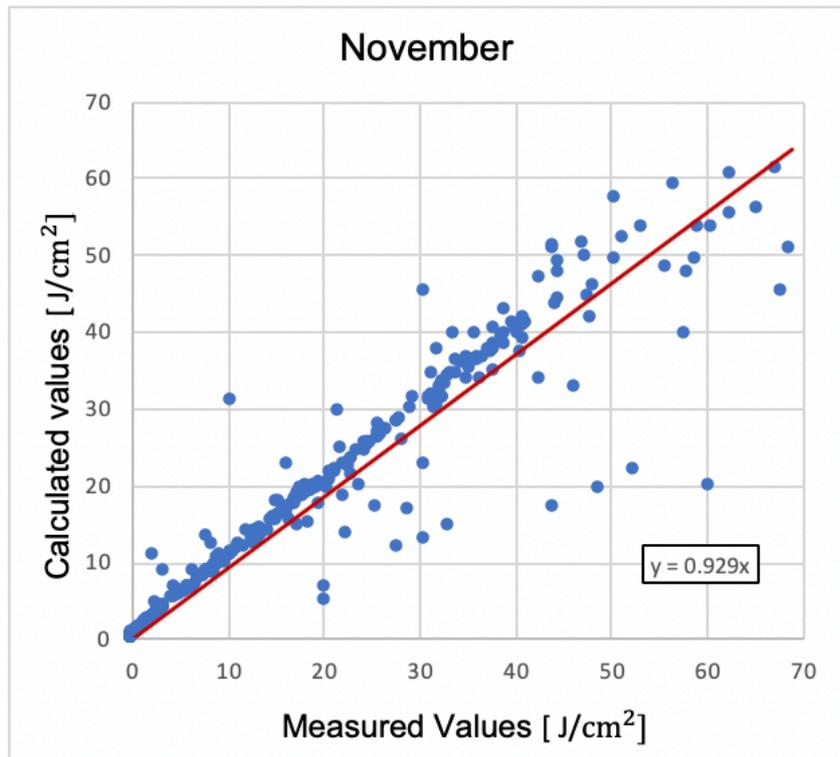


Figure 19: Diffuse radiation on a horizontal plane; measured data vs calculated data for November

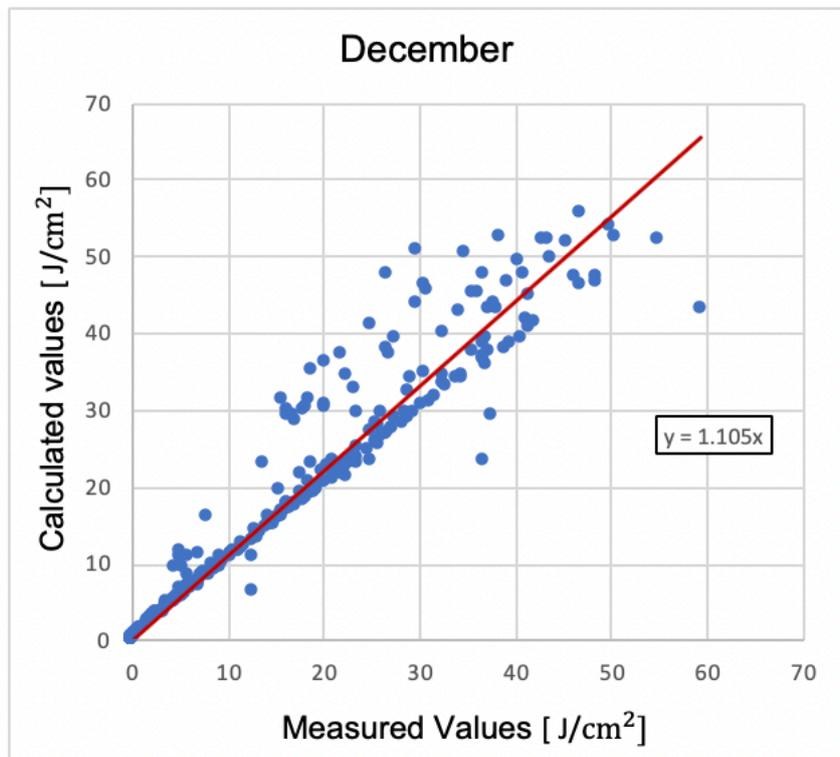


Figure 20: Diffuse radiation on a horizontal plane; measured data vs calculated data for December

## 7.2 Correlation between measured value and calculated value

Table 2: Coefficient of determination for monthly data

<b>Month</b>	<b>R<sup>2</sup>[-]</b>
January	0.969
February	0.936
March	0.893
April	0.813
May	0.780
June	0.705
July	0.789
August	0.891
September	0.822
October	0.889
November	0.981
December	0.987

The  $R^2$  value ranges from 0 to 1, with 1 defining perfect match between the calculated and measured values. It quantifies the correlation between the calculated values by using the Erbs model and an ideal model that would align the measured data. The lower value indicates a weaker correlation between the calculated and measured values.

### 7.3 Calculated direct radiation on horizontal plane

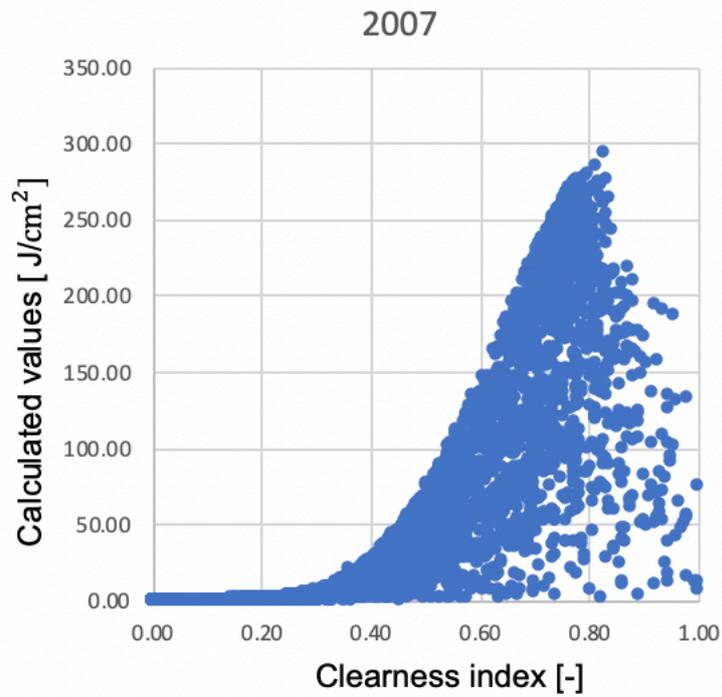


Figure 21: Calculated direct radiation energy related to clearness index

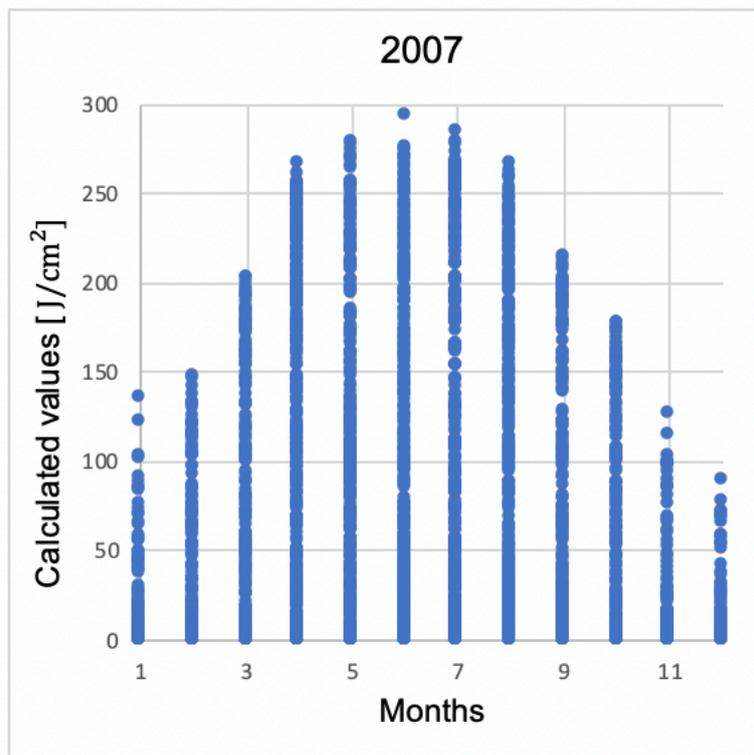


Figure 22: Direct radiation energy on a horizontal plane; calculated data for each month

Correlation of clearness index and diffuse fraction describes the direction of values due to the changes within days, weeks, etc., of the atmospheric physical condition for average cloudiness in the sky, seasonal impacts, chemical pollutant presence, and so on. The range of clearness index lies between 0 to 0.8, for every 0.2 intervals there is a different formula to find the diffuse fraction. The range between 0 to 0.2 for clearness index is determined as low clearness index range, range between 0.20 to 0.50 determines as medium-range and 0.50 to 1 is considered as the high clearness index range. Calculated direct irradiance on a horizontal surface is correlated in terms of clearness index graph shows the direction of direct irradiance in different atmospheric conditions. High clearness index leads to having more direct irradiance on the surface. The graph, distribution of direct irradiance on a horizontal surface for each months are shown above.

## 8 Discussion

The graphs above help to determine the efficiency of the Erbs model in predicting the diffuse irradiance and direct irradiance. It is noted from the calculation that the clearness index values are in low range during the hours close to sunrise and sunset, with values between 0.02 and 0.30. Whereas the diffuse fraction has very high values in such hours which is between 0.95 and 0.99. It is due to the solar irradiance received at the surface, during this period it consists mainly of diffuse components [1]. During the local noon, that means the Sun is overhead, the values of the clearness index is high which lie between 0.30 and 0.8, the diffuse fraction has a low value in this period. From the above graphs it is seen that from May to July, the Erbs model overestimates the diffuse fraction which results in having large diffuse irradiance from the global irradiance.  $R^2$  values for these months have a lower value which shows a weaker correlation between the calculated and measured values, whereas for the other months like January, February, November, and December, the differences are really not that high which is almost aligning to the measured values.

The calculated diffuse irradiance for these months is underestimated compared to the measured irradiance but the correlation is better with the measured data. When the clearness index value is more than 0.8, the diffuse fraction has a constant value which is 0.165 using the Erbs model. When the clearness index is between range 0.6 and 0.8, the estimated diffuse irradiance is often underestimating or overestimating the measured value. This is the reason why month like May, June, and July has low  $R^2$  value compared to the other months. This can result in having more direct irradiance for that period. Also, when we compare the calculated and the measured diffuse irradiance using the Erbs model, in some days of the year, the Erbs model cannot estimate diffuse irradiance above 40 [ $\text{J}/\text{cm}^2$ ] while the measured value can reach unto 70 [ $\text{J}/\text{cm}^2$ ], which is due to the high clearness index value.

January and December have a lower value of diffuse irradiance compared to other months. The Erbs model makes use of a decreasing linear feature within the low clearness index range; that means the higher diffuse fraction will have a low range clearness index. So the sunny day will have less diffuse fraction which results in more direct irradiance than the diffuse irradiance on a horizontal plane.

It is here seen that for values of the clearness index less than 0.80, there is good correlation between the measured and calculated values. For values of clearness index higher than 0.8, the prediction that the diffuse fraction has a constant value is not in a good agreement with the measured value.

As it was described in the theory chapter, the diffuse irradiance is measured by a pyranometer which requires a special shading ring attached to it, that should be manually moved to continuously prevent any direct irradiance arrive on the device. This can bring some errors into the measured data, for example, if there is an interruption in the control of the ring which can happen due to various reasons. The accuracy of measured data can vary because of this.

The direct irradiance on a horizontal surface is high when the clearness index is above 0.6 according to the graph as less irradiance is scattered in the air. Direct irradiance on a horizontal surface is low for January and December as the global irradiance is less for these months. The direct irradiance has high values during the month of June and July. As some of the direct irradiance data can be overestimated or underestimated depending on the calculated diffuse irradiance accuracy.

## 9 Conclusion

Measurements of Sun components require instruments that won't be feasible to a lot of growing countries. To offer a complete Sun radiation dataset, one of the strategies carried out is to find the diffuse and direct irradiance using mathematical models. In this article, we are calculating the solar irradiance data by using the Erbs model.

The second chapter gives a background about the solar geometry, this concept will help to understand the component of the atmosphere that obstruct the sunlight from entering the Earth's surface. And also how the angle can help to determine the irradiance on a surface. These geometrical studies will help to calculate some parameters like the clearness index, declination angle, and zenith angle, which are three important parameters in this model to find the diffuse and direct irradiance. The experiment was done for the location of Třeboň.

The Erbs model can be used to compute direct and diffuse irradiance from the total irradiance. The experiment exploits the model's vulnerability when the clearness is low, medium, and high range. Scattered graphs of measured data and calculated values are provided for determining the difference between the two. The more data scatter outwards the linear line the more error in a calculated value. This data was calculated in  $[\text{J}/\text{cm}^2]$  units. The range of clearness index lies between 0 to 0.8, for every 0.2 intervals there is a different formula to find the diffuse fraction.

The clearness index above 0.8 has a constant value for diffuse fraction, hence results in low accuracy during this range. The model performs better in predicting the diffuse irradiance when the clearness index is in medium range. The Erbs model primarily experimented in the American cities which had clearness index in the intermediate. This means the model might include some more parameters like relative humidity, etc into the calculations to improve the accuracy. Overall the Erbs model performs well in predicting the diffuse irradiance from the global irradiance. There have been a lot of studies going on to find a common model that can be used for any location. The quality of the solar data is helpful and makes a huge difference in the solar energy sector.

One of the possible applications of the decomposed solar data can be to use them as inputs for building performance simulations. In this context, the results obtained from the current research might be tested on a set of building models. Building simulation results obtained from the calculated and measured solar data can be then compared. This could be an interesting topic for future work.

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