Estimation of Angular Deviations in Precise Magnetometers

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Abstract—Capabilities for calibrations of angular deviations of sensor triplets in precise magnetometers were evaluated in a 2.5-meter, triaxial Helmholtz-coil facility. The coil system is located in a magnetically quiet environment at SANS Space Science in Hermanus, South Africa. The angular calibration results obtained from the “thin-shell” calibration procedure were compared with direct measurements on a non-magnetic tilting/rotational platform. One-year expanded uncertainty of angular deviation calibrations is estimated as 6x10^-5 degrees of arc; 3x10^-4 degrees coil orthogonality is possible when doing a numerical re-calibration and correction on a short-term basis. In addition, an approach for obtaining body-to-sensor angular calibrations is presented, allowing for speed-up of the calibrations and possibly increasing their accuracy and repeatability by avoiding alignment to the coils with a laser beam and leveling.

Index Terms—Magnetic instruments, magnetometer calibrations, precision, uncertainty.

I. INTRODUCTION

Precise triaxial magnetometers require careful calibration to establish all nine parameters (gains, offsets, non-orthogonal angles) [Olsen 2003]; if the magnetometer is intended for navigation and data fusion with another physical sensor, three further parameters are needed to describe their (mutual) attitude [Primdahl 2002], [Vcelak 2009], [Figaro 2011].

SANS Space Science in Hermanus, South Africa, operates a square, 2.5 m triaxial Helmholtz coil system for magnetic sensor calibrations - see Fig. 1. With the help of a LEMI-025 magnetometer at a distance of 40 m, it is possible to suppress local magnetic field variations occurring during the calibration run due to the high homogeneity of the Earth’s magnetic field at the location (the site houses a magnetic observatory). Moreover, the on-site magnetic noise is less than 10 pT/√Hz at 1 Hz, even during the day.

The coil system is mechanically leveled and calibrated on a periodic basis; the magnetic direction of the EW axis is aligned with a reference laser.

We present our current approach of calibrating the angular deviations, the results, estimation of the uncertainty, and a novel method of estimating the body-frame related calibration.

II. MAGNETOMETER MODEL AND CALIBRATION PROCEDURE

A. Magnetometer model

To express non-orthogonalities between magnetometer axes, we use the typical model as described by Olsen [2003] - see Fig. 2.

Fig. 1. The square triaxial coil system at SANS.

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In this case, the X axis is assumed as reference, the non-orthogonal Y' axis is assumed to be in plane, only rotated by an angle u1 from the X axis; i.e. the XY (XY') plane is the reference plane. Then the Z' axis is established by two non-orthogonal angles

u_2 and u_3 deviating from the ideal Z axis orthogonal to XY plane. The effect of non-orthogonality can be then expressed with a matrix P containing the angular deviations u_1, u_2 and u_3 [Olsen, 2003]:

\[
P = \begin{bmatrix}
1 & 0 & 0 \\
-sin(u_1) & cos(u_1) & 0 \\
-sin(u_2) & sin(u_3) & \sqrt{1-sin^2(u_2) - sin^2(u_3)}
\end{bmatrix}
\]

(1)

We can then establish the magnetic field vector b_{mag} from the magnetometer output vector e_{mag} by multiplying it by the inverse non-orthogonality matrix \( P^{-1} \) and the inverse sensitivity matrix \( S^{-1} \), after subtracting the offset vector \( e_o \) in arbitrary (engineering) units.

\[
b_{mag} = [b_{magx} b_{magy} b_{magn}]^T = P^{-1}S^{-1}(e_{mag} - e_o)
\]

(2)

So far the calibration is considered to an x-axis referenced frame ("sensor frame"), which can differ from the mechanical enclosure of the magnetometer ("body frame") - see Fig. 3 - which is positioned in the frame of the coil system ("global frame").

\[
\begin{align*}
\text{SENSOR FRAME} \\
\text{GLOBAL FRAME} \\
\text{BODY FRAME}
\end{align*}
\]

Fig. 3. Definition of the magnetometer sensor frame, magnetometer body frame and global frame (=coil frame).

To be able to fully describe the measured field with reference to the magnetometer body frame, we need to add an additional rotational matrix R describing the rotation of the sensor frame to the body frame. \( R, P^{-1}, \) and \( S^{-1} \) can be combined to a single matrix A:

\[
b_{mag} = R^{-1}P^{-1}S^{-1}(e_{mag} - e_o) = A(e_{mag} - e_o)
\]

(3)

B. Calibration Procedure

The calibration procedure relies generally on solving an overdetermined system of equations (2), i.e. the \( b_{mag} \) is created by the coil system, \( e_{mag} \) is measured, and the \( R^{-1}P^{-1}S^{-1} \) matrix can be established, or even the components of \( P \) matrix (2) individually to obtain the non-orthogonal angles \( u_1, u_2 \) and \( u_3 \). The test field vector \( b_{mag} \) is usually generated with an (almost) constant magnitude but different vector orientations to cover all possible spherical angles.

This "thin-shell" calibration procedure employed with the SANSA Helmholz coil system uses the spherical harmonic analysis method (SHM), and is described in detail by Risbo [2001, 2003]. The magnetometer is currently aligned with the coil system using a laser beam aligned to the magnetic axes of the coil system, reflecting off a mirror attached to magnetometer enclosure. The resulting "sphere" of magnetic field vectors are decomposed using SHM and least squares minimization. The result is a 3x3 matrix related to magnetometer body-frame containing the \( A = R^{-1}P^{-1}S^{-1} \) matrix.

To obtain non-orthogonal angles from the \( A \) matrix, we used "QR" decomposition to obtain the orthogonal and upper triangular matrix [Anderson 1992]. To obtain the \( P^{-1} \) components, we also used the "scalar-calibration" procedure described in Olsen [2003] on the same thin-shell data (omitting the \( R \) matrix). In this method, the minimization criteria to find the \( P^{-1}S^{-1} \) matrix (or its components) is the root-mean-square-error RMSE between the scalar magnitude of the applied vector in the coil system \( b_{coil} \) and the scalar magnitude of estimated vector \( || b_{mag} || \):

\[
\text{RMSE} = \frac{1}{N} \sum_{i=1}^{N} (|| b_{mag}(i) || - b_{calc}(i))^2
\]

(4)

We verified our results with a different calibration procedure, which is described in Brauer [1999], and Merayo [1999] and we did not find any significant difference in the results of these methods.

III. ANGULAR CALIBRATIONS

A. Calibration Results

The results were obtained on a single magnetometer type LEMI-011B [ISR Lviv 2019], serial numbers 319 and 379, respectively – see Table 1. The angles, obtained by the method of Olsen [2003], were compared to a direct measurement using an Askania circle with about ±1.5° = 2.5×10^{-2} degrees accuracy, and with a two-axis tilting jig with modified optical encoders (Heidenhain ERO-1324-3600, estimated total system accuracy about ±100'' = 3×10^{-2} degrees) – see Fig. 4. The direct measurements were done by minimizing response at the respective axis when energizing the orthogonal coil (by rotation/tilting), and then doing the same for the second axis in pair.

\[
\begin{array}{c|c|c|c|c}
\text{LECNO} & \text{u_1 [°]} & \text{u_2 [°]} & \text{u_3 [°]} & \text{remark} \\
\hline
\text{LEMIO11B #319} & \text{08/2013} & -1.71 & -0.51 & 4.94 & 5 years old \\
22/10/2018 & -1.68 & -0.47 & 4.83 & \\
22/10/2018 & -1.64 & -0.53 & 4.75 & \text{direct meas.} \\
24/10/2018 & -1.68 & -0.47 & 4.84 & \\
\text{LEMIO11B #379} & \text{05/2017} & 1.39 & -1.42 & -0.84 & 1 year old \text{direct} \\
15/10/2018 & 1.38 & -1.42 & -0.85 & \text{coils misaligned} \\
18/10/2018 & 1.39 & -1.41 & -0.87 & \\
18/10/2018 & 1.33 & - & & \text{direct} \\
19/10/2018 & 1.41 & -1.36 & -0.94 & \\
20/10/2018 & 1.42 & -1.37 & -0.93 & \\
22/10/2018 & 1.41 & -1.37 & -0.94 & \\
22/10/2018 & -1.32 & -0.90 & & \text{direct meas.} \\
\end{array}
\]

We can see that the short-time spread of calculated angles of
about ±0.01° (LEMI-011B #319) increases up to 0.1° for the 5-year period, which is more than anticipated. Also, the comparison to the direct measurement was within 0.1°, although the instruments are by far more accurate. In the next section, we will try to derive the sources of this uncertainty. It is evident that the coil calibration is an issue, which can be seen in the LEMI-011B #379 results - on 19/10/2018 the coils have been misaligned accidentally, which manifested itself in the angular calibration results.

We could verify the coil misalignment by doing a subsequent calibration of the coil system with an Overhauser magnetometer using a modified scalar-calibration procedure [Olsen 2003]. Further details are found in [Risbo 2003, p. 677]. The non-orthogonality of the coils was up to 6×10⁻⁴ degree and could be suppressed below 3×10⁻⁴ degree with the abovementioned recalibration - see Table 2.

### Table 2. Result of coil system re-calibration

<table>
<thead>
<tr>
<th></th>
<th>u₁ [°]</th>
<th>u₂ [°]</th>
<th>u₃ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>before cal.</td>
<td>10/2018</td>
<td>9.7×10⁻³</td>
<td>6.4×10⁻³</td>
</tr>
<tr>
<td>after cal.</td>
<td>07/2019</td>
<td>2×10⁻⁴</td>
<td>-3×10⁻⁴</td>
</tr>
</tbody>
</table>

### B. Estimating the Uncertainty

To establish the uncertainty of our calibration, we performed fifteen consecutive test runs and calculations on a single sensor - the space-qualified LEMI-011S (Fig. 1). The resulting histogram for the estimation of the three angles is shown in Fig. 5 - standard deviation was found below 6.6×10⁻⁴ degree. As the measurements were performed over a 12-hour span, these statistics also cover the effects of on-site noise and imperfections of the Earth's field cancellation in the coil system. We can consider the standard deviation as a type-A measurement uncertainty $U_A$ [JGCM 2008].

As shown previously, we can experience coil non-orthogonality and its instability - see Table 2. This would be the source of type-B calibration uncertainty $U_B$ for both the thin-shell method and direct measurement. The combined uncertainty $U$ (k=2, 95% probability coverage) is then [JGCM 2008]:

$$U = 2 \sqrt{U_A^2 + U_{B coils}^2 + U_{Binst}^2}$$  \hspace{1cm} (5)$$

where $U_{B coils}$ is the type-B measurement uncertainty due to coils calibration, $U_A$ is the standard deviation of the results calculated above and $U_{Binst}$ is the uncertainty of the instrument used in the direct comparison if applicable (either Askania or tilting device).

For numeric calculations from the thin-shell run, we assume the worst-case observed standard deviation being $U_A$ and coil misalignment with assumed triangular distribution being $U_A$:

$$U_{calc} = 2\sqrt{(6.6 \times 10^{-4})^2 + \left(\frac{0.04}{3}\right)^2} = 5.2 \times 10^{-2}$$  \hspace{1cm} (6)$$

For the direct measurement, where we have the instrument uncertainty in addition (assuming a uniform distribution of scale error), thus we can write:

$$U_{mean} = 2\sqrt{(6.6 \times 10^{-4})^2 + \left(\frac{0.04}{3}\right)^2 + \left(\frac{0.03}{3}\right)^2} = 6.2 \times 10^{-2}$$  \hspace{1cm} (7)$$

In Fig. 6, the calculation and measurement results for $u_2$ on LEMI-011B #379 are plotted, together with uncertainties. The other angles are not displayed because of similarity of the results. We see that our measurements of $u_2$ fit well within the established uncertainty.

### C. Statistics on a single magnetometer type

We demonstrate the necessity of angular calibrations on the example of a set of 57 magnetometers (LEMI-011B). From the results in Fig. 7 we see that the datasheet value [LEMI011B] of max. 2° non-orthogonality is met within one standard deviation. The maximum observed value was 44.5°. This is due to the fact that the precise fluxgate magnetometer sensors are mostly hand-assembled.
IV. BODY-FRAME RELATED CALIBRATIONS

A. Current method

The alignment of the DUT to the global (coil frame) is performed with a laser aligned to the magnetic axis of the horizontal coil and a precise mirror glued to the magnetometer body. This requires a skilled operator, thus we propose a novel method for body-frame calibration.

B. Proposed procedure

The procedure relies on magnetometer calibration in 4 (or even just 3) particular attitudes. The first sensor attitude can be arbitrarily chosen. The three remaining are attitudes with the sensor rotated along its body axis X, Y, Z respectively - see Fig. 8. In each step, a thin-shell calibration is performed and a rotation matrix \( R \) is obtained from the calibration matrix result (by QR decomposition). The main advantage of using the reference plane and block to perform the rotation is that once the 4-step method is executed for relative rotations:

\[
R_{SB} = R_{BC} R_{SB} \]

then rotated according to Fig. 8, the plane - see Fig. 9. The magnetometer in its square enclosure was then calibrated at the same time. After that, only 1-step calibration in one magnetometer, the reference plane/block (in arbitrary attitude) is calibrated at the same time. The spread of rotation axes are perpendicular. Due to arithmetic imprecision and manufacturing precision, \( 6 \times 10^{-4} \) degrees would be possible.

The last row are the differences between alignment to the coil axes as obtained by the current method (leveling and laser alignment) and the new method. We see that both methods agree within 0.1°.

V. CONCLUSION

We show that our one-year expanded (k=2) uncertainty of angular deviation calibrations is about \( 6 \times 10^{-2} \) degrees of arc which we found as the coil-system non-orthogonality, which can improve down to \( 3 \times 10^{-3} \) degrees with a numerical coil re-calibration. The numerical results were comparable with direct measurements within this uncertainty. We also see that with hand-assembled fluxgate magnetometers, it is crucial to calibrate the orthogonal angles.

The proposed method to obtain body-frame related magnetometer calibration was verified. Its advantage is not only time-saving, but avoiding of tilting and leveling of the device under test, to align it with the coil system, which brings further uncertainties. Even with a non-ideal reference block and magnetometer enclosure, the body-frame referenced calibration resulted in a spread of \( \pm 0.1° \). Also, the agreement to the current procedure with leveling and laser alignment is within \( 0.1° \), which also corresponds to inclinometer resolution. In order to improve the results, a more precisely machined reference block and reference enclosure is required – with 10 \( \mu \)m manufacturing precision, \( 6 \times 10^{-3} \) degrees would be possible.
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