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Chatter suppression in finish turning of thin-walled cylinder: model of tool workpiece interaction and effect of spindle speed variation

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Abstract

Finishing operation of a thin-walled cylindrical workpiece may suffer from regenerative vibration due to high compliance of the workpiece and tool-workpiece interaction on a tool nose. In this case the machining stability cannot be simply controlled by chip width.

The main goal of this study is to find a suitable harmonic spindle speed variation which would lead to a stable cutting process and at the same time is energetically economical, i.e. requires only low amplitude and frequency of the variation. This paper presents linear stability analysis of the cutting process with varying spindle speed using semi-discretization. This model of tool-workpiece interaction incorporates the effect of moving force on a cylindrical shell, the effect of spindle speed variation and contact of tool nose with the work-piece. A nonlinear model of the interaction is also presented and compared with experimental data. The procedure is applied to an industrial case and validated experimentally.

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Nomenclature

r	tool nose radius
$K_{e,c}$	edge and cutting coefficients
h	undeformed chip thickness
w	chip width
λ	inclination angle
$\Gamma(s)$	parametric curve describing the cutting edge
g	indicator function of the cutting edge engagement
$[T]$	transformation matrix between local and global CS
ω_n	dominant mode natural frequency
ζ	dominant mode damping
m	dominant mode mass
\mathbf{f}	empirical cutting force model
τ	time delay
$\Omega(t)$	time dependent spindle speed
A	spindle speed variation amplitude
T_v	spindle speed variation period
T_0	mean period of spindle revolution
φ	angle of revolution
ξ	displacement of the workpiece in radial direction
f_e	feed per revolution

1. Introduction

Self-excited machine tool vibrations are one of the most serious issues that limit productivity and compromise quality of the surface finish. The most widely accepted explanation of the phenomena is the regeneration principle developed by Polacek and Tlustý [1] in 1950s. The most common method for chatter suppression based on the theory is lowering chip width/cutting depth or changing tool orientation in order to avoid a potentially destabilizing regeneration effect. However this solution is not always practicable or efficient. One of the long established methods of chatter suppression is spindle speed variation (SSV) developed in the 1970s [2]. In the time-domain the mathematical description of machining dynamics leads to a system of delay differential equations (DDE). In the case of SSV the time delay is time dependent which makes calculation more difficult.

The semidiscretization method is used in this article for transforming the delay differential equation into a set of ordinary differential equations. Though the method can be used considering the variable delay as shown by Insperger [4], it was found more convenient to use transformation into a constant delay problem using the intrinsic time of the system as proposed by Tsao [3] and recently demonstrated by Otto [7].

The goal of this research is chatter suppression in finish turning of a large thin-walled cylinder demanded by our industrial

partner (see figure 1a). In this case stability cannot be simply controlled by changing depth of cut or tool orientation because the radial force variation due to vibration occurs only at the radial nose of the tool ($r = 0.8$ mm).

The first part addresses an interesting problem of non-linear chatter observed during the machining (see chatter marks at the figures 1b,c). The model of tool-workpiece interaction contain process damping due to velocity direction perturbation. It generalizes the approach presented by Das and Tobias[5] - an effect of vibrations perpendicular to cutting edge normal plane is included into the cutting force model. Another effect taken into account is change of tool workpiece contact along the cutting edge due to vibration. A similar effect using a different modelling approach to cutting force has been taken into account by Eynian and Altintas [9].

The second part describes the SSV and discusses appropriate choices of its parameters.

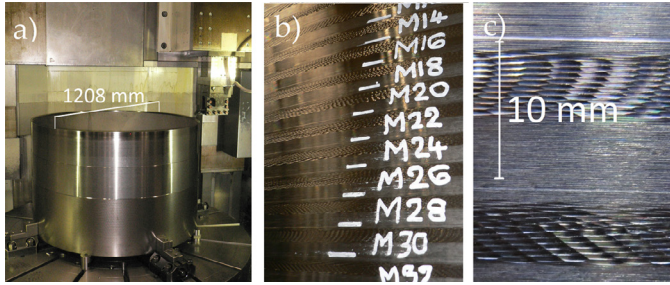


Fig. 1. (a)Cylindrical workpiece on the vertical lathe. (b,c) Machined surface and its detail.

2. Model of the tool-workpiece interaction

A model of the interaction of compliant workpiece and rigid tool will be presented in this section. The interesting feature of the observed chatter is that amplitude of its steady state vibrations is significantly lower than the radial depth of cut a_p , i.e. the system stabilizes itself on a large amplitude oscillations without any contact loss between the tool and workpiece. The scheme of the tool-workpiece contact is in figure 2. Dynamics of the compliant workpiece is described by the following equation

$$\ddot{\xi} + 2\zeta\omega_n\dot{\xi} + \omega_n^2\xi = \frac{1}{m}\mathbf{e}_r \cdot \mathbf{F}(\xi, \dot{\xi}, \xi_\tau) \quad (1)$$

$$\mathbf{F} = \int_{\Gamma} g(s, \xi) [T(s, \dot{\xi})] \mathbf{f}(h(\xi, \xi_\tau), \lambda(\dot{\xi}), \dots) \frac{dw}{ds}(s, \dot{\xi}) ds .$$

where \mathbf{e}_r is a unit vector in the radial direction, $g(s)$ is characteristic function which equals to 1 if the element of the cutting edge at the parameter s is in cut and 0 if not. The transformation matrix $[T] = [e_r, e_n, e_b]$ between the local coordinate system and global one is affected by the deflection velocity $\dot{\xi}$. The vectors e_r, e_n, e_b create orthonormal basis for the element of the cutting edge which is based on the local cutting velocity vector (subscript t) and local surface normal vector (subscript n). The model of the force acting on an element of the tool/workpiece contact \mathbf{f} depends on chip thickness affected by the regeneration, but it also depends on tool geometry (e.g. inclination angle λ or rake angle α) and chip width $w(s)$, which all depend on the actual velocity direction, i.e. on the displacement rate.

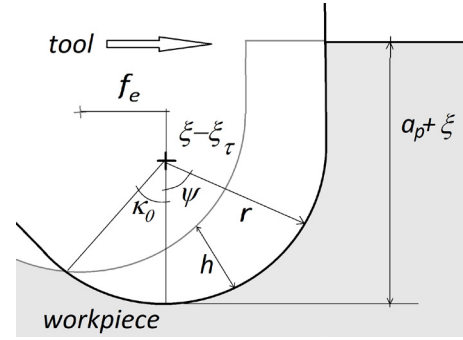


Fig. 2. Scheme of the tool workpiece interaction

In our case the integral along the tool edge is described in polar coordinates using angle parameter ψ . The chip thickness is expressed by the following formula which takes into account several previous tool paths (similar approach as in [8])

$$h \approx \min_{k=1\dots N} \left(k f_e \sin \psi + \frac{(k f_e)^2}{2r} \cos^2 \psi + (\xi - \xi_{k\tau}) \cos \psi \right) \quad (2)$$

In case of the fully immersed round nose ($a_p > r$) the characteristic function g can be introduced into the integral through its limits. On the tool tip the condition for the lower limit is given by equality $h(\psi) = 0$.

As the problem has only one degree of freedom in the radial direction, it is not necessary to calculate the whole transformation matrix with respect to velocity direction variation due to the displacement $\mathbf{v} = (0, \dot{\xi}, v_0)^T$.

Similarly it can be shown that the displacement rate does not affect the chip width in the first order of magnitude with respect to velocity variation $\frac{dw(s)}{ds} = 1 + O(\|\dot{\xi}/v_0\|^2)$. The last thing needed for the formulation is the model of force for the element of the tool. The model was assumed in a form $\mathbf{f}(h, \lambda) = -(\mathbf{K}_e + \mathbf{K}_c(\lambda)h)w$ where λ is the inclination angle, $\mathbf{K}_e = (K_{et}, K_{en}, 0)$ is the edge coefficient vector and $\mathbf{K}_c(\lambda) = (K_{ct}, K_{cn}, K_{cb}\lambda)$ is the cutting coefficient vector. The inclination angle can be calculated from the inner product of cutting edge tangent and the perturbed cutting velocity, i.e. $\lambda \approx \frac{\dot{\xi}}{v_0} \sin \psi$. The effect of the rake angle on the cutting coefficients is neglected.

The resulting integral over the contact can be expressed as an integral along the circular edge and linear edge

$$\mathbf{F}_r = - \int_{\kappa_0}^{\kappa_1} \left(\frac{\dot{\xi}}{v_0} \cos \psi, -\sin \psi \right)^T \cdot (\mathbf{K}_e + \mathbf{K}_c h) r d\psi + \left(\frac{\dot{\xi}}{v_0}, 0, -1 \right)^T \cdot (\mathbf{K}_e + \mathbf{K}_c f_e) \max(0, a_p - r + \xi) \quad (3)$$

where the limits are given by the non-zero chip thickness on one side and the contact between the tool and the outer surface of the workpiece on the other, i.e.

$$\kappa_1 = \min\left(\frac{\pi}{2}, \arccos \frac{r - a_p - \xi}{r}\right)$$

$$\kappa_0 = \min\left(\kappa_1, \max_{k=1\dots N} \left(-\frac{(k f_e)^2}{2r} - \arctan \frac{\xi - \xi_{k\tau}}{k f_e}\right)\right) \quad (4)$$

The formula for linearised variation of force with respect to

a small displacements is

$$\Delta F_r \approx -(\xi - \xi_\tau) \left(\underbrace{\frac{\pi r}{4} K_{cn}}_I + \underbrace{K_{en} \frac{r}{f_e}}_II \right) - \underbrace{\frac{\xi}{v_0} (K_{ct} - K_{cb}) a_p f_e}_{III} \quad (5)$$

The term (I) is the cutting process stiffness used in standard stability analysis, the second term (II) contains the effect of variation of the contact with displacement. This term is inversely proportional to a ratio of feed and nose radius, which could also explain the observed tendency of circular inserts to chatter[6].

The third term (III) contain additional damping caused by a projection of the cutting forces into the radial direction. This term is not the exact result of the linearization but some terms of it were neglected in order to make the formula clear under the assumption that $K_{ct} \ll K_{cb} \ll K_{cn}$. Physically this term can be interpreted as a projection of friction force on the contact between the tool and workpiece into the radial direction. It suggests that machining stability may be enhanced by increasing feed, decreasing cutting velocity or increasing the radial depth of cut.

2.1. Simulation

The dominant mode was selected according to observed chatter frequency. The identified modal parameters were $f_n = 343.8$ Hz, $\zeta = 0.00022$ and $m = 16$ kg at the upper part of the workpiece before machining and $f_n = 336.74$ Hz, $\zeta = 0.00022$ and $m = 120$ kg at the point of the last measurement (240 mm bellow the upper edge of the workpiece). The frequency response functions (FRF) were measured both by a laser Doppler vibrometer and an accelerometer. The most compliant mode was fitted by one degree of freedom (1DOF) approximation. For simplicity the chip thickness was calculated from the previous cut only. The complex definition of the thickness (2) was needed mainly for finding the integration limits. This simplified analytical calculation of the integral and accelerated the calculation significantly. Three additional multiples of the minimum delay ($N = 4$) were used for calculation of the lower limit in the equation (4).

The material of the workpiece was steel C45. Experimentally identified cutting coefficients were $K_e = (45, 50, 0)^T$ N mm⁻¹, $K_c = (1920, 800, 430\lambda)^T$ N mm⁻², feed $f_e = 0.15$ mm, radial depth $a_p = 1$ mm and the insert was Sandvik DNMG 15 06 08R ($r = 0.8$ mm). The spindle speed was 37 min⁻¹ so the corresponding minimum time delay was set to $\tau = 1.6$ s and the cutting velocity is $v_0 = 2.35$ m s⁻¹. The result of the simula-

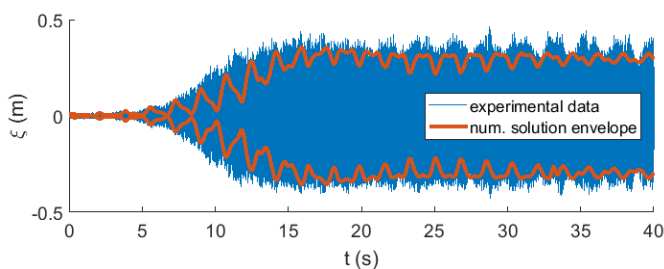


Fig. 3. Simulated and measured displacement during the chatter

tion together with measured displacement from machining near

the top are at figure 3. This shows that the results are qualitatively and quantitatively comparable although there is rather high uncertainty in the inputs. Moreover the displacement is measured at the top of the workpiece which makes its amplitude slightly higher. When the infinitesimal forces were applied along the cutting in the usual way, which considers only the unperturbed velocity and surface normal, the simulations showed exponential growth until the tool loses its contact with the workpiece. The nonlinearity caused by the contact angle $\kappa_0(\xi, \xi - \tau, \dots)$ between the tool and workpiece led to significantly higher oscillations than observed during the experiment.

There is another reflection based on the experiment that gives support to the described approach. The experiment on the SSV was stopped when the system became more or less stable without the SSV. This can be seen as a stability limit measurement (not planned so not very precise). The FRF in this part of the workpiece was measured, so that stability with tool nose radius as a parameter can be calculated, however the theoretical stability would be reached only if the tool nose radius was lower than ca 0.3 mm. It means the machining should still be unstable. However, we may see the term (III) in equation (5) as an additional proportional damping which can be expressed as $\zeta_{CP} \approx \frac{(K_{ct}-K_{cb})f_e a_p}{2mv_0\omega_n}$. Adding the frictional damping $\zeta_{CP} = 0.0002$ means practically doubling the overall damping of the system. This in turn means doubling the estimated limit radius to 0.6 mm which is closer to the actual radius 0.8 mm.

3. Spindle speed variation model

The SSV recommendation is based on a calculation of the relative effect of SSV on stable chip width \tilde{w} (nondimensional) in the basic non-dimensional model of machining stability

$$\ddot{\xi}(t) + 2\zeta\dot{\xi}(t) + \xi(t) = -\tilde{w}(\xi(t) - \xi(t - \tilde{\tau}(t))) \quad (6)$$

where $\tilde{\tau}(t)$ is non dimensional time delay, \tilde{w} is nondimensional chip width, ξ is displacement, ζ is proportional damping. Using the approach proposed by Tsao [3], the problem can be formulated using angle of revolution as an intrinsic time of the system (assuming that it is strictly positive).

$$\frac{1}{\Omega} \frac{\partial}{\partial \varphi} \left(\frac{1}{\Omega} \frac{\partial \xi}{\partial \varphi} \right) + 2\zeta \frac{1}{\Omega} \frac{\partial \xi}{\partial \varphi} + \xi = -\tilde{w}(\xi(\varphi) - \xi(\varphi - 2\pi)) \quad (7)$$

The SSV is assumed to be sinusoidal $\Omega(t) = \Omega_0 \left(1 + A \sin\left(\frac{2\pi}{T_v} t\right) \right)$, where A is the amplitude of SSV ($0 \leq A < 1$), T_v is its period, Ω_0 is mean spindle speed and T_0 corresponding period. The equation (7) assumes the spindle speed as a function of φ so we need to find an inversion of $\varphi(t)$. The inversion of the relation is impossible in a closed form. However if the amplitude is small $A \ll 1$, the time can be expanded in A . The spindle speed as a function of φ can be approximated $\Omega(\varphi) \approx \Omega_0 \left(1 + A \sin\left(\frac{T_0}{T_v} \varphi + A \cos\left(\frac{T_0}{T_v} \varphi\right) \right) \right)$.

Another condition needed for calculation of a monodromy matrix is periodicity of $\Omega(\varphi)$, i.e. the ratio $\frac{T_v}{T_0}$ is rational. The problem given by equation(7) and the relation for $\Omega(\varphi)$ can be solved by semidiscretization over the least common period of

the spindle speed revolutions and spindle speed variation.

3.1. Numerical results and comparison with experiment

The results are presented as a ratio of limit width for machining with and without SSV for three nondimensional $\tilde{\omega} = \{0.05, 0.02, 0.002\}$ and proportional damping $\zeta = 0.005$.

The simulation results (see figure 4) calculated on a (non-uniform) grid 28x40 has shown that the limit width greatly increases if the spindle speed is much lower than the dominant natural frequency and in this case even small amplitude of SSV have a significant impact on stability. In order to make the variation energy efficient it is reasonable to choose a longer period of SSV rather than higher amplitude. Moreover it should be reminded that the simulation is based on an assumption of small amplitude of modulation.

The experiment tested 17 combinations of amplitudes and periods of spindle speed variation marked at the figure 4c by the circles. All the spindle speed variations suppressed chatter. This is due to the fact that the ratio of spindle speed and dominant frequency of the workpiece vibration is so small that even a small variation of the spindle speed disrupts the regenerative effect.

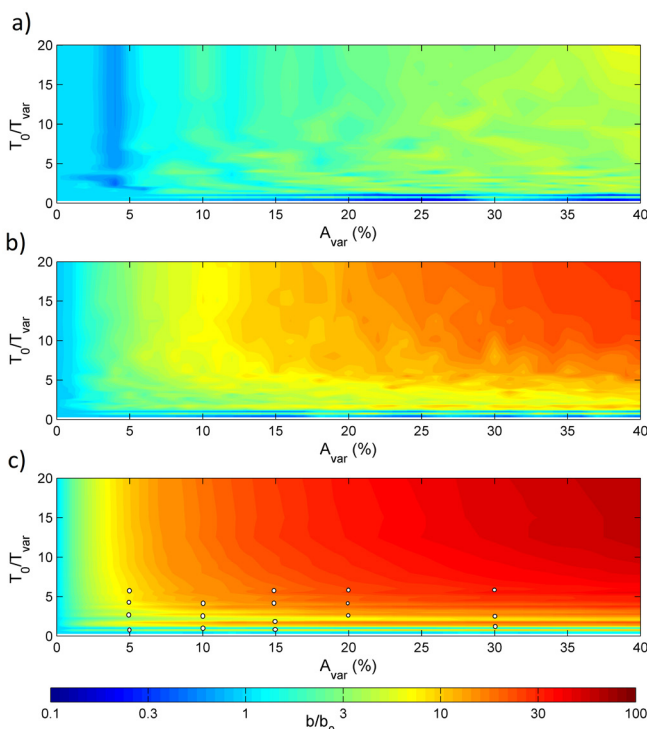


Fig. 4. Limit chip width of time varying case relatively to limit chip width for a constant spindle speed. Nondimensional spindle speed (relatively to a dominant frequency) is (a) $\tilde{\omega} = 0.05$ (b) $\tilde{\omega} = 0.02$ (c) $\tilde{\omega} = 0.002$. The last case is comparable with the measurement and the small circles show parameters of spindle speed variations used in the experiment.

4. Conclusion

In the first part of the study the model of tool-workpiece dynamics was considered. It was found that inclusion of frictional side force is necessary in order to keep the amplitude of the oscillation in accordance with the measurement. This frictional force model follows from the static force model if the local

cutting force is projected into the global workpiece coordinate system with respect to the actual velocity direction, which is sum of tangential speed due to the workpiece rotation and the displacement rate.

Linearisation of the overall force with respect to the small displacement leads to two additional terms which are often neglected in the standard linear stability analyses. The first (see (II) in the equation(5)) is due to linearisation of the displacement dependent contact between the tool and workpiece. This term negatively affects stability if the edge coefficient K_{en} is positive (and thermodynamically it should be positive due to energy needed for a creation of a new surface). This effect is proportional to the radius of the tool tip and inversely proportional to feed. The second term (see (III) in the equation (5)) can be seen as a projection of a friction force on the rake face into the direction perpendicular to cutting edge normal plane. This term dampens oscillations in this direction and is analogical to the process damping introduced by Das and Tobias [5] in a cutting edge normal plane.

The second part of the study dealt with effect of SSV on machining stability. It seems reasonable to choose moderate amplitude ca 20 % and long period of SSV. One might expect that there should be a limit, where further increase of the period have a negative impact on the limit of stability, but it would be rather computationally expensive to solve it numerically, especially for very low relative spindle speed $\tilde{\omega}$.

The authors are aware that the presented model is not generally applicable and needs to be formulated and analysed for a general displacement in 3D, tool geometries and cutting force models. The model's validity is going to be tested experimentally with higher precision.

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