Evaluation of the bachelor's thesis entitled

Approximating by Fuzzy Conjunctions

by Alexej Popovič,

Supervisor: prof. Ing. Mirko Navara, DrSc.

Thesis deals with an interesting problem of fitting triangular norms to given data (in other words, approximation of t-norms was examined) . As mentioned in the introduction of the thesis, databases containing data information concerning t-norms are quite rare till now, and thus for illustration of the proposed approximation methods, it was necessary first to create relevant data sets. Next, approximation of data based on parametric families of (continuous Archimedean) t-norms was studied. Basic approach considered here was the standard least square method. As an alternative approach, in Section 5 the author has considered fitting of additive (multiplicative) generators to the considered data sets.

The topic of thesis is rather challenging and the author did a good job. There is a minimal number of misprints (e.g., by description of Hamacher t-norms, a correct use of words minimum and minimal, etc.), formal level of this thesis is good. The next comments are aimed to help the author to improve his work in the case of his continuation of research in this topic. Namely, if the author will continue in this topic, it would be preferable to deal also with some other families of t-norms, in particular with the Schweizer-Sklar t-norms (they contain all 4 basic t-norms, and also extremal Hamacher t-norm). When describing strict t-norms (page 5), continuity should be supposed (the same for nilpotent t-norms). The idea of generating data as copula data is interesting, but it should be later discussed deeply – in general, copulas are not associative (also not symmetric, but the proposed method will generate symmetric data) and hence author's work opens an interesting problem – how „far" are copulas from t-norms? Discussion in Section 3 concerns possible non-monotonicity and non-symmetry of data. It would be good to spend few words also concerning possible non-associativity (while first two properties are visible from data, this is not the case of associativity). With generators and their uniqueness, it would be good to mention that additive generators of a given continuous Archimedean t-norm differ only in a positive multiplicative constant (for multiplicative generators, difference could be in positive exponent only). Example on page 26, the inverse of Lukasiewicz's additive generator was not correctly applied.

Note that it would be good to compare results obtained in this thesis also with some works of Beliakov, in particular from his paper Construction of k-Lipschitz Triangular Norms and Conorms From Empirical Data from 2009.

I have few questions:

- Also when approximation was done by means of generators, we should have some constraints on considered generators. What was author's approach?

- Can you relate your results with approximation of idempotent elements (and, subsequently, with fitting ordinal sums of t-norms to data sets)?

- In evaluation of the quality of obtained approximation, only global aspect was considered; can you tell something concerning the quality of approximation related to 3 considered properties (monotonicity, symmetry, associativity)?

Note that these questions could be considered as possible new directions of continuation of this thesis.

Summarizing, the thesis has dealt with nontrivial problems, the author has shown good orientation in the problematics of triangular norms, his results are interesting and worth continuation in this spirit. I propose to accept this thesis for defence, and after successful defence to award its author by grade A.

Prof. RNDr. Radko Mesiar, DrSc.

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