



CZECH TECHNICAL UNIVERSITY IN PRAGUE  
Faculty of Electrical Engineering  
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**Real-time estimation of some physical  
parameters of trams relevant for onboard  
prediction of braking distance**

Master's thesis

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## II. Master's thesis details

Master's thesis title in English:

**Real-time estimation of some physical parameters of trams relevant for onboard prediction of braking distance**

Master's thesis title in Czech:

**Odhadování vybraných fyzikálních parametrů tramvaje v reálném čase relevantních pro palubní predikci zábrzdě dráhy**

Guidelines:

The goal of the project is to enhance the existing onboard algorithms for prediction of tram braking distance with a capability to update the estimates of some relevant physical parameters in real time. In particular, accurate estimates of the mass of the tram (given by the number of passengers), the railway track gradient (with some initial estimate obtained from a map) and the adhesion coefficient (accounting for slippery rails due to snow, ice, morning dew or leaves) need to be available to the onboard predictor of braking distance. Nonetheless, critical discussion of the very assignment is expected in the thesis. Namely, the mass of payload of the tram might be available to the onboard computer and the slope of the track might be measured once and for all and encoded in a map or a lookup table (hence no need to estimate it onboard). Data obtained in real experiments with trams will be provided. The resulting algorithm(s) should be implemented in Matlab/Simulink. Demonstration of functionality through extensive simulations (combined with the measured data) is sufficient but some experimental demonstration can also be targeted (in collaboration with the industrial partner).

Bibliography / sources:

- [1] D. Simon, Optimal State Estimation. Hoboken, NJ, USA: John Wiley & Sons, Inc., 2006.
- [2] L. Do, „V2V communication-enabled collision avoidance for railroad vehicles,” Master thesis, Czech Technical University in Prague, Prague, Czech Republic, 2019.
- [3] “IEEE Guide for the Calculation of Braking Distances for Rail Transit Vehicles,” IEEE Std 1698-2009, pp. C1-31, Nov. 2009, doi: 10.1109/IEEESTD.2009.5332051.

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## Declaration

I hereby declare that this thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

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Bc. Petr Hrych

21.5.2020



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I would like to thank my supervisor, doc. Ing. Zdeněk Hurák, PhD., for his guidance and questions, which forced me to investigate each problem more deeply. My thanks also go to Herman systems for organizing experiments in Ostrava and Český úřad zeměměřický a katastrální for providing an altitude map of Ostrava. My last and greatest thanks go to my family for their support during my studies.



## Abstract

This diploma thesis deals with the problem of estimating parameters of a tram model to estimate the braking distance. The thesis introduces the model of a tram and all its parameters. These parameters are split into three groups based on their properties. Each parameter is described in detail, as well as its effect on the model and behavior of the tram. For the purpose of this diploma thesis, an experiment on a standard tram line was performed. During this experiment, the acceleration and the velocity of the tram were measured. The algorithms developed for estimating the values of the parameters were evaluated on data gained from this experiment. In addition to algorithms for estimating parameter values, the thesis also presents an algorithm for estimating the input from measured data. The results of this thesis can contribute to the accuracy of the estimation of tram braking distance and improve the anti-collision system.

**Keywords:** tram, modeling of dynamic systems, parameter estimation

## Abstrakt

Diplomová práce se zabývá problémem odhadování parameterů matematického modelu tramvaje za účelem odhadování brzdné dráhy. V práci je přestaven model tramvaje včetně všech jeho parameterů. Jednotlivé parametry jsou podle jejich vlastností rozděleny do tří skupin. Každý parametr je detailně popsán, stejně tak i jeho vliv na model a chování tramvaje. Pro účely této diplomové práce byl proveden experiment, během kterého probíhalo měření zrychlení a rychlost tramvaje při běžém provozu. Algoritmy vytvořené pro účely odhadování parameterů byly použity a otestovány na naměřených datech. Kromě algoritmu pro odhadování hodnot parameterů modelu je představen i algoritmus pro zpětné odhadování vstupu systému z naměřených dat. Výsledky této práce mohou přispět k zpřesnění odhadu brzdné dráhy tramvaje a vylepšení antikolizního systému.

**Klíčová slova:** tramvaj, modelování systému, odhadování parameterů



# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Motivation . . . . .	1
<b>2</b>	<b>Model and Parameters</b>	<b>3</b>
2.1	Model of Tram Dynamics . . . . .	3
2.1.1	Model Equations . . . . .	3
2.2	Parameters of Motion Model . . . . .	6
2.2.1	Known Time-invariant Parameters . . . . .	6
2.2.2	Unknown Time-invariant Parameters . . . . .	6
2.2.3	Time-varying Parameters . . . . .	7
<b>3</b>	<b>Estimation of Inclination</b>	<b>9</b>
3.1	Data from OpenStreetMap . . . . .	9
3.2	Altitude Map . . . . .	9
3.2.1	Křovák Projection . . . . .	12
3.3	Estimation of Inclination . . . . .	13
3.3.1	Smoothing and Filtering . . . . .	14
<b>4</b>	<b>Experiments</b>	<b>19</b>
4.1	Measurement Setup . . . . .	19
4.2	Measured Data . . . . .	22
4.2.1	Acceleration Measurements . . . . .	22
4.2.2	Velocity Measurements . . . . .	23
4.2.3	Input . . . . .	23
4.3	Synchronization . . . . .	23
4.4	Inclination . . . . .	25
<b>5</b>	<b>Input Estimation</b>	<b>29</b>
5.1	Motors of Tram . . . . .	29
5.2	Types of Tram Brakes . . . . .	30
5.2.1	Electrodynamic Brake . . . . .	30
5.2.2	Parking Brakes . . . . .	30
5.2.3	Friction Rail Brake . . . . .	31
5.3	Tram Control . . . . .	31
5.4	Input of Model . . . . .	32
5.4.1	Offset Filtering . . . . .	33

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5.4.2	Acceleration Related to the Gravity . . . . .	36
5.4.3	Estimating Strategy . . . . .	36
5.4.4	Simulations with Estimated Input . . . . .	39
<b>6</b>	<b>Estimation of Resistance</b>	
	<b>Parameters</b>	<b>41</b>
6.1	Estimation . . . . .	42
<b>7</b>	<b>Estimation of Weight</b>	
	<b>and Motor Parameters</b>	<b>45</b>
7.1	Used Functions and Data Preparation . . . . .	45
7.2	Estimation of $k_+$ . . . . .	45
7.3	Estimation of $k_-$ . . . . .	46
7.4	Estimation of Weight . . . . .	47
<b>8</b>	<b>Conclusion</b>	<b>51</b>
8.1	Future Work . . . . .	52
	<b>Appendices</b>	<b>53</b>
	<b>A Motion Model</b>	<b>55</b>
	<b>References</b>	<b>57</b>



# 1 | Introduction

In this thesis, I will follow up on my colleague's work in which he estimated the braking distance of a tram. In his thesis, he was mainly focused on methods for estimating the braking distance. Problems with the parameters of the model and their estimation were left open. In my thesis, I will introduce the model of the tram dynamics and describe its parameters. Further on, I will try to introduce methods for estimating some parameters of this model. The evaluation of developed methods will be done on the measured data from experiments done in Ostrava.

## 1.1 Motivation

Over the last few years, larger cities have begun to reduce the car traffic in city centers, either to lower the emissions or to improve the overall traffic situation. These restrictions go hand in hand with the support of public transportation, including trams. Unlike the car traffic, the contact between trams and pedestrians can be much closer. People are watching the passing cars when crossing the road. However, trams, in many cases, can go through places where people are not so careful (e.g. squares). Trams are also usually much quieter than cars. This and many other aspects put high demands on the attention of tram drivers. Tram accidents, either with pedestrians, cars, or other trams, are not unique events. The number of accidents for trams in Prague between 2008 and 2017 is in table 1.1.

These days, new cars are equipped with systems that are able to alert the driver to an obstacle and even start braking, in critical situations. The logical question arises as to whether such a system could be developed for trams as well. A system that would be able to evaluate the distance between the tram and the obstacle could significantly reduce the number of accidents. For any system that estimates the braking distance, it is necessary to know the motion model and its parameters with the highest possible accuracy. In my thesis, I will describe the model of a tram, describe its parameters, and develop several methods for estimating these parameters.

Year	Total number of accidents	Accidents caused by tram driver	Accidents caused by other drivers	Accidents with pedestrians
2008	1441	219	1055	76
2009	1421	225	1021	91
2010	1432	242	1040	76
2011	1279	177	959	69
2012	1282	204	943	62
2013	1302	204	938	82
2014	1394	226	999	90
2015	1359	194	984	99
2016	1353	183	1026	75
2017	1572	231	1164	111

**Table 1.1:** Tram accidents in Prague between years 2008 and 2017.

## 2 | Model and Parameters

In this chapter, I will introduce a mathematical model for simulating tram dynamics, discuss its properties, and inaccuracies. Further on, I will introduce the main parameters of this model, describe their importance for estimating braking distance and main ideas for their estimation or identification.

### 2.1 Model of Tram Dynamics

The roughest model of a tram can be described by Newton's second law, but in real life, we are facing other aspects that are influencing train dynamics, such as friction between wheels and rails, the resistance of the air, slip of the wheels, etc. All these and many more aspects have an impact on the tram behaviour. For the acceptably accurate model, it is required to describe these aspects and include them in the model.

Many scientific articles are aiming at describing the dynamic of railway vehicles, such as [1]. These models are mainly describing the dynamic of trains, but it shows up that the dynamics of trains and tram is sufficiently similar.

One of the main goals of my thesis is to estimate parameters of a tram, for estimating the braking distance [3]. Therefore, the model used in this thesis is based on the model used in the related paper [2].

#### 2.1.1 Model Equations

At first, let me introduce all the parameters used in the model. The detailed description is in the section 2.2.

Parameter	Description	Unit
$M$	Tram weight	kg
$g$	Gravitational acceleration	$\text{m s}^{-2}$
$N_{\text{tr\_wh}}$	Number of traction wheels	-
$N_{\text{wh}}$	Total number of wheels	-
$\theta$	Inclination of rails	rad
A,B,C	Resistance parameters	-
$a_a, b_a, c_a, d_a$	Adhesion parameters	-
$m_{\text{wh}}$	Weight of the wheel	kg
$r$	Wheel radius	m
$P_{\text{max}}$	Maximum power	W
$p$	Notch position	-
$k_+$	Traction parameter for positive notch position	-
$k_-$	Traction parameter for negative notch position	-

**Table 2.1:** All parameters used in the tram model.

For creating a mathematical model of a tram, it is necessary to consider the dynamics of two main masses, that is the body of the tram and wheels. When creating the model, following simplifications were considered:

1. The body of a tram is considered as a point of a mass.
2. All wheels are considered to be the same, homogeneous disks.
3. Only the longitudinal dynamics is considered.
4. In the formula 2.3 for the resistance force, the parameter C is considered to be 0 due to lower tram speeds in comparison to trains (more in Chap 6).
5. Dynamics in curves is not considered.

The motion equation for the tram body can be described by Newton's second law.

The dynamics of the tram body can be expressed as

$$\frac{dv}{dt} = \frac{1}{M}(F_{\text{ad}}(v) - F_{\text{res}}(v) - F_G(\theta)), \quad (2.1)$$

The gravitational force  $F_G(\theta)$  [N] acting on the tram is

$$F_G(\theta) = Mg\sin(\theta). \quad (2.2)$$

For the resistance force  $F_{\text{res}}(v)$  [N] I consider the following formula

$$F_{\text{res}}(v) = A + Bv(t) + Cv^2(t). \quad (2.3)$$

Later in this thesis in Chap 6 I came up with arguments for neglecting quadratic term. The final form of the resistance formula is

$$F_{\text{res}}(v) = A + Bv(t). \quad (2.4)$$

The next formula describes adhesion force  $F_{\text{ad}}(v)$  [N]. In the equation 2.1 this force has positive sign. It represents the force which is pushing the body of a tram forward. If wheels would be only slipping, value of this force would be zero and therefore the tram would not be accelerating.

$$F_{\text{ad}}(v) = \frac{Mg\mu(v_s)}{N_{\text{wh}}} \quad (2.5)$$

The term  $\mu(v_s)$  for the friction coefficient was taken from [4] and has following form

$$\mu(v_s) = c_a e^{-a_a v_s} - d_a e^{-b_a v_s}, \quad (2.6)$$

where  $v_s$  [m/s] is the slip velocity

$$v_s(v, \omega) = \omega r - v. \quad (2.7)$$

The dynamics of each wheel can be also described by Newton's second law

$$\frac{d\omega}{dt} = \frac{1}{J}(T - rF_{\text{ad}}(v_s)N_{\text{tr\_wh}}), \quad (2.8)$$

where  $J$  is the moment of inertia of the wheel

$$J = \frac{1}{2}m_{\text{wh}}r^2. \quad (2.9)$$

From equation 2.8 we can see that input to the model is requested torque. In reality this torque corresponds with current going through the tram. Unfortunately, sensors for measuring the current were not available. Because of this the model uses the position of the control lever as an input. The position is an integer from  $-7$  to  $7$ . Lever position is transformed to torque using traction parameters  $k_+$  and  $k_-$ .

$$T(p) = \begin{cases} \min(k_+p, P_{\text{max}}), & \text{if } p \geq 0 \\ k_-p, & \text{if } p < 0 \end{cases} \quad (2.10)$$

The transformation of lever position to requested torque is described in Chap 5.

## 2.2 Parameters of Motion Model

In this section, I will describe the main parameters used in the motion model and divide them into three groups, based on their properties.

### 2.2.1 Known Time-invariant Parameters

This group contains parameters whose values can be measured, can be found in data sheets or are well-known constants. These parameters are easiest for identification.

The following parameters belong to this group:

$\mathbf{g}$	Gravitational acceleration
$\mathbf{N}_{\text{tr wh}}$	Number of traction wheels
$\mathbf{N}_{\text{wh}}$	Total number of wheels
$\mathbf{m}_{\text{wh}}$	Weight of the wheel
$\mathbf{r}$	Wheel radius
$\mathbf{P}_{\text{max}}$	Maximum power

Changes of these parameters are rather small and caused mainly by some mechanical failures (e.g. loss of a wheel mass). I consider that their value is constant, and their properties are not changing.

### 2.2.2 Unknown Time-invariant Parameters

This group includes parameters which are somehow determined by a tram or its motors, but their values cannot be determined by simple measurements.

**Resistance parameters  $\mathbf{A}$  and  $\mathbf{B}$**  (2.4) belong to this group. They represent all resistive forces acting on a tram, such as friction or air-drag. Some studies [5] focused on trams estimate these parameters from some form of the Davis formula. However, this formula was originally developed for trains and contained values estimated empirically, so I have decided to identify resistance parameters experimentally from measured data (Chap 6).

**Traction parameters  $k_+$  and  $k_-$**  (2.10) are describing transformation between the position of control lever and the requested torque. In the ideal scenario, they would not be necessary, but since measurements of a current through the motor are not available, they have to be identified from the measured data.

### 2.2.3 Time-varying Parameters

The main attribute of parameters in this group is that they change value during the ride. They can significantly affect the braking distance of the tram, as can be seen in Figs 2.1, 2.2, 2.3, and they must be either estimated from measured data or obtained by other methods. The velocities and braking distances in the figures were estimated from the model. However, the values of the parameters were estimated manually, and they serve only as an illustration.

**Tram weight (M)** is the first of the time-varying parameters. Its value can differ in tens of percents (e.g. empty Tram Tatra T3 weights 16 000kg, and 27 500kg when fully loaded). When estimating its value, we can take into account that it can change only in stations, and while driving, this value is fixed.

The second time-varying parameter is the **inclination of rails ( $\theta$ )**. Like in the previous case, this parameter also has some restrictions that can be used for easier estimation. Cities are putting limits on rails inclinations (e.g. in Brno, the maximum rail inclination is 70‰<sup>1</sup>). The second important restriction is the unchanging character of the rails. This allows me to find the inclination profile in advance and create the look-up table for each tram line.

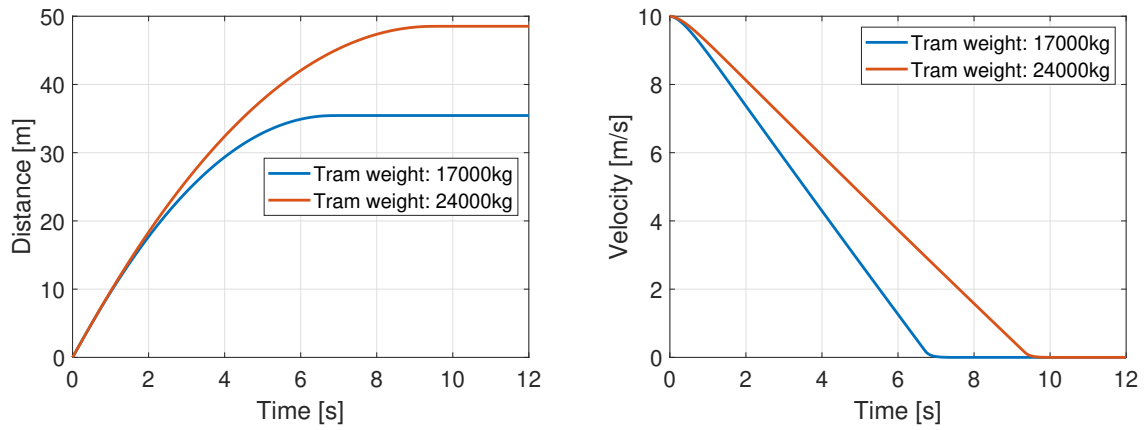
The last time-varying parameter in the motion model, and arguably the hardest one to estimate, is **adhesion**. Adhesion in the model is expressed by the set of four parameters ( $a_a, b_b, c_c, d_d$ ). These parameters have fixed values for different adhesion conditions. Adhesion condition can be graded from 0 (ideal conditions) to 3 (very weak conditions such as icing).

Parameter \ Adhesion	Adhesion			
	0	1	2	3
$a_a$	0.54	0.54	0.54	0.05
$b_b$	1.2	1.2	1.2	0.5
$c_c$	1.0	0.2	0.1	0.08
$d_d$	1.0	0.2	0.1	0.08

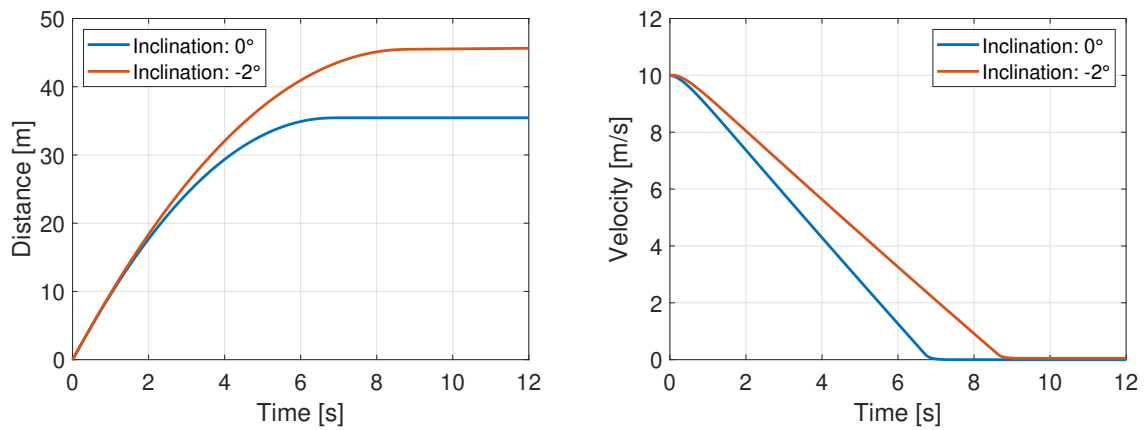
**Table 2.2:** Values of adhesion parameters.

Following figures are showing how much can the braking distance and the braking time differ for empty and fully loaded tram, while braking downhill or during weak adhesion conditions. Values of braking distances are not accurate and serves only for demonstration.

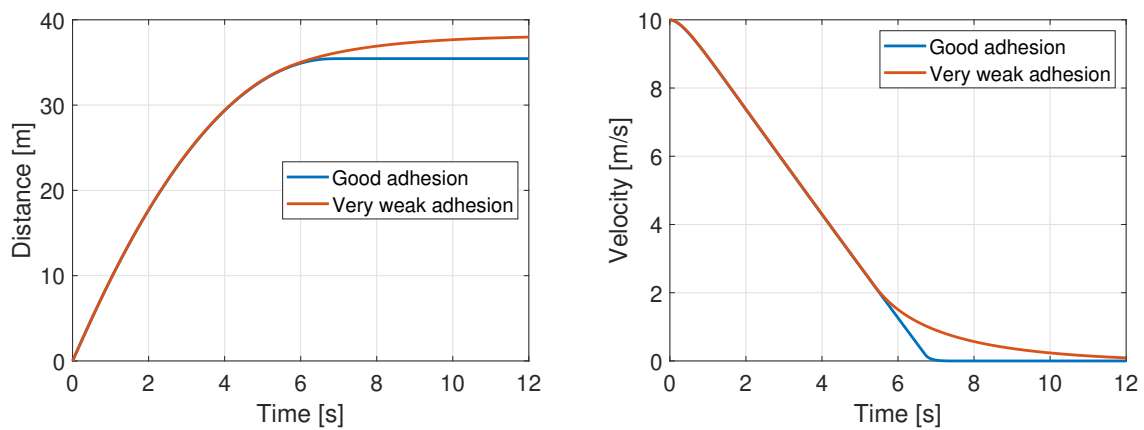
<sup>1</sup>Směrnice T-08, Provozně technické podmínky tramvajových sítí na území města Brna a jeho okolí



**Figure 2.1:** Comparison of braking distances for different weights.



**Figure 2.2:** Comparison of braking distances for different inclination.



**Figure 2.3:** Comparison of braking distances for different adhesion conditions.



## 3 | Estimation of Inclination

In this chapter, I will introduce my solution for estimating the inclination of rails. The inclination is a time-varying parameter. The fact that the tram line along which the tram rides is known allows me to estimate the inclination profile in advance. Assuming that the tram has some localization device, the inclination can be obtained from this profile. Having fewer parameters to estimate during the actual ride can provide better accuracy.

The biggest problem I faced when estimating the inclination was the fact that maps contain either information about the altitude or the position of rails, not both. For this reason, I created an algorithm which can merge information from two different types of maps. The algorithm has two inputs. The first input is a list of GPS coordinates forming the rails. This type of data can be obtained from multiple sources. I used data from **OpenStreetMap** <sup>1</sup>. The second input for the algorithm is an altitude map.

### 3.1 Data from OpenStreetMap

OpenStreetMap (OSM) is a project which allows users to create and edit maps and also create so-called **relations**. Relations are objects containing other objects such as nodes (points in the map) or ways (a group of multiple points). One of the public relations is **PID** (Pražská Integrovaná Doprava) <sup>2</sup>. In this relation, multiple users created all lines of public transport in Prague, including trams, buses, and metro.

Data from relations can be extracted and converted to more suitable forms. Map of all tram lines in Prague is shown in Fig 3.1. The tram line 20 from Sídlíště Barrandov to Divoká Šárka is shown in Fig 3.2. This tram line is used further in this chapter.

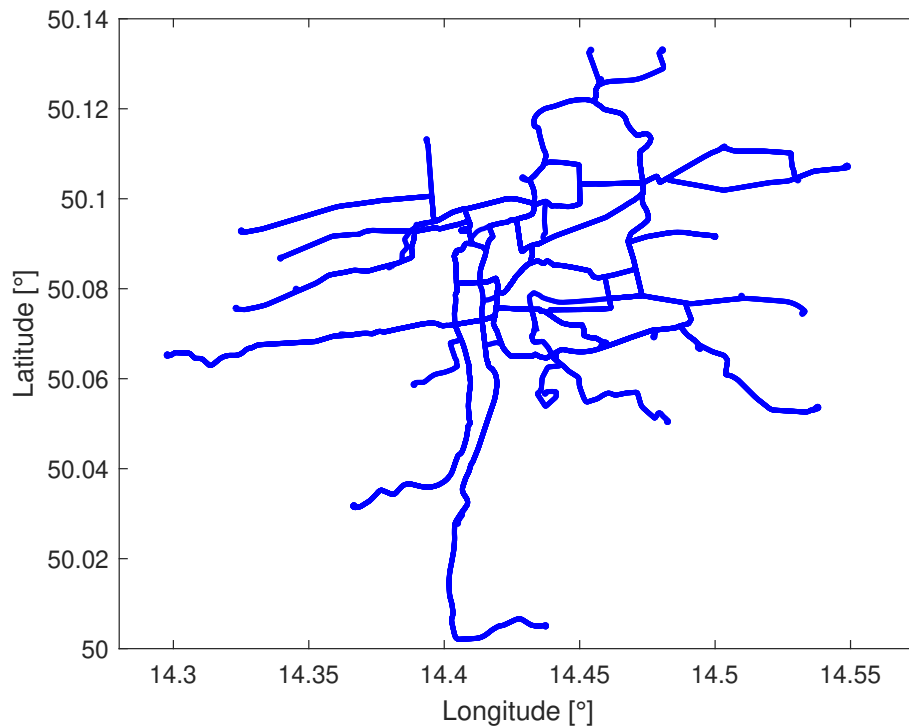
### 3.2 Altitude Map

The second input to developed algorithm is an altitude map. The format of this map is either a list of coordinates and their altitudes or a grid. In both cases, coordinates have to be in WGS84 format (the format used by GPS). For demonstration I used the altitude map

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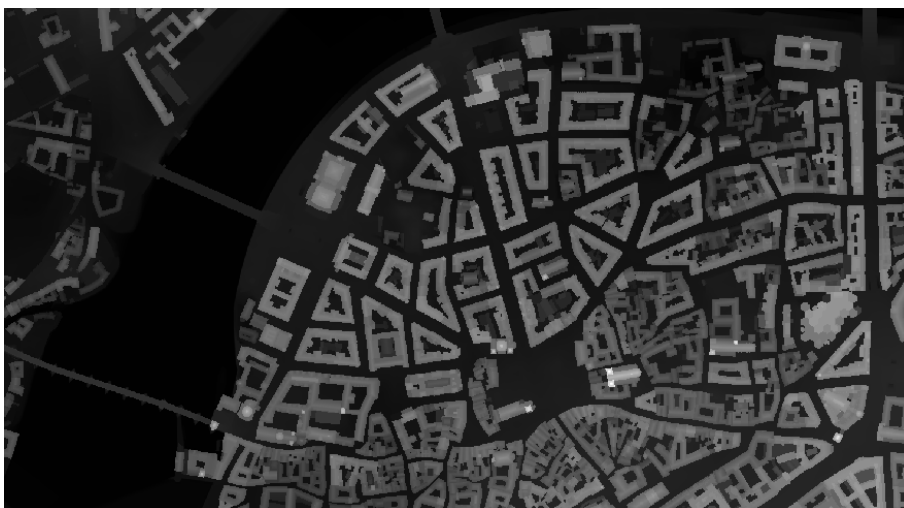
<sup>1</sup><https://www.openstreetmap.org/>

<sup>2</sup><https://www.openstreetmap.org/relation/1596861>



**Figure 3.1:** Map of all tram lines in Prague.

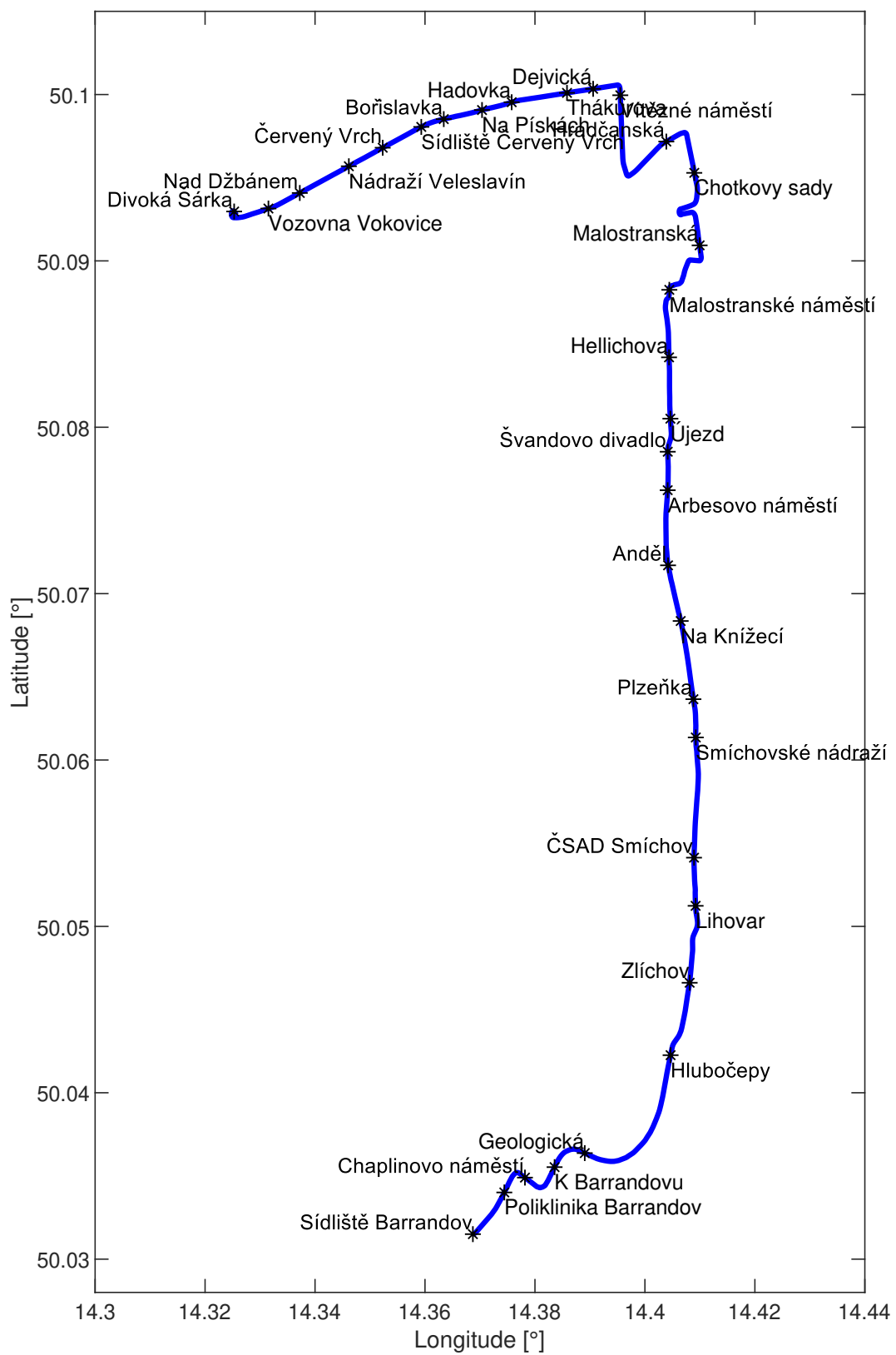
of Prague<sup>3</sup>. The resolution of this map is  $1 \times 1$ m. This map, as well as many others for the Czech Republic, is using the Křovák projection. In Fig 3.3 is shown a small portion of this map - the Old Town.



**Figure 3.3:** Part of the altitude map of Prague - the Old Town.

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<sup>3</sup>[https://www.geoportpraha.cz/cs/data/otevrena-data/FAEDA90C-F5CB-4B1E-BA41-419B982DBA3A](https://www.geoport Praha.cz/cs/data/otevrena-data/FAEDA90C-F5CB-4B1E-BA41-419B982DBA3A)



**Figure 3.2:** Tram line 20 from Sídliště Barrandov to Divoká Šárka.

### 3.2.1 Křovák Projection

In 1922, ing. Josef Křovák developed a projection, which minimizes inaccuracies caused by the spherical shape of the Earth, in order to maximize the accuracy of maps for the Czech Republic and Slovakia. This projection is also known as S-JTSK (Systém jednotné trigonometrické sítě katastrální).

The main principles are well described by [6].

*"The Conformal Oblique Conic Projection of Czechoslovakia was prepared by Josef Krovak in the year 1922 for construction of cadastral maps and topographical maps of medium scales for the civil geodetic service of Czechoslovakia. The Bessel ellipsoid of 1841 ( $a = 6\,377\,397.155\text{ m}$ ,  $1/f = 299.152812853$ ) which is widely adopted in Central Europe, is used.*

*Longitude  $\lambda$  refers to the Ferro meridian (an island of the Canaries), not the Greenwich meridian. The projection is conformal, so that in the projection plane meridians and parallels intersect at right angles. Round value  $17^\circ 40'$  is used for cadastral and topographic mapping in the Czech Republic/Slovakia and Austria. The rectangular plane grid S-JTSK has an origin in Finnish Basin, near Tallinn. The X-axis normally coincides with the meridian  $42^\circ 30'$  from Ferro increasing south. The Y-axis is perpendicular to the X-axis and increasing to west."*

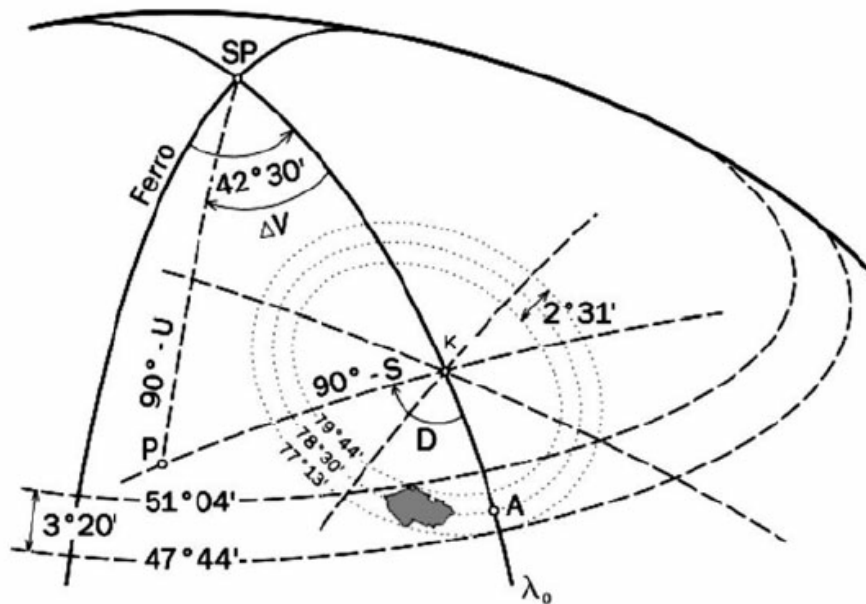


Figure 3.4: Localization of the Czech Republic on the Bessel Ellipsoid[7].

### 3.3 Estimation of Inclination

With data about the position of the rails and altitudes, I could finally develop the algorithm for estimating the altitude along the tram line. At first, I converted data to format WGS84 (World Geodetic System 1984). This standard is used all around the world. Since GPS coordinates from OSM relation are already in WGS84 format, it was necessary to convert only the altitude map. For transforming coordinates from S-JTSK (Krovak) to WGS84, I used the algorithm described in [8]

When I had both datasets in one format, I matched points forming the rails to the altitude map. For each point from the list forming the rails the algorithm proceeds as follows. At first, it picks 9 closest points (neighbours) by the Euclidean norm in the altitude map. Then it removes all points with the altitude significantly different from others (e.g. point is on the building or in the river). For the final altitude, I designed this formula

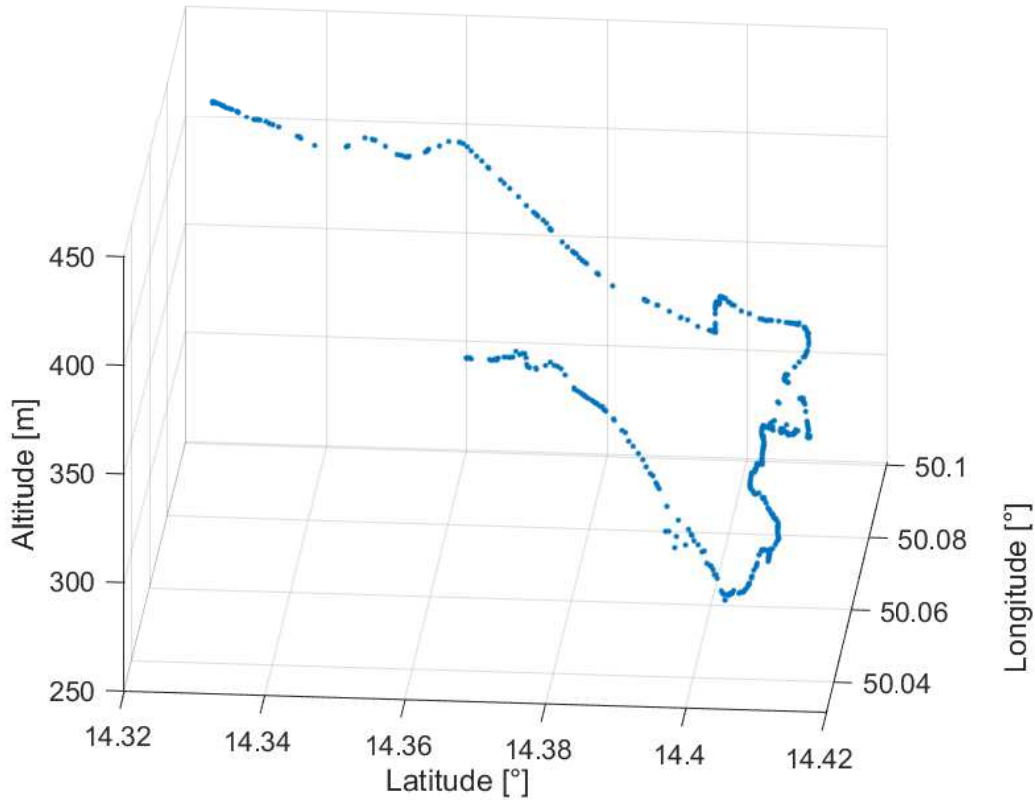
$$a = \sum_{i=1}^n n_{a_i} \cdot \frac{1}{\mu}, \quad (3.1)$$

$$\mu = \sum_{i=1}^n \frac{1}{n_{d_i}}. \quad (3.2)$$

Used variables:

<b>a</b>	final altitude of the point
<b>n</b>	number of neighbours (default 9)
<b>n<sub>a</sub></b>	altitude of the neighbour
<b>n<sub>d</sub></b>	distance of the neighbour from the point
<b>μ</b>	normalization factor

In Fig 3.5 is shown the result of this algorithm for tram 20 from Sídliště Barrandov to Divoká Šárka. The result corresponds to my expectations. However, it is still noisy.



**Figure 3.5:** Altitudes of points for tram line 20.

### 3.3.1 Smoothing and Filtering

After assigning the altitude value to each point on the rail, I calculated the altitude and the inclination profile of the tram line. For computing the inclination between two points, I used the standard formula

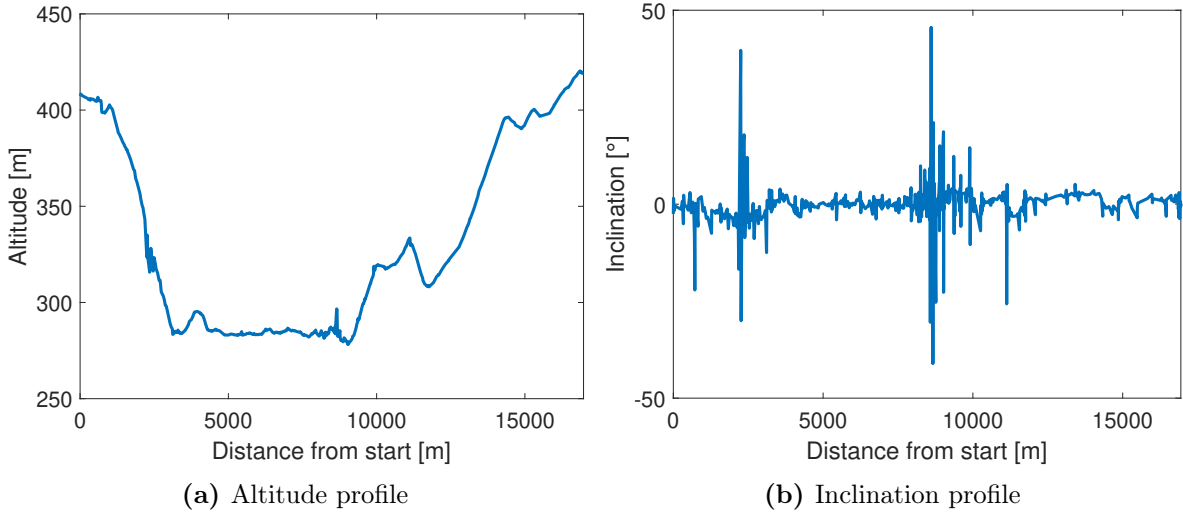
$$i_{12} = \arctan \left( \frac{a_2 - a_1}{d_{12}} \right). \quad (3.3)$$

Used variables:

$i_{12}$	Inclination between points 1 and 2 [rad]
$a_1$	Altitude of the first point [m]
$a_2$	Altitude of the second point [m]
$d_{12}$	Distance between points 1 and 2 [m]

The altitude profile along the whole route (distance zero is at the Sídlíště Barrandov station)

is shown in Fig 3.6a and the inclination profile along the same route is shown in Fig 3.6b. The profiles, especially the inclination profile, are noisy. This would lead to significant errors in the simulation.



**Figure 3.6:** The tram line 20 from Sídlíště Barrandov to Divoká Šárka.

For filtering the inclination profile, I tried three different approaches. In the first approach, I applied a low-pass filter on altitude data. In the second one, I applied it on inclination data, and in the third one, I designed procedure based on the knowledge about the usual profile of the rails.

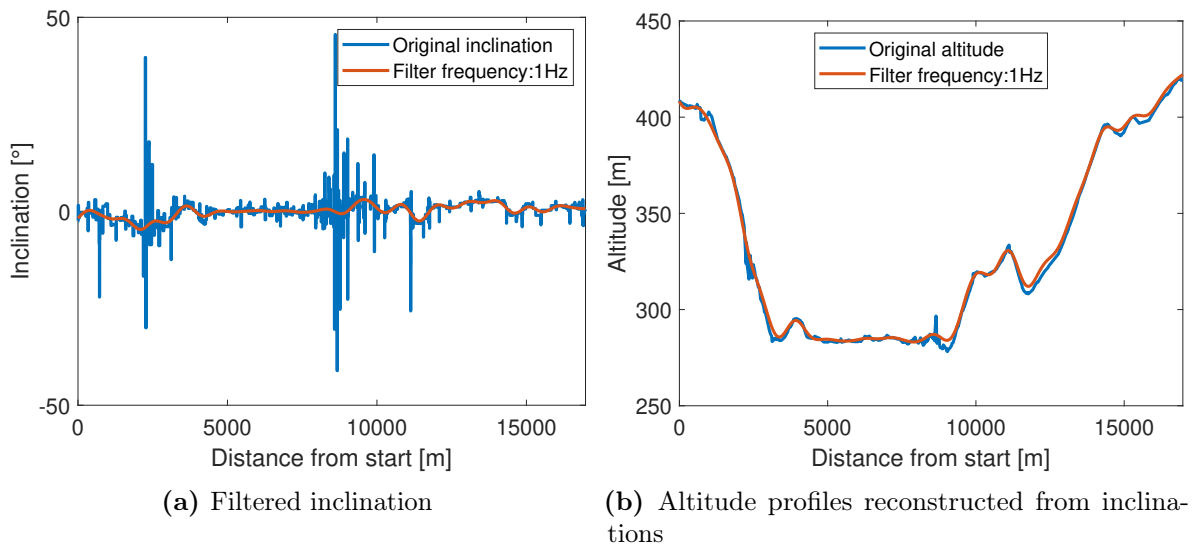
Before applying the low-pass filter, I had to resample data, since distances between points were not uniform. The rails are usually not shorter than 1m. Therefore, I have decided to use this value as a new fixed distance. As a low-pass filter, I used the **IIR** (Infinite Impulse Response) filter of the third order, with the sample rate 1000 samples/second and the passband frequency 1Hz.

The results of a first approach (filtering inclination) can be seen in Fig 3.7. To verify the result, I calculated the altitude profile back from the filtered inclination. The comparison can be seen in Fig 3.7b.

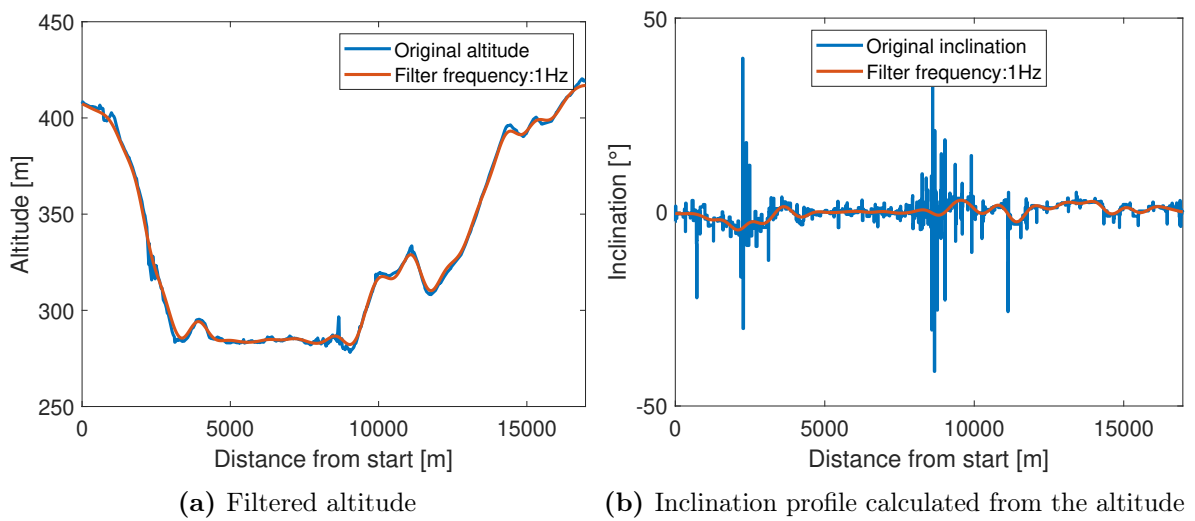
In the second approach, I did the same thing as in the first one, but the filter was applied to the altitude. The result is shown in Fig 3.8 and is very similar to the result of first approach.

From physical restrictions, it is known that changes in the altitude and, therefore, in the inclination are rather slow. With this knowledge, I designed the algorithm based on fitting points close to each other with the lines.

Each  $n$ -th point and  $k$  following points are approximated by a line. If multiple lines approximate the point, its final altitude value is the mean of all values. I used this algorithm with  $n = 5$  and  $k = 20$  on the altitude profile. After that, I calculated the inclination profile

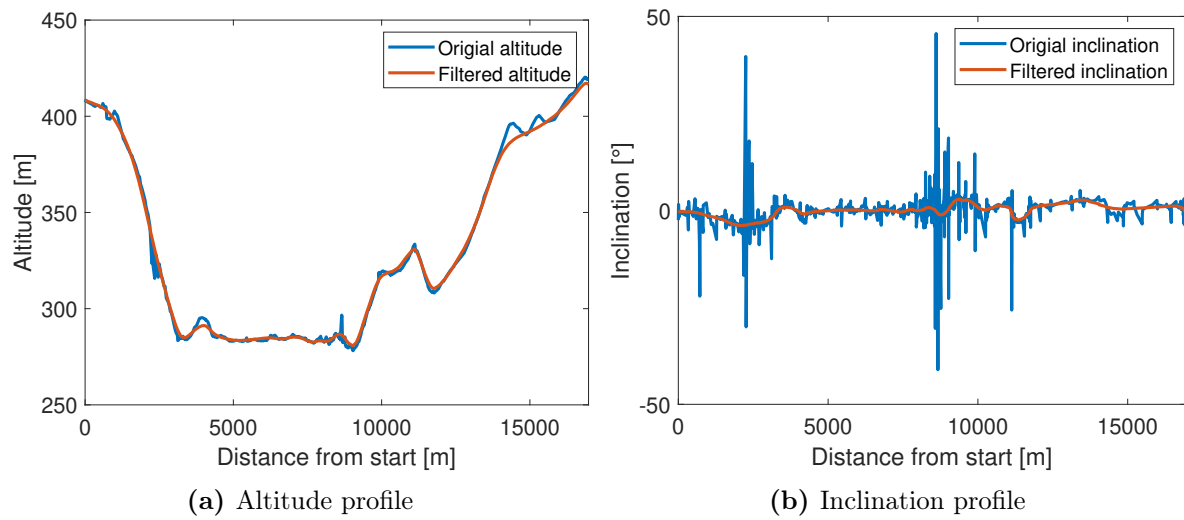


**Figure 3.7:** The result of the first filtering method.



**Figure 3.8:** The result of the second filtering method.





**Figure 3.9:** The result of the third filtering method.

and used the filter again. The inclination profile was then used for recreating the altitude profile for the comparison, which can be seen in Fig 3.9.



## 4 | Experiments

The methods for estimating individual parameters can be designed and tested on simulated data. However, models contain only known dynamics. They can reflect only phenomena that were already observed and described. This and many other problems are the reason for performing experiments on real devices and with real hardware. In this chapter, I will describe experiments I did in Ostrava. Further on, I will show measured data and introduce methods for their evaluation.

Due to a stable collaboration of our department with the company **Herman systems**<sup>1</sup>, I could perform measurements in the real tram during the regular ride.

Herman systems cooperates with the transport company in Ostrava (Dopravní podnik Ostrava). Some trams in Ostrava have on their board installed Universal Communication Unit UCU 5.0X. This unit can, besides other things, measure acceleration and velocity of the tram. My colleagues did some experiments with these units before, and based on their experience, I have also decided to bring the additional external sensors of velocity and acceleration. The comparison of data measured by the UCU unit and external sensors are shown further on in this chapter.

### 4.1 Measurement Setup

Experiments were done 3.3.2020, in Ostrava on the tram line 3 from Poruba vozovna to Dubina. The tram from these experiments can be seen in Fig 4.1. It is a standard tram VarioLF.

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<sup>1</sup><http://www.herman.cz>



**Figure 4.1:** VarioLF tram used for experiments in Ostrava.

For measuring acceleration I used **Precision MEMS IMU Module - ADIS16465** from Analog Devices <sup>2</sup>. This sensor was attached to armrest by double-sided tape, as can be seen in Fig 4.2a. Before that, I checked that armrest is not tilted in any direction. The accelerometer provides data with sampling frequency 2kHz. The output from this sensor for two-hour measurement would be big, and it would make post-processing much more complicated. Because of that, my colleague was stopping measurement in each tram stop. Unfortunately, some stops were unexpectedly short, which led to a delay in measurement. For the next experiments, I would recommend asking the driver to make stops a bit longer or ask him which in which stops he expects only a short break.

For the velocity measurement, I used **ZED-F9P** module from U-Blox company <sup>3</sup>. The GPS module is magnetic, and it is recommended to place it outside. Through the window, I managed to put it on top of the tram (Fig 4.2b). The U-Blox unit has sampling frequency only 10Hz, so I could measure velocity during the whole ride without the necessity to separate individual measurements.

One of the estimated parameters in the model is the weight. The measurement of the weight is not possible, but the information about it is necessary. Because of that, I was manually counting people in the tram and then estimated the total weight of the tram. The average weight of people in the Czech Republic is roughly 75kg, but the experiment was done in the

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<sup>2</sup><https://www.analog.com/media/en/technical-documentation/data-sheets/adis16465.pdf>

<sup>3</sup>[https://www.u-blox.com/sites/default/files/ZED-F9P\\_DataSheet\\_%28UBX-17051259%29.pdf](https://www.u-blox.com/sites/default/files/ZED-F9P_DataSheet_%28UBX-17051259%29.pdf)

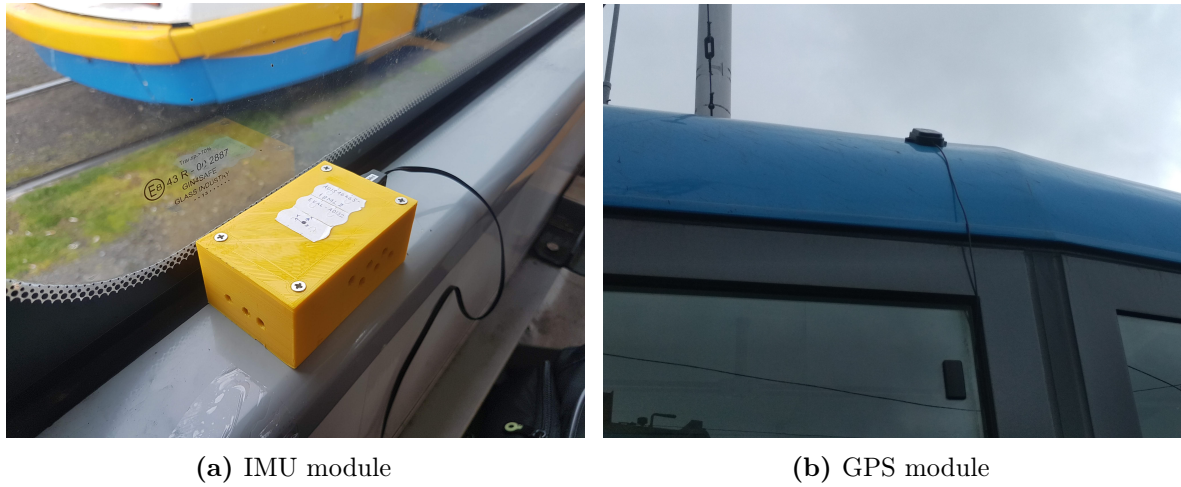


Figure 4.2

early afternoon, and most of the passengers were children going home from school, so I have decided to lower this value to 65kg. The weight of the empty tram was 21200kg. The total estimated weight of the tram during the experiment is in Fig 4.3.

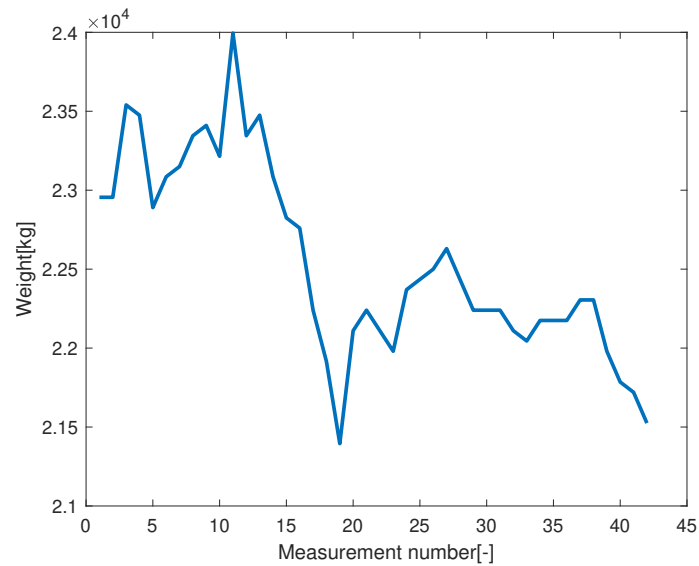
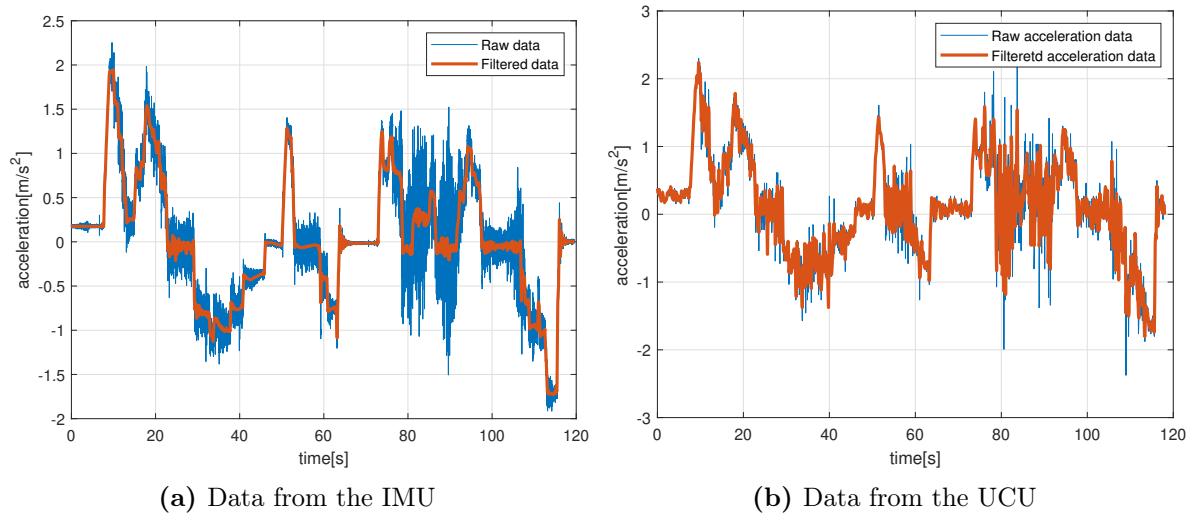


Figure 4.3: The estimated weight of the tram.

In the tram model is the adhesion force discretized to four levels. It is very hard to make a threshold between them, so for this variable, I will use the opinion of the tram driver. Even though rails were slightly wet, he described adhesion as ideal. During the ride, he did not register any slips or other problems.

Measurements have been taken from two rides in the opposite direction. The setup was unchanged during both rides.



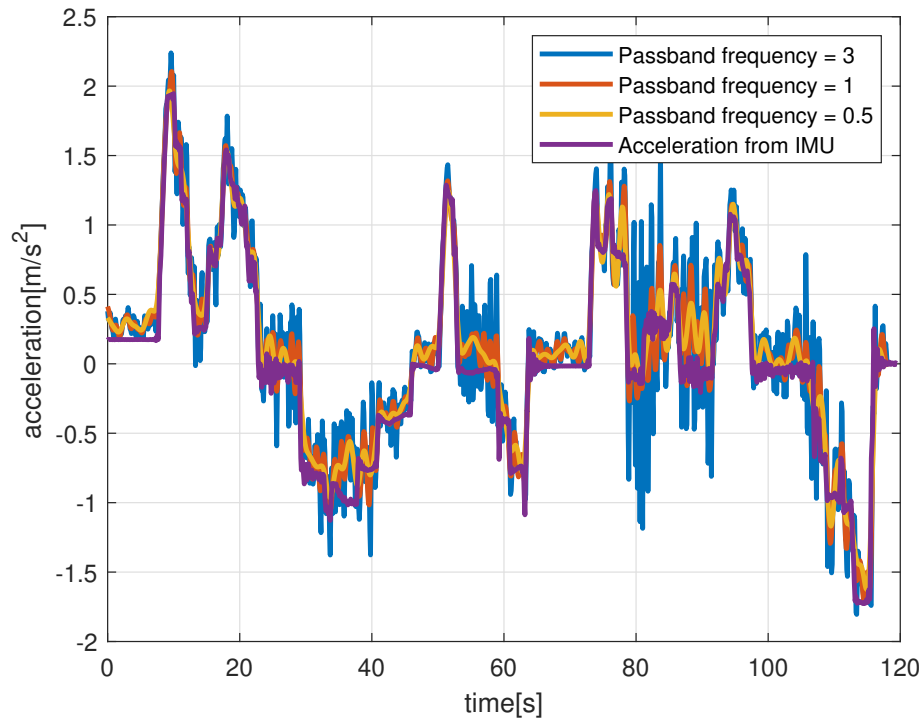
**Figure 4.4:** Acceleration data from experiments in Ostrava.

## 4.2 Measured Data

As I have already mentioned in the previous section, data were measured by two external sensors (IMU and GPS module). In addition to these sensors measuring unit from Herman systems was collecting the data about acceleration and velocity. The measurement started in the tram depot, but by that time, we were still making some adjustments. I will consider the first tram stop as the starting point. Unfortunately, the sensors from Herman systems are limited in size, and due to miscommunication, I got data only from the last 20 minutes. However, as it is discussed later on, the external sensors had a higher sampling rate, and I would use them for the estimation anyway.

### 4.2.1 Acceleration Measurements

Raw data of acceleration measured from both sensors as well as filtered data are shown in Fig 4.4. For filtering, I used the low-pass filter. For the data from the IMU, I used passband frequency 3Hz. Results were satisfying, as can be seen in the figure. Filter the data from the UCU was more complicated due to the small sampling period (10Hz, the IMU has 2kHz). The figure shows the result for the same passband frequency. This result is not good. I tried filters with smaller passband frequency. Results and comparison with the data from the IMU can be seen in Fig 4.5. From this figure, it is clear that the data from the IMU are more suitable for my problem.



**Figure 4.5:** Comparison between acceleration measured by IMU and UCU

### 4.2.2 Velocity Measurements

Similarly to the acceleration measurements, the velocity was measured by the external GPS and the UCU. The changes in velocities are slower, and the lower sampling period in the UCU is not that problematic. This can be seen in Fig 4.6.

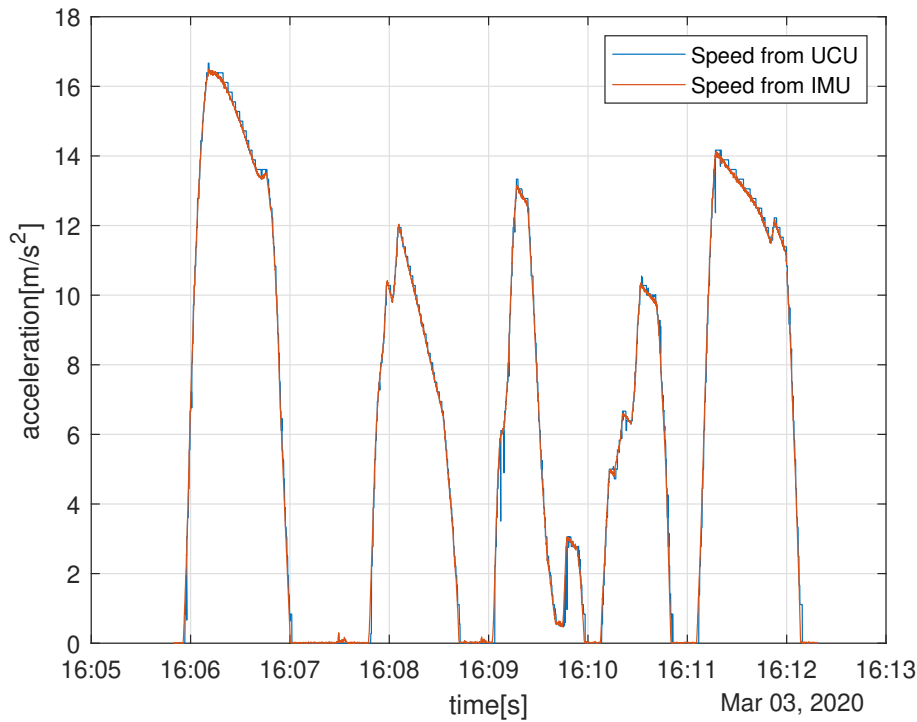
The data from the external GPS have higher sampling frequency, and by closer inspection, some inaccuracies can be found in the data from the UCU (e.g. around 16:09). This was the reason for working mainly with data from the external GPS unit.

### 4.2.3 Input

Unfortunately, there was no possible way how to measure the input of the system, which is current. In the later experiments, Herman systems may be able to provide this type of measurement.

## 4.3 Synchronization

The acceleration measurement had to be, due to the high sampling frequency (2kHz), separated into multiple files. Each file contained information about acceleration from the



**Figure 4.6:** Comparison between velocity measured by external the GPS module and the UCU.

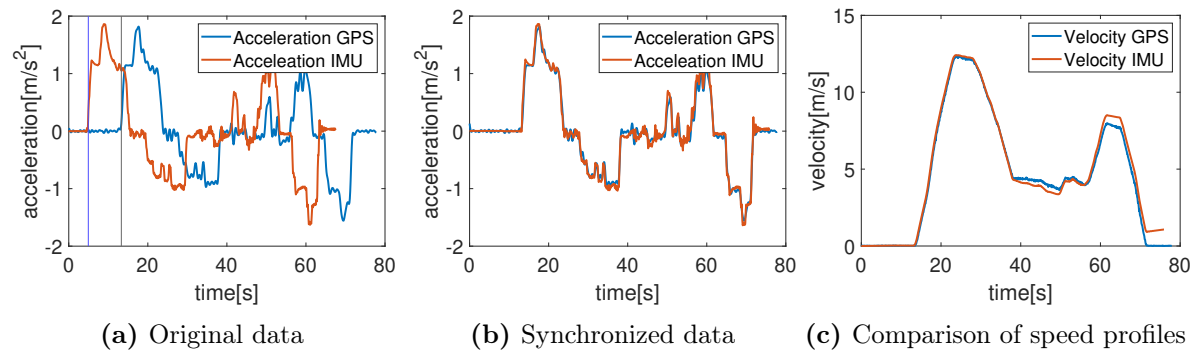
one tram stop to the other. The velocity measurement could be stored in two files. Each file contained information about one ride (from Poruba,vozovna to Dubina and back).

The first step in the synchronization was to match a section in the velocity measurement and the corresponding acceleration file. In total, I had 53 files with acceleration measurements, so I did this part manually. By that time, I also removed all damaged or corrupted data.

The second step was precise synchronization. Synchronizing data sets while using their original values (velocity and acceleration) would be rather hard. Because of this, I converted the measured velocity to acceleration and synchronized these profiles. In each profile, I found the first noticeable change and marked it as a starting time. Then I moved the profile with the smaller value of starting time in a way that it corresponds with the other profile.

The results of this algorithm can be seen in Fig 4.7. Original accelerations are shown in Fig 4.7a. Vertical lines are showing detected starting times. Acceleration measurements from the IMU were effected by the offset caused by inclination. This offset was constant at the start (the tram was not moving). For easier comparison, I shifted the acceleration to have a start at zero. The Fig 4.7c shows the comparison of velocities obtained from synchronized profiles.



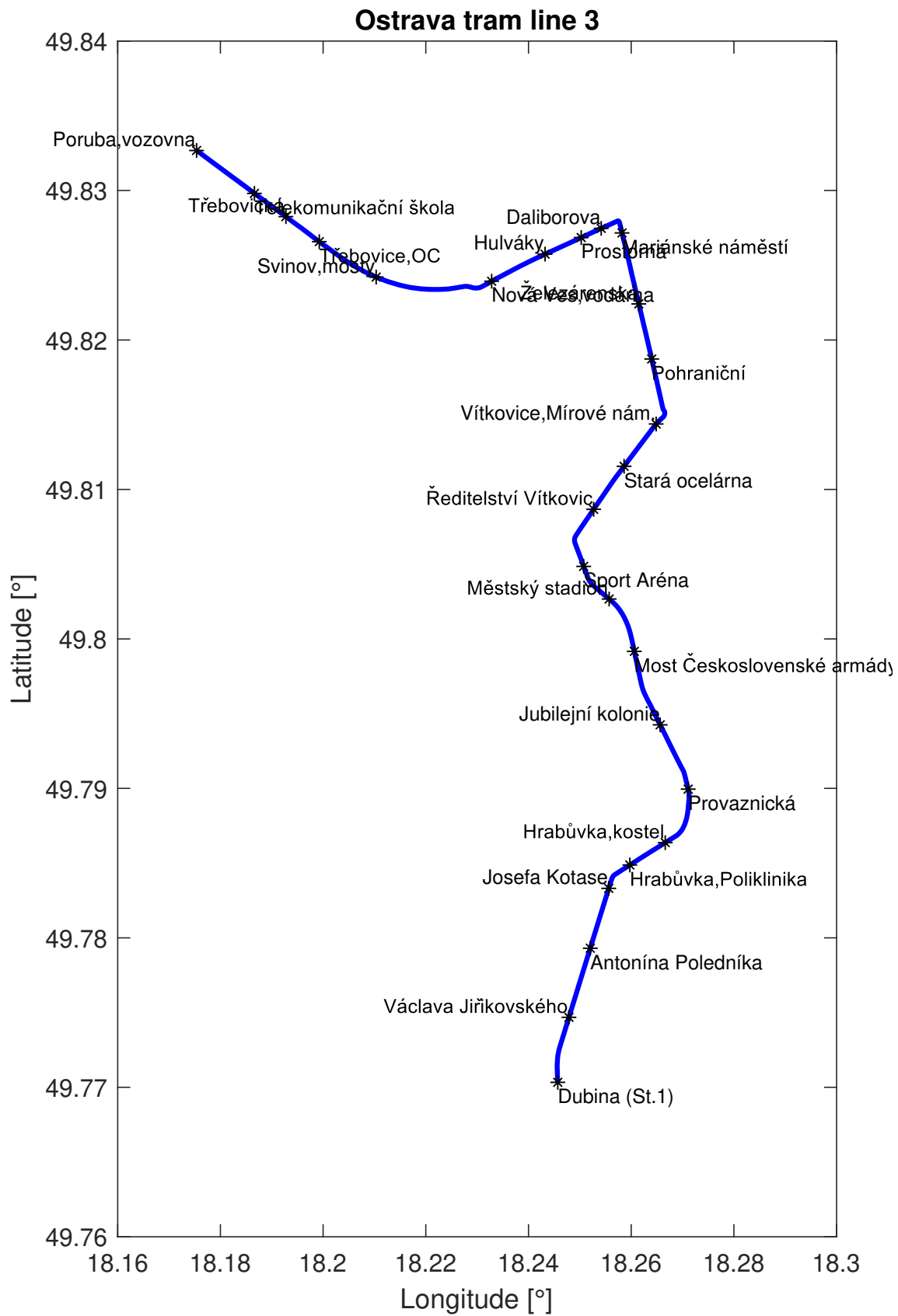


**Figure 4.7:** Demonstration of the data synchronizing algorithm.

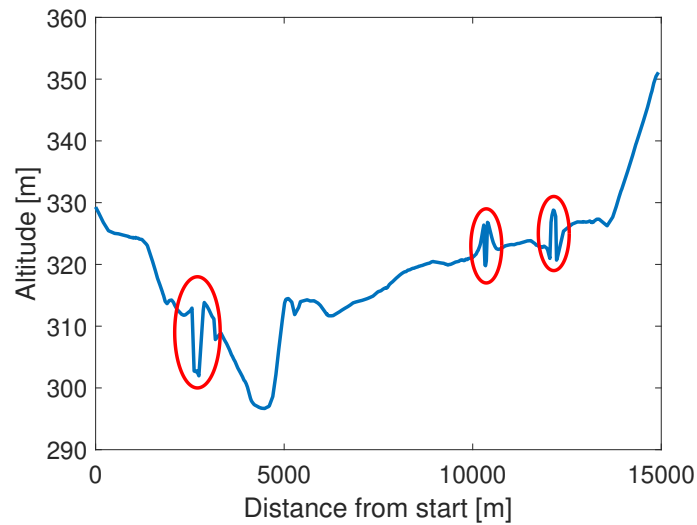
## 4.4 Inclination

In chapter 3, I described how important the information about the inclination is and introduced the algorithm for estimating the inclination for a tram line. The algorithm requires data with coordinates of the rails and the altitude map. For the coordinates of rails, I could use the same data source as for the example, OpenStreetMap. The OSM contains the relation about the tram line, on which I was doing the experiment <sup>4</sup>. The coordinates obtained from this relation are shown in Fig 4.8.

<sup>4</sup><https://www.openstreetmap.org/relation/917553>



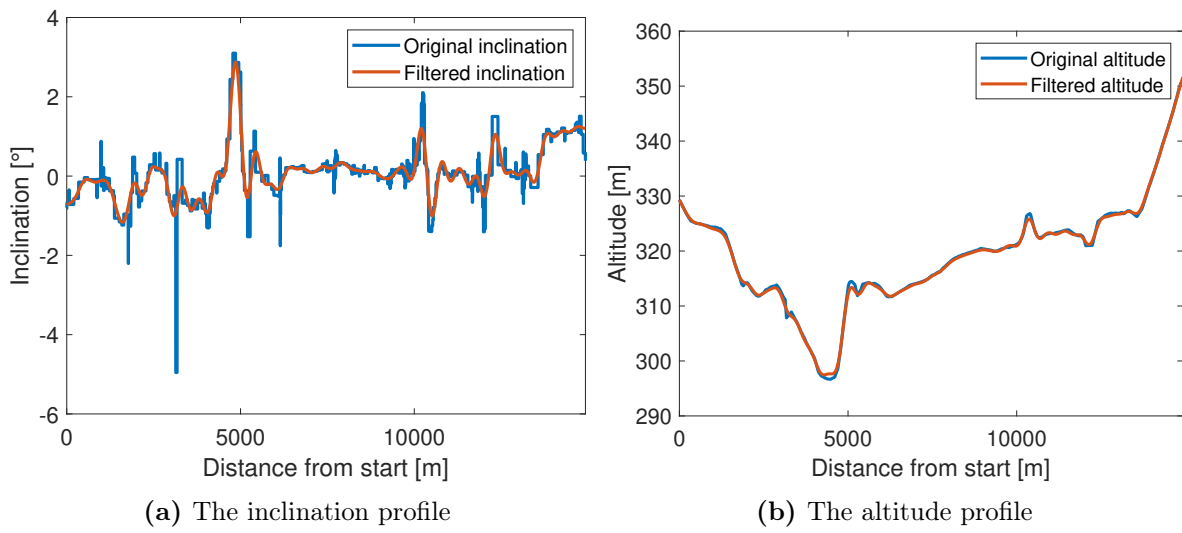
**Figure 4.8:** Map of tram line 3 in Ostrava: Dubina - Poruba, vozovna.



**Figure 4.9:** The altitude profile of the tram line 3.

As the altitude data, I used the map from <https://geoportal.cuzk.cz/>. This data set contains points in the Křovák projection and the corresponding altitude value. The points form a rough grid (distance between points is not uniform, usually is around 1 meter).

The estimated altitude profile is shown in Fig 4.9. The profile looks believably, except for the parts in red circles. After taking a closer look at the map, I noticed that these parts are either bridges (first two parts) or a tunnel (third part). With this information, I have decided to repair these parts before applying any filter manually. In section 3.3.1 I introduced three methods for filtering the inclination and the altitude profile. For this data set, I chose the first method, which is filtering inclination. The result, as well as, the recalculated altitude profile is in Fig 4.10.



**Figure 4.10:** Profiles of the tram line 3.

## 5 | Input Estimation

In this chapter, I will introduce the basic principles of the motors, the brakes of the tram, and show how they are related to the model. Further on, I will describe and discuss some properties of measured data and introduce a procedure for estimating the input from them.

### 5.1 Motors of Tram

The first trams in the 19th century were drawn by horses. Later on, horses were replaced by the steam engine. At the end of the 19th and beginning of the 20th century, some cities used gas trams. Finally, in 1880 the first electric tram line has been build in Sestroretsk near Saint Petersburg. After the oil crisis in 1973, trams became a considerable part of the public transport, especially in Europe. Nowadays, almost all trams are powered exclusively by the electric traction motors.

Electric motors powering the trams and their regulators also evolved during the time. The older trams such as Tatra T3 (Fig 5.1a) are using the synchronous motors with shunts. They allow to control the total resistance of the tram and regulate current and thus acceleration and velocity. Connecting and disconnecting resistors causes quantization in the acceleration values. The other methods of regulation described further on are not using resistors. However, the quantization effect is replicated even for them. I will use this phenomenon to identify the input. The most significant disadvantage of this type of regulation is the fact that all excess current is transformed into the heat. Some of the newer trams are using this excess heat to warm up the interior of a tram. Nowadays, trams with this type of regulation are not produced. However, some older tram models with this kind of control are still used.

In the early seventies, tram manufacturers started using the power regulators based on the thyristors. For the current regulation pulse width modulation (PWM) is used. Trams with thyristors are capable of recuperating the energy while braking. This is a significant improvement over the previous method. Trams using this method (such as Tatra T6A5 in Fig 5.1b) have much higher efficiency than trams with resistors, although their construction is more complicated. Despite being significantly lower than the losses on resistors, the main disadvantages of these regulators are the losses on thyristors. Another problem is the necessity to filter the high-order harmonic frequencies, which are disturbing other electronic devices. This effect is caused by the fast switching of thyristors.

The newest trams (e.q. Škoda T15 in Fig 5.1c) are using regulators with IGBT (Insulated-



**Figure 5.1**

Gate Bipolar Transistor). This technology allows fast current switching with much higher efficiency than the standard thyristors. It is also designed for switching high voltages (typical voltage from tram network is 600V). Some cities are upgrading their tram fleet with this technology due to high energy savings.

## 5.2 Types of Tram Brakes

While the methods for the power regulation systems can differ, braking concepts are similar to all trams. Each tram has three types of brakes that can work independently and use different physical principles.

### 5.2.1 Electrodynamic Brake

The electrodynamic brake, also known as the operational brake, is the most frequently used brake. Electrodynamic brakes are transferring the kinetic energy to electric power. Based on the type of mechanism for the power regulation, the electric power is then either transferred to heat (for trams with resistors) or is recuperated back to the power network (regulators with thyristors and IGBT). The operational brake is used for the standard braking. For the small velocities, the efficiency of this brake decreases.

### 5.2.2 Parking Brakes

The parking brakes (or the disc brakes) are mechanical brakes used for low velocities (usually below  $5 \text{ km}\cdot\text{h}^{-1}$ , depending on the tram). This brake is also active if the tram is at the tram stop for disabling the spontaneous movement of the tram. These brakes are transferring the kinetic energy of the tram into heat via friction. The braking pads are pressed against the

rotor, and created friction is slowing down the tram. In case of bad adhesion conditions, friction between wheels and pads can be significantly reduced which can lead to a wheel slip. This brake is used automatically when the velocity of the tram is too low for the electrodynamic brake or during the emergency braking.

### 5.2.3 Friction Rail Brake

The last type of brakes is the friction rail brake (in Czech "Kolejnicová brzda"). This brake is used only in an emergency. It is a non-adhesion type of brake. The friction rail brake is a beam with a huge electromagnet. In a normal state, this beam is above the rail. When the brake is activated, this beam is released on the rail. The current flowing through the electromagnet creates a magnetic field which attracts beam to the rail. Friction between the beam and the rail then forces the tram to slow down. The current is generated from the tram battery, so it can be activated even when the power line is damaged. The rail brakes are also used in trains. However, the principle of braking is slightly different. The trains are not using friction, but eddy currents to slow down. These brakes are called magnetic rail brakes.

Additionally, trams usually have sandboxes. If the tram driver concludes that the adhesion conditions are too bad, he can release some sand from these boxes to avoid the slip of the wheel.

## 5.3 Tram Control

During a standard ride, the tram is controlled by the control lever. The position of the control lever is transferred to the torque and, therefore, the acceleration. The majority of trams in the Czech Republic have control lever with seven positions for acceleration, seven positions for deceleration, the position for the coasting (analogy to the neutral in a car), and two positions for the emergency braking. In Fig 5.2 can be seen the control lever from the VarioLF tram, which was used during the experiments in Ostrava. The control lever in the figure is in the position for coasting, moving the control lever up, towards the label JÍZDA causes acceleration, moving it down causes deceleration. The positions for emergency braking are labeled K1 and K2.

Note that the mentioned positions for acceleration and deceleration are not visible. Unlike the shift lever in a car, the control lever in a tram can be moved fluently without noticing any changes. Even the driver was not aware of these positions. Although when we tried slowly moving the lever up and down, we heard silent clicks when the control lever changed a position.



**Figure 5.2:** The control lever from the VarioLF tram used in Ostrava experiments.

During the ride the tram is using the electrodynamic brakes. For the low velocity, the parking brakes are used instead, since the electrodynamic brakes are not that efficient. During the emergency braking, the driver pulls the control lever to the lowest position and activates the friction rail brakes. All types of brakes are also activated when any passenger pulls down the emergency levers inside a tram.

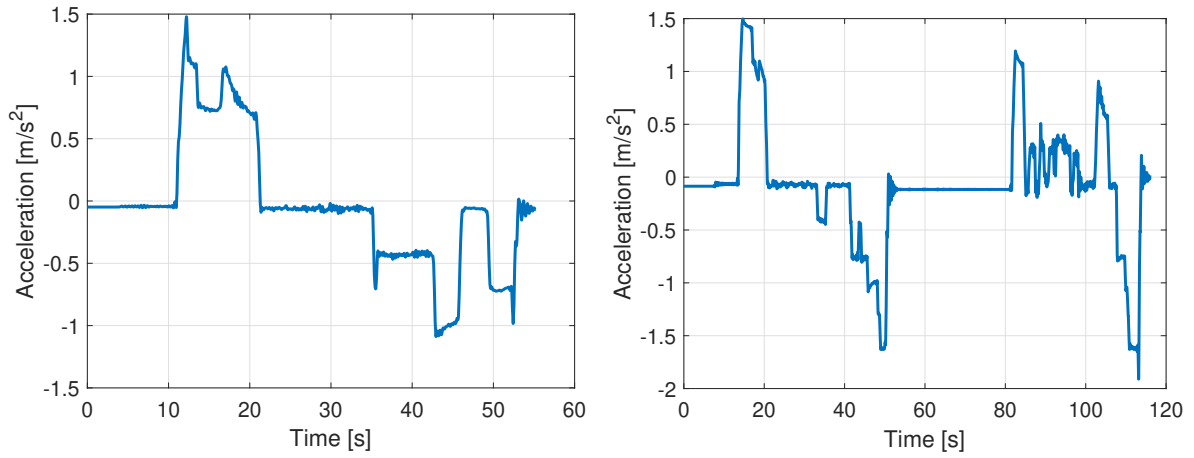
For the input identification, I will consider braking with the electrodynamic brakes. The parking brakes are used only for low velocities. They are not used for a long portion of time (when the velocity is low, it usually means that the tram is going to stop), I will consider their dynamics to be similar to the electrodynamic brakes. The friction rail brakes are used only during the emergency braking. My goal is to estimate the parameters for the tram operating in the normal mode. For this reason, I will not consider them at all.

## 5.4 Input of Model

The input of the model is the position of the control lever. In total, the control lever can be in fifteen positions (seven for acceleration, seven for deceleration, one for coasting). In the model, I assigned that the coasting position will have value 0, values for acceleration will be from 1 to 7. Similarly, deceleration values will be from -1 to -7.

In Fig 5.3 are shown two records of acceleration from the experiments. On both of them, the quantization levels are noticeable.





**Figure 5.3:** Examples of acceleration measurements.

#### 5.4.1 Offset Filtering

Before developing the estimation strategy, I had to deal with the problem caused by the accelerometer. As the tram moves, the accelerometer is inclined. This inclination causes the offset in the measured data. There are several possibilities for filtering this offset. One of them is to use knowledge about the inclination to estimate the offset. Another one is to use the GPS measurement, which is not affected by inclination.

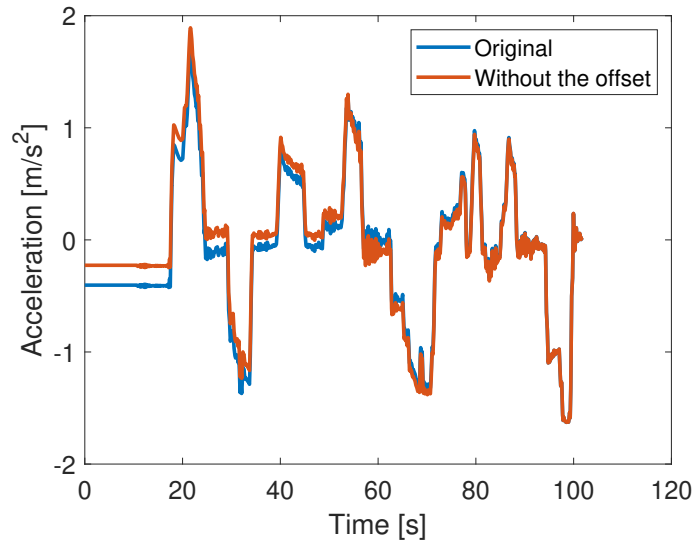
At first, I tried to use the known inclination profile to filter the offset. For localizing the tram during the experiments, I used the measurements from the GPS module. Along with the measurement of velocity, it also saved the GPS position in each step. With this information, I could create the time profile of inclination for each data set. The value of the offset can be calculated as

$$a = g \cdot \sin(\theta), \quad (5.1)$$

where  $a$  is acceleration,  $g$  is the gravitational acceleration, and  $\theta$  is inclination.

This method successfully removed the offset from some of the datasets, but in some of them, the inclination was probably not accurate enough. The result of unsuccessful filtration is shown in Fig 5.4. The expected result would have zero acceleration at the start.

In my second approach, I used the data measured by the GPS. The GPS module is not affected by the inclination and therefore is without the offset. In Fig 5.5a is the comparison between acceleration measured by the IMU and the acceleration computed from the velocity measured by the GPS module. The Fig 5.5b shows the measured velocity used for calculating the acceleration. The typical example of the offset is between seconds 15 and 20. The

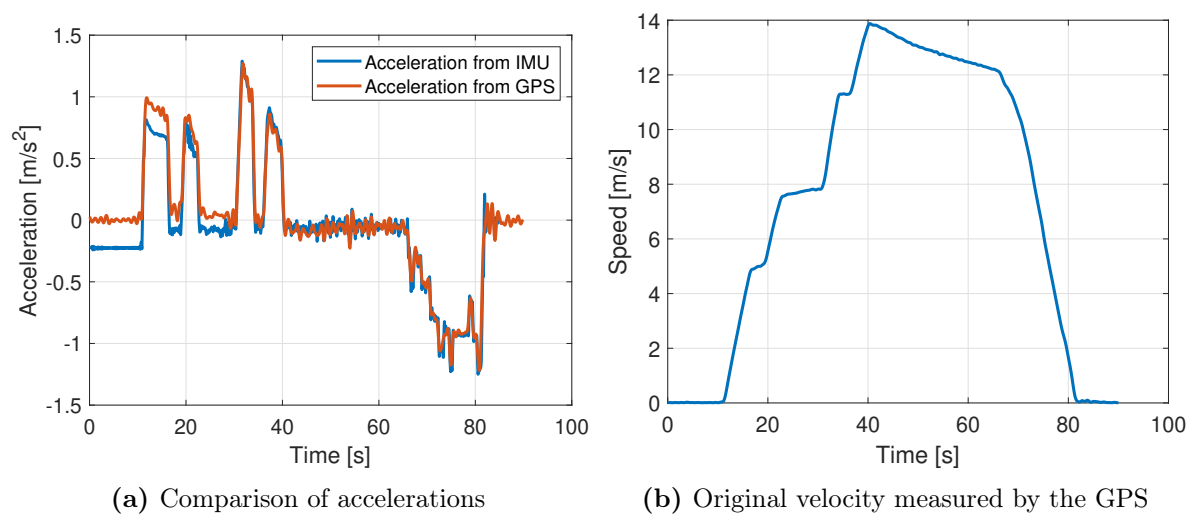


**Figure 5.4:** Removing the offset with informations about the inclination.

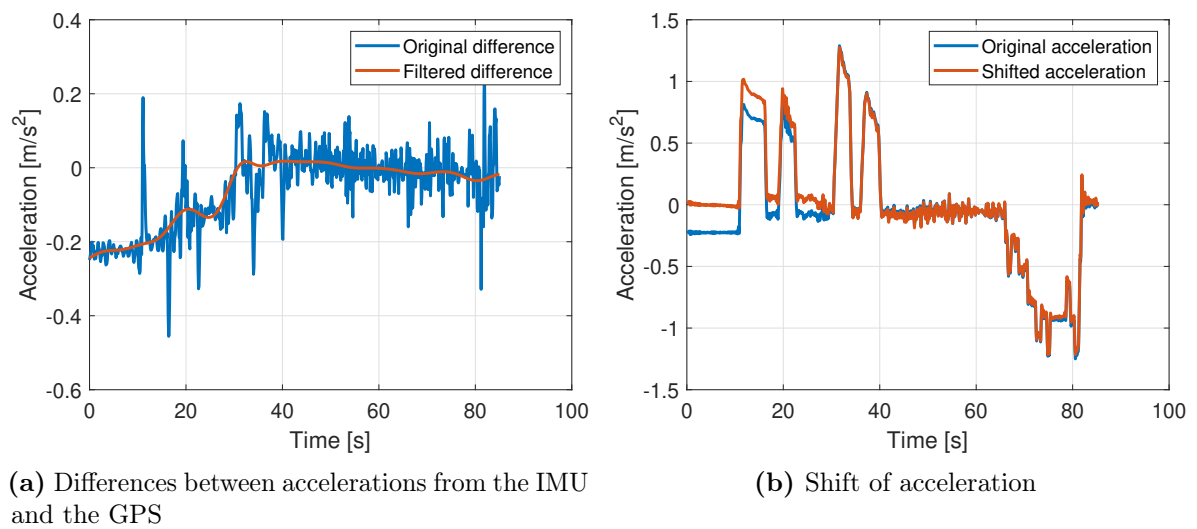
acceleration from the IMU is negative, which would imply that the tram is slowing down, but the velocity measurement shows that the tram is slightly accelerating.

I have decided to remove this offset by calculating the difference between acceleration measured by the IMU and the acceleration calculated from the velocity. At first, I resampled the accelerations (their sampling period was different) and calculated the difference. Because the offset is caused by the inclination, and thus it cannot change fast, I applied the low-pass filter with the passband frequency 0,1Hz. The result can be seen in Fig 5.6. The first figure shows the calculated difference. The second one shows the comparison between the original and the shifted acceleration.

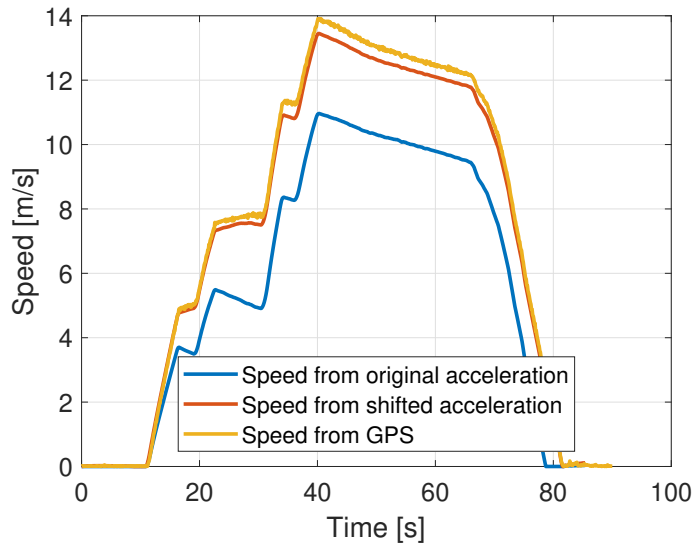
The significant improvement can also be seen when comparing the velocity obtained from the shifted acceleration. The velocity calculated from the original acceleration is affected by the offset. The velocity from the shifted acceleration is much closer to the velocity measured by the GPS module.



**Figure 5.5:** Comparison of the acceleration measured by the IMU and the acceleration calculated from the GPS velocity measurement.



**Figure 5.6:** Removing the offset from acceleration measurements.



**Figure 5.7:** The comparison of velocities.

#### 5.4.2 Acceleration Related to the Gravity

The during a ride the tram is affected by the gravitational force. This effect could cause a problem in the input estimation. I will illustrate that problem with an example.

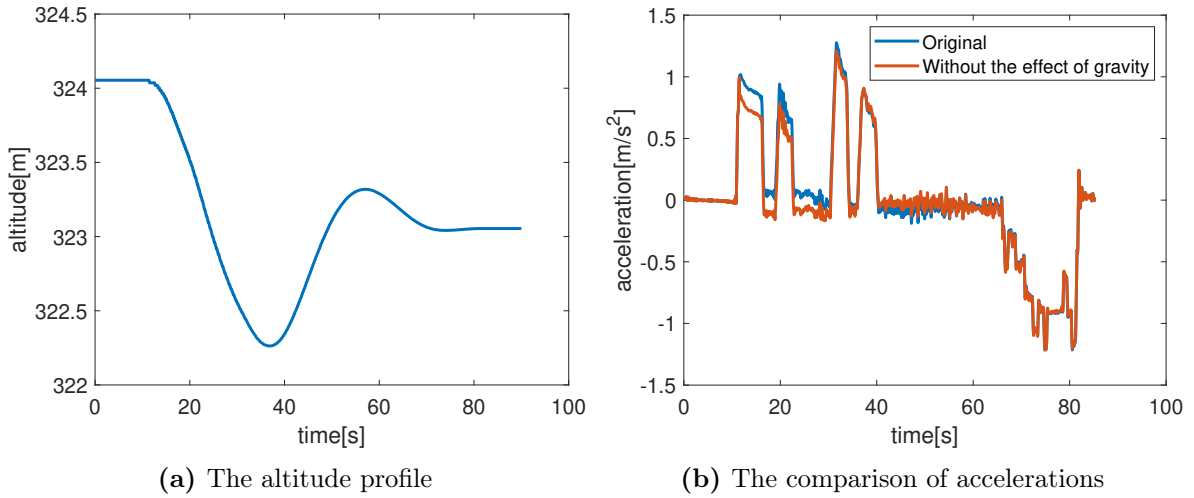
In the first case, the tram rides on rails with zero inclination and constant acceleration. Because the inclination is zero, the acceleration is generated by the tram's motors, and, therefore, the input is non-zero. In the second case, the tram rides on the rails with negative inclination (tram is going downhill). The gravitational force causes the tram to accelerate, even if the input is zero.

To suppress this effect, I have decided to calculate the acceleration caused by the gravitational force. After that, I can separate the acceleration (or deceleration) generated by the tram's motors. For calculating the acceleration caused by the gravitational force, I used the formula 5.1. The result is shown in Fig 5.8. In the first 40 seconds, the tram went downhill. The acceleration without the effect of gravity is lower. On the other hand, between seconds 40 and 50, the tram went uphill, and the new acceleration is higher.

#### 5.4.3 Estimating Strategy

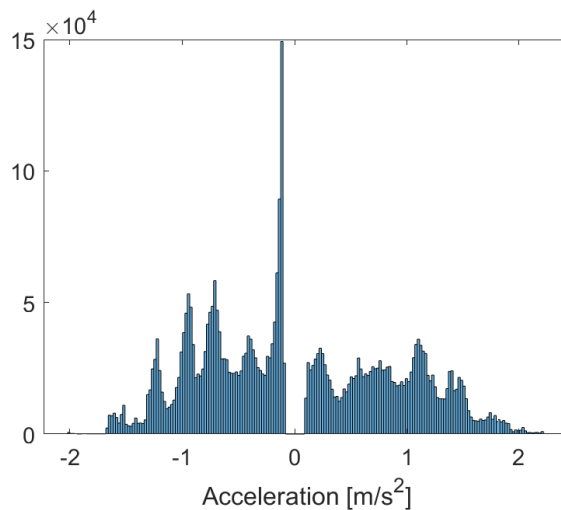
The first step in the estimating strategy is assigning the positions of control lower to acceleration values. The second step is to make the input more realistic.

Knowing that the control lever can have only 15 positions, I have decided to assign one acceleration value to each position. This allows me to immediately assign all measured acceleration values to the positions of the control lever.



**Figure 5.8:** Removing the effect of gravity from acceleration measurements.

The first approach to finding these acceleration values was the histogram. The main idea was that these values should appear more frequently than the others. In Fig 5.9 can be seen the histogram of the acceleration values from all measurements. The majority of values were in the range from -0.1 to 0.1, so to make the histogram more illustrative, I removed all these values. Some peaks can be observed, especially for negative values. However, the identification of all 15 values just from this histogram would not be very precise, so I have decided to use another method. Dividing values into the known number of groups seemed like a problem that could be solved by the **K-means** algorithm.



**Figure 5.9:** The histogram of measured accelerations without values from -0.1 to 0.1.

The **K-means** clustering algorithm was firstly described in 1967 by James MacQueen [9], although the original concept comes from Hugo Steinhaus [10]. The concept of the K-means

clustering is simple. Each cluster is represented by a centroid. The initial positions of the centroids are random. The algorithm assigns each measured value to the closest centroid. In the next step, the values of centroids are recalculated to be the mean value of the points assigned to the centroid. This loop repeats until the centroid positions are changing. This problem can be expressed as the minimization of the following objective function:

$$J = \sum_{j=1}^k \sum_{i=1}^n \|x_i^{(j)} - c_j\|^2, \quad (5.2)$$

where  $k$  is the number of clusters,  $n$  is the number of points assigned to the cluster (can differ for each cluster), and  $\|x_i^{(j)} - c_j\|^2$  is a distance between the measured point and the centroid.

In this case, measured values are accelerations, and the centroids represent the acceleration (or deceleration) values for each control lever position.

The most straightforward approach now would be to merge all datasets and use the K-means algorithm with 15 clusters. This approach would not guarantee that the resulted values of centroids will have 7 positive values, 7 negative values, and 1 value around zero. The more sophisticated approach could use two data sets, one with positive values and one with negative values. For each run would use 8 centroids, and then the centroids for values around zero would be merged together. This would remove the problem mentioned before. However, even this approach does not take into account the fact that the tram reached the full acceleration and deceleration only in a few measurements. In fact, the full deceleration has been achieved only once.

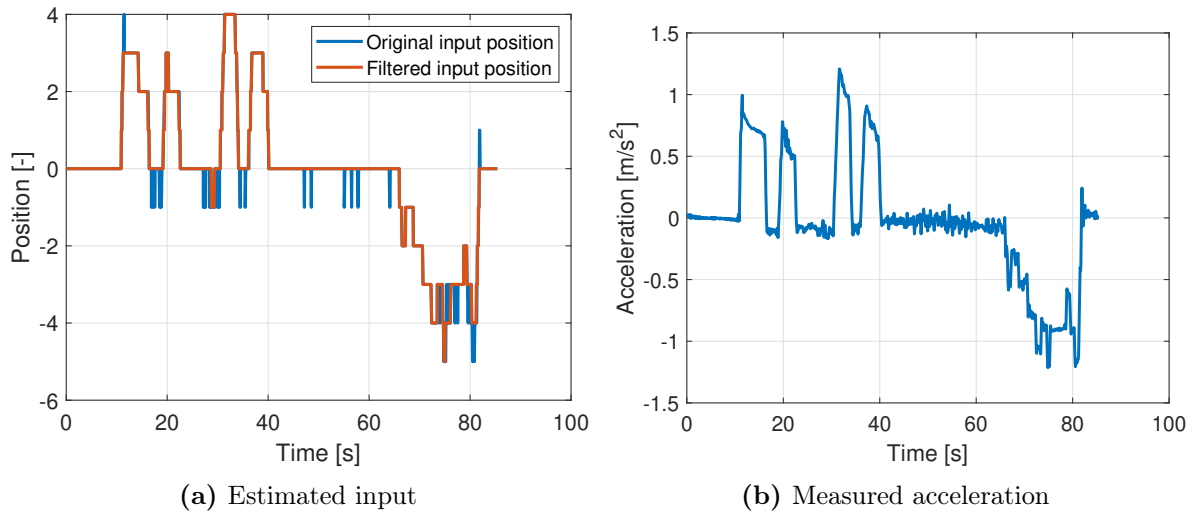
To avoid the problems mentioned above, I used the K-means two times for each data set (for negative and positive values separately). The highest acceleration value determined the number of clusters for each data set. From the histogram, I estimated that the distance between centroids should be around 0.3. For each data set, I saved the values of found centroids and assigned them to the appropriate position control lever. At the end, I merged values for centroids, which represented the position for acceleration around zero. As a result, each position had several centroid values. The final value is their mean. The acceleration values for each position are shown in table 5.1.

Position	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
Acceleration [m/s <sup>2</sup> ]	-1.98	-1.59	-1.29	-1.04	-0.78	-0.52	-0.24	-0.01	0.30	0.59	0.84	1.09	1.36	1.65	1.95

**Table 5.1:** The estimated average acceleration value for each position of the control lever.

With the acceleration values representing each position, I could estimate the input. Each measured acceleration value is assigned to the closest position.

In some situations, the value of input oscillates between two positions. This effect does not correspond with reality. To remove this effect, I found all changes in the input, which

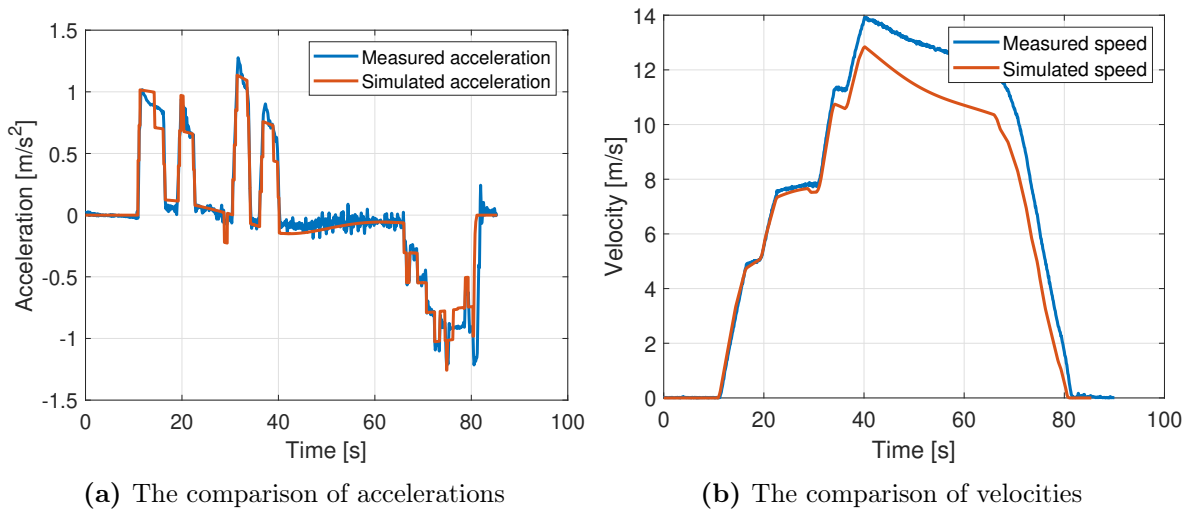


**Figure 5.10:** Results of the input estimation algorithm.

lasted less than 0.5s and removed them. After that, I used a slow low-pass filter (passband frequency 1Hz) and then re-assigned the positions. The estimated input and the filtered input can be seen in Fig 5.10a.

#### 5.4.4 Simulations with Estimated Input

To validate the results, I simulated the tram ride with the estimated input and compared it with measured data. The model still has some unknown parameters, so this serves only as a validation that the simulated output is close to the expected values. Unknown parameters were for simulation purposes estimated manually. The results of the simulation and the comparison with the measured values can be seen in Fig 5.11.



**Figure 5.11:** The comparison between the measured values and the simulated values for the estimated input.



## 6 | Estimation of Resistance Parameters

In this chapter, I will describe the formula for the resistance force and its parameters. Moreover, I will provide some values estimated by other researchers and introduce their methods. At last, I will show my solution for estimating parameters in the resistance formula.

The resistance of railway vehicles has been studied over one hundred years. The formula for describing the resistance force was described in 1926 by W.J.Davis [11]. The most general form of the Davis formula is

$$F_{\text{res}} = A + Bv + Cv^2, \quad (6.1)$$

where  $F_{\text{res}}$  is the resistance force,  $v$  is velocity, and  $A, B, C$  are coefficients greater than zero. These coefficients have their physical representation.

The coefficient  $A$  represents the mechanical resistance, and thus it is mass dependant.

The coefficient  $C$  represents the aero-dynamic drag. It is usually dependent on things such as the coefficient of air penetration, the frontal surface of the vehicle, and the air density. This coefficient is more important at high speeds.

The physical representation of the  $B$  coefficient is not that unambiguous. Some sources such as [14] claim, that it describes air-drag forces not covered in coefficient  $C$  and excludes mass dependency. Another source [15], describes this coefficient as a velocity-dependent non-aerodynamic drag. Some of them are splitting it into two parts, where one is dependent on mass, and the other is not.

Over the years, several methods for estimating values of parameters  $A, B, C$  were developed. Davis himself came up with following formula:

$$F_{\text{res}} = \left(1.3 + \frac{29}{m}\right) + 0.045v + \frac{0.0005a}{mn}v^2, \quad (6.2)$$

where  $F_{\text{res}}$  is the resistance force in  $\frac{\text{lb}}{\text{ton}}$ ,  $m$  is the weight per axle in US tons (1US ton  $\sim$  907kg),  $v$  is the vehicle velocity in mph,  $a$  is the train cross-sectional area in square feet and  $n$  is the number of axles per train.

Despite the differences in procedures, the main structure remained unchanged. The resistance

formula had constant part (parameter A) linearly dependant on weight, part linearly dependant on velocity (parameter B) and the part which was dependant on velocity quadratically. More information about this formula and its development over the years, as well as a more sophisticated background, which exceeds this thesis, can be found in [12].

More current approaches for estimating parameters of the Davis formula are described in [13]. As can be already seen, research in this area is mainly focused on trains. Trams and trains are similar in many ways. Therefore, resistance formulas should be similar. The most significant difference is the average and maximal speed. The maximum permitted tram speed depends on the city. Usually it is around 50km/h. The average velocity of regular trains can easily exceed 100km/h. Several studies agree that the aero-dynamics resistance has a considerable part in the total resistance. However, they usually consider speeds over 100km/h. For this reason, I have decided to neglect the parameter C and work with the simplified version of equation 6.1.

$$F_{\text{res}} = A + Bv \tag{6.3}$$

## 6.1 Estimation

By simplifying the resistance formula, the estimation problem has been reduced to finding two values. During my research, I came up with two approaches. I could use a tram's physical parameters such as weight, frontal surface, coefficient of resistance, etc., and use some formula developed in other studies. This approach would lead to some resistance formula, however since the majority of these formulas were derived for trains, therefore for higher speed and slightly different vehicles, accuracy would be debatable. I used a different approach, which does not consider the tram's physical properties at all and is based only on measured data.

At first, let me remind the formula for the acceleration of a tram used in my model.

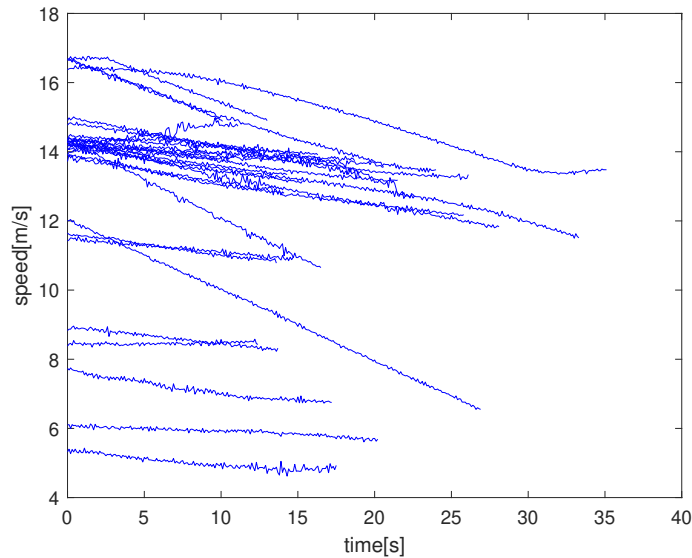
$$\frac{dv}{dt} = \frac{1}{M} (F_{\text{ad}}(v) - F_{\text{res}}(v) - F_G(\theta)) \tag{6.4}$$

The main idea of my approach was to find sections, on which the only force affecting the tram would be the resistance force. With these sections, I could estimate resistance parameters by estimating the parameters of the differential equation.

The first term in equation 6.4 is related to the input. In the model, the input is the position of the control lever, which is then transformed into the torque. Simulations proved that for good adhesion conditions and the zero input, the value of term  $F_{\text{ad}}(v)$  is sufficiently close to zero. The error caused by this simplification is bellow 0.1%.

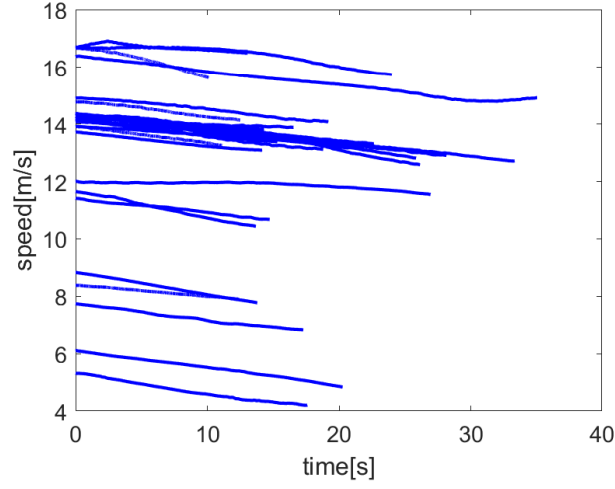
In addition to the zero input restriction, it was necessary to add a few more restrictions. The acceleration and the input are naturally zero when the tram is not moving, so the other restriction was non-zero velocity. The next restriction was related to time. For estimation it is always better to observe the effect for longer time period. I decided that the section has to be at least 10s long. Finally, measurements were more noisy when the tram was approaching the tram stop. Because of that, the sections found at the end were not considered.

From all measurements, only 28 sections fulfilled all restrictions (zero input, non-zero velocity, more than 10s long and not from the end of the measurement). Velocity profiles of these sections are shown in Fig 6.1. The velocities are from the GPS measurements and are affected by the gravitational force.



**Figure 6.1:** Velocity profiles affected by gravity.

For removing the influence of this force and therefore neglect the  $F_G(\theta)$  term, I used the result from section 5.4.2. In that section, I removed the acceleration caused by the gravitational force from the measured acceleration. I used this acceleration to estimate how the tram would slow down without the influence of gravity. The integration of this acceleration, together with the initial value, gave a velocity profile, which should not be affected by gravity. The resulting velocity profiles are shown in Fig 6.2. It is possible to observe a much greater similarity of individual profiles than in the previous figure.



**Figure 6.2:** Velocity profiles not affected by gravity.

The equation 6.4 after removing terms  $F_{ad}(v)$  and  $F_G(\theta)$  can be expressed as

$$\frac{dv}{dt} = \frac{1}{M} (-F_{res}(v)) \quad (6.5)$$

Substituting 6.1 into 6.5 gives following expression

$$\frac{dv}{dt} = \frac{1}{M} (-A - Bv). \quad (6.6)$$

This differential equation has an explicit solution

$$v(t) = M \left( ce^{-Bt} - \frac{A}{B} \right). \quad (6.7)$$

With the velocity profiles and weights, I could estimate parameters A and B. For the estimation I used the Levenberg–Marquardt algorithm for least square curve fitting. I estimated parameters A and B for each profile. The final value of each parameter is the mean of all estimated values. The estimated resistance formula is

$$F_{res} = 0.005797M + 90.20v \quad (6.8)$$

# 7 | Estimation of Weight and Motor Parameters

The last unidentified parameters in my model are the traction parameters  $k_+$ ,  $k_i$ , and the weight of the tram. I identified these parameters using a similar approach, with methods implemented in the System identification toolbox for Matlab[17]. In this chapter, I will describe that approach, used methods, and show the results.

## 7.1 Used Functions and Data Preparation

The System identification toolbox (SIT) provides, among other things, new functions to Matlab for modeling and identification of dynamic systems. All information about functions, the data structures and the methods provided by this toolbox are available online <sup>1</sup>. For the identification I used function **nlgreyest** (p.786). This function estimates parameters of the non-linear grey-box model. The input to the function is a system model (idnlgrey object with the information about the system equations, the initial states, the initial parameter values etc. - p.530), the measured data (object iddata - p.411) and the input.

Data for identification are formed by four parts (acceleration, speed, inclination, and input). For creating the iddata object, I had to make sure that they all have the same size, therefore the same sampling period. I decided to resample all parts so that the new sampling period is 1ms.

The model used for the identification is described in chapter 2 with outputs velocity and acceleration.

As a solving method, I chose the lsqnonlin option. This option is implementing a Trust-region-reflective algorithm.

## 7.2 Estimation of $k_+$

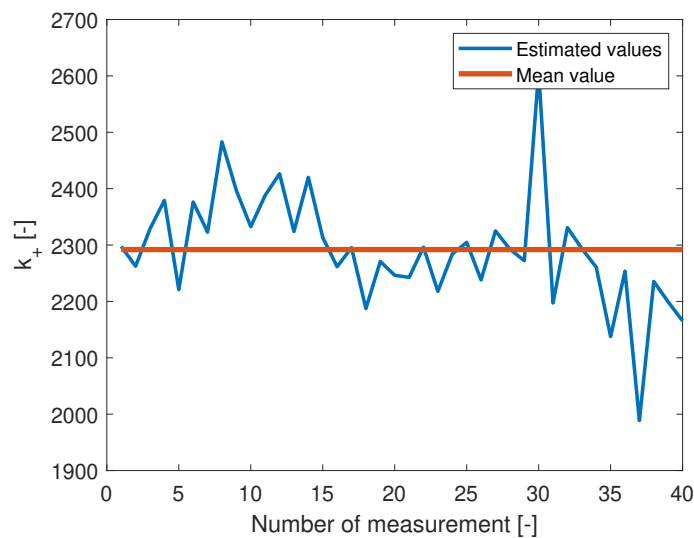
The traction parameter  $k_+$  describes transformation between the control lever position and the requested torque for the positive input. For the estimation of  $k_+$ , I decided, similarly

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<sup>1</sup>[https://www.mathworks.com/help/pdf\\_doc/ident/ident\\_ref.pdf](https://www.mathworks.com/help/pdf_doc/ident/ident_ref.pdf)

to the resistance parameters, separate it from other parameters as much as possible. This parameter affects the tram movement when accelerating. Because of this, the estimation is done on a section from the start of the measurement (measurements always started at the tram stop), to the point where the tram stopped accelerating. On this section, the tram is always accelerating, so the parameter  $k_-$  never affects the estimation. The value of weight is known from the measurements. Therefore, it is possible to estimate the  $k_+$  parameter alone. The estimated values from each measurement can be seen in Fig 7.1. The final value of this parameter is mean of all estimated values

$$k_+ = 2292. \quad (7.1)$$



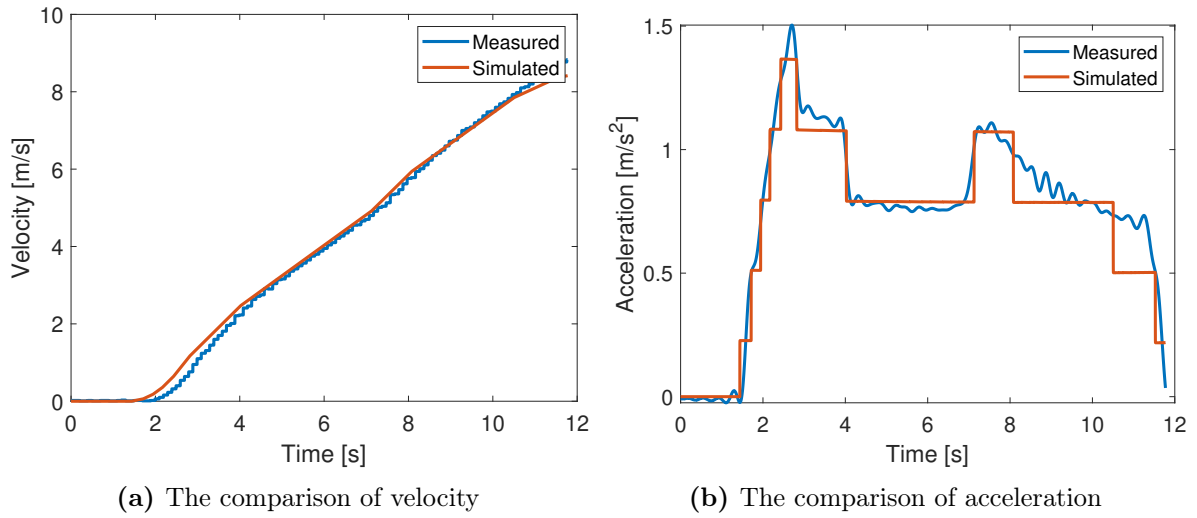
**Figure 7.1:** Estimated values of  $k_+$  parameter.

The comparison of measured values and simulated values on one section is shown in Fig 7.2.

### 7.3 Estimation of $k_-$

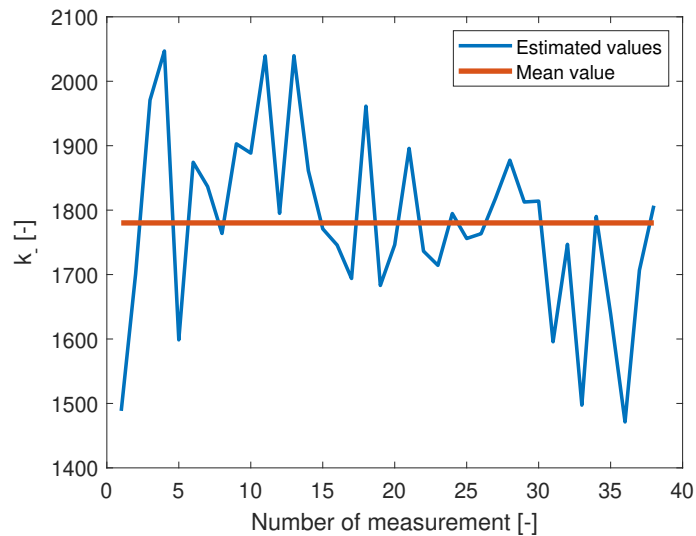
The parameter  $k_-$  describes the transformation between the position of the control lever and the torque for the negative input. As in the previous section, I tried to isolate the parameter  $k_-$  from other parameters as much as possible. On each measurement, I looked for the section with the longest continuous braking time. On these sections, I estimated the parameter  $k_-$ . The estimated values are shown in Fig 7.3. The final value is again the mean of all estimated values.

$$k_- = 1780 \quad (7.2)$$



**Figure 7.2:** The comparison of measured data and simulated data for the estimation of parameter  $k_+$ .

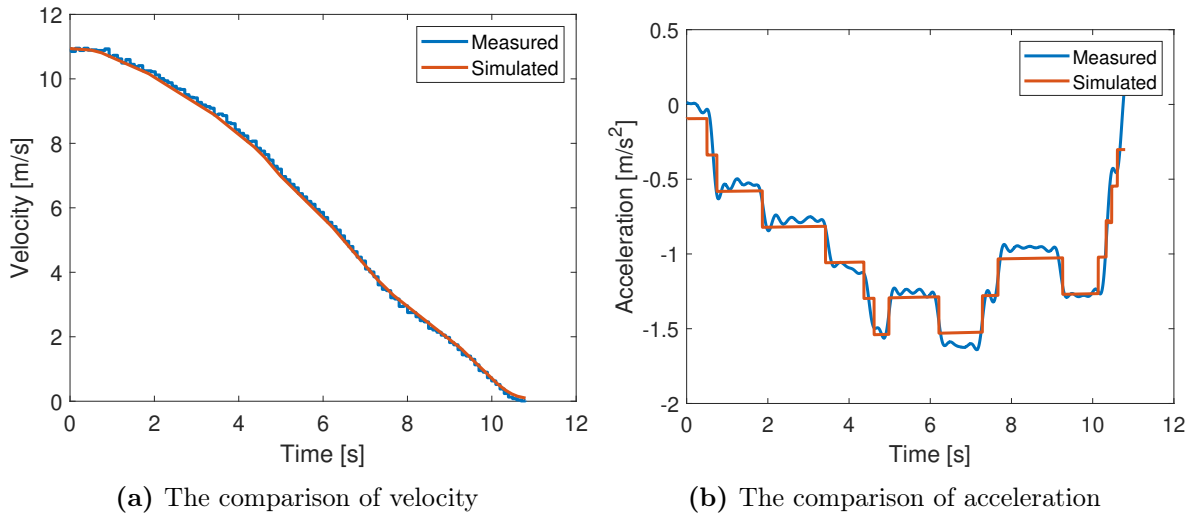
The comparison of measured values and simulated values on one section is shown in Fig 7.4.



**Figure 7.3:** Estimated values of  $k_-$  parameter.

## 7.4 Estimation of Weight

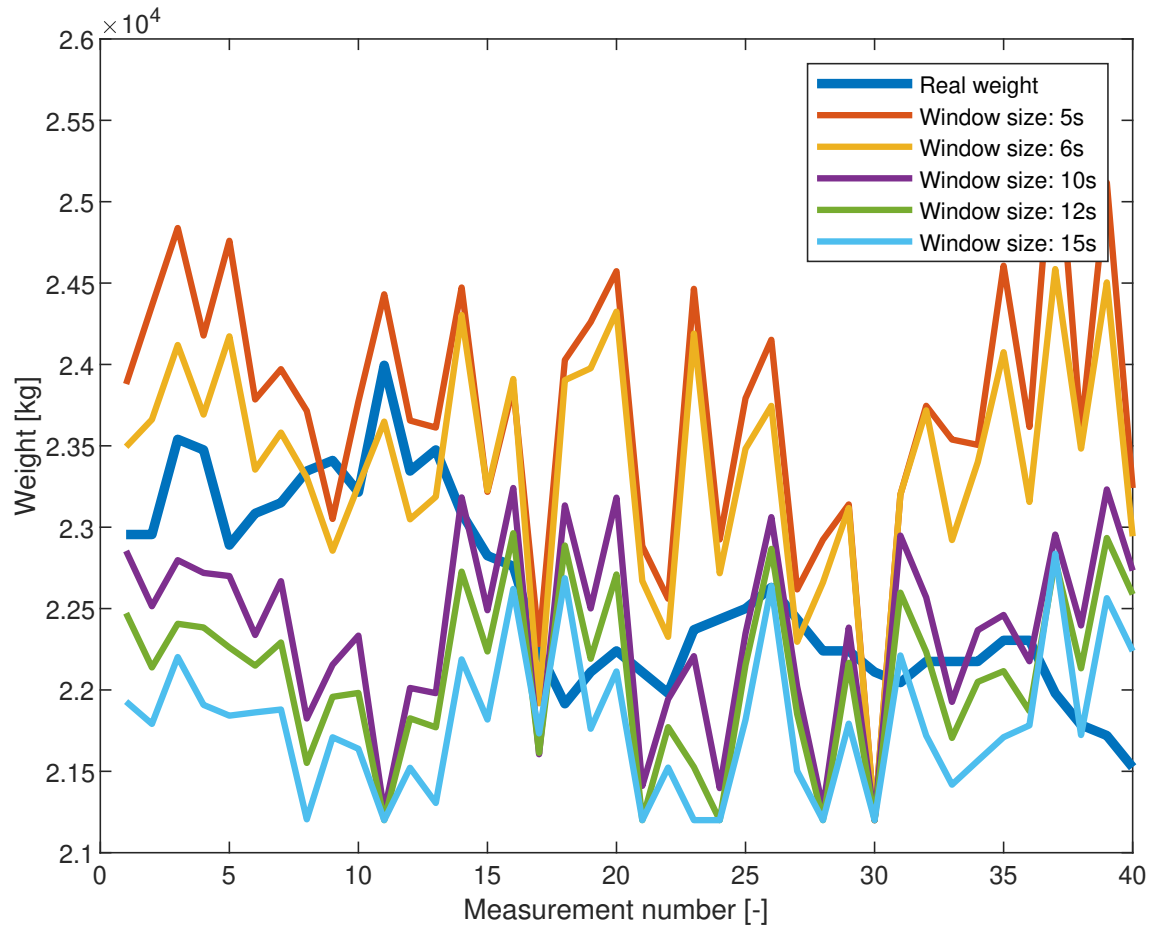
Unlike parameters  $k_+$  and  $k_-$ , the weight of a tram is a time-varying parameter. It is not possible to choose a specific section for its estimation. However, the weight could not change during one measurement. The information about the weight is necessary for the estimation of the braking distance, so the weight is estimated from the start of the ride. The time



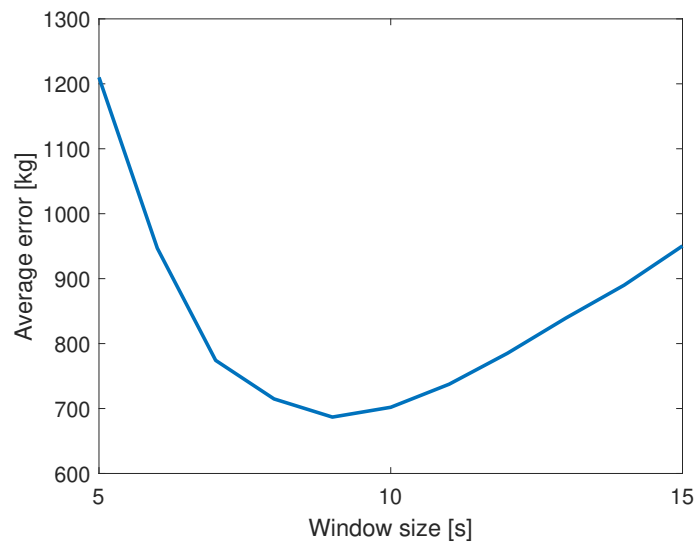
**Figure 7.4:** The comparison of measured data and simulated data for the estimation of parameter  $k_-$ .

window specified the length of the section for estimation. For the estimation, I also made the assumption that the weight of the tram cannot change rapidly. It is very unlikely that in one tram stop, more than 25 people get on or exit the tram at the same time. Therefore I limited the change of the weight to 2000 kg from the last estimated value. The estimated values, as well as real values, are shown in Fig 7.5. I did multiple simulations with different sizes of the time window. In Fig 7.6 can be seen that the longer time window is not always better. For the 9 s window the average error is almost 700 kg. From the model, this would lead to error from 0.1 m for the initial speed 10 km h<sup>-1</sup> up to 1.5 m for the initial speed 40 km h<sup>-1</sup>. However, it must be noted that the values of braking distances are based only on the model and consider braking only with the electrodynamic brake.





**Figure 7.5:** The estimated weights for different window sizes.



**Figure 7.6:** Average errors for each time window.

The fact that the longer time window and, therefore, more information does not improve the estimate is, in my opinion, caused by the inaccuracies of other parameters. As can be

seen in the previous sections, the optimal values of traction parameters differed in some cases in tens of percent. Missing input measurements cause the necessity to estimate these parameters. In the future, the information about the current should be available, and the weight estimation should be more precise.

## 8 | Conclusion

The idea for estimation parameters of the tram model came up from the work of my colleague Loi Do. He tried to estimate the braking distance of the tram. The braking distance is highly dependant on parameters such as weight or inclination. In Chap 2, I introduced the motion model of a tram, described all used parameters, and sorted them to three different groups based on their properties.

In Chap 3, I examined the first parameter of the model, the inclination. I developed the algorithm for estimating the inclination profile with the assumption that the position of rails is fixed. The algorithm has two inputs. The first one is a list of GPS coordinates with information about the position of the rails. The second one is an altitude map. The result of this algorithm was demonstrated on the tram line from Sídliště Barrandov to Divoká Šárka in Prague.

The Chap 4 describes the experiment from in Ostrava. Due to the collaboration with company Herman systems, I could perform measurements during the standard tram ride. In this chapter I described the measuring devices and their setup. Further on, I describe how the data were processed and showed a comparison of multiple sensors. At the end of this chapter, I used the algorithm developed in the previous chapter to estimate the inclination profile of the tram line used for the experiment.

At the time I was doing the measurements in Ostrava, it was not possible to obtain the information about the input. However, because this information was necessary for my work, I had to estimate it. The whole procedure is described in Chap 5. From the basic principles of tram motors, I concluded that the input could be quantized, and the quantization levels are related to the acceleration. With this knowledge, I designed a procedure which uses the K-means algorithm to estimate the input during the ride.

The parameters in the equation for the resistance of the tram were estimated in Chap 6. In this chapter, I introduced the method for estimating parameters of the Davis equation based on measured values.

At the end of my thesis, in Chap 7, I estimated the traction parameters  $k_+$  and  $k_-$ , as well as the weight of the tram. I used methods for the gray-box model parameter estimation from the System identification toolbox in Matlab.

The results of this thesis proved that developed methods can be used to estimate the values of parameters for the model of a tram. I also proved that it is not necessary to have information about current. However, I assume that with that information, the estimation would be more

precise.

In the model, as well as in my colleague's model, the adhesion parameter can have four values, based on the weather conditions. This model was developed for trains. To estimate the adhesion parameter, I would have to validate this model first. Unfortunately, during the experiments, and also during the experiments, my colleague did earlier, the adhesion conditions were the same. Because of this, the problem with estimating adhesion was not solved. For the time being, because the conditions do not change much during the ride, the adhesion parameter can be set at the start and remain unchanged.

## 8.1 Future Work

My algorithms for parameter estimation were developed without any knowledge about input values. This information would definitely improve the estimation of all parameters. Moreover, it would be useful to do more measurements on the same tram line to increase the accuracy and eventually detect some periodical phenomena. The problem with estimating the adhesion coefficient and the validation of the adhesion model remained unsolved and requires further research.

# Appendices



# A | Motion Model

The motion model of a tram used in this thesis:

$$\begin{aligned}
 \frac{dv}{dt} &= \frac{1}{M}(F_{ad}(v) - F_{res}(v) - F_G(\theta)) , \\
 \frac{d\omega}{dt} &= \frac{1}{J}(T - rF_{ad}(v_s)N_{tr\_wh}) , \\
 F_G(\theta) &= Mgsin(\theta) , \\
 F_{res}(v) &= A + Bv(t) , \\
 F_{ad}(v) &= \frac{Mg\mu(v_s)}{N_{wh}} , \\
 \mu(v_s) &= c_a e^{-a_a v_s} - d_a e^{-b_a v_s} , \\
 v_s(v, \omega) &= \omega r - v , \\
 J &= \frac{1}{2}m_{wh}r^2 , \\
 T(p) &= \begin{cases} \min(k_+p, P_{\max}), & \text{if } p \geq 0 \\ k_-p, & \text{if } p < 0 \end{cases} ,
 \end{aligned} \tag{A.1}$$

Parameter	Description	Unit
M	Tram weight	kg
g	Gravitational acceleration	m s <sup>-2</sup>
$N_{tr\_wh}$	Number of traction wheels	-
$N_{wh}$	Total number of wheels	-
$\theta$	Inclination of rails	rad
A,B,C	Resistance parameters	-
$a_a, b_a, c_a, d_a$	Adhesion parameters	-
$m_{wh}$	Weight of the wheel	kg
r	Wheel radius	m
$P_{\max}$	Maximum power	W
p	Notch position	-
$k_+$	Traction parameter for positive notch position	-
$k_-$	Traction parameter for negative notch position	-

**Table A.1:** All parameters used in the tram model.





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