Czech Technical University in Prague
Faculty of Civil Engineering

Department of Concrete and Masonry Structures

Diploma Thesis

Design methods for reinforced concrete deep beams

2019 Bc. Martin Slavata

ČESKÉ VYSOKÉ UČENÍ TECHNICKÉ V PRAZE
Fakulta stavební
Thákurova 7, 166 29 Praha 6

ZADÁNÍ DIPLOMOVÉ PRÁCE

I. OSOBNÍ A STUDIJNÍ ÚDAJE

Příjmení: Slavata Jméno: Martin Osobní číslo: 438944

Zadávající katedra: Katedra betonových a zděných konstrukcí

Studijní program: Stavební inženýrství

Studijní obor: Konstrukce pozemních staveb

II. ÚDAJE K DIPLOMOVÉ PRÁCI

Název diplomové práce: Metody navrhování železobetonových stěnových nosníků

Název diplomové práce anglicky: Design methods for reinforced concrete deep beams

Pokyny pro vypracování:
Diplomant zpracuje přehledný souhrn výpočetních metod a přístupů k návrhu železobetonových stěnových nosníků. Dále provede návrh vyztužení železobetonového stěnového nosníku u vybraného objektu při uplatnění různých výpočetních metod. V závěru práce zpracuje rozbor a porovnání získaných výsledků.

Seznam doporučené literatury:
KONG, F.K & Co. Reinforced Concerete Deep Beems. Taylor & Francis Books, Inc., 2002
BAŽANT, Zdeněk. Plošné betonové konstrukce, nádrže a zásobníky, Brno, 2010.

Jméno vedoucího diplomové práce: Ing. Josef Novák, Ph.D.

Datum zadání diplomové práce: 21.6.2019 Termín odevzdání diplomové práce: 5.1.2020
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Podpis vedoucího práce Podpis vedoucího katedry

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S P E C I F I K A C E   Z A D Á N Í

Jméno diplomanta: Bc. Martin Slavata

Název diplomové práce: Design methods for reinforced concrete deep beams

Základní část: K133 podíl: 100 %

Formulace úkolů: Předmětem úvodní části diplomové práce je zpracovat přehledný souhrn výpočetních metod a přístupů k návrhu železobetonových stěnových nosníků. Přehled bude obsahovat pravidla a zásady navrhování stěnových nosníku podle evropské a americké normy a dále studii výpočetních metod vhodných pro konstrukční návrh stěnových nosníků. Druhá část práce se bude věnovat návrhu vybraného stěnového nosníku při užití různých výpočetních metod. Přesnost výpočetních metod bude porovnána s výsledky nelineární numerické simulace vybraného prvku. Výstupem konstrukčního návrhu bude výkres výztuže řešeného nosníku.

Podpis vedoucího DP:.. Datum: ....

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Podpis konzultanta:.. Datum: ....

Poznámka:

I declare that this thesis and the work presented in it are my own and have been worked out by me only with the expert assistance of the supervisors, Ing. Josef Novák, Ph.D. and Proffesor Hayder A. Rasheed.

I confirm that all sources used in this work are given in the list of literature.

In Manhattan, Kansas on December 16th, 2019

……………………………

Bc. Martin Slavata

I would like to thank Ing. Josef Novák, Ph.D. for his guidance, comments, and helpful advice throughout the process of writing this thesis.

I would like to thank Dr. Hayder A. Rasheed for his expertise and assistance during my stay at Kansas State University, USA.

Design methods for reinforced concrete deep beams

Abstract:

The aim of this diploma thesis is to investigate design methods of deep beams. In brief, approaches of Czech technical standard (ČSN EN 1992-1-1) and American Concrete Institute (ACI 318-14) as well as recommendations from technical literature are presented herein. All design methods are introduced in detail and then compared. The results show that some methods are inaccurate and therefore unsuitable for the design of deep beams. The design method which was evaluated as the best one is then applied to design a selected deep beam.

Keywords: deep beam, D-region, reinforced concrete, non-linear analysis, strut-and-tie method, stringer-panel method

Anotace:

Cílem této diplomové práce je analýza výpočetních modelů pro sténové nosníky. Ve stručnosti jsou představeny přístupy řešení stěnových nosníků dle Českých technických norem (ČSN EN 1992-1-1) a Amerického institutu pro beton (ACI 318-14) a doporučení uvedená v odborné literatuře. Jednotlivé výpočetní modely jsou popsány a porovnány. Z výsledků je patrné, že některé metody jsou nepřesné a proto jsou pro řešení stěnových nosníků nevhodné. Návrh vybraného stěnového nosníku je proveden výpočetním modelem, který byl při porovnání vyhodnocen jako nejvhodnější.

Klíčová slova: stěnový nosník, D-oblast, železobeton, nelineární analýza, metoda náhradní příhradoviny, metoda prut-stěna

Content

1 Introduction

This thesis aims to compare the design methods of reinforced concrete deep beams and design the selected deep beam.

The design of deep beams is one of the specific tasks for which there are no certain computational methods. Current standards and regulations provide general information and recommended approaches for the design of these members, but detailed information and design principles are missing.

Presented work summarizes available information on the design of deep beams.

1.1 Motivation

A motive for a selection of the topic of this diploma thesis was a previous study when it was not given much attention to D-regions and non-linear analysis. The main reason was to increase my knowledge and understanding of these issues.

In the previous semester, I designed a superstructure which is statically independent of an original building and it is supported by columns. It would be possible to clear a space at ground level by applying a deep beam. The deep beam and the load of this building were used during an analysis of the design process of deep beams.

1.2 Objectives

The diploma thesis aims to present, analyze, compare and evaluate the design methods of reinforced concrete deep beams. The next objective of this thesis is to accurately design the selected deep beam. My goal was also to gain new knowledge of the problem of designing deep beams and D-regions in general. The outcome of the work is the text and drawings.

2 Deep beams

A deep beam is a planar element which is loaded in its plane [8]. According to the definition of the standard valid in the Czech Republic, ČSN EN 1992-1-1, a beam is considered to be the deep beam if its span is equal to or less than three times its height [3]. Standards of other countries (for example ACI 318-14, which is valid in the United States of America) state a different span to height ratio, but the meaning of the definition is the same [5]. An advantage of deep beams is the possibility to clear space on the lower floors. On the other hand, a disadvantage is that deep beams are so-called D-regions – discontinuity or also failure regions [9]. D-regions do not meet the conditions of linear deformation of a cross-section.

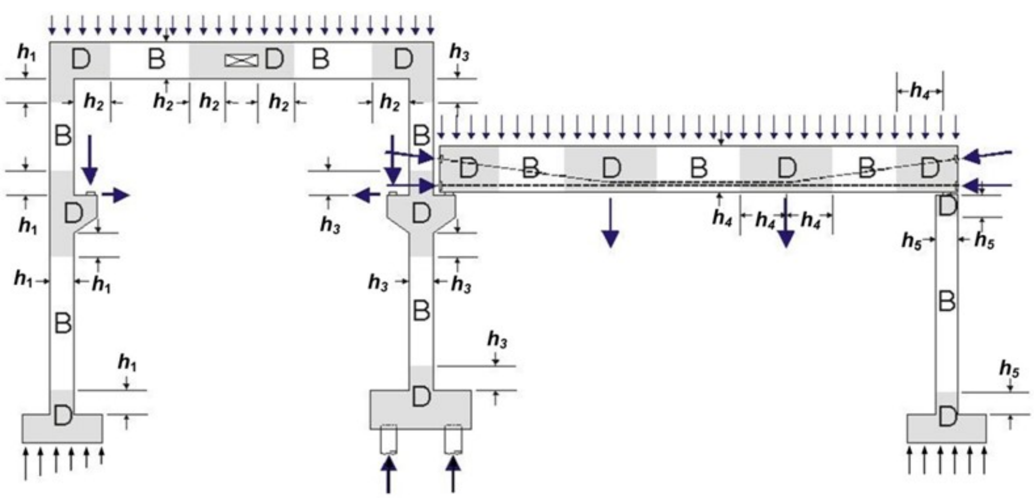


Figure 1: Different D-regions of a structure [17]

A difference between beams and deep beams is in a curve of stress along the height of the cross-section in a linear elastic analysis [8, 9]. The Bernoulli-Navier hypothesis of maintaining the flatness of the cross-section after a deformation is valid and the curve of the stress is linear for beams. Beams have usually a large moment arm of internal forces compared to the height of the cross-section. Deep beams do not meet the Bernoulli-Navier hypothesis of maintainingthe flatness of the cross-section after the deformation, the curve of the stress-strain relationship is non-linear, and the value of extreme tensile stress is significantly higher than the value of extreme compressive stress. Deep beams also have a small moment arm of internal forces compared to the height of the cross-section (Figure 2).

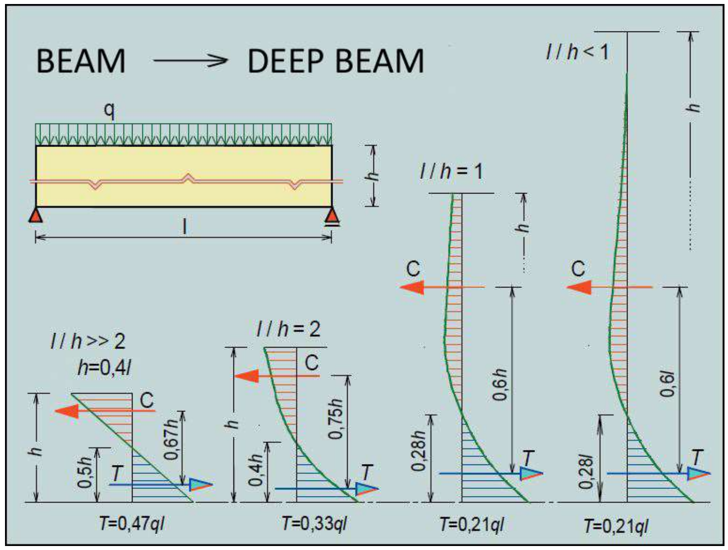


Figure 2: The curve of the stress along the height of the cross-section according to the span to ratio of the beam [9]

A position of a load is an important aspect in a process of designing deep beams. If the deep beam is loaded at an upper surface, then the compressive stress goes directly to supports and the perpendicular tensile stress causes vertical cracks [9]. The high concentration of the compressive stress is reached in areas above the supports. The concrete is stressed by a high compression in support areas. Therefore, it is necessary to properly reinforce the support areas in addition to the horizontal tensile reinforcement located between the supports to prevent a concrete from crushing (bearing failure).

If the load is applied at a lower surface, then the load causes an arch effect. The arch is supported at the edges and the perpendicular horizontal tensile stress at the lower surface represents a tie of the arch [9].

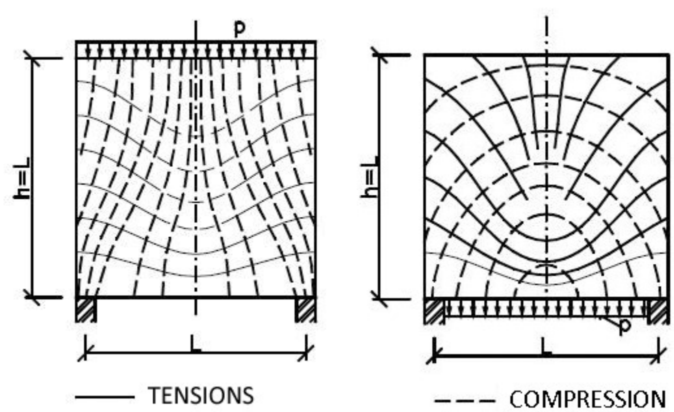


Figure 3: The stresses according to the position of the load [8]

Because an author of this diploma thesis comes from the Czech Republic and at the same time the thesis was worked out during a study exchange in the United States of America, attention is given primarily to recommendations and approaches for the design of deep beams according to the standards valid in these two countries.

The Czech Republic is a member of the European Union and the standard ČSN EN 1992-1-1 applies to the design of concrete building structures.

In the United States, the standard ACI 318-14 applies to the design of concrete building structures.

Although these two standards mentioned above are independent on each other, they provide neither the specific guidance nor the procedure for the design and the assessment of deep beams.

2.1 ČSN EN 1992-1-1

The standard ČSN EN 1992-1-1 considers an element to be a deep beam if its span is equal to or less than three times of its height [3].

This standard provides only general recommendations with no guidance for the design of deep beams. Deep beams are considered to be so-called D-regions, also known as discontinuity regions or failure regions. These areas should be designed by a strut-and-tie method according to the standard ČSN EN 1992-1-1. The strut-and-tie method is described in chapter 3.2.

Deep beams should have an orthogonal reinforcement along both surfaces with a minimum area $A_{s,dbmin}$, this value can be adjusted by national annexes. The recommended value of the minimum reinforcement area is $0.001A_c$ and at the same time a minimum of 150 mm^2/m for each surface and direction. The distance between two bars should not be greater than the smaller of twice of the thickness of the deep beam or 300 mm.

It is necessary to fully anchor the reinforcement bars. The position of the bars should be the same as the position of ties in the strut-and-tie model. If there is not enough space for an anchor length between a node considered in the strut-and-tie model and the end of the deep beam, it is recommended to anchor the reinforcement by bending the bars, using U-clips or anchor devices.

2.2 ACI 318-14

According to ACI 318-14, the element is a deep beam if its clear span is not more than four times its height or the concentrated load is within a distance of two heights of the deep beam from the edge of the support [5].

The American standard ACI 318-14, as well as ČSN EN 1992-1-1, has no specific guidance, but it provides (together with Commentary ACI 318R-14) more information about the design of deep beams. The standard ACI 318-14 recommends considering a non-linear strain distribution over the height of the cross-section. However, it does not mention any specific method for the design of deep beams.

Dimensions of deep beams should be selected to meet the following condition [5, 6]:

$$V_u \leq ø 10 \sqrt{f_c^{'}}\, b_w\, d$$

where V_u is a design shear force, ø is a safety factor (ø=0.75), $f_c^{'}$ is a cylindrical compressive strength of the concrete, b_w is a thickness of the deep beam and d is a distance of a compressed surface to a center of primary reinforcement. If this condition is met, crushing of the concrete and a formation of excessively wide cracks should not occur [6].

A minimum required area of primary reinforcement $A_{s,min}$ is determined in the same way as for a classical shallow beam. Stress in a longitudinal reinforcement is more uniform and high values can occur at the ends of the bars and not just in the middle of the span like for classical shallow beams. Because of this reason, the longitudinal reinforcement must be sufficiently anchored.

Reinforcement along the vertical surface should be at least such that it meets the following conditions:

- Area of reinforcement perpendicular to longitudinal axis of deep beams A_v should be at least $0.0025 b_w s$, where s is the spacing of the transverse reinforcement.
- Area of reinforcement parallel to the longitudinal axis of deep beams A_{vh} should be at least $0.0025 b_w s_2$, where s_2 is the spacing of the longitudinal reinforcement.
- Bar spacing should not exceed the smaller of $b_w/5$ and 12 inches (300 mm).

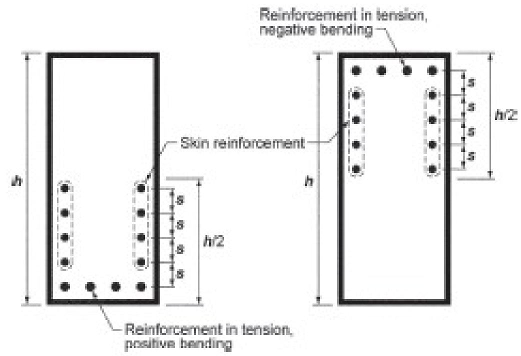


Figure 4: The position of the main reinforcement and the additional reinforcement according to ACI 318-14 [6]

The skin reinforcement should be placed along vertical surfaces in the tensile zone to limit the formation and development of cracks in the area above the main tension reinforcement. The amount of the additional reinforcement is not determined, but experience suggests that it is more suitable to select more bars of a smaller diameter.

2.3 Technical literature

The problem of designing deep beams is not discussed in Czech literature. Most publications suggest using the strut-and-tie method and a finite element method.

An exception is the university textbook of doc. Bažant, which mentions the possibility to use a simplified method [8]. This method is described in chapter 3.1.

Other technical articles in magazines deal with principles of reinforcement concerning existing experience and so complete the general rules stated in the standards. Professor Procházka and Ing. Šmejkal recommend in their article to place the lower horizontal reinforcement to the height of $0.1k$ to $0.2k$, where k is the smaller of the height or the span of the deep beam [9]. This position of the reinforcement, which is not just at the lower surface as for the classic beams, reduces the crack width. It is necessary to place the additional horizontal and vertical reinforcement in the support area because of lateral stress. It is also necessary to check the stress in the contact joint of supports.

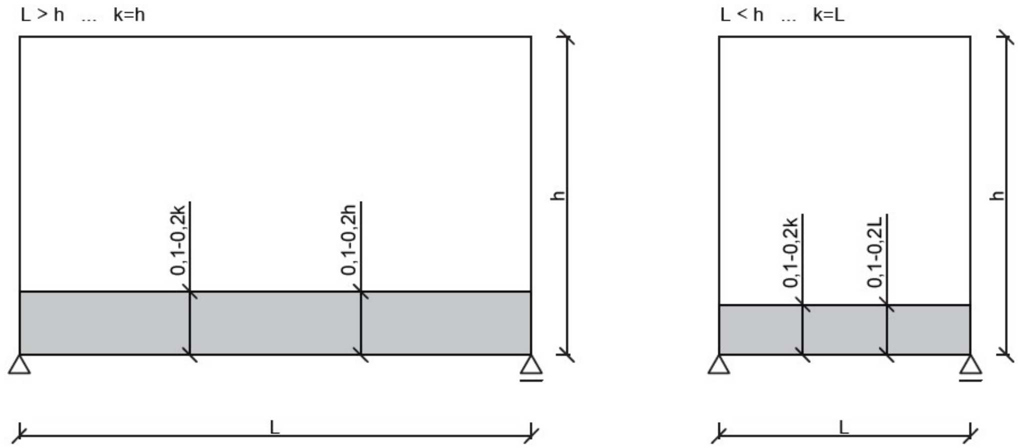


Figure 5: The position of the main reinforcement of the simply supported beam

It is possible to find more information about the problem of designing deep beams in the English-language technical literature. The most comprehensive and detailed publication is *Deep Beams.* However, it was published in 2002 so it can lack the newest findings and information.

3 Design methods

3.1 Simplified method

According to doc. Bažant, the simplified method for designing of deep beams can be used, if design bending moments are determined in the same way as for the classical shallow beams [8]. It is recommended to increase values of reactions at the outer supports by 15% to limit the failure caused by concentric stress when using this method [8].

The required area of reinforcement is determined by the following equation:

$$A_{s,rqd} = \frac{M_{Ed}}{z_c \times f_{yd}}$$

where $A_{s,rqd}$ is the required area of reinforcement, M_{Ed} is a value of a design bending moment, z_c is the moment arm of internal forces and f_{yd} is yield strength of reinforcement.

The moment arm of internal forces z_c is determined for a simply supported deep beam by the following equation:

$$z_c = 0.2\ (L + 2h)$$

where L is a theoretical span and h is the height of the deep beam. At the same time, the value of the moment arm of internal forces z_c must not exceed the value of 0.6h (or 0.6L if the height of the deep beam is greater than the span) [8].

For a continuous deep beam, the moment arm of internal forces (above the supports and in the clear span domain of the beam) is determined according to the span to height ratio, but its value must not exceed the value of 0.5h [8].

- if $h/L \leq 0.5$: $z_c = 0.7h$
- if $h/L > 0.5$: $z_c = 0.2\ (L + 1.5h)$

The main reinforcement of the simply supported deep beam must run from one support to the other support and be properly anchored. It is recommended to use more bars of smaller diameter and to bend the reinforcement at the ends and anchor to the opposite surface of the deep beam.

The reinforcement shall be distributed along both surfaces of the deep beam in the height of v, which is not higher than $0.25L$ [8].

$$v = \min\ (0.25h - 0.05L;\ 0.25L)$$

The rules for the position of the main tensile reinforcement result from the distribution of the horizontal tensile stress. It can be observed that an upper part of the deep beam is no longer subjected to the tension.

In addition to the main reinforcement, an orthogonal (vertical and horizontal) reinforcement must be designed. Again, it is recommended to use more bars of smaller diameter. The distance between the bars in both directions must not exceed 150 mm [8]. The area of orthogonal reinforcement for one direction and both surfaces is determined by the following equation:

$$A_{s2} = 5t\ [\mathrm{mm^2/m}]$$

where A_{s2} is the area of orthogonal reinforcement and t is the section width of the deep beam. In the support area, the amount of the orthogonal reinforcement is double [8]. The area of the orthogonal reinforcement, A_{s2} is limited by a maximum value of 600 $\mathrm{mm^2/m}$.

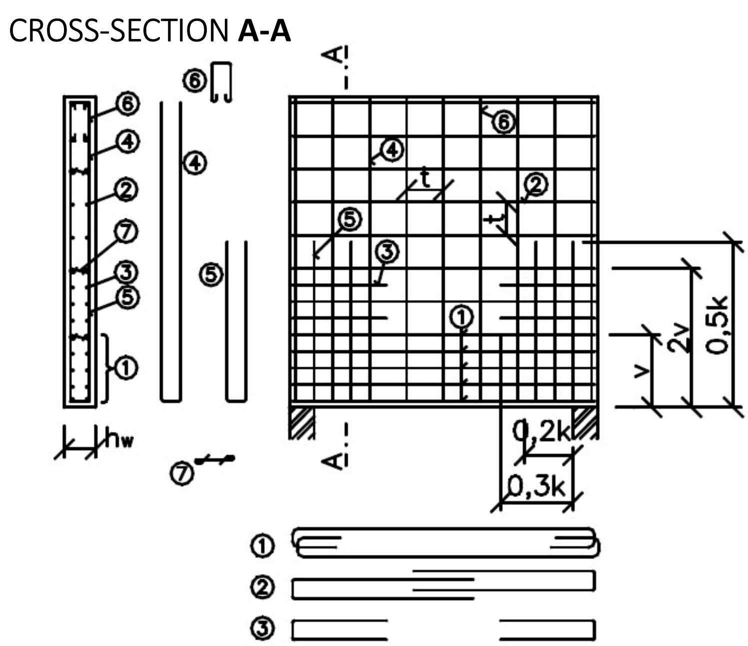


Figure 6: The scheme of the reinforcement of the simply supported deep beam according to the simplified method [8]

3.2 Strut-and-tie method

Deep beams are usually designed according to the strut-and-tie method. The strut-and-tie model is based on the distribution of the main stresses. This is a common and standard method for designing any D-region for the ultimate limit state. The method can be also used to verify the serviceability limit state if the provided model is sufficiently compatible (the position of important struts should correspond to the linearly elastic theory) [3].

The following assumptions apply to this method [16]:

- only axial forces in ties and struts are considered
- there is a balance of forces in all nodes
- reinforcement in ties becomes effective after cracks occur in the concrete
- redistribution of internal forces occurs after cracks develop
- in ties, the yield strength of reinforcement is reached before the concrete strength is reached in struts

The strut-and-tie model is developed from the distribution of main stresses and consists of struts, ties, and nodes. Struts and ties may only cross in the nodes. The struts represent the compressive forces transmitted by concrete and the ties represent the tension forces transmitted by the steel reinforcement. The value of the forces in the members is determined from the condition of equilibrium with the external load in the nodes.

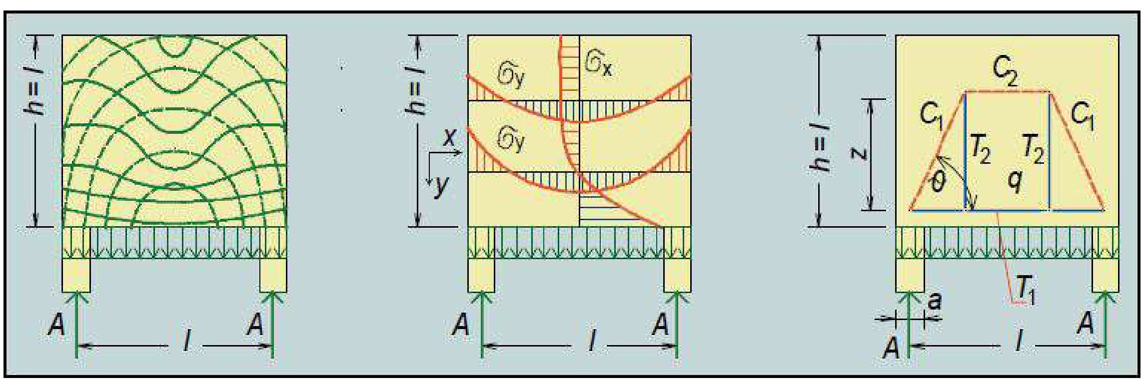


Figure 7: An example of strut-and-tie model derived from the distribution of main stresses [source: 9]

Nodes are considered in places of concentric loads, reinforcement bars and in supports or corners of the structure. We distinguish four basic types of nodes – CCC (3 struts), CCT (2 struts and 1 tie), CTT (1 strut and 2 ties) and TTT (3 ties) [16]. An angle between the individual members (struts and ties) in the nodes should be at least 25 degrees [10].

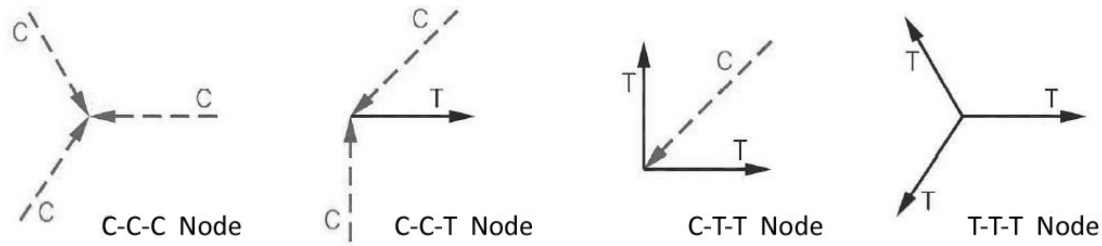


Figure 8: Types of nodes [16]

The calculation process is briefly explained. In the first step, all D-regions have to be defined and isolated. The resulting forces acting at the boundary of each D-region are then calculated.

A suitable strut-and-tie model is chosen based on the stress distribution. All forces in the struts and ties are calculated.

In the next phase, the struts, ties, and nodes are designed to have sufficient strength. The width of struts and nodes is determined depending on the effective strength of the concrete. The bars represent reinforcement, which must be anchored in the nodes or areas behind them.

3.2.1 Reinforcement according to strut-and-tie method

Unlike ČSN 1992-1-1, the American standard ACI 318-14 deals more with the strut-and-tie method and provides some recommendations for reinforcement.

The orthogonal reinforcement shall be designed to hold the lateral tension in the struts. The area of that reinforcement is determined according to the following equation:

$$\frac{A_{si}}{b_s \times s_i} . \sin \propto_i \geq 0.003$$

where A_{si} is the area of reinforcement in a given direction, s_i is the distance of reinforcement bars in the given direction, α_i is the angle between the reinforcement and the strut and b_s is the width of the strut (Figure 9). This reinforcement reduces the crack width and increases the compressive strength of the struts [6].

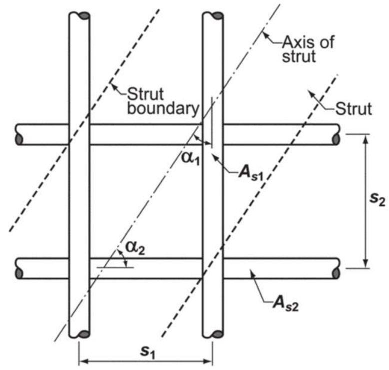


Figure 9: The reinforcement of the deep beam according to ACI 318-14 [6]

If compression reinforcement is needed, it should be parallel to the axis of the respective strut and longitudinally held by closed stirrups or wrapped by a spiral. The stirrup distance *s* should not be greater than the smallest of the following values [5]:

- lesser cross-section dimension of the strut
- 48 of stirrup diameters
- 16 of diameters of compression reinforcement

The first stirrup should be placed within *0,5s* of the node at both ends of the strut [5].

The tension reinforcement can be designed non-prestressed and prestressed. For the construction of deep beams, non-prestressed reinforcement is usually used. The position of the tension reinforcement axis should be identical with the tie axis in the strut-and-tie model [16]. The reinforcement must be sufficiently anchored.

3.3 Stringer-panel method

The stringer-panel method (SPM) represents a simplified method for the design of deep beams. This method is very specific, and it is relatively unknown not only in the Czech Republic but also in the United States of America. It is used only in Denmark (Danish National Annex to Eurocode 2, DK NA 2013). The national annex mentions that the stringer-panel method can be used for the design of structures with a non-planar cross-section after a deformation [11].

A development of this method occurred in the 1950s when it was used to model aircraft wings by aerial engineers. The stringer-panel method was first used by researchers from the Technical University of Denmark in 1971 for purposes of civil engineering [11].

A significant progress in the development of this method in the field of civil engineering was achieved in the 1990s when the package of software SPanCAD was developed. This software can analyze the stringer-panel models both by linear and non-linear analysis [11]. Unfortunately, it was the last improvement. Since then the development of this method has stopped. The stringer-panel method is almost unknown and is used only marginally nowadays.

A principle of this method consists in dividing a 2D structure into two main elements. First elements are stringers, which transmit axial forces in tension or compression. The stringers can be arranged horizontally and vertically. If the structure has a variable height, stringers can be inclined. The width of stringers is not exactly recommended, but it should not exceed one-fifth of the width of an adjacent panel [11]. The stringers are located on edges of the structure, around openings, above supports and under places of a concentrated load.

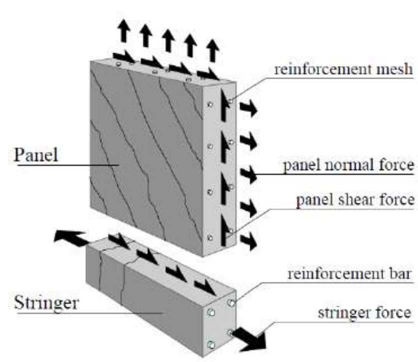


Figure 10: The dividing of the structure using the stringer-panel method and acting forces [11]

The second elements are panels, which represent planar elements transferring a shear. Panels are normally placed between four stringers and have a rectangular shape. Panel´s behavior is based on the membrane theory, thus ignores the influence of bending moments and considers only the influence of normal and shear forces [11]. The shear forces of panels must be in a balance with the axial forces of adjacent stringers.

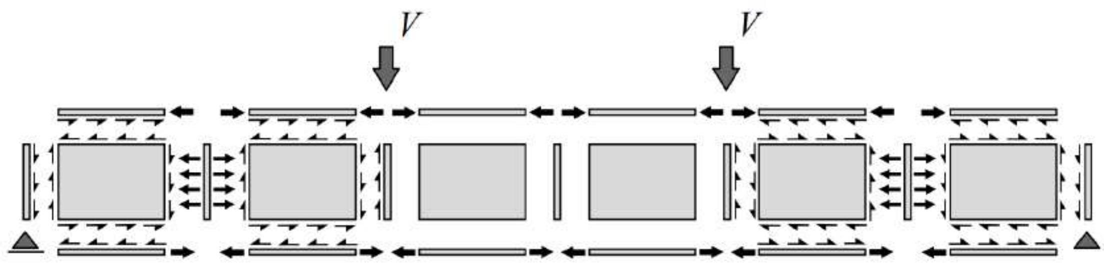


Figure 11: The example of dividing the structure into stringers and panels [11]

The principle of the stringer-panel method is somewhere between the finite element method and the strut-and-tie method. While the finite element method applies the finest possible mesh, the stringer panel method tries to apply the roughest mesh for a given geometry and loading.

Internal forces can be determined by hand calculations for simple problems. The principle is shown in chapter 3.4.3 on a problem of a simply supported deep beam loaded with the point load in the middle of the span, because this method is practically unknown.

3.3.1 Reinforcement according to stringer-panel method

It is necessary to consider the values of internal forces at the ultimate limit state for the design of reinforcement. It should be emphasized that the design values of internal forces may not be the same for all countries. In Brazil, for example, a safety factor should be used for D-regions according to the standard ABNT [11]. Contrary, the standard valid in the Czech Republic does not mention any such factor.

The design of the stringers is simple. It is assumed that concrete does not act in tension and all tensile stress is transmitted only by steel reinforcement [11]. This implies an equation for determinating the required area of reinforcement:

$$A_{s,rqd} = \frac{F_{tension}}{f_{yd}}$$

The designed reinforcement should be placed throughout the whole length and properly anchored at the ends of the deep beam.

For stringers in compression, it is necessary to check that the compressive stress does not exceed the compressive strength of the concrete. If this condition is not met, it is possible to increase the compressive strength by reinforcement in the form of stirrups [11].

We assume that the panel acts as a membrane (it transmits pure shear, normal forces are zero) and the main stress angle is 45°. The reinforcement is the same in both directions for common problems.

3.3.2 Non-linear analysis

Non-linear analysis can be performed using a sophisticated software SPanCAD (Stringer Panel Computer Aided Design). SPanCAD is a plugin for AutoCAD, which was developed in the 1990s at the Delft University of Technology in the Netherlands. It is a simple tool for non-linear analysis which is defined by just five parameters [11]:

- Young´s modulus of elasticity of concrete
- compressive strength of concrete
- tensile strength of concrete
- Young´s modulus of steel
- tensile strength of steel

The analysis consists of three steps. In the first step, elastic analysis is carried out and reinforcement is designed according to it. In the second step, non-linear analysis is performed to specify the reinforcement and to verify a deformation and cracks opening. The third step simulates the behavior of the model until the limit loading is reached [11].

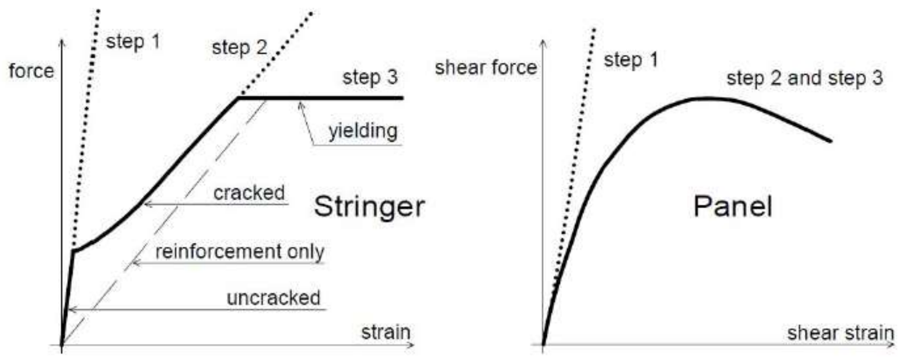


Figure 12: The behavior of stringers and panels during the non-linear analysis [source 11]

Unfortunately, the author of this work was not able to run SPanCAD properly. This is because the development of this software has stopped in the 1990s and it is no longer compatible with present operating systems and AutoCAD software.

3.4 Sample problem

Design methods differ in a process of a calculation, use of simplifications and accuracy of results. Their principles are shown on the sample problem of a simply supported deep beam without opening with a point load in the middle of the span. Self-weight is not considered. This type of problem can be solved by all presented methods without any sophisticated design software. The geometry and the load of the deep beam are shown in the following figure (Figure 13), the thickness is 200 mm.

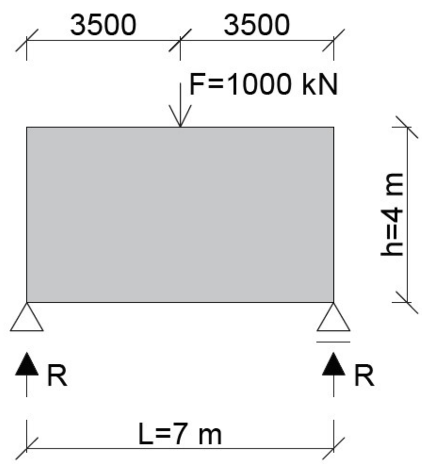


Figure 13: The deep beam for a prenentation of design methods

3.4.1 Simplified method

First, it is necessary to determine the value of vertical reactions in supports:

$$R = \frac{F}{2} = \frac{1\,000}{2} = 500\ kN$$

Afterward, the maximum value of the design bending moment M_{Ed} can be calculated:

$$M_{Ed} = R \times \frac{L}{2} = 500 \times \frac{7}{2} = 1\,750\ kNm$$

In the next step, it is necessary to determine the arm of internal forces z_c:

$$z_c = 0.2(L + 2h) = 0.2(7 + 2 \times 4) = 3\ m$$

The value of the arm of internal forces, when using the simplified method, is limited by the following condition:

$$z_c \leq 0.6h \ldots if\ h \leq L$$
$$z_c \leq 0.6 \times 4 = 2.4\ m$$

Because this condition is not met, the value of the arm of internal forces is limited by the maximum allowed value:

$$z_c = 3\ m > 2.4\ m\ \rightarrow\ z_c = 2.4\ m$$

It is possible to determine the required area of reinforcement:

$$A_{s,rqd} = \frac{M_{Ed}}{z_c \times f_{yd}} = \frac{1\ 750 \times 10^3}{2.4 \times (435\ \times 10^6)} = 1.68 \times 10^{-3}\ m^2 = 1\ 680\ mm^2$$

The reinforcement is placed regularly along both surfaces in the height of v from the bottom:

$$v = \min\ (\ 0.25h - 0.05L; 0.25L$$
$$v = \min\ (\ 0.25 \times 4 - 0.05 \times 7; 0.25 \times 7)$$
$$v\ = \min\ (0.65; 1.75) = 0.65\ m$$

3.4.2 Strut-and-tie method

In the beginning, it is necessary to determine the right strut-and-tie model. Due to the simplicity of the selected example (the model has been also verified several times in the literature [10]), it is possible to reliably estimate the stress distribution without the use of the software. Based on the type of load and the stress distribution, it is appropriate to select the model, which consists of two inclined struts and one horizontal tie at the lower edge of the deep beam.

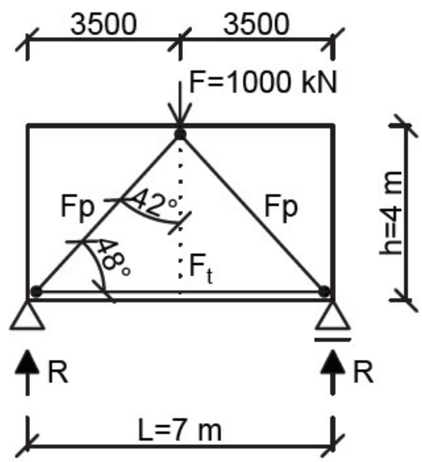


Figure 14: The calculation model of strut-and-tie method

From the conditions of the equilibrium at the point load, the value of forces in struts can be easily determined:

$$F_{strut} = \frac{1}{2} \times \frac{F}{\cos\beta} = \frac{1}{2} \times \frac{1\,000}{\cos 42°} = 672.8\ kN$$

For struts, it is necessary to check the compressive strength of the concrete to prevent crushing of the concrete:

$$F_{strut} \leq A_c \times f_c$$

If the compressive strength of the concrete is exceeded, the compression reinforcement must be designed according to the equation:

$$A_{s,2} = \frac{F_{strut} - A_c \times f_c}{f_{yd}}$$

The width of the struts is not determined for problem example, and so this condition is not considered.

The force in the tie can be simply determined thanks to the conditions of equilibrium in nodes B or C:

$$F_{tie} = F_{strut} \times \cos\alpha = 672.8 \times \cos 48 = 450.2\ kN$$

The required area of reinforcement is designed according to the equation:

$$A_{s,rqd} = \frac{F_{tie}}{f_{yd}} = \frac{450.2 \times 10^3}{435\ \times 10^6} = 1.04 \times 10^{-3}\ m^2 = 1\ 040\ mm^2$$

It is appropriate to use the sophisticated design software for more complicated structures. The software can verify the stress distribution or calculate the forces in the strut-and-tie model (e.g. Scia Engineer or CAST). However, it is always necessary to choose the right strut-and-tie model. The recommended strut-and-tie models for the most common cases (simply supported deep beam, short cantilever, etc.) can be found in technical literature.

3.4.3 Stringer-panel method

The deep beam is divided into stringers and panels in the first step. Due to the geometry and load, it is appropriate to divide the structure below the point force in the middle of the span into two panels surrounded by seven stringers (Figure 15).

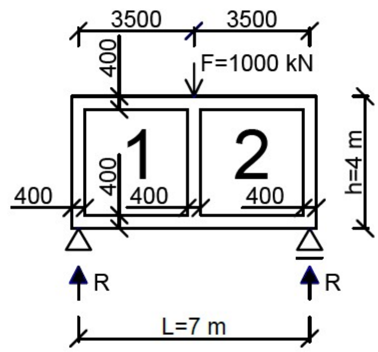


Figure 15: The calculation model of the stringer-panel method

The value of the reactions in supports can be easily determined from the conditions of equilibrium of the entire structure:

$$R = \frac{F}{2} = \frac{1\,000}{2} = 500\ kN$$

The symmetry of the whole structure allows dealing only one half of the deep beam, which consists of one panel and three stringers after the dividing. The middle stringer is ignored in the next phases of the calculation.

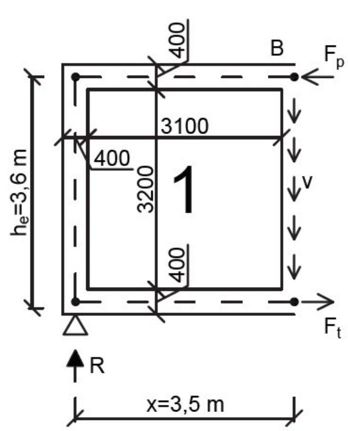


Figure 16: Section 1 of the calculation model for the determination of internal forces

The horizontal condition implies that the axial forces in the upper and the lower stringers F_p and F_t have the same value but the opposite direction.

$$F_p = F_t$$

The value of forces in horizontal stringers can be determined from the moment of equilibrium around point B:

$$0 = R \times x - F_t \times h_e$$
$$F_t = \frac{R \times x}{h_e} = \frac{500 \times 3.1}{3.6} = 430.6\ kN$$

The stress in a compressed stringer should not exceed the compressive strength of the concrete:

$$\sigma_c = \frac{F_p}{A_{stringer}} \leq f_{cd}$$

If the compressive stress in the stringer exceeds the value of the compressive strength of the concrete, a compressive reinforcement must be designed.

Based on an obtained tensile force in the lower stringer, it is possible to design the required area of reinforcement:

$$A_{s,rqd} = \frac{F_t}{f_{yd}} = \frac{430.6 \times 10^3}{435\ \times 10^6} = 9.9 \times 10^{-4}\ m^2 = 990\ mm^2$$

The continuous shear force v_l at the edge of the divided deep beam acts on the effective height h_e, which is defined as a distance between the axes of the upper and the lower stringer.

The value of the continuous shear force can be obtained from the vertical condition of equilibrium:

$$v = \frac{R}{h_e} = \frac{500}{3.6} = 138.9\ kN/m$$

Now, it is possible to obtain shear stress acting on the panel according to the following equation:

$$\tau_p = \frac{v}{t} = \frac{138.9 \times 10^3}{0.2} = 0.7\ MPa$$

In order to prevent a concrete from crushing, the stress must be limited to the value of an effective compressive strength:

$$\sigma_c = -2 \times |\tau_p| \leq f'_{cd}$$

It is recommended to assign shear forces to certain panels for statically indeterminate structures (models with more than one panel per section). The choice of these panels is entirely up to the designer [11]. For more complex constructions, it is also advisable to use the sophisticated design software SPanCAD.

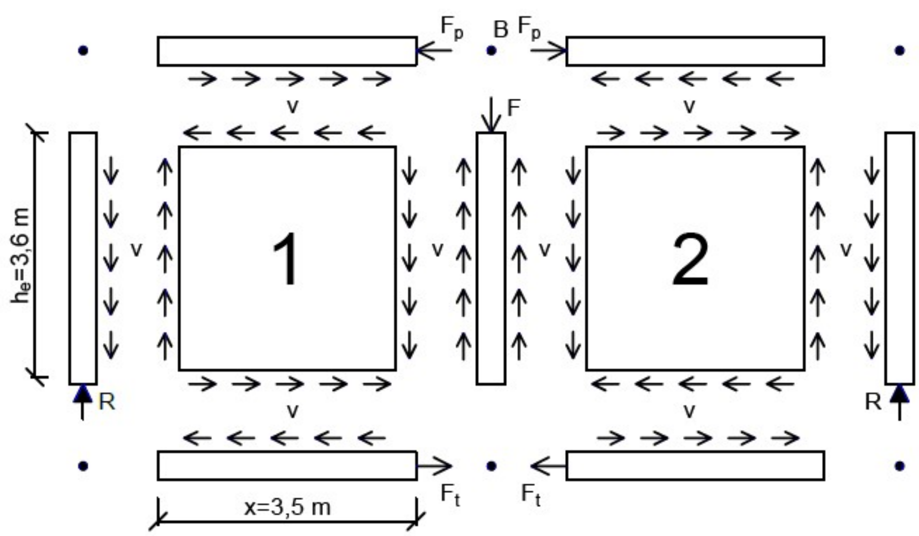


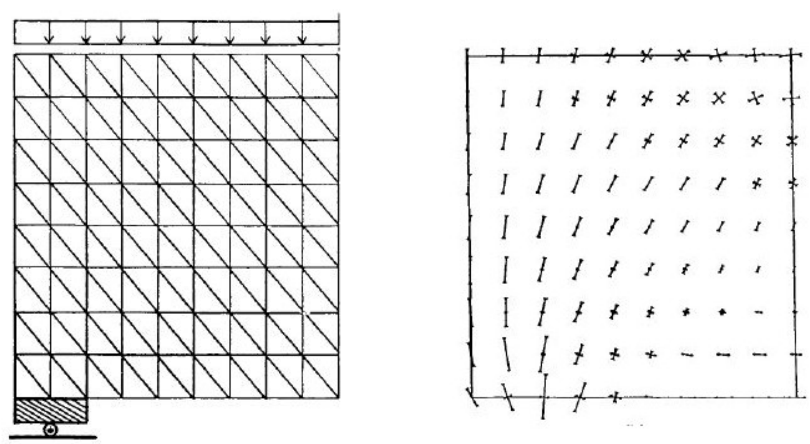
Figure 17: Stringers and panels of the deep beam and acting forces

3.5 Finite element method

The finite element method (FEM) is a numerical method that can be used to solve various problems. An advantage of this method is its versatility and ability to describe complicated and large problems [19]. On the other hand, the finite element method has high computational demands. Because of this reason, the method was expanded only with the arrival of modern technologies.

A principle of this method is to divide the structure into a large number of small areas of a simple shape – finite elements [10]. Desired parameters are determined at node points. A deformation variant of the finite element method using Lagrange´s variational principle is the most common in civil engineering [19]. In practice, the finite element method is most often used to find out internal forces or to check designed structures for a serviceability limit state.

The finite element method is a versatile method, which can be used to analyze problems with difficult geometry. It is appropriate for solving deep beams with openings, inclined edges or deep beams with a variable thickness [10].



(a) (b)

Figure 18: The idealisation by triangular elements (a) and the elastic stress distribution (b) of the simply supported deep beam with a uniformly distributed line load at the top [10]

3.6 Conclusion

The simplified method implements some questionable assumptions like the determination of the design bending moment or the value of the arm of internal forces. Thanks to these simplifications, the calculation is fast and easy. It can be performed without any sophisticated structural design software. This method is sufficiently described in the textbook of doc. Bažant [8], which provides all the necessary information about the method. On the other hand, the simplified method is suitable only for deep beams with simple geometry. There is also only limited experience with the use of this method.

The strut-and-tie method is currently the most common approach for the design of deep beams. The most important aspect is the right choice of the strut-and-tie model. This model should be based on the stress distribution according to the linear calculation. However, there is no universal model and it differs according to the deep beam geometry and the load layout. The strut-and-tie method is well-known as it was described and extensively discussed in the technical literature. The use of this method is not limited by the geometry, it can be used even for more complicated structures such as deep beams with openings. Calculations can be carried out without the calculation software, but it is appropriate to use the software for more difficult cases.

The stringer-panel method is an alternative approach for a design of deep beams. Contrary to the strut-and-tie method, a calculation model is easier to develop. An analysis of simple geometry deep beams can be performed by hand. However, structural software must be used for cases with more complicated geometry. The biggest disadvantage is the lack of information and experience with this method – for example, the procedure for the determination of the width of stringers is not adopted, only the maximum value is given.

Method	**Simplified**	**Strut-and tie**	**Stringer-panel**
$A_{s,rqd}$	1 680 mm^2	1 040 mm^2	990 mm^2

Table 1: The comparison of the required area of reinforcemt for the tie along bottom edge according to design methods

The required area of reinforcement found by using the simplified method is larger in comparison with those determined by either the strut-and-tie method or the stringer-panel method. This fact likely results from the conservative approach of this method. The results obtained by the strut-and-tie method and the stringer-panel method were more or less identical.

4 Structural design of the deep beam

4.1 Basic information of the building

The presented study on the design of a deep beam follows up on the work carried out within the Structural design project course in the previous semester. The building consists of an original silo and a new superstructure, which is statically completely independent of the original building. The objective of a term paper in the Structural design project course was to conduct a detailed study on slender columns of the proposed building. There are an entrance and a reception on the first floor as well as a lift to the upper floors. The rest of the silo is not used. The superstructure has four floors with offices. The building is located in Olomouc, Czech Republic.

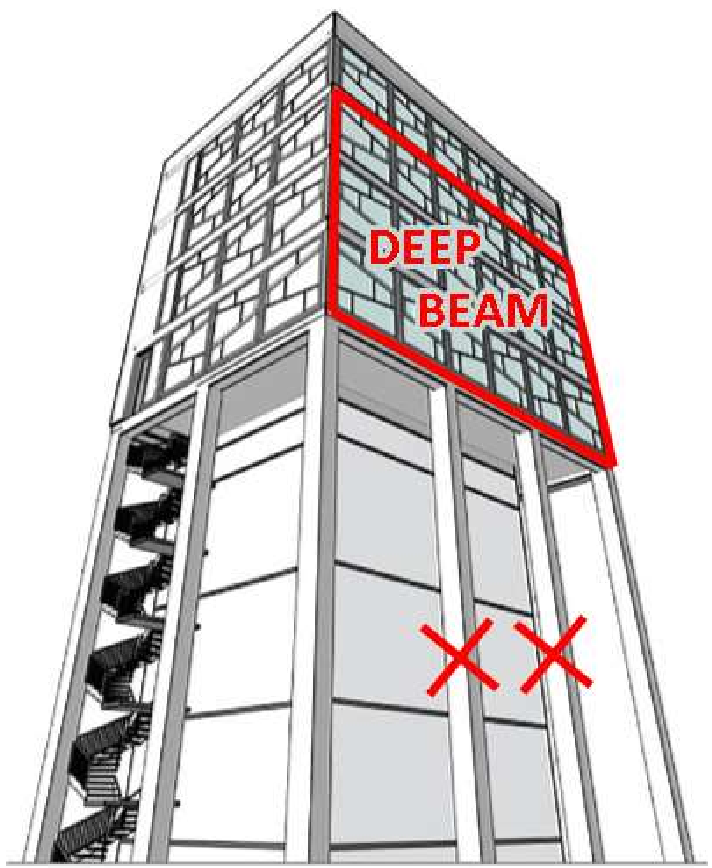


Figure 19: The 3D model of the building with proposed changes [18, modified]

The layout of the superstructure was slightly changed for the purpose of the diploma thesis. It was required to clear the space in front of the entrance and to create a presentable space in the superstructure – a gallery.

4.1.1 Layout

The original silo has ground plan dimensions 12.1 x 10.2 m and it is located in the back of the building. The external columns situated outside the silo create the main supporting structure of the superstructure placed above the original silo.

On the first floor of the building is the entrance, the reception, space for guests and toilets. The rest of the silo is not used, only the lift shaft goes through it.

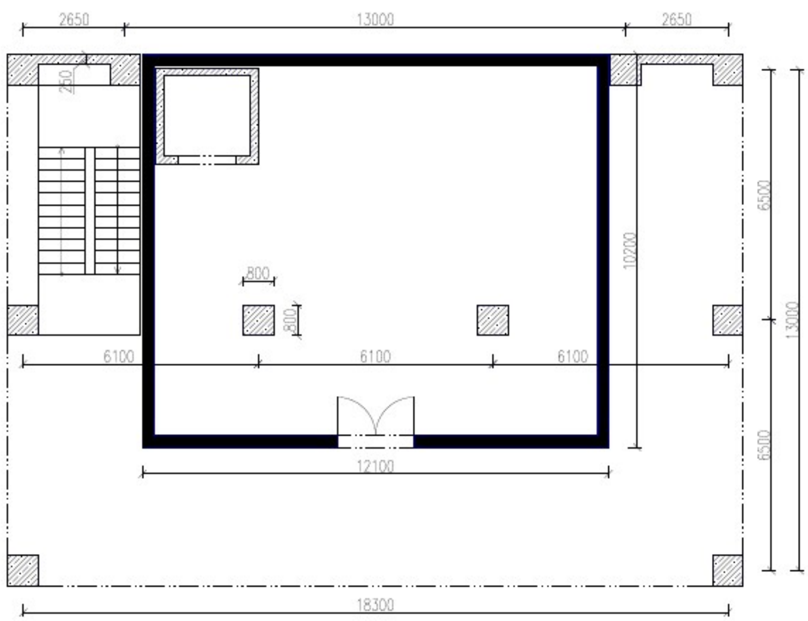


Figure 20: The scheme of a structural system of the first floor (an existing structure of the silo is marked by a solid hatch)

The new upper part of the building has four floors which have ground plan dimensions 19.1 x 13.8 m. The first floor of the superstructure is located at a height of 21.5 m above a ground level. On all four floors are offices, a kitchen, and toilets. In addition to the lift, all floors of the superstructure are connected by a staircase. An escape staircase is situated outside the building.

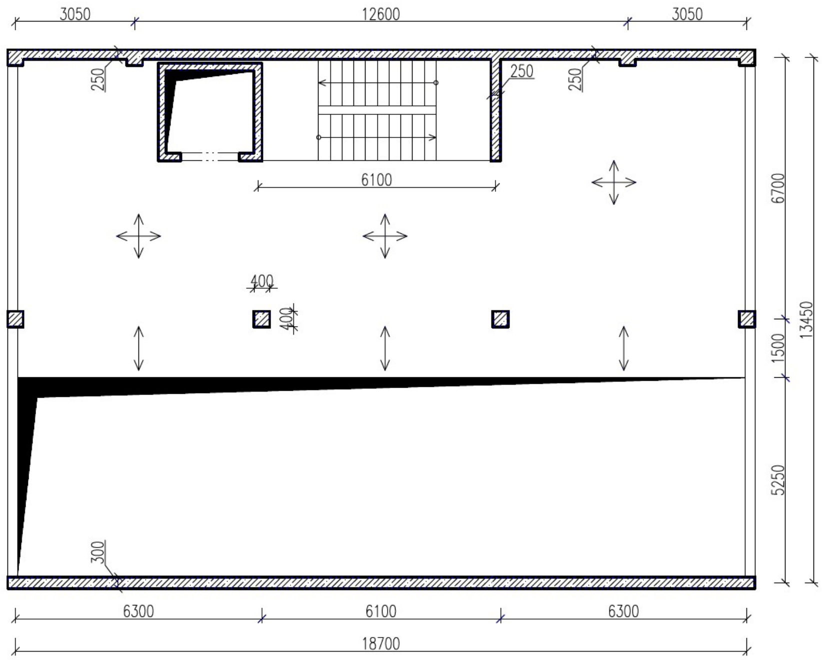


Figure 21: The scheme of a structural system of the 1st, 2nd and 3rd floor of the superstructure

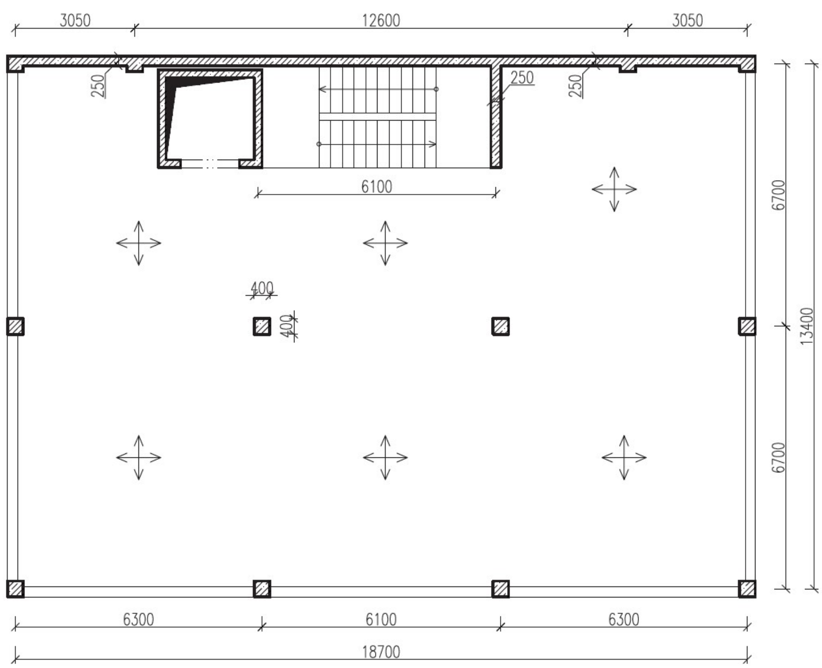


Figure 22: The scheme of a structural system of the 4th floor of the superstructure

The roof has two levels. The lower part is designed for a relaxation of employees and it is accessible by the staircase and the lift. The higher part (above the staircase and the lift) is a non-accessible flat roof except for normal maintenance and repair.

4.1.2 Structural system

Structural design documentation of the original silo was not available during the working on the diploma thesis. A presented description of a structural system is based on general information gained from unofficial sources and a consideration of the author. A construction survey was not carried out for the purpose of the thesis. The structural system of the silo consists of perimeter walls which lie on strip foundations. The original silo is not assessed because it is only proposed to remove an original roof which will reduce the load. No other changes to the structure are made. The sufficient load-bearing capacity has been proved by a previous operation.

The upper floors of the superstructure are supported by the columns, which are founded on piles. The structural system consists of the columns with a regular raster and a stiff lift shaft which goes through the entire height of the building. The ceiling slabs are designed as locally supported monolithic reinforced concrete with a thickness of 240 mm. The staircase is designed as a double-arm with a landing from the monolithic reinforced concrete. The stiffness of the building is ensured by a combination of massive reinforced concrete columns, the stiff slabs, and the reinforced concrete lift shaft. Construction height of a floor is 3 700 mm.

4.1.3 Load

In the process of the preliminary design of the main structural elements were taken into consideration the characteristic values of the following loads.

- Load from the self-weight: 25.0 kN/m^3 (structures of reinforced concrete)
- Load from the floor composition: 1.88 kN/m^2
- Load from the non-bearing walls: 0.5 kN/m^2
- Live load: 3.0 kN/m^2, 5.0 kN/m^2 (the gallery)

4.1.4 Materials

The structure is made of reinforced concrete.

- Piles: reinforced concrete, concrete C25/30 XC2 - Cl 0.2 - D_{max} 16 - S3
- Columns supporting the superstructure: reinforced concrete, concrete C30/37 XC4, XF1 - Cl 0.2 - D_{max}16 - S4
- Columns, deep beams, beams and slabs: reinforced concrete, concrete C30/37 XC1 - Cl 0.2 - D_{max}16 - S4
- Reinforcement: steel B500B

4.2 Introduction

The application of various methods for the design of a deep beam will be presented on the selected deep beam of the administrative building, which was designed by the author in the previous study. The layout of the building was changed to create the deep beam for the thesis. Compared to the original design of the superstructure, the middle columns on the front side of the building were removed. This modification allowed to clear the space in front of the building and to create the deep beam (Figure 23). The gallery has been created inside the building behind the deep beam running through three floors.

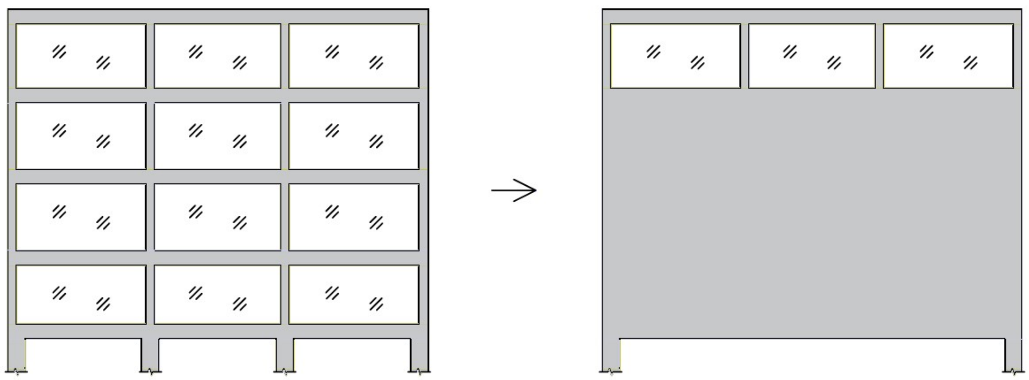


Figure 23: The replacement of middle columns of the superstructure by the deep beam

The selected deep beam is loaded by four point load and uniformly distributed line load at the upper and lower edge. The analysis of the deep beam is performed. Three design methods are used. Obtained results are then compared with the true stress of the deep beam, which is obtained from non-linear analysis using the finite element method.

4.3 Determination of load

First, it is necessary to determine the load that is applied to the deep beam. The geometry of the deep beam and the load layout is shown in Figure 25. The deep beam has a width of 19.1 m, a height of 11.34 m and a thickness is 300 mm. The supports of the deep beam are massive columns of reinforced concrete with a cross-section 800 x 800 mm. The clear span is 17.5 m and the theoretical span is 18.3 m.

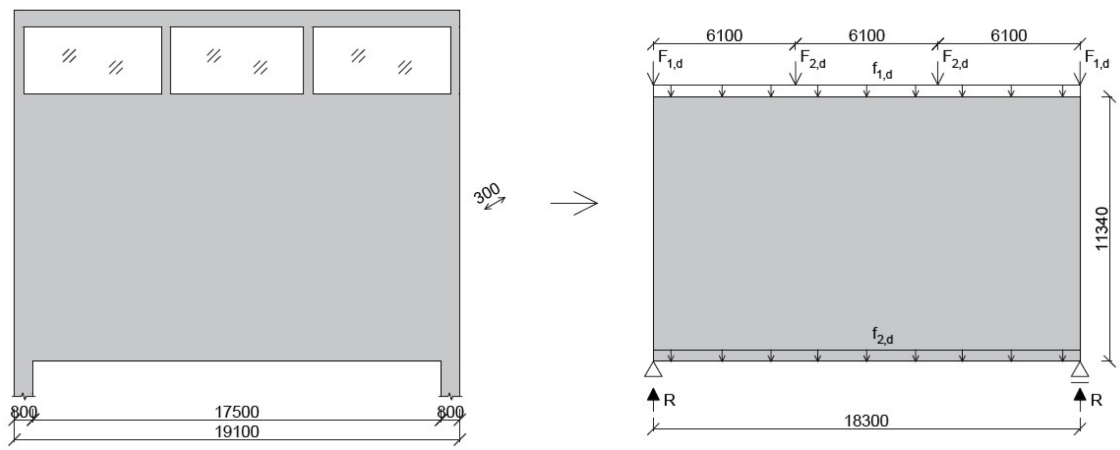


Figure 24: The static scheme, geometry and load of the deep beam

The point load $F_{1,d}$ and $F_{2,d}$ represent the load injected into the deep beam by outer and middle reinforced concrete columns of the 4th floor of the superstructure. Uniformly distributed line load $f_{1,d}$ represents the self-weight and the load of the slab above the 3rd floor, which includes the load of the floor-composition and non-bearing walls and the live load. Continuous uniform load $f_{2,d}$ represents the load of the slab under the 1st floor, which includes the load of the floor-composition and non-bearing walls and the live load.

The value of the point load $F_{1,d}$ and $F_{2,d}$ were obtained from the 3D model of the whole superstructure in Scia Engineer, which was used in the Structural design project course in the previous semester.

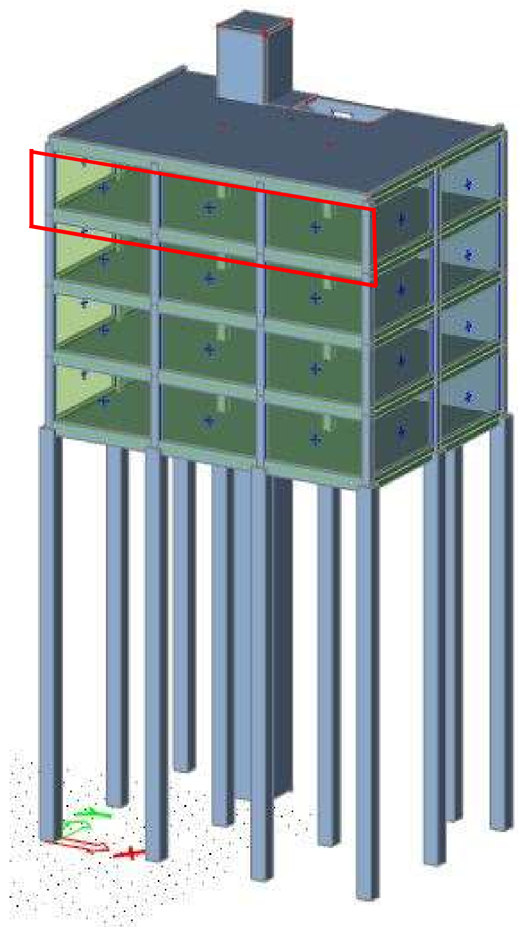


Figure 25: The 3D mode of the structurel in Scia Engineer

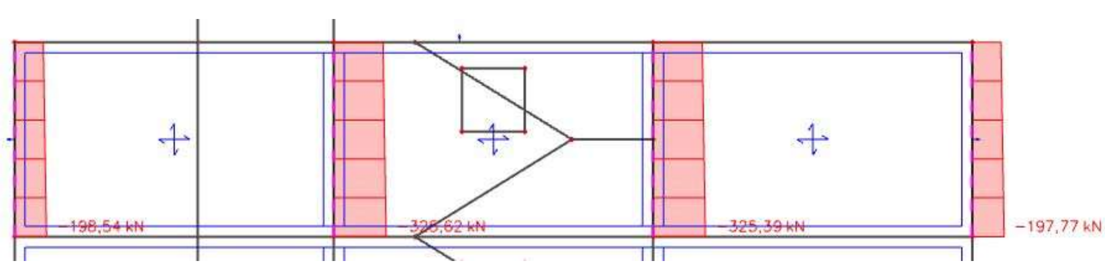


Figure 26: The part of the superstrucure (front side of the 4th floor) with normal forces

$$F_{1,d} = 199\ kN$$

$$F_{2,d} = 326\ kN$$

The value of the tributary width b for the calculation of the continuous loads $f_{1,d}$ and $f_{2,d}$ was determined according to the scheme (Figure 27).

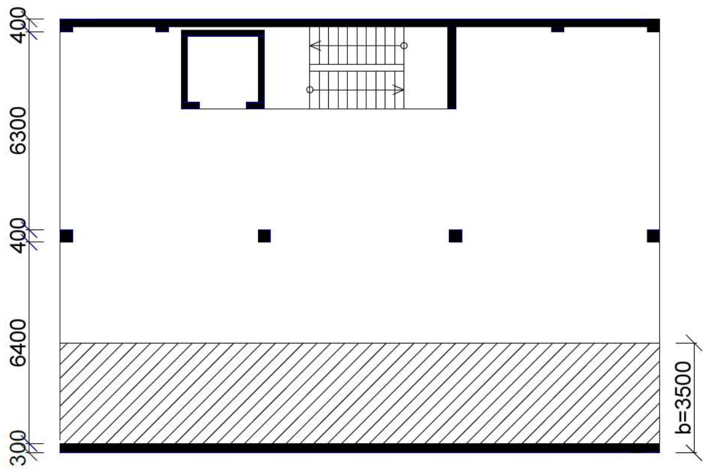


Figure 27: The scheme for the determination of the loading width b

The value of the uniformly distributed line load $f_{1,d}$ consists of the self-weight of the deep beam and the load $f'_{1,d}$, which represents the load from the slab above the 3rd floor, the load of the floor-composition, the load of non-bearing walls and the live load. Its value was calculated by hand:

		f_k [kN/m]	**γ_f**	**f_d [kN/m]**
RC slab, thickness 240 mm	$0.24 \times 25 \times 3.5\ m$	21.0	1.35	28.3
floor-composition	$1.88 \times 3.5\ m$	6.6	1.35	8.9
non-bearing walls	$0.5 \times 3.5\ m$	1.8	1.35	2.4
live load	$3.0 \times 3.5\ m$	10.5	1.50	15.8
			$f'_{1,d}$ =	**55.4**

The total design value of the uniformly distributed line load $f_{1,d}$ is then as follows:

$$f_{1,d} = f'_{1,d} + 0.5 \times b_{deep} \times h_{deep} \times 25 \times \gamma_{dead}$$

$$f_{1,d} = 55.4 + 0.5 \times 0.3 \times 11.34 \times 25 \times 1.35$$

$$f_{1,d} = 112.8\ kN/m$$

The value of the continuous load $f_{2,d}$ consists of the self-weight of the deep beam and the load of the slab under the 1st floor, the load of the floor-composition, the load of non-bearing walls and the live load. Its value was calculated by hand:

		f_k [kN/m]	γ_f	f_d [kN/m]
RC slab, thickness 240 mm	$0.24 \times 25 \times 3.5\ m$	21.0	1.35	28.3
floor-composition	$1.88 \times 3.5\ m$	6.6	1.35	8.9
non-bearing walls	$0.5 \times 3.5\ m$	1.8	1.35	2.4
live load	$5.0 \times 3.5\ m$	17.5	1.50	26.3
			$f'_{2,d}$ =	**65.9**

$$f_{2,d} = f'_{2,d} + 0.5 \times b_{deep} \times h_{deep} \times 25 \times \gamma_{dead}$$
$$f_{2,d} = 65.9 + 0.5 \times 0.3 \times 11.34 \times 25 \times 1.35$$
$$f_{2,d} = 123.3\ kN/m$$

It is necessary to transfer the uniformly distributed line load into point load when some design methods are used in this chapter. Namely, it concerns the strut-and-tie method and the stringer-panel method.

The value of the tribunary widths for transferring the uniformly distributed line load to point load is shown in Figure 28.

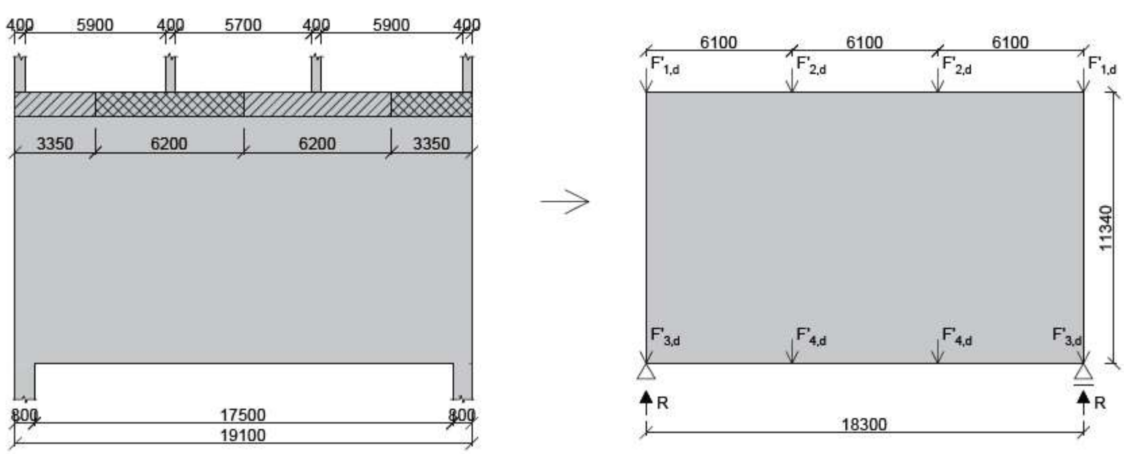


Figure 28: The scheme for the determination of tribunary widths b1 and b2

$$F'_{3,d} = b_1 \times f_{2,d} = 3.35 \times 123.3 = 413.1\ kN$$
$$F'_{4,d} = b_2 \times f_{2,d} = 6.2 \times 123.3 = 764.5\ kN$$

It is necessary to add together forces $F'_{3,d}$ and $F'_{4,d}$ for some cases (the model A and B of the strut-and-tie method).

$$F'_{5,d} = F'_{3,d} + F'_{4,d} = 413.1 + 764.5 = 1\ 177.6\ kN$$

To simplify the following calculations, the uniformly distributed line load and the point load injected by the columns at the upper edge, are added together:

$$F'_{1,d} = F_{1,d} + b_1 \times f_{1,d} = 199 + 3.35 \times 112.8 = 576.9\ kN$$
$$F'_{2,d} = F_{2,d} + b_2 \times f_{2,d} = 326 + 6.2 \times 112.8 = 1\ 025.4\ kN$$

4.4 Analysis – simplified method

The detailed information about the simplified method developed by doc. Bažant was presented in chapter 3.1. How to use the method for a structural design of deep beams was demonstrated on the simply supported deep beam exposed to the point load at its mid-span in chapter 3.4.1. Contrary to the task in chapter 3.4.1, the given deep beam has a different geometry and a different load. The calculation process is still the same. Figure 29 shows the calculation model.

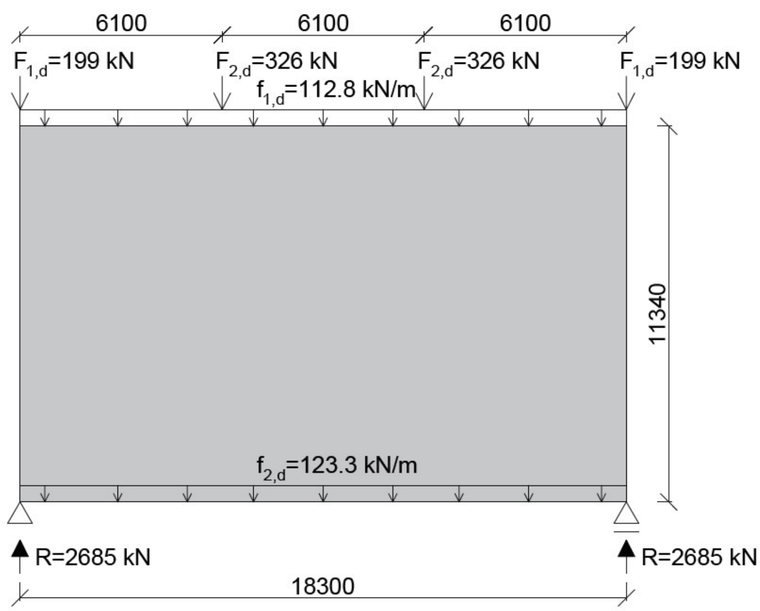


Figure 29: The calculation model for the simplified method

First, it is necessary to calculate the maximum value of the design bending moment M_{Ed}. Since it is a linear calculation, the principle of a superposition can be used:

$$M_{Ed} = R \times 9.15 - F_1 \times 9.15 - F_2 \times 3.05 - \frac{1}{8} \times f_d \times L^2$$

$$M_{Ed} = 2\,685 \times 9.15 - 199 \times 9.15 - 326 \times 3.05 - \frac{1}{8} \times (112.8 + 123.3) \times 18.3^2$$

$$M_{Ed} = 11\,869\ kNm$$

It is necessary to determine the arm of internal forces z_c according to the following equation:

$$z_c = 0.2 \times (L + 2h) = 0.2 \times (18.3 + 2 \times 11.34) = 8.2\ m$$

The value of the arm of internal forces is limited by the following condition when using the simplified method:

$$z_c \leq 0.6h \ldots if\ h \leq L$$
$$z_c \leq 0.6 \times 11.34 = 6.8\ m$$

It can be observed below, that this condition is not met, so the value of the arm of internal forces is limited by the maximum allowed value:

$$z_c = 8.2\ m > 6.8\ m\ \rightarrow\ z_c = 6.8\ m$$

Now, it is finally possible to determine the required area of reinforcement:

$$A_{s,rqd} = \frac{M_{Ed}}{z_c \times f_{yd}} = \frac{11\,869\ \times 10^3}{6.8\ \times (435\ \times 10^6)} = 4.02 \times 10^{-3}\ m^2 = 4\,020\ mm^2$$

The calculated reinforcement is placed uniformly along both surfaces of the deep beam from the bottom up to the height of v:

$$v = 0.25h - 0.05L = 0.25 \times 11.34 - 0.05 \times 18.3 = 1.92\ m$$

4.5 Analysis – strut-and-tie method

The strut-and-tie method is currently the most common calculation approach for a structural design of D-regions including deep beams. The most important and crucial is the right choice of a strut-and-tie model. As mentioned in Chapter 3.2, ties and struts should correspond to the main stresses. Because of this reason, a linear analysis of the deep beam in Scia Engineer was performed in the first step.

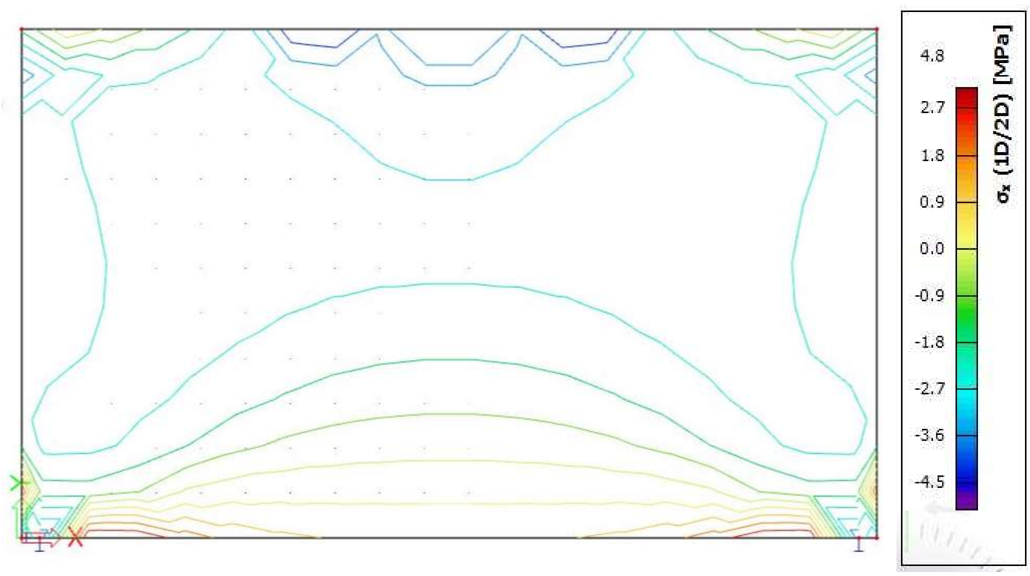


Figure 30: The stresses in direction x (horizontal)

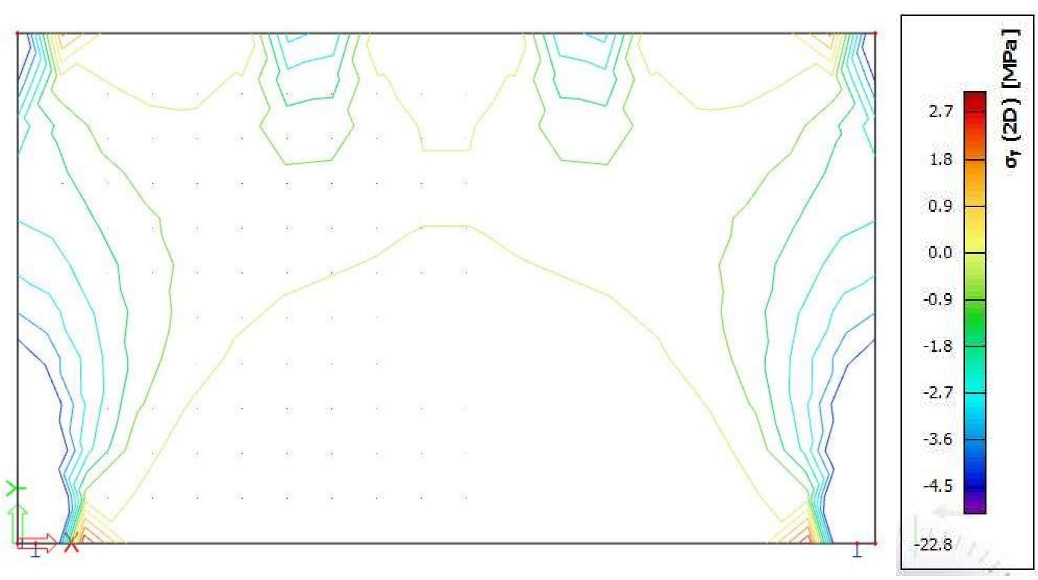


Figure 31: The stresses in direction y (vertical)

Based on the distribution of stresses two different strut-and-tie models were developed, model A and model B. Later on, two other strut-and-tie models were added, model C and model D in order to develop a more precise model.

This fact clearly illustrates the greatest disadvantage of this method. The most critical moment is the choice of the right strut-and-tie model. When an engineer selects a wrong model, all results obtained by the following calculations are wrong. There are recommended models for the most common D-region problems (e.g. a short corbel, a deep beam with a uniformly distributed line load). However, there is no general strut-and-tie model that could be used for all kinds of structures. As a consequence, entirely new models have to be developed for atypical structures with complex geometry or loading layout.

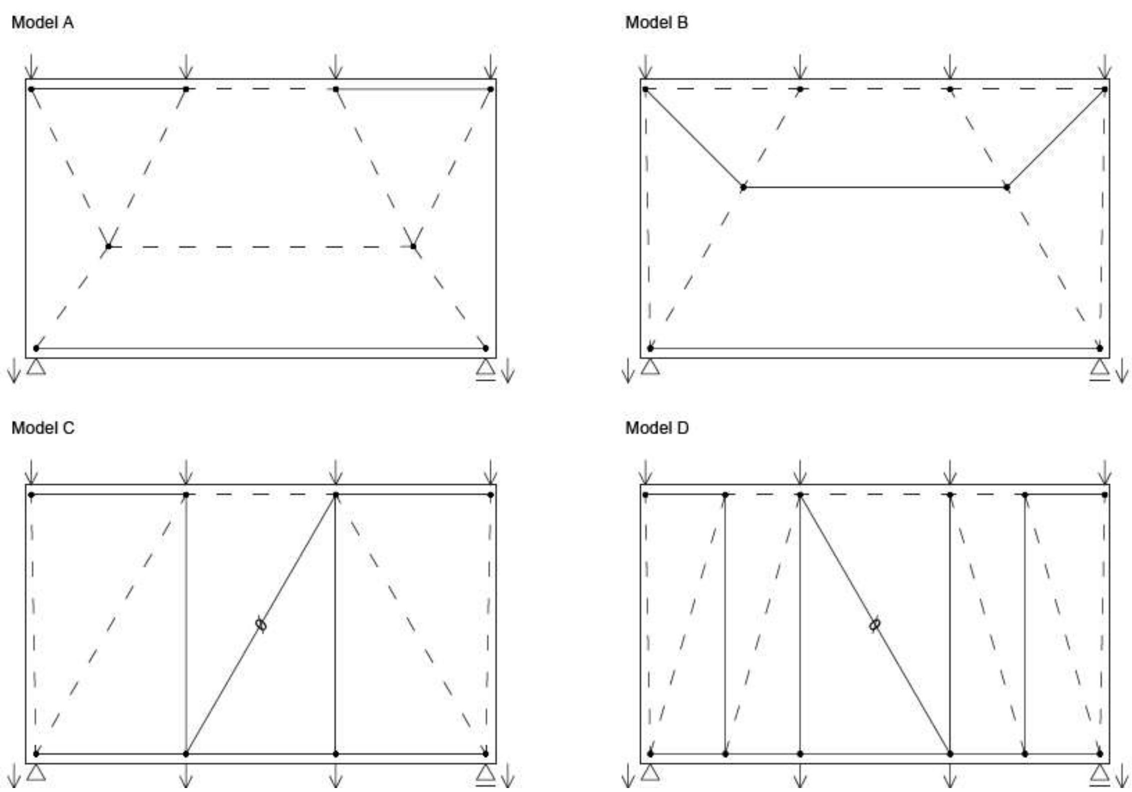


Figure 32: Four strut-and-tie models selected for the comparison

With the aim to show and compare the computational accuracy of the proposed models, each model was used for a design of the selected deep beam. First, the assumption about the position of ties and struts in each model was made. The calculation of the forces in the truss was always done by hand and then checked using the calculation software Scia Engineer. The process was chosen to avoid a possible calculation error.

4.5.1 Model A

The geometry of the model A is shown in Figure 33. The position of the ties and struts was chosen according to the stress distribution. The results obtained with Scia Engineer (Figure 34) are in accordance with the calculation by hand.

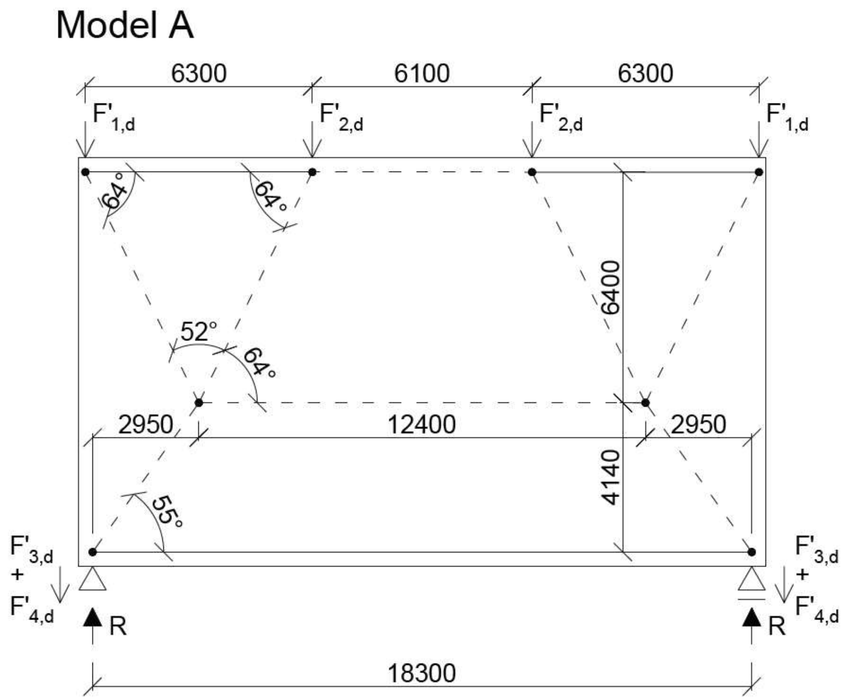


Figure 33: The geometry and the load of the model A

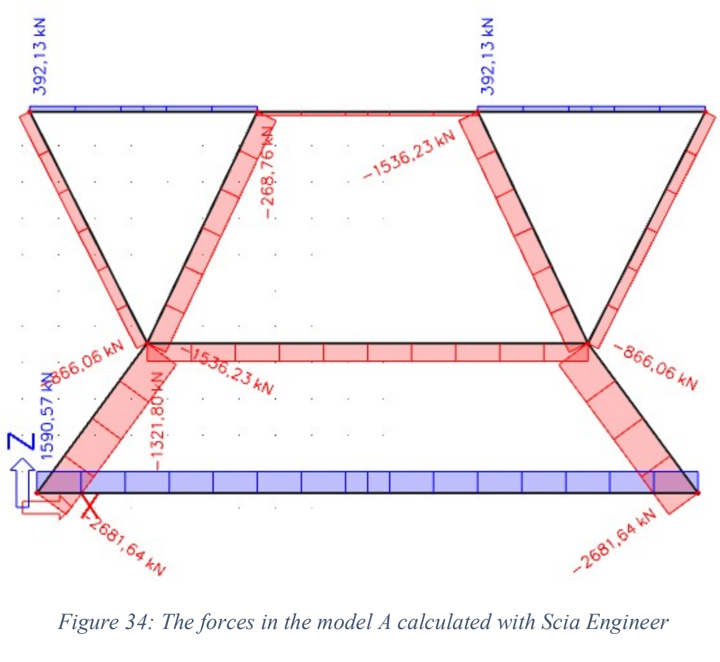


Figure 34: The forces in the model A calculated with Scia Engineer

The next step would be to verify the ties, struts, and nodes. This verification will be shown later in this work during the design of the selected deep beam in chapter 5.

If any strut fails (a stress exceeds the concrete strength), it would be necessary to design the reinforcement that would twist the concrete in compression and so prevent the lateral tension.

The greatest tensile force, and thus decisive for the design of the deep beam, is in the lower tie. It is necessary to design reinforcement:

$$A_{s,rqd} = \frac{F_t}{f_{yd}} = \frac{1\,591 \times 10^3}{435 \ \times 10^6} = 3.66 \times 10^{-3}\ m^2 = 3\,660\ mm^2$$

According to the model A, it is necessary to design the main reinforcement at the bottom edge of the deep beam. This is in accordance with the conclusion of the simplified method. The required area of reinforcement is about 10 % lower to the simplified method.

4.5.2 Model B

The geometry of the model B is shown in Figure 35. In comparison to the model A, the most significant change is a vertical strut under the load $F_{1,d}$. The load aims to transmit by the shortest distance and this strut fulfills this condition. The strut from the load $F_{2,d}$ directs straight to the support now. Because of these changes, the inclined strut does not transmit all load and so does not overestimate the force in the tie along the bottom edge of the deep beam. The results obtained with Scia Engineer (Figure 36) are in accordance with the calculation by hand.

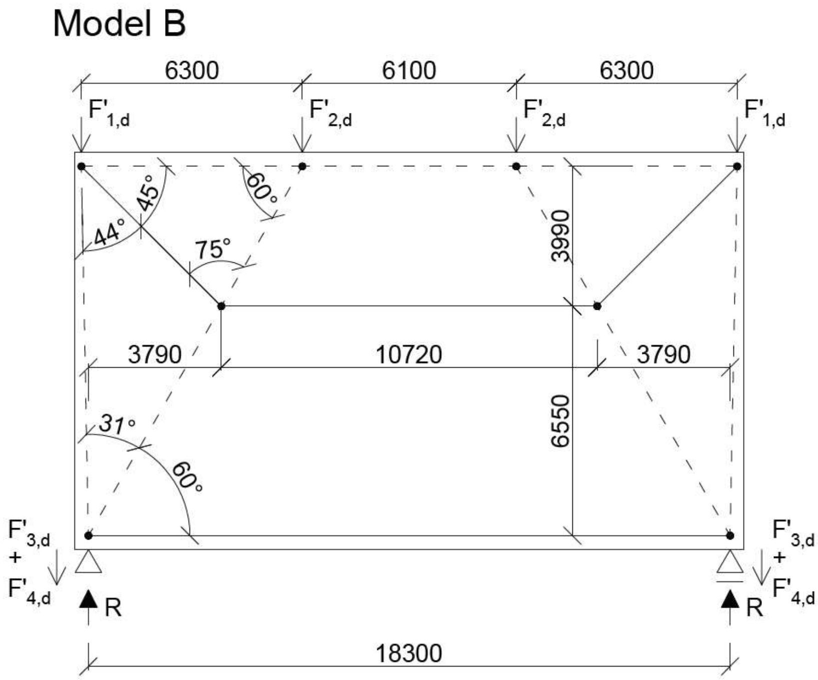


Figure 35: The geometry and the load of the model B

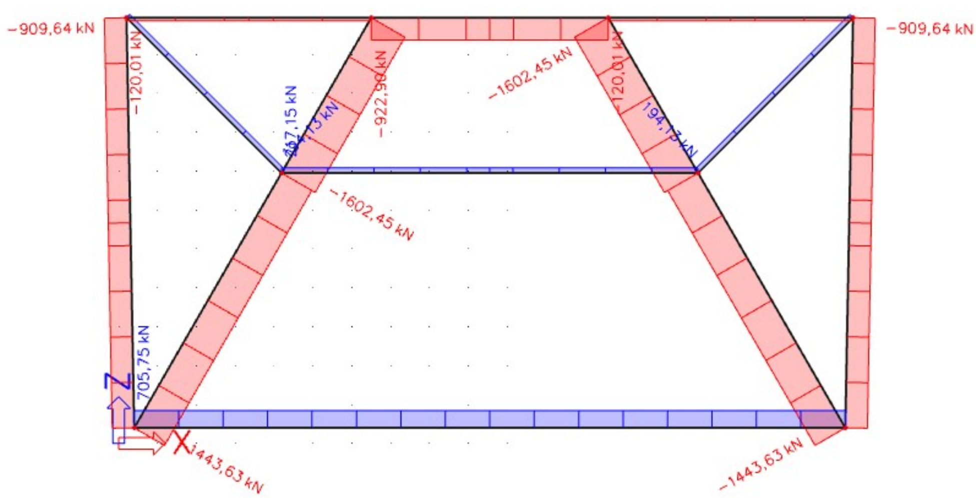


Figure 36: The forces in the model B calculated with Scia Engineer

When calculating the model B, the middle horizontal strut is located at a distance of 6.95 m from the bottom edge of the deep beam. According to the simplified method, the tensile reinforcement for a simply supported deep beam should be placed up to the hight of 1.92 m from the bottom edge of the deep beam. At a distance of 6.95 m, there is already compression, and therefore it is not appropriate to place there a reinforcement. According to this statement, the model B is inadequate.

The next step would be to verify the ties, struts, and nodes. Because the aim is to show the comparison of different strut-and-tie models, the verification is shown later in this work.

The greatest tensile force which is decisive for the design of the deep beam is again the lower horizontal tie. However, its value is different because the inclined struts directed into the supports have a different angle with the lower tie. It is necessary to design reinforcement:

$$A_{s,rqd} = \frac{F_t}{f_{yd}} = \frac{706 \times 10^3}{435 \ \times 10^6} = 1.63 \times 10^{-3} \ m^2 = 1\ 630 \ mm^2$$

According to the model B, it is necessary to design the main reinforcement at the bottom edge of the deep beam. This is in accordance with the conclusion of the simplified method and the model A. The required area of reinforcement is less than half of that of the model A.

4.5.3 Model C

The geometry of the model C is shown in Figure 37. Two new nodes in a third of the span were added along the bottom edge of the deep beam. Thanks to these nodes it is possible to consider the effect of the load at the bottom edge of the deep beam. The results obtained with Scia Engineer (Figure 38) are in accordance with the calculation by hand.

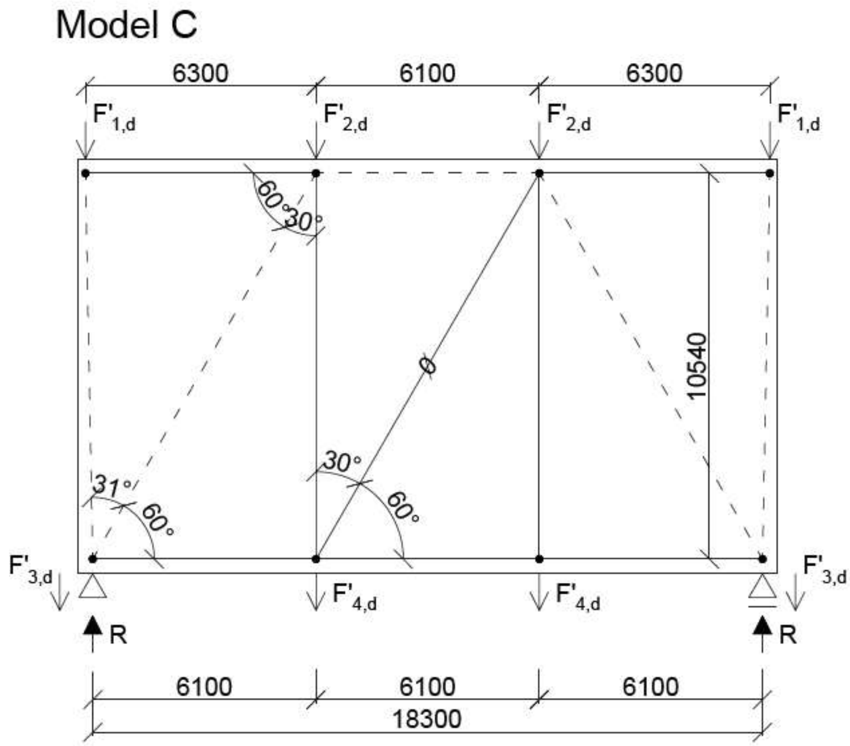


Figure 37: The geometry and the load of the model C

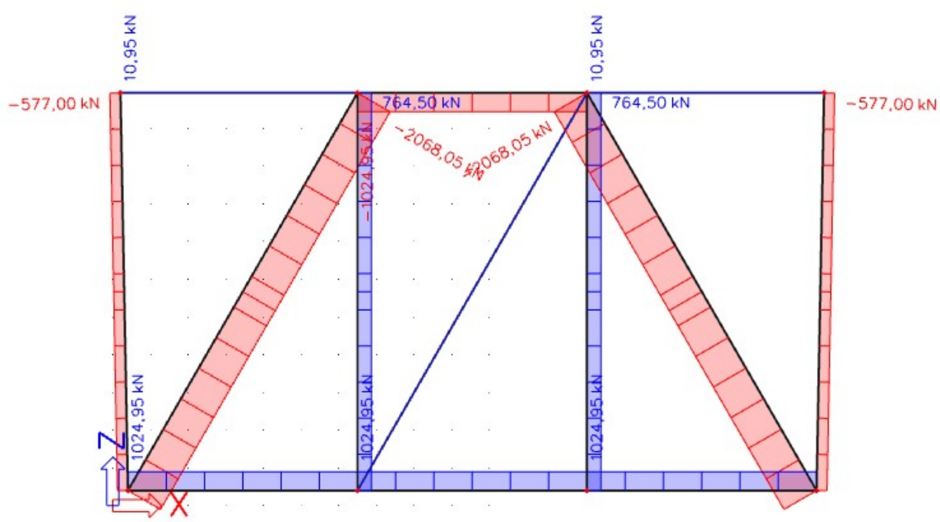


Figure 38: The forces in the model C calculated with Scia Engineer

The next step would be to verify the ties, struts, and nodes. This verification will be shown later in this work during the design of the selected deep beam in Chapter 5.

The greatest tensile force, and thus decisive for the design of the deep beam, is in the lower tie. It is necessary to design the reinforcement:

$$A_{s,rqd} = \frac{F_t}{f_{yd}} = \frac{1\,025 \times 10^3}{435\ \times 10^6} = 2.36 \times 10^{-3}\, m^2 = 2\,360\, mm^2$$

According to the model C, it is necessary to design the main reinforcement at the bottom edge of the deep beam. This is in accordance with the conclusion of the simplified method and models A and B. The required area of reinforcement is about half that of the model A and about one third higher of the model B.

4.5.4 Model D

The geometry of the model D is shown in the Figure 39. In comparison to the model C, the model C has another vertical tie between the load $F_{1,d}$ and $F_{2,d}$. The results obtained with Scia Engineer (Figure 40) are in accordance with the calculation by hand.

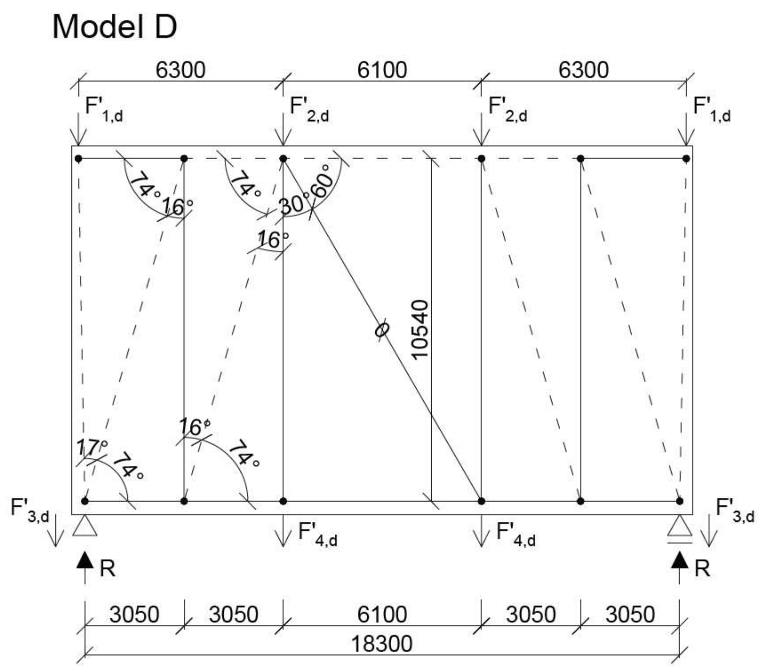


Figure 39: The geometry and the load of the model D

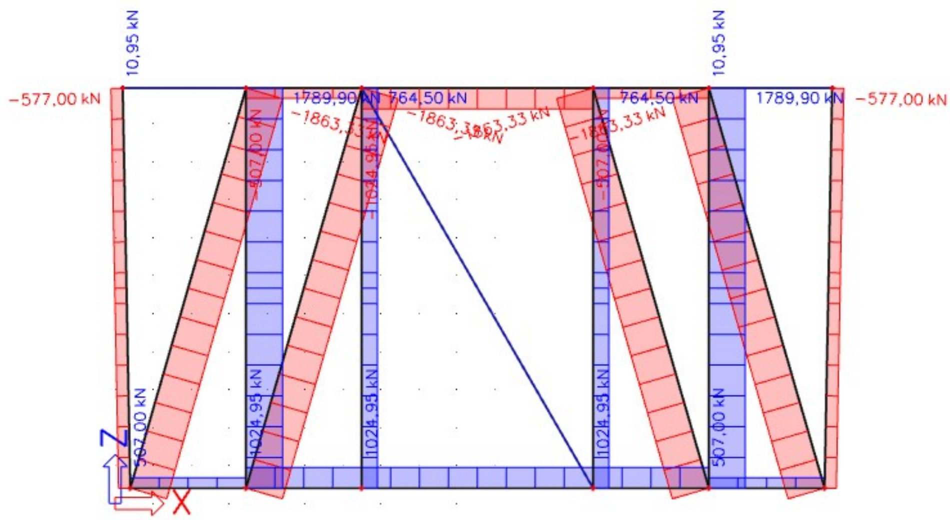


Figure 40: The forces in the model D calculated with Scia Engineer

The next step would be to verify the ties, struts, and nodes. This verification will be shown later in this work during the design of the selected deep beam in Chapter 5.

The greatest tensile force which is decisive for the design of the deep beam is again the lower tie. Unlike the previous models, however, the force in this tie is not constant along the entire length of the deep beam, but it is higher in the middle of the span. This is caused by the different geometry of the model D. It is necessary to design the reinforcement:

$$A_{s,rqd} = \frac{F_t}{f_{yd}} = \frac{1025 \times 10^3}{435 \ \times 10^6} = 2.36 \times 10^{-3}\ m^2 = 2\ 360\ mm^2$$

According to the model D it is necessary to design the main reinforcement at the bottom edge of the deep beam. Compared to the previous models, the tensile force at the bottom surface of the deep beam is lower at edges than in the middle of the span. The amount of the required reinforcement in the middle is exactly the same as for the model C.

4.5.5 Comparison of models A, B, C and D

As was mentioned several times, the most important and at the same time the most difficult moment when using the strut-and-tie method is to choose the right model. Some important findings have been demonstrated on the four models in chapters 4.5.1 – 4.5.4.

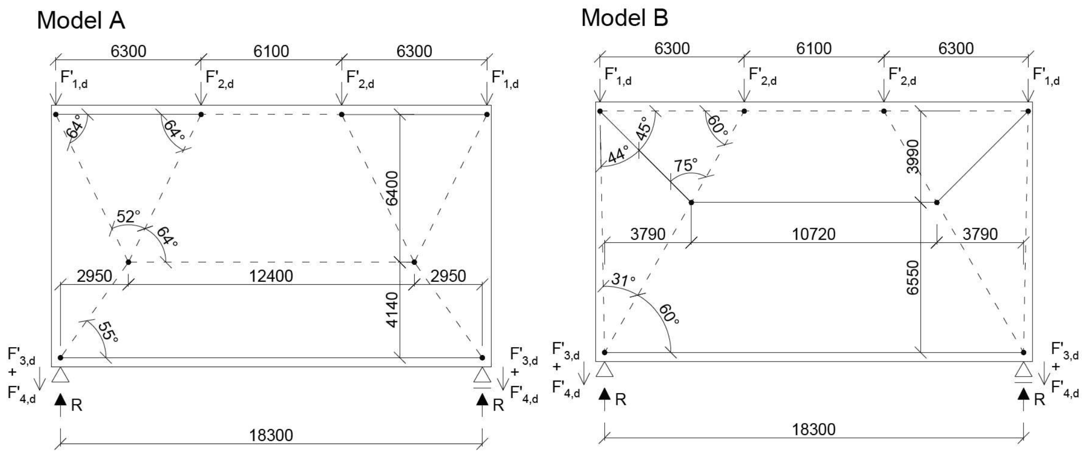


Figure 41: Comparison of the geometry of models A and B

The model A is missing the vertical strut from the outer point load, and thus transmits all load only by the inclined struts. That is the reason why the required area of reinforcement is so much higher than for the other models. The model B transmits the load by the vertical and the inclined struts, which corresponds to the expected behavior. Model B, in my opinion, is more corresponding to the stresses, so I consider it more appropriate. However, it assumes the compression at the outer top edge of the deep beam and according to the linear analysis there should be the tension.

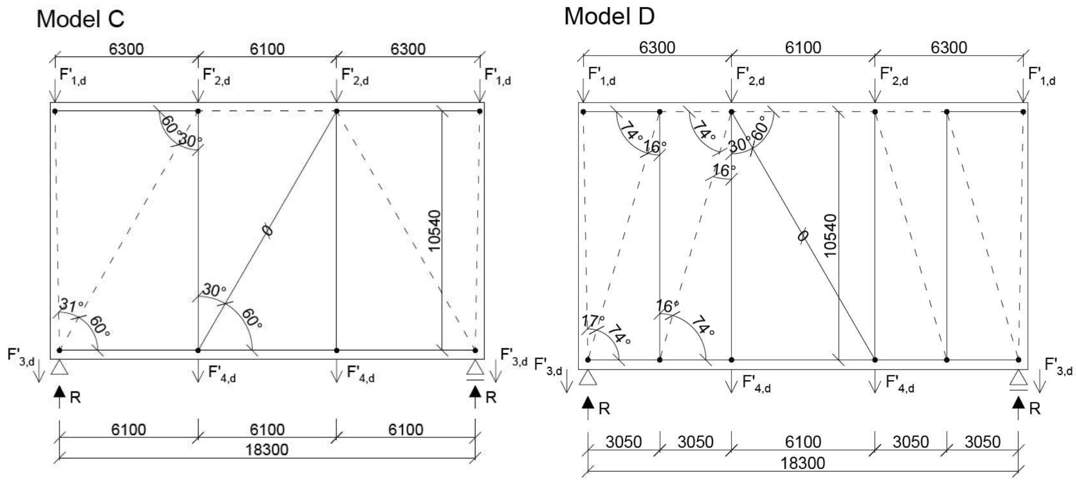


Figure 42: Comparison of models C and D

Models C and D are very similar to each other. The model C is also quite similar to the model B but it better considers the load at the bottom edge. The model D has a variable tensile force at the lower edge, which corresponds more to the behavior of the classic beam than the deep beam.

Due to the lack of experience, it is difficult for the author of this diploma thesis to choose the best model. The most appropriate seem to be models B and C. However, the final decision will be made after the non-linear analysis and a verification of the models.

Strut-and-tie model	**A**	**B**	**C**	**D**
The area of the required reinforcement	3 660 mm^2	1 630 mm^2	2 360 mm^2	2 360 mm^2 / 1 170 mm^2

Table 2: Comparison of the required area of reinforcement of strut-and-tie models

4.6 Analysis – stringer-panel method

Because of the simple geometry and loading, calculations by hand can be performed using the stringer-panel method (SPM). An alternative to the calculations by hand or to check the results is to use SpanCAD. SPanCAD is a specialized structural design software, which allows to calculate internal forces, a deformation, etc.

First, it is necessary to divide the deep beam into stringers and panels (Figure 43).

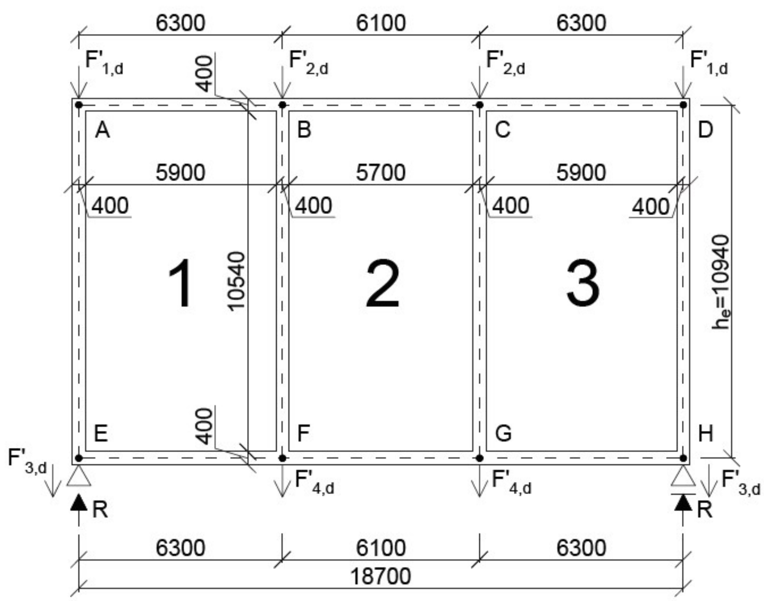


Figure 43: The calculation model of the deep beam according to the stringer-panel method

Due to the four point load, the deep beam was divided into four vertical stringers (under each point load), six horizontal stringers at the lower and upper edges between the vertical stringers; and three panels. The width of stringers was chosen 400 mm as it corresponds to the width of columns of the 4th floor and at the same time it meets the condition of the maximum width (1/5 of the panel width). Reactions in the supports were determined from the vertical condition of the equilibrium with an external load.

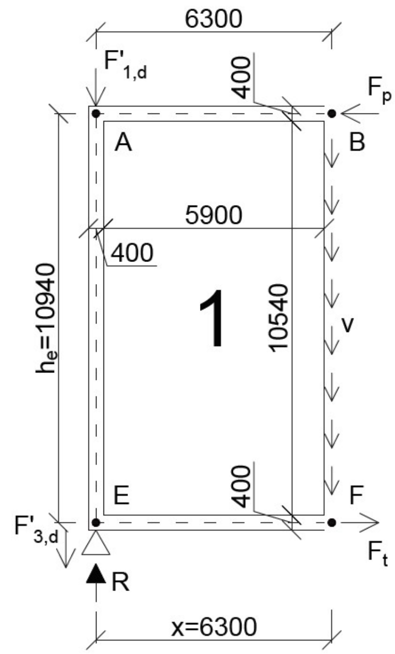


Figure 44: The section of the calculation model for the determination of internal forces

The structure was divided to calculate the internal forces in the individual members. Only a section according to Figure 44 was considered.

The value of the tensile force at the lower edge is decisive for the design of the deep beam. The easiest way to obtain the value of this tensile force is thanks to the bending condition around point B:

$$0 = F_t \times h_e + F'_{1,d} \times x + F'_{3,d} \times x - R \times x$$

From this condition, it is possible to express the value of the tensile force:

$$F_t = \frac{R \times x - F'_{1,d} \times x - F'_{3,d} \times x}{h_e} = \frac{2800 \times 6.3 - 576.9 \times 6.3 - 413.1 \times 6.3}{10.94} = 1\ 042\ kN$$

Based on the obtained tensile force, it is now possible to determine the required area of reinforcement:

$$A_{s,rqd} = \frac{F_t}{f_{yd}} = \frac{1\,042 \times 10^3}{435\ \times 10^6} = 2{,}4 \times 10^{-3}\ m^2 = 2\,400\ mm^2$$

The verification of panels is not done, because it is not necessary for the following comparison of the design methods. However, it would have to be carried out for the design of deep beams using the stringer-panel method.

According to the stringer-panel method, it is necessary to design the main reinforcement at the lower edge of the deep beam. This is in an accordance with the previous methods. The required area of reinforcement is almost exactly the same as for the model C of the strut-and-tie method.

The advantage of this method compared to the strut-and-tie method is the simple choice of the computation model. Generally, we can say that stringers are always placed on the vertical and horizontal edges of the deep beam, the other vertical stringers are below the point load.

4.6.1 Non-linear analysis

SPanCAD is a plugin for AutoCAD and it is not a separate software. SPanCAD was developed in the 1990s and, unfortunately, the latest version is designated for AutoCAD Release 14, which dates back to 1997 and it is no longer used in a practice.

After installing SPanCAD for AutoCAD 2015, a menu appeared, but functions of it were not working. It was not possible to draw the structure – stringers, panels, supports, etc. or to run any action at all.

AutoCAD Release 14 cannot be run on modern operating systems (e.g. Windows 10). For the purpose of the diploma thesis, Windows 98 and AutoCAD Release 14 were installed. This combination is recommended by the authors of the software SPanCAD.

Now it was possible to draw the structure including supports and loads, but an unknown error appeared when trying to run the calculation (Figure 45).

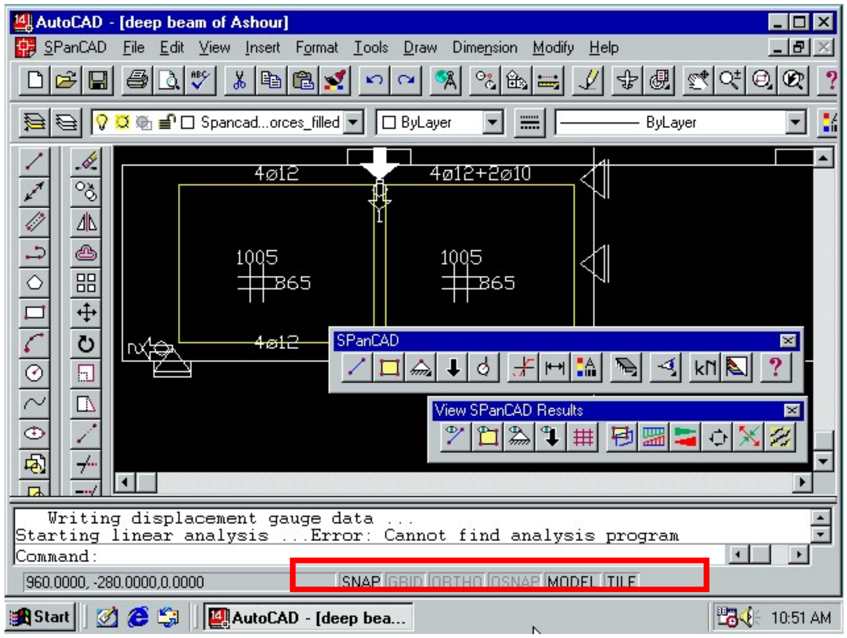


Figure 45: The error message when trying to run the calculation

The latest update of SPanCAD took place in 2001 and the authors of SPanCAD were not able to help with this problem. Since it was not possible to run the calculation in SPanCAD, the non-linear analysis of the deep beam was not carried out by SPanCAD.

4.7 Non-linear analysis

The non-linear analysis was performed in the software ATENA 2D to verify the results and evaluate design methods. ATENA 2D is a sophisticated software for the analysis of reinforced concrete structures. This software is base on the finite element method and it can be used to simulate the behavior of concrete structures, including verification of cracks or stress of the reinforcement.

Design values of materials (concrete and reinforcement) and load were used for the non-linear analysis. The design values were preferred because of safety reasons.

It is necessary to choose a model that is as close as possible to the reality for the most accurate simulation of the real behavior of the structure. In the non-linear analysis, four point load correspond to the load of the columns of the 4th floor of the superstructure. The uniformly distributed line load at the upper and lower edge correspond to the load of the slab above the 3rd floor of the superstructure, respectively the load of the floor of the 1st floor of the superstructure. The self-weight of the deep beam is generated automatically.

It is convenient to use the symmetry of the structure and model only a half of the deep beam to speed up the calculation. The calculation model consists of four macroelements. Macroelement 1 represents a reinforced concrete deep beam. The material is the concrete C30/37, the finite element mesh is quadrangular, and the size of the element edge is 300 mm, which is the thickness of the structure. Macroelements 2, 3 and 4 represent reinforced concrete columns. Because of the simplicity, the columns were replaced by steel plates.

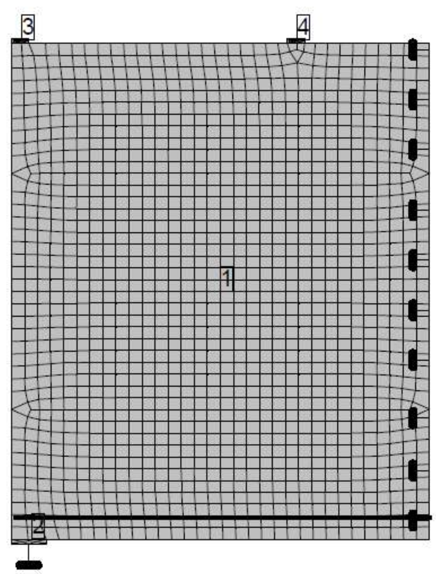


Figure 46: Calculation model of the deep beam in ATENA 2D with displayed supports and the finite element mesh

The reinforcement was defined as one bar. The area of the reinforcement was selected according to the results of the design methods. The non-linear analysis was performed for the reinforcement according to the simplified method, four models of the strut-and-tie method and the stringer-panel method.

First, non-linear analysis with the reinforcement according to the simplified method was performed. When using this method, it is recommended to place the main reinforcement for this deep beam up to 1 950 mm from the bottom edge. Therefore, the axis of the reinforcement was placed at a height of 975 mm from the bottom edge.

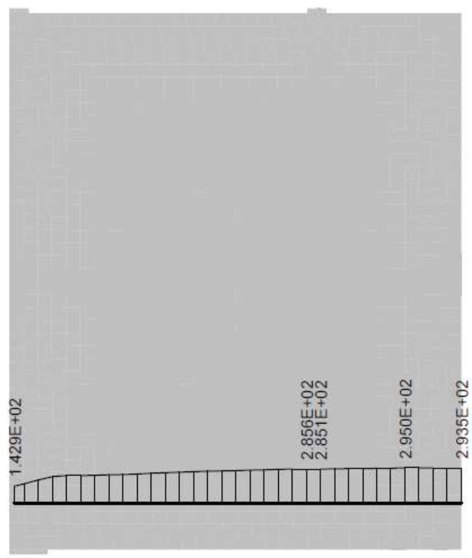


Figure 47: Stress of the reinforcement according to the simplified method (unit MPa)

The stress of the reinforcement is uniform over the entire length (Figure 47). This is in accordance with the assumptions of the simplified method. However, the value of the stress is significantly lower than the expected value of 435 MPa. The required area of the reinforcement is overestimated.

Next, the non-linear analysis was performed with the reinforcement obtained by four models of the strut-and-tie method. The axis of the reinforcement was placed at a height of 400 mm from the bottom edge. This height corresponds to the position of the ties of all models.

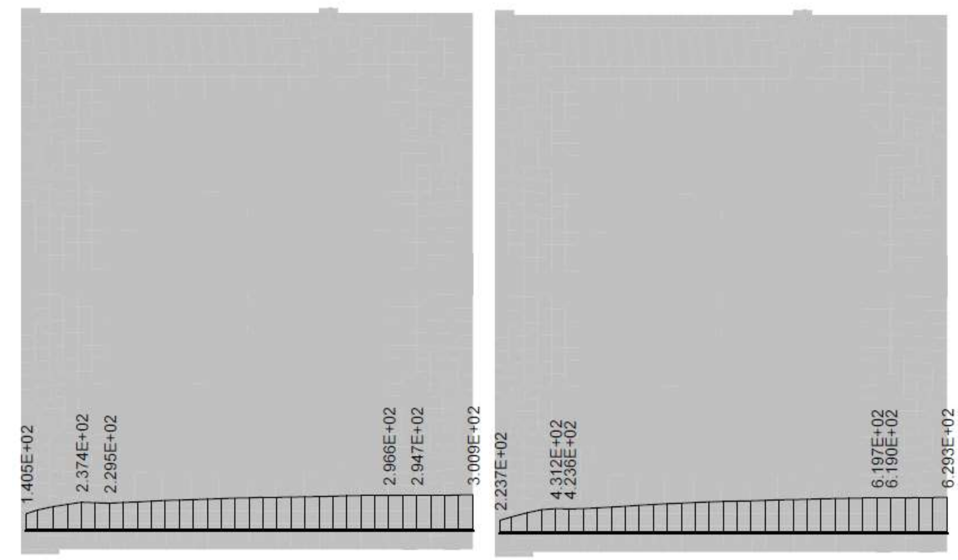


Figure 48: Stress of the reinforcement according to the strut-and-tie model A (left) and B (right; unit MPa)

Models A and B are inaccurate as it was expected in chapter 4.5 (Figure 48). Model A overestimates the amount of the required reinforcement due to the absence of a vertical strut. Model B underestimates the amount of the required reinforcement because all the load at the lower edge, including the half of the self-weight of the deep beam, is introduced into the joints in the supports and so does not increase the strain of the deep beam.

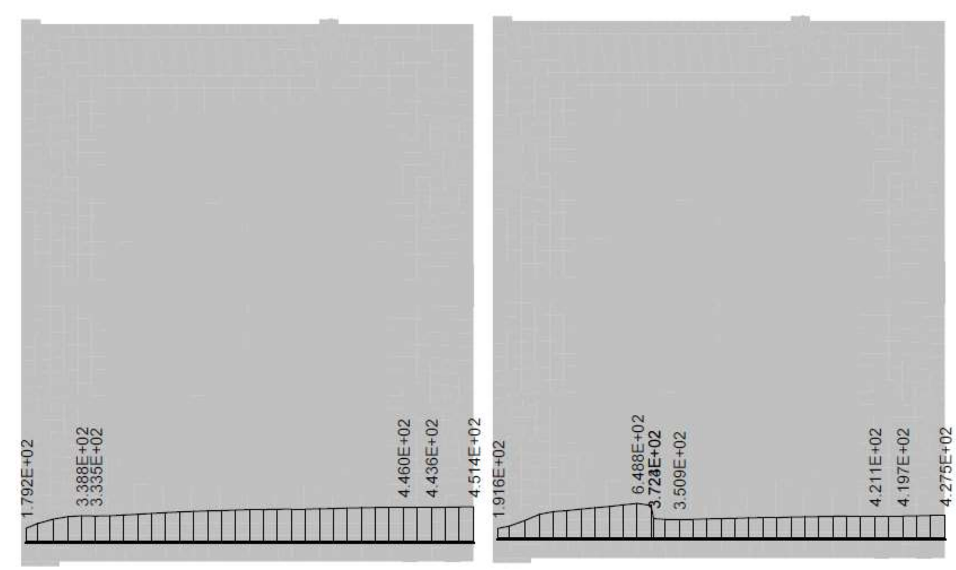


Figure 49: Stress of the reinforcement according to the strut-and-tie model C (left) and D (right; unit MPa)

Model C is close to expectations. The stress of the reinforcement is uniform over the entire length and its value is close to the expected value of 435 MPa. Model D assumed uneven force in the tie at the bottom edge. Because of this reason, more reinforcement was defined in the middle of the span and less at the edges. The results of the non-linear analysis show that model D was wrong because the stress at the edge far exceeds the expected value of 435 MPa.

Finally, the non-linear analysis was performed with the reinforcement obtained by the stringer-panel method. The reinforcement was placed at a height of 200 mm from the bottom edge, which corresponds to the axis of the lower horizontal stringer.

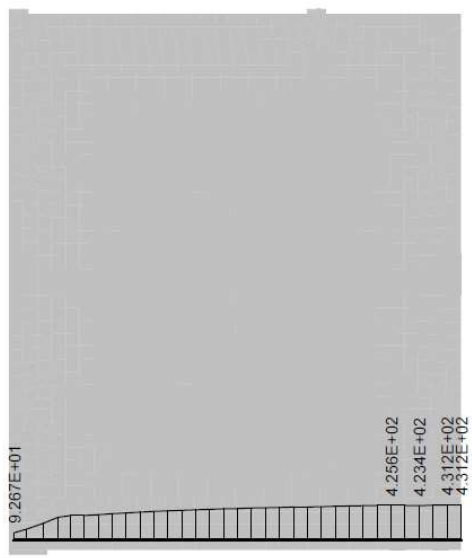


Figure 50: Stress in the reinforcement according to the stringer-panel method (unit MPa)

In accordance with the assumptions, the stress of the reinforcement is uniform over the entire length. The value of the stress corresponds to the expected value of 435 MPa.

The results obtained by the non-linear analysis are summarized in table 3.

Method	$A_{s,rqd}$ [mm^2]	Position from the bottom [mm]	Stress [MPa]	Accuracy [%]
Simplified	4 020	975	295.0	67.8
STM – model A	3 660	400	300.9	69.2
STM – model B	1 630	400	629.3	144.7
STM – model C	2 360	400	451.4	103.8
STM – model D	2 360 / 1 170	400	427.5 / 648.8	98.3 / 149.1
Stringer-panel	2 400	200	431.2	99.1

Table 3: Comparison of the stress of the reinforcement according to the design methods

The non-linear analysis confirmed some assumptions and statements in technical literature. It has been proved that the value of stress of the reinforcement is uniform over the entire length of the deep beam. The value of the stress of the reinforcement is significantly greater in the middle of the span for a shallow beam.

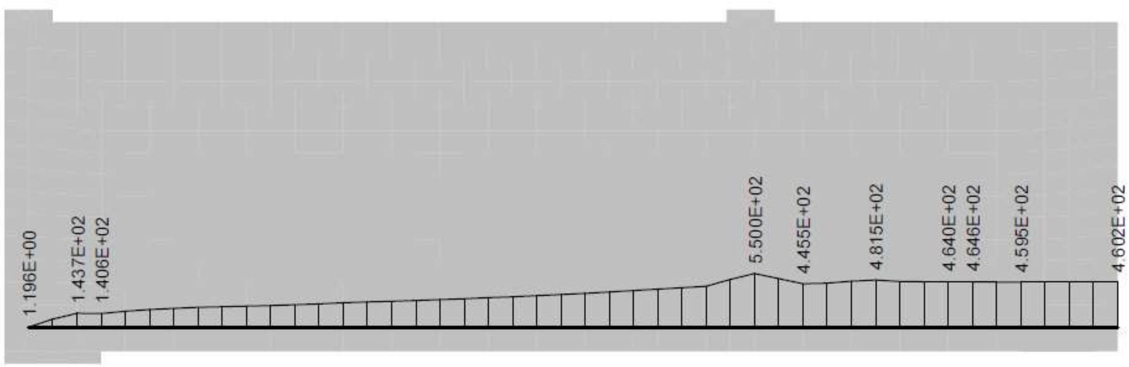


Figure 51:Stress in the reinforcement for the shallow beam (unit MPa)

It was possible to verify the accuracy of the design methods and strut-and-tie models because of the non-linear analysis. The conclusion is in the following chapter.

4.8 Comparison of design methods

In the previous chapters, the deep beam was analyzed by three different methods. Each method has certain advantages and disadvantages.

The simplified method developed by doc. Bažant is proposed for deep beams with the simple geometry and the load. The structural design of the deep beam produced by using this method was time-efficient and simple. Moreover, all calculations within the design were possible to make by hand without any sophisticated software. On the other hand, the required area of reinforcement was overestimated in comparison with the value obtained from the non-linear analysis. This method overestimates the required area of reinforcement even for deep beams with the simple geometry and the load as it was shown in chapter 3. The simplified method is conservative, the design of the deep beam is safe but not economic.

The key moment when using the strut-and-tie method is the choice of the right strut-and-tie model. The calculations were carried out without the calculation software, but it is appropriate to use the software for more difficult cases or for checking the results. The advantage is the often application of this method and so a lot of available experience and information.

According to the non-linear analysis, model C is the most suitable. Model A is inadequate because of the missing vertical strut. Model B is inadequate because it does not consider the load of the bottom edge of the deep beam. Model D is not completely wrong, but it assumes various tensile force in the lower tie, which is in contrary to the behavior of deep beams. This conclusion was supported by the non-linear analysis in chapter 4.7.

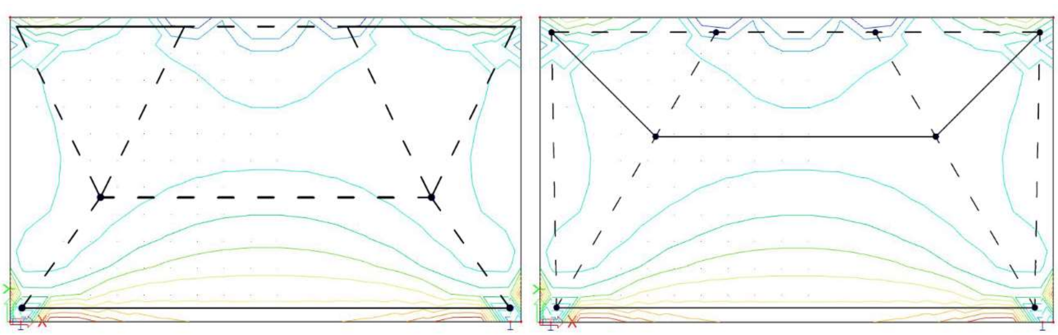


Figure 52: Comparison of models A and B to horizontal stress (direction x)

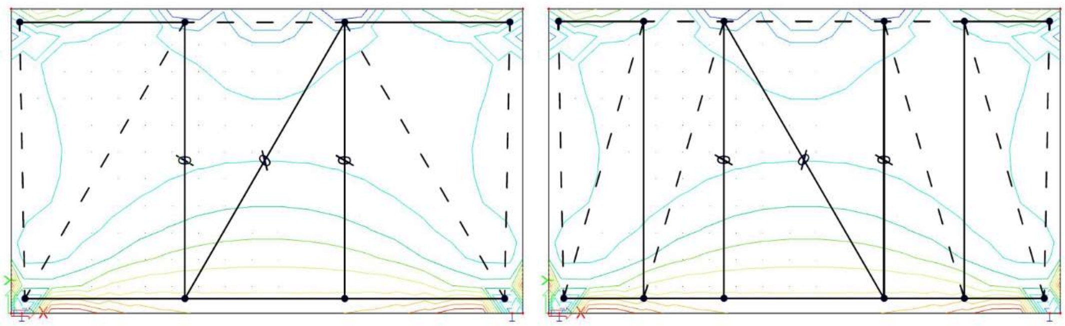


Figure 53: Comparison of models C and D to horizontal stress (direction x)

The stringer-panel method is an alternative approach, which provides similar results as the strut-and-tie method (model C). The calculations were performed by hand thanks to the simple geometry. The results can be affected by the lack of information about this method. Specifically, there is no rule or recommendation for determining a width of stringers. It could be useful to explore this method more into detail.

Method	**Simplified**	**Strut-and-tie (model C)**	**Stringer-panel**
The reinforcement	4 020 mm^2	2 360 mm^2	2 400 mm^2

Table 4: Comparison of the required area of reinforcement according to design methods

The conclusion of the analysis of the deep beam is the following. It can be stated that the strut-and-tie method (if chosen right model) is the most suitable and the most reliable method. The model C was evaluated as the most adequate one. Contrary, models A, B, and D do not reflect the stress completely right and the results are not correct.

The stringer-panel method gave very similar results to the strut-and-tie method. The only problem is the lack of experience, otherwise, this method can be recommended.

The simplified method required a higher amount of reinforcement. It is caused by the simplification and the need to stay on the safe side of the design. This method seems to be too conservative for the design of the deep beam.

5 Design of the deep beam

The design of the deep beam of the silo superstructure is carried out by the strut-and-tie method, which was assessed as the most suitable in chapter 4.

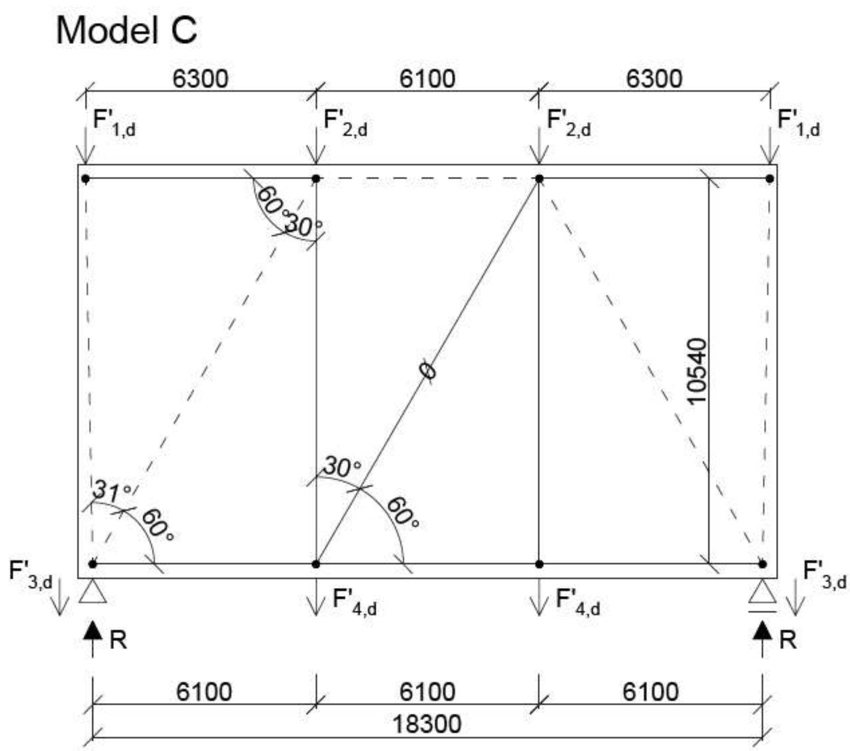


Figure 54: Preliminary strut-and-tie model

First, it is necessary to precisely determine the position of nodes and dimensions of struts and ties of the strut-and-tie model. A preliminary design using the strut-and-tie method was carried out in chapter 4. The position of the nodes was estimated, now it is necessary to determine the exact position of the nodes.

The width of the tie T1 should be determined first. In literature, it is generally recommended not to place the main reinforcement only at the bottom edge of the deep beam. To determine this height was used the equation from the simplified method of doc. Bažant for the area where the main reinforcement should be placed:

$$T1 = 0.25h - 0.05L = 0.25 \times 11.34 - 0.05 \times 18.3 = 1.92\ m$$

The value of the tie width obtained from the presented formula is 1.92 m. However, to install provided reinforcement in such wide tie would lead to the relatively low reinforcement ratio of the tie given the tie cross-section area. As a consequence, the tie width was reduced from 1.92 m to 1 m with the aim to concentrate reinforcement bars to the region where the highest tensile stresses are expected to occur.

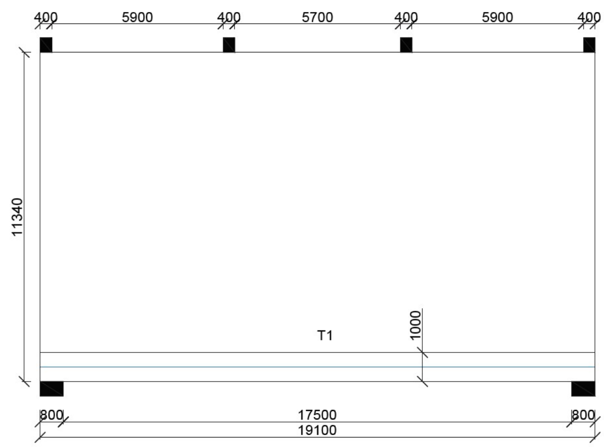


Figure 55: Highlighting of the tie T2

It is possible to determine the width of the tie T2, which is equal to the width of the column above the tie (400 mm). The axis of the tie T1 and the axis of the tie T2 form a node 1.

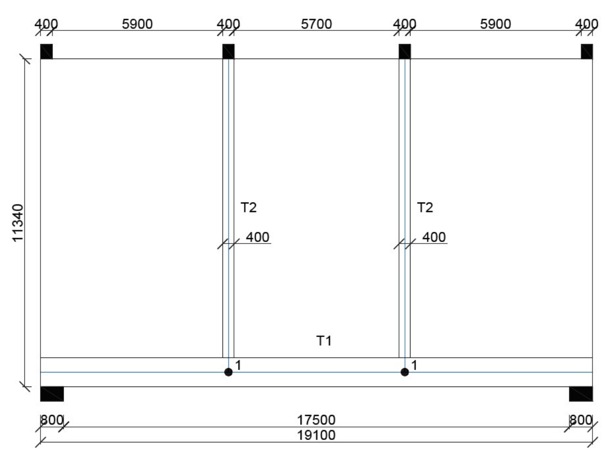


Figure 56: Highlighting of the tie T2

The width of the strut C1 is chosen to provide enough anchorage length for the reinforcement of the tie T2. This width also corresponds to the horizontal stress distribution according to the non-linear analysis in ATENA 2D. As the reinforcement is not known at this time, the anchorage length is conservatively estimated to be 1 m (sufficient anchorage length for 20 mm diameter rods). The axis of the tie T2 and the axis of the strut C1 form a node 2.

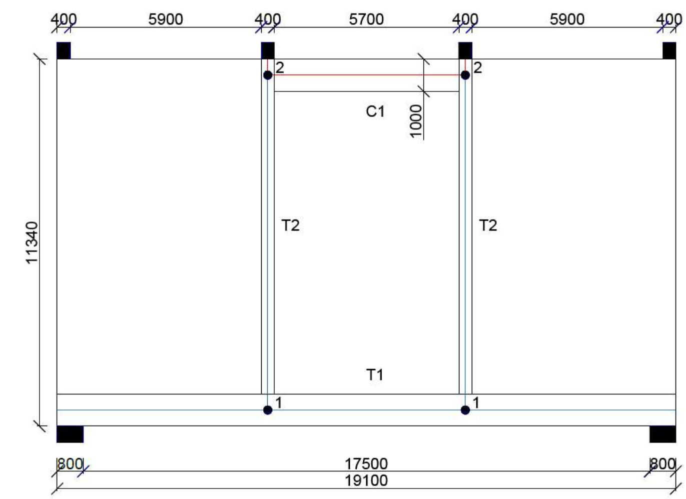


Figure 57: Highlighting of the strut C1

The axis of the tie T1 and the axis of the support form a node 3. The inner edge of the support and the inner edge of the column of the 4th floor of the superstructure form a strut C2.

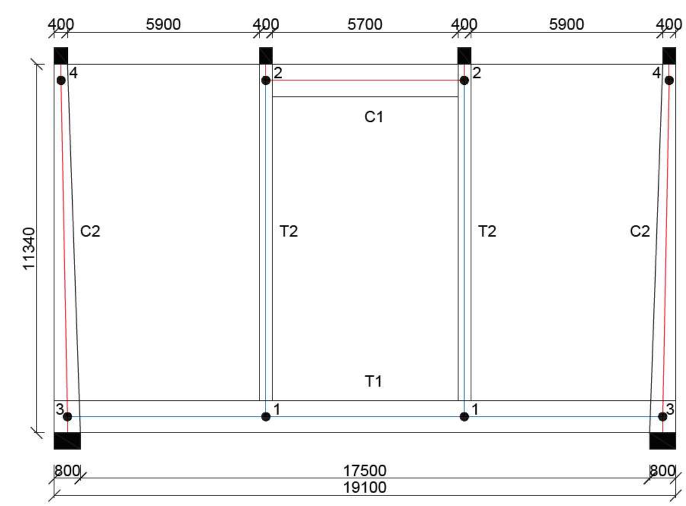


Figure 58: Highlighting of the strut C2

The strut C3 is formed by connecting the edges of the nodes 2 and 3.

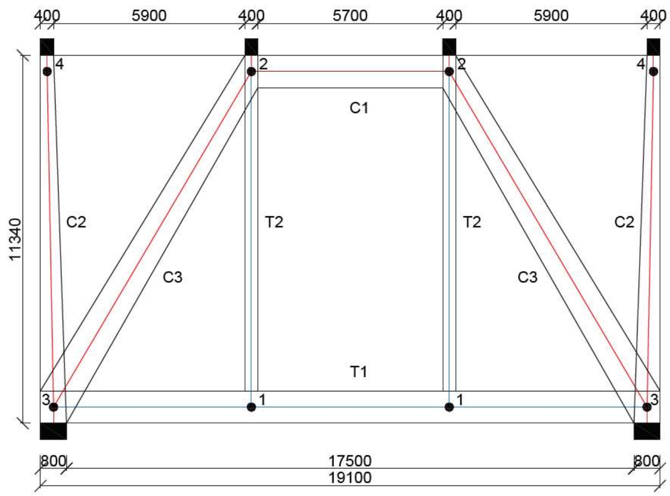


Figure 59: Highlighting of the strut C3

The calculation must be performed for the obtained geometry of the strut-and-tie model. Forces compared to the preliminary calculation in chapter 4 have changed only slightly (Figure 60). This is due to the fact that the geometry of the strut-and-tie model has changed only a little (the distance of the upper and lower nodes is now 200 mm less).

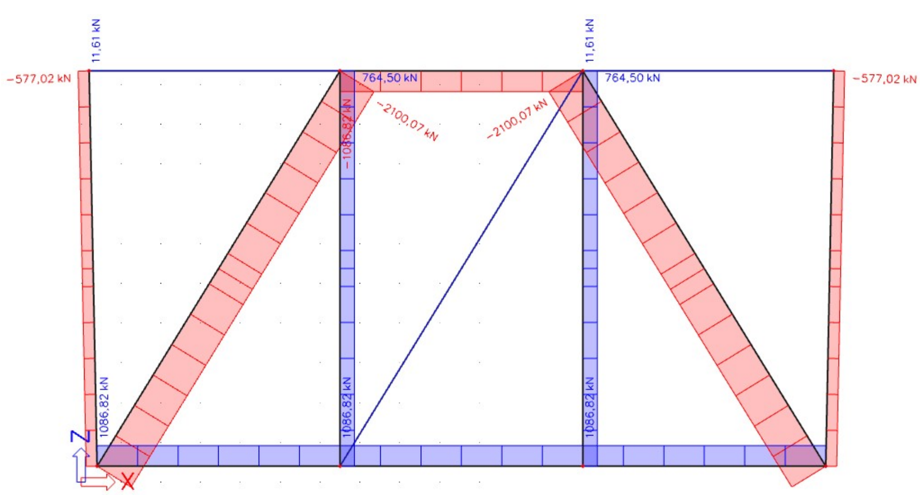


Figure 60: The forces in the strut-and-tie model calculated with Scia Engineer

It is necessary to design the reinforcement of the ties and to check the struts and nodes. The force of the ties at the edge at the upper surface is almost zero, so these ties are not taken into account in the following calculations.

In the next step, the non-linear analysis of the deep beam with the area of the reinforcement according to the final strut-and-tie model was performed. The same conditions as in chapter 4.7 were applied.

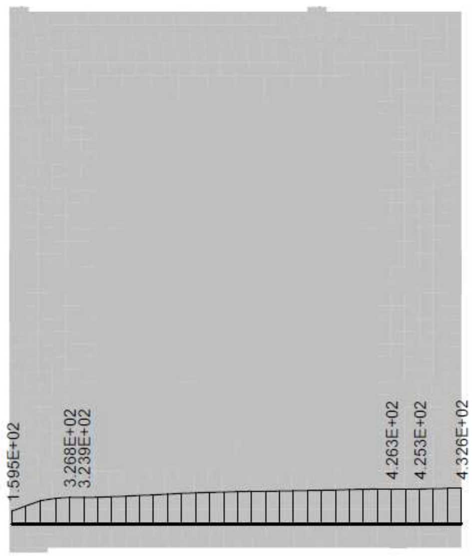


Figure 61: Stress of the reinforcement according to the final strut-and-tie model (unit MPa)

The stress of the reinforcement is 432.6 MPa according to the non-linear analysis. This value is very close to the expected value of 435 MPa. The goal to improve the strut-and-tie model to be more accurate was successful and it can be said that this strut-and-tie model is suitable for the given geometry and the load of the deep beam.

5.1 Verification of stress in nodes

<u>Node 4 – C-C node</u>

- node with compressive forces

$\sigma_{Rd,max} = k_1 \times v \times f_{cd} \qquad k_1 = 1.0 \;\; ; \;\; v = 1 - \frac{f_{ck}}{250} \;\; ; \;\; f_{cd} = 20\ MPa$

$\sigma_{Rd,max} = 1.0 \times 1 - \frac{30}{250} \times 20 = 17.6\ MPa$

- width of strut $w_{c,4F} = w_{c,43} = 400$ mm (width of the column above the strut)

$$\sigma_{Sd,4F} = \sigma_{Sd,43} = \frac{577 \times 10^3}{400 \times 300} = 4.81\ MPa \ \leq 17.6\ MPa = \sigma_{Rd,max}$$

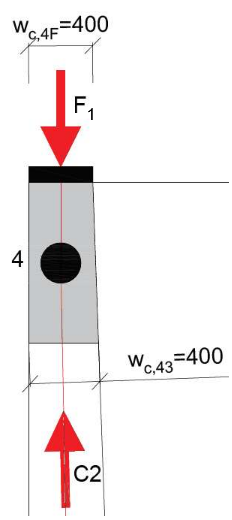


Figure 62: Node 4 of the strut-and-tie model

Node 3 – C-C-C-T node

- node with compressive forces and a tie in one direction

$$\sigma_{Rd,max} = k_2 \times v \times f_{cd} \qquad k_2 = 0.85 \;\; ; \;\; v = 1 - \frac{f_{ck}}{250} \;\; ; \;\; f_{cd} = 20\,MPa$$

$$\sigma_{Rd,max} = 0.85 \times 1 - \frac{30}{250} \times 20 = 14.96\,MPa$$

- width of strut $w_{c,3R}$ = 800 mm (width of the column in a support)
- width of strut $w_{c,34}$ = 720 mm
- width of strut $w_{c,32}$ = 1200 mm
- width of tie $w_{t,1}$ = 1000 mm

$$\sigma_{Sd,3R} = \frac{2780 \times 10^3}{800 \times 300} = 11.58\,MPa \;\leq 14.96\,MPa = \sigma_{Rd,max}$$

$$\sigma_{Sd,34} = \frac{577 \times 10^3}{720 \times 300} = 2.67\,MPa \;\leq 14.96\,MPa = \sigma_{Rd,max}$$

$$\sigma_{Sd,32} = \frac{2100 \times 10^3}{1200 \times 300} = 5.83\,MPa \;\leq 14.96\,MPa = \sigma_{Rd,max}$$

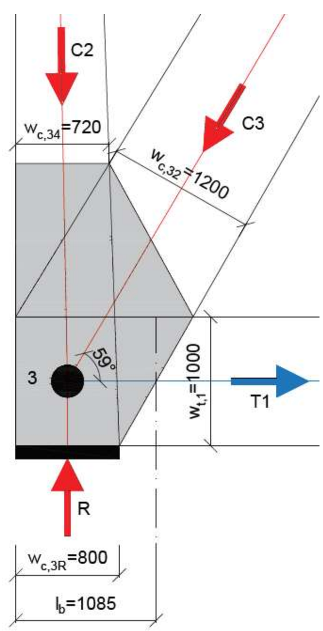


Figure 63: Node 3 of the strut-and-tie model

Node 2 – C-C-C-T node

- node with compressive forces and a tie in one direction

$\sigma_{Rd,max} = k_2 \times v \times f_{cd} \qquad k_2 = 0.85 \;\; ; \;\; v = 1 - \dfrac{f_{ck}}{250} \;\; ; \;\; f_{cd} = 20\,MPa$

$\sigma_{Rd,max} = 0.85 \times 1 - \dfrac{30}{250} \times 20 = 14.96\,MPa$

- width of strut $w_{c,2F}$ = 400 mm (width of column above)
- width of strut $w_{c,22}$ = 1000 mm
- width of strut $w_{c,23}$ = 858 mm
- width of tie $w_{t,2}$ = 400 mm (width of column above)

$$\sigma_{Sd,2F} = \frac{1025 \times 10^3}{400 \times 300} = 8.54\,MPa \le 14.96\,MPa = \sigma_{Rd,max}$$

$$\sigma_{Sd,22} = \frac{1087 \times 10^3}{1000 \times 300} = 3.62\,MPa \le 14.96\,MPa = \sigma_{Rd,max}$$

$$\sigma_{Sd,23} = \frac{2100 \times 10^3}{858 \times 300} = 8.16\,MPa \le 14.96\,MPa = \sigma_{Rd,max}$$

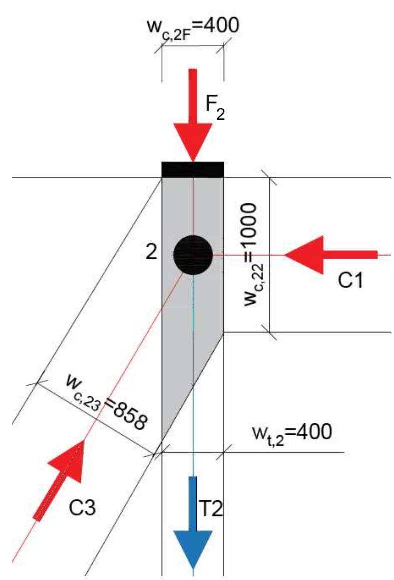


Figure 64: Node 2 of the strut-and-tie model

5.2 Verification of stress in struts

<u>Strut C2 (2-2)</u>

$F_{c,1}$ = 1 025 kN

- strut with no cracks

$\sigma_{Rd,max} = v_2 \times f_{cd} \qquad v_2 = 1.0 \;\; ; \;\; f_{cd} = 20\ MPa$

$\sigma_{Rd,max} = 1.0 \;\; \times 20 = 20.0\ MPa$

- width of strut C2 is 1000 mm

$$\sigma_{Sd,12} = \frac{1087 \times 10^3}{1000 \times 300} = 3.62\ MPa \;\leq 20.0\ MPa = \sigma_{Rd,max}$$

<u>Strut C2 (4-3)</u>

$F_{c,2}$ = 577 kN

- strut with cracks of normal width

$\sigma_{Rd,max} = v \times v_2 \times f_{cd} \qquad v_2 = 0.6 \;\; ; \;\; v = 1 - \dfrac{f_{ck}}{250} \;\; ; \;\; f_{cd} = 20\ MPa$

$\sigma_{Rd,max} = 0.6 \times 1 - \dfrac{30}{250} \;\; \times 20 = 10.56\ MPa$

- minimum width of strut C2 is 400 mm (strut C2 widens from the load at the upper edge towards the support at the bottom edge)

$$\sigma_{Sd,12} = \frac{577 \times 10^3}{400 \times 300} = 4.81\ MPa \;\leq 10.56\ MPa = \sigma_{Rd,max}$$

<u>Strut C3 (2-3)</u>

$F_{c,3}$ = 2 100 kN

- strut with cracks of normal width

$\sigma_{Rd,max} = v \times v_2 \times f_{cd} \qquad v_2 = 0.6 \;\; ; \;\; v = 1 - \dfrac{f_{ck}}{250} \;\; ; \;\; f_{cd} = 20\ MPa$

$\sigma_{Rd,max} = 0.6 \times 1 - \dfrac{30}{250} \;\; \times 20 = 10.56\ MPa$

- minimum width of strut C3 is 858 mm (strut C3 widens from the load at the upper edge towards the support at the bottom edge)

$$\sigma_{Sd,12} = \frac{2100 \times 10^3}{858 \times 300} = 8.16\ MPa \;\leq 10.56\ MPa = \sigma_{Rd,max}$$

5.3 Design of reinforcement of ties

<u>Tie T1 (3-3)</u>

$F_{t,1}$ = 1 087 kN

$$A_{s,rqd} = \frac{1\,087 \times 10^3}{435} = 2\,499\ mm^2 < A_{s,prov} = 2\,814\ mm^2\ (14x\ \emptyset 16\ mm)$$

The reinforcement must be properly anchored in the node. Anchoring can be achieved by providing enough anchorage length or by bending the bars.

Limit stress of cohesiveness:

$$f_{bd} = 2.25 \times \eta_1 \times \eta_2 \times f_{ctd} = 2.25 \times 0.7 \times 1.0 \times 1.35 = 2.12 \text{ MPa}$$

Stress of reinforcement:

$$\sigma_{Sd} = \frac{A_{s,rqd}}{A_{s,prov}} \times f_{yd} = \frac{2\,499}{2\,814} \times 435 = 387\ MPa$$

Basic anchorage lentgth:

$$l_{b,rqd} = \frac{\emptyset}{4} \times \frac{\sigma_{Sd}}{f_{bd}} = \frac{16}{4} \times \frac{387}{2.12} = 731\ mm$$

<u>Tie T2 (1-2)</u>

$F_{t,2}$ = 765 kN

$$A_{s,rqd'} = \frac{765 \times 10^3}{435} = 1\,759\ mm^2$$

The reinforcement will be placed along the whole length of the deep beam because the load at the bottom edge is uniformly distributed line load.

$A_{s,rqd'} = 1\,759\ mm^2$ ... for the half of the deep beam

$A_{s,rqd} = \frac{A_{s,rqd'}}{L/2} = \frac{1\,759}{9.55} = 185\ mm^2/m < A_{s,prov} = 200\ mm^2$ ($\emptyset 8\ mm$ à $250\ mm$)

The reinforcement must be properly anchored in the node. Anchoring can be achieved by providing enough anchorage length or by bending the bars.

Limit stress of cohesiveness:

$$f_{bd} = 2.25 \times \eta_1 \times \eta_2 \times f_{ctd} = 2.25 \times 0.7 \times 1.0 \times 1.35 = 2.12 \text{ MPa}$$

Stress of reinforcement:

$$\sigma_{Sd} = \frac{A_{s,rqd}}{A_{s,prov}} \times f_{yd} = \frac{1\,759}{1\,884} \times 435 = 406\ MPa$$

Basic anchorage length:

$$l_{b,rqd} = \frac{\emptyset}{4} \times \frac{\sigma_{Sd}}{f_{bd}} = \frac{8}{4} \times \frac{406}{2.12} = 383\ mm$$

5.4 Design of vertical and horizontal reinforcement

According to ČSN EN 1992-1-1 it is possible to do not design vertical shear reinforcement if two following conditions are met.

First, it is necessary to verify the condition of the non-reduced shear force:

$$V_{Ed} \leq 0.5 \times b_w \times d \times v \times f_{cd}$$

$$V_{Ed} = 2\,780\ kN \ \leq 0.5 \times 300 \times 10\,840 \times 0.6 \times \left(1 - \frac{30}{250}\right) \times 20 = 17\,170\ kN$$

The design value for the shear resistance $V_{Rd,c}$ must be greater then the design shear force:

$$V_{Ed} \leq V_{Rd,c} = [C_{Rd,c} \times k \times (100 \times \rho_1 \times f_{ck})^{\frac{1}{3}} + k_1 \times \sigma_{cp}] \times b_w \times d$$

$$V_{Rd,c} = [C_{Rd,c} \times k \times (100 \times \rho_1 \times f_{ck})^{\frac{1}{3}} + k_1 \times \sigma_{cp}] \times b_w \times d$$

$$C_{Rr,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{1.5} = 0.12$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{10\,840}} = 1.13 \leq 2.0$$

$$\rho_1 = \frac{A_{s1}}{b_w \times d} = \frac{2\,814}{300 \times 10\,840} = 0.0009 \leq 0.02$$

$$k_1 = 0.15$$

$\sigma_{cp} = 0$ (non-prestressed reinforcement)

$$V_{Rd,c} = \left[0.12 \times 1.13 \times (100 \times 0.0009 \times 30)^{\frac{1}{3}} + 0.15 \times 0\right] \times 300 \times 10\,840 = 613\ kN$$

with a minimum of

$$V_{Rd,c} = \left(v_{min} + k_1 \times \sigma_{cp}\right) \times b_w \times d$$

$$v_{min} = 0.035 \times k^{3/2} \times f_{ck}^{1/2} = 0.035 \times 1.13^{3/2} \times 30^{1/2} = 0.21$$

$$k_1 = 0.15$$

$\sigma_{cp} = 0$ (non-prestressed reinforcement)

$$V_{Rd,c} = (0.21 + 0.15 \times 0) \times 300 \times 10\,840 = 683\ kN$$

$V_{Ed} = 2\,780$ kN $\nleq 683$ kN $= V_{Rd,c}$ → It is necessary to design vertical shear reinforcement.

$$A_{s,rqd} = \frac{2\ 780 \times 10^3}{435} = 6\ 390\ mm^2$$

The reinforcement is placed in the distance of 0.75 × a_v = 0.75 × 5 500 = 4 125 mm from the inner edge of the support.

$A_{s,prov} = 6\ 560\ mm^2$ – 29 two-legged stirrups Ø12 mm à 150 mm

The orthogonal reinforcement for the generated transverse tension is designed according to the computational model with one inclined strut. The length of the concrete strut is h=12 000 mm, the minimum width of the strut is a=858 mm and the force of the strut is F=2 100 kN.

Transverse tension of the concrete strut is:

$$2 \times F_t = 0.5 \times \left(1 - 0.7 \times \frac{a}{h}\right) \times F = 0.5 \times \left(1 - 0.7 \times \frac{858}{12\ 000}\right) \times 2\ 100 = 998\ kN$$

Transverse tension is divided into a vertical part:

$$F_v = 998 \times cos59 = 514\ kN$$

And into a horizontal part:

$$F_h = 998 \times sin59 = 856\ kN$$

The total vertical reinforcement is designed for the force of 2 780+1.2×514=3 397 kN:

$$A_{s,rqd} = \frac{3\ 397 \times 10^3}{435} = 7\ 810\ mm^2$$

$A_{s,prov} = 9\ 274\ mm^2$ – 41 two-legged stirrups Ø12 mm à 150 mm

The horizontal reinforcement is designed to the force of 1.2×856 kN:

$$A_{s,rqd} = \frac{1.2 \times 856 \times 10^3}{435} = 2\ 362\ mm^2$$

$A_{s,prov} = 2\ 412\ mm^2$ – 24 two-legged stirrups Ø8 mm à 470 mm

→ it does not meet a requirement of the maximum bar spacing (300 mm), horizontal reinforcement is designed according to the design principles and the geometry of the deep beam

$A_{s,prov} = 3\ 517\ mm^2$ – 35 two-legged stirrups Ø8 mm à 295 mm

Note: Due to the large length of the reinforcement bars, straight bars will be used instead of stirrups. These straight bars will be complemented by U-clamps with sufficient overlap at the edges of the deep beam.

Drawings of the reinforcement are given in Annex 2.

6 Summary

The aim of this thesis was to provide an overview of computational methods for the design of reinforced concrete deep beams and to carry out a design of the selected deep beam. The prime motive in the selection of this topic was the interest in the study of D-regions and non-linear simulations of reinforced concrete structures which were included in the academic program only exceptionally.

At the beginning information from technical literature focused on a design and behavior of deep beams were summarized. These information indicate that the behavior of deep beams is different compared to shallow beams. A curve of the stress along the height of the cross-section according to elastic analysis is not linear. Deep beams do not meet the Bernoulli-Navier hypothesis of maintaining the flatness of the cross-section after the deformation.

Deep beams are very specific structures and standards valid in the Czech Republic and the United States of America provide no exact design approach but only some rules for reinforcement detailing. Both standards recommend using the strut-and-tie method. Other methods suitable for the design of deep beams can be found in technical literature. These are mainly a finite element method, a stringer-panel method and a simplified method developed by doc. Bažant.

Basic information and rules for applying the design methods were presented in the diploma thesis. The design methods were applied to the design of the selected deep beam. Obtained results were compared with the real behavior of the deep beam according to the non-linear analysis. The non-linear analysis was performed using the finite element method. The design methods differ in a rate of simplification, a computational procedure, time demands and an accuracy of results.

Findings show that the simplified method developed by doc. Bažant is very easy and fast. On the other hand, the accuracy of results is poor, and it overestimates the required area of reinforcement.

The stringer-panel method is time undemanding as well. Unfortunately, a development of this method has stopped, and it is not widespread nowadays. This situation influences the procedure of calculation. Specifically, there is no exact rule for determining a width of stringers. It is also not possible to run the non-linear analysis in SPanCAD using this method. The non-linear analysis would be helpful for the design of complicated deep beams and for checking the serviceability limit state. However, obtained results show that the accuracy of this method is very good, and it could have great potential.

The strut-and-tie method is the most common approach for the design of deep beams. The correct choice of the strut-and-tie model is essential. A problem is that there is no universal strut-and-tie model for deep beams with a specific geometry and loading layout. It is recommended to perform a linear analysis first and then to design the strut-and-tie model based on the stress distribution. The results obtained by this method were accurate, but it is quite time-consuming.

A problem can be also checking of the serviceability limit state (SLS). According to available information, it is assumed that the SLS is met if the rules for reinforcement are fulfilled. However, no standard or technical literature confirms this statement. It can be necessary to carry out the non-linear analysis using the finite element method. The non-linear analysis requires a sophisticated structural design software which is expansive, and it does not have to be accessible to all engineers.

It is not clear which values of material properties and forces should be used for the non-linear analysis. The non-linear analysis performed in this thesis assumed design values because of safety. However, the results of a simulation of the real behavior are not accurate then. It would be more suitable to use characteristic value or even mean values when using the non-linear analysis.

The thesis had a personal benefit for me in terms of gaining new knowledge in this issue. I have learned about the simplified method and the stringer-panel method which I did not know before. Thanks to this thesis I can understand the behavior of the deep beam and its design.

The non-linear analysis was a great experience for me. In the previous study, I have met the non-linear analysis only marginally. The biggest problem was to set up a correct computational model that would correspond to reality enough and not be too much time-consuming. I was able to make a suitable model for the verification of results obtained by different design methods.

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http://mi21.vsb.cz/sites/mi21.vsb.cz/files/unit/metoda_konecnych_prvku_stavebni_mechanika_interaktivne.pdf

List of used programs

Scia Engineer v18.1.54

Scia Engineer v19.1.0031 (necessary update during work)

ATENA 2D v5

AutoCAD 2015

AutoCAD Release 14

SPanCAD

Microsoft Word 2016

Microsoft Excel 2016

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