CZECH TECHNICAL UNIVERSITY IN PRAGUE

FACULTY OF MECHANICAL ENGINEERING

BACHELOR THESIS

COMPARISON OF SELECTED NOZZLES FOR HYBRID ROCKET ENGINE

Prague, 2019

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BACHELOR'S THESIS ASSIGNMENT

I. Personal and study details

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Study program: **Theoretical Fundamentals of Mechanical Engineering**
Branch of study: **No Special Fields od Study**

II. Bachelor's thesis details

Bachelor's thesis title in English:

Comparison of selected nozzles for hybrid rocket engine

Bachelor's thesis title in Czech:

Srovnání vybraných trysek hybridního raketového motoru

Guidelines:

To pursue a thesis:
1) Prepare a general review of rocket engine nozzles with emphasis on the hybrid rocket engines
2) For the particular hybrid rocket engine, design a set of nozzles with various geometries
3) Characterize the influence of these nozzles on the general performace of the engine

Bibliography / sources:

G. P. Sutton, O. Biblarz: Rocket propulsion elements, 8th ed., Wiley, 2010, ISBN 978-0-470-08024-5
M. Chiaverini, K. Kuo: Fundamentals of Hybrid Rocket Combustion and Propulsion - AIAA, 2007, ISBN 978-1-56347-703-4
+ další literatura dle doporučení vedoucího / additional sources recommended by supervisor

Name and workplace of bachelor's thesis supervisor:

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Date of bachelor's thesis assignment: **30.04.2019** Deadline for bachelor thesis submission: **02.08.2019**

Assignment valid until: ____________

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III. Assignment receipt

The student acknowledges that the bachelor's thesis is an individual work. The student must produce his thesis without the assistance of others, with the exception of provided consultations. Within the bachelor's thesis, the author must state the names of consultants and include a list of references.

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Abstract:

This thesis deals with nozzles for a particular small hybrid rocket engine. The thesis covers theoretical calculation, simulation, technical design and manufacturing of nozzles with various geometries. The impact of these nozzles on the performance of the rocket engine is predicted and compared.

Anotace:

Tato práce se zabývá tryskami pro daný malý hybridní raketový motor. Práce pokrývá teoretické výpočty, simulace, návrh konstrukce a výrobu trysek s různou geometrií. Vliv těchto trysek na výkony raketového motoru je vypočten a porovnán.

Declaration

I declare that this thesis has been composed solely by myself, under the guidance of thesis supervisor Mgr. Jaroslav Kousal. Except where stated otherwise by reference or acknowledgment, the work presented is entirely my own. I have no objections against using this educational work in the sense of § 60 of the Act.no.121/2000 Sb.
(Law of authorship).

……………………………….

Acknowledgement

I would first like to thank my supervisor Mgr. Jaroslav Kousal for his great deal of support and guidance. I would also like to thank Vítek Putna for his assistance and permission to use his rocket engine. I'm also thankful to Mr.Fíl for manufacturing the nozzles.

I am forever grateful to my family for their undying support and assistance. You are always there for me. I would also like to thank my friends for their support and happy distraction for my mind outside of my studies.

Contents

1 Introduction

The Nozzle is a part of a rocket engine which generates thrust. It is used to convert the thermal energy of the gas flowing through it into kinetic energy, thereby accelerating the flow and creating thrust. Nozzles are categorized by their geometry/configuration. The configuration of the nozzle influences the performance of the rocket engine. There are several different nozzle configurations in use today.

In this thesis, two different nozzle configurations are considered. The first one is a conical nozzle. It is the oldest and simplest configuration. It is easy to fabricate and is widely in use. Three sets of conical nozzles are considered, rated for 50,55 and 60N. They have been fabricated for the given hybrid rocket engine.

The second nozzle is the Bell-shaped or Contour nozzle. It is the most common nozzle in use today. Since it is hard to fabricate, the CFD software Ansys Fluent is used to simulate its flow conditions.

This thesis aims to study the influence of nozzle geometry on the performance of the rocket engine. Comparison of the rocket engine parameters with the nozzles will be made for the above.

2 Theory
The theory section was written using [5].

2.1 Rocket engines
The only method to get to space today is through rockets. It carries its propellant and expels it at high velocities to produce thrust. Rocket engines are reaction engines, they produce thrust per newtons third law. Most Rocket engines use stored propellants as reaction mass for producing a high-velocity propulsive jet of fluid. Non-combusting rocket engines such as cold gas thrusters and nuclear thermal rockets also exist. Compared to other types of jet engines, rocket engines are the lightest and have the highest thrust but are the least efficient.

Principle

Rocket engines produce thrust by expelling exhaust fluids that have been highly accelerated by the nozzle. The fluid is usually a gas created by high-pressure combustion of the propellants, consisting of the oxidizer and fuel components in the combustion chamber. As the gases expand in the nozzle, they are accelerated to supersonic speeds by the nozzle, which pushes the rocket in the opposite direction. Combustion is most frequently used, as high temperatures and pressure are desirable for maximizing performance.

Propellant

Rocket propellant is the reaction mass of the rocket. It is usually stored in some form of a propellant tank or within the combustion chamber itself, before being ejected from the nozzle in the form of a fluid jet to produce thrust. Chemical rocket propellants are most commonly used, which undergo exothermic chemical reactions which produce hot gas which is used by a rocket for propulsive purposes.

Types

Rockets can be briefly subdivided into 2 categories. Thermal rockets and Chemical rockets. Thermal rockets use an inert propellant, as opposed to being internally heated by a reaction. Although not widely used for propulsion today, with vast amounts of research being done, thermal rockets are set to become mainstream in the coming decade. Chemical rockets are powered by exothermic reduction-oxidation chemical reactions of the propellant. The rocket used in this thesis is a chemical rocket.

2.2 Chemical Rocket Engines
Chemical rockets use fuel and an oxidizer as propellants. As the propellants react in the combustion chamber, the chemical reaction produces hot gases at high pressures. It is the ejection of these rapidly expanding gases that produce thrust.

Chemical rockets can be grouped by the phase of its propellant. Solid rockets use propellant in the solid phase, liquid fuel rockets use propellant in the liquid phase, gas fuel rockets use propellant in the gas phase, and hybrid rockets use a combination of solid and liquid or gaseous propellants.

2.2.1 Solid-fuel rockets

A solid-propellant rocket uses solid fuel and oxidizer. The propellants are already stored in the combustion chamber or case. The solid propellant is called the grain and it contains all the chemical elements for complete combustion. The solid grain mass burns in a predictable fashion to produce exhaust gases. The grain is packed in the combustion chamber with a cylindrical hole in the middle. Once the igniter combusts the surface of the propellant, the cylindrical hole acts as the combustion chamber.

Solid rockets cannot be shut down once ignited but have a simple construction and are cheaper than liquid rockets. They can also be stored for a long duration without much propellant degradation and are hence used in military application extensively.

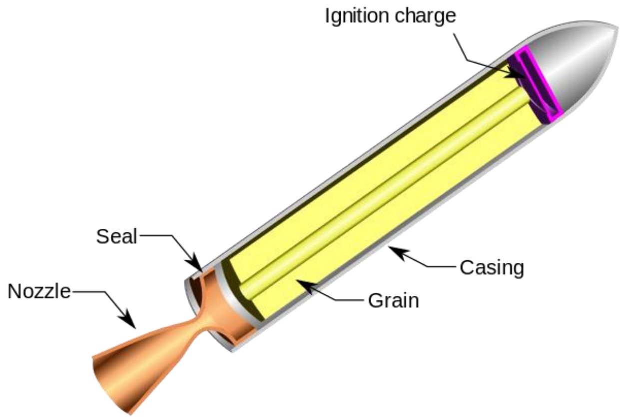

Figure 1 Solid rocket engine schematics [1]

2.2.2 Liquid fuel rockets

A liquid rocket engine uses liquid propellants which are fed under pressure from separate tanks to the thrust chamber. As they are sprayed into the combustion chamber through injection nozzles, they rapidly mix and react before being ejected. Liquid rocket engines are more complicated but offer several advantages over solid rocket engines. They can be throttled and shut down on command.

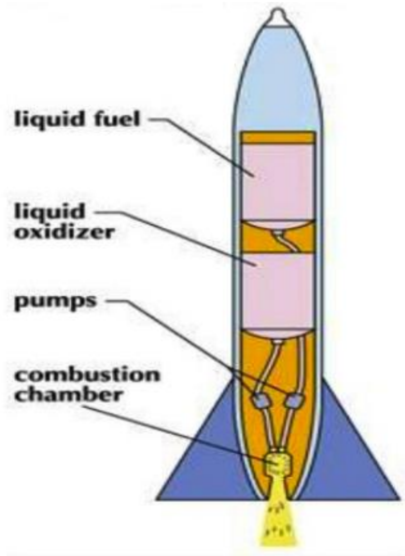

Figure 2 Liquid rocket engine schematics [2]

2.2.3 Hybrid Rocket Engine

Hybrid propellant rocket propulsion systems use both liquid and solid propellant storage. Hybrid rocket engines avoid some disadvantages of solid rockets like the dangers of propellant handling while also avoiding some disadvantages of liquid rocket engines like their complexity. They can be shut down and throttled like liquid rocket engines. Generally, hybrid rocket engines have the liquid propellant as the oxidizer and the solid propellant as the fuel because solid oxidizers are more dangerous and lower performing than liquid oxidizers.

The hybrid rocket engine used in this thesis uses Delrin (Polyoxymethylene) as the solid fuel and O_2 as the liquid oxidizer.

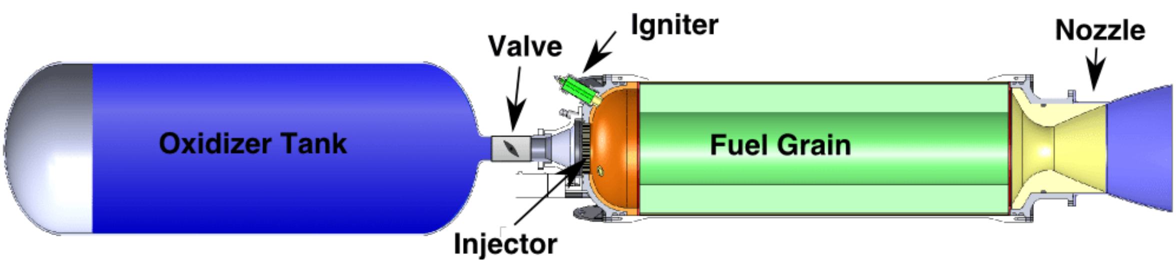

Figure 3 Hybrid rocket engine schematics [3]

2.3 Rocket engine parameters

2.3.1 Thrust

Thrust is the force produced by the rocket acting at the vehicle's center of mass. It is a reaction force, caused by the ejection of propellant at high velocities.

$$F = \dot{m}v_2 + (p_2 - p_3)A_2 \qquad (1)$$

The first term in the momentum thrust. It is given by the product of the propellant mass flow rate and the exhaust velocity. The second term represents the pressure thrust. It consists of the product of the difference between the gas pressure at the exit of the nozzle and the ambient pressure, and the cross-sectional area of the nozzle exit.

The subscript '1' represents the beginning of the nozzle, 't' represents the throat or section of minimum area of the nozzle, '2' represents the end of the nozzle and '3' represents the ambient conditions outside the nozzle.

Rocket nozzles are generally designed so that the exhaust pressure is equal or slightly greater than the ambient pressure. When the exhaust pressure is equal to the ambient pressure, the thrust is represented only by the momentum thrust.

$$F = \dot{m}v_2 \tag{2}$$

This condition isn't achievable throughout the trajectory of the rocket since the ambient pressure varies as a function of the altitude. Since the ambient pressure decreases with increasing altitude, the thrust and specific impulse will increase as the vehicle reaches higher altitudes.

Max thrust is achieved in the vacuum of space where p3=0.

$$F = \dot{m}v_2 + p_2A_2 \tag{3}$$

Nozzles are designed so that the exhaust pressure will equal the ambient air pressure at a point at or above sea level. For a nozzle of fixed geometry, this point is referred to as nozzle operation at optimum expansion ratio.

2.3.2 Exhaust Velocity

It is the velocity, relative to the rocket, at which the exhaust gases leave the nozzle of the rocket engine.

$$c = v_2 + (p_2 - p_3)\frac{A2}{\dot{m}} \tag{4}$$

where v2 is the average actual nozzle exhaust velocity of the propellant gases. When p2=p3, the value of c equals v2. But even when p2 ≠ p3 and c ≠ v2, the second term is relatively small and hence the effective exhaust velocity is always close in value to the actual exhaust velocity. At the surface where c=v2, the thrust can be written as

$$F = \dot{m}c \tag{5}$$

2.3.3 Specific Impulse

Specific Impulse is a measure of how effectively a rocket uses propellant. A propulsion system with a higher specific impulse uses the mass of the propellant more effectively in creating forward thrust. It implies that the rocket is more effective at gaining altitude, distance, and velocity.

$$I_s = \frac{c}{g} \tag{6}$$

Where I_s is the specific impulse (s),

c is the average exhaust velocity along the axis of the engine (m/s),

g is the acceleration due to gravity (m/s^2)

Rocket Engine	Manufacturer	Specific Impulse
Merlin	SpaceX, USA	282s
RD-180	NPO Energomash, Russia	311s
SSME	Rocketdyne, USA	366s
Raptor	SpaceX, USA	334s
Vikas-2	ISRO, India	290s

Table 1: The specific impulse of a few commercial rocket engines

2.4 Nozzle theory

A nozzle is used in a rocket engine to expand and accelerate the combusted propellant gases so that the exhaust gases exit at supersonic/hypersonic velocities. After the complete combustion takes place, the nozzle accelerates the flow of gases to supersonic velocities by converting the thermal energy of the flow to kinetic energy. The most commonly used nozzle is the de Laval nozzle. It consists of a diverging section after the combustion chamber where the velocity of the flow is subsonic (M<1), the section of minimum area called the throat of the nozzle where the flow in sonic (M=1) and the diverging section in which the flow is supersonic (M>1).

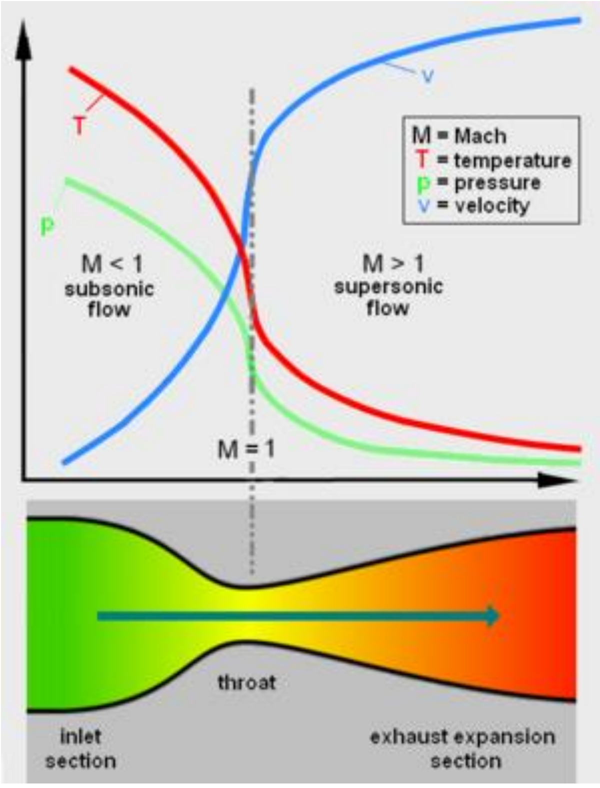

Figure 4 Cross section of a de Laval nozzle [4]

When designing a nozzle, ideal rocket parameters are used. The measured actual parameters turn out to be between 1 and 6% below the ideal calculated values. “An ideal rocket propulsion unit is defined as one for which the following assumptions are valid:

1. The working fluid (which usually consists of chemical reaction products) is homogeneous in composition.

2. All the species of the working fluid are treated as gaseous. Any condensed phases (liquid or solid) add a negligible amount to the total mass.

3. The working fluid obeys the perfect gas law.

4. There is no heat transfer across any and all gas-enclosure walls; therefore, the flow is adiabatic.

5. There is no appreciable wall friction and all boundary layer effects may be neglected. 6. There are no shock waves or other discontinuities within the nozzle flow.

7. The propellant flow rate is steady and constant. The expansion of the working fluid is uniform and steady, without gas pulsations or significant turbulence.

8. Transient effects (i.e., start-up and shutdown) are of such short duration that may they be neglected.

9. All exhaust gases leaving the rocket nozzles travel with a velocity parallel to the nozzle axis.

10. The gas velocity, pressure, temperature, and density are all uniform across any section normal to the nozzle axis.

11. Chemical equilibrium is established within the preceding combustion chamber and gas composition does not change in the nozzle (i.e., frozen composition flow).

12. Ordinary propellants are stored at ambient temperatures. Cryogenic propellants are at their boiling points."[5]

2.5 Fluid parameters

Specific heat

The design of the nozzle depends predominantly on the specific heat of the gas. It is the ratio of the specific heat at constant pressure C_p and the specific heat at constant volume C_v.

$$k = C_p/C_v \tag{7}$$

$$C_p - C_v = R/J \tag{8}$$

Where,

J is the mechanical equivalent of heat which is utilized only when thermal units(eg btu and calorie) are mixed with mechanical units (eg joule). In SI units (kg, m, s) its value is one.

R is the gas constant. It's the ratio of the universal gas constant R' and the molecular mass ɱ of the gas mixture.

$$R = \frac{R'}{ɱ} \tag{9}$$

$$R' = 8314.3 \text{ J/Kg–mol–K}$$

For any isentropic flow process, the following relation holds between any two sections of the nozzle x and y

$$\frac{T_x}{T_y} = \frac{p_x}{p_y}^{\frac{(k-1)}{k}} = \frac{v_y}{V_x}^{k-1} \tag{10}$$

Mach number

The Mach number M is a dimensionless quantity and is the ratio of the flow velocity v to the local acoustic velocity a.

$$M = \frac{v}{a} \tag{11}$$

Where a = $\sqrt{\mathrm{kRT}}$

$$M = \frac{v}{\sqrt{kRT}} \tag{12}$$

When the Mach number is one, the flow is said to be sonic. This is an important parameter while determining the diameter of the nozzle throat as we'll see later. Mach numbers less than one corresponds to subsonic flow while Mach numbers greater than one corresponds to supersonic flow.

The ratio between the cross-sectional area of two arbitrary locations along the nozzle can be expressed in terms of the Mach number.

$$\frac{A_y}{A_x} = \frac{M_x}{M_y}\sqrt{\{\frac{1+\left[\frac{k-1}{2}\right]{M_y}^2}{1+\left[\frac{k-1}{2}\right]{M_x}^2}\}(k-1)/(k+1)} \tag{13}$$

At the throat of the nozzle, where $A_x = A_t$ and $M_x = 1$, the equation can be written as

$$\frac{A_y}{A_t} = \frac{1}{M_y}\sqrt{\{\frac{1+\left[\frac{k-1}{2}\right]{M_y}^2}{\left[\frac{k+1}{2}\right]}\}(k-1)/(k+1)} \tag{14}$$

Velocity

The velocity of the exiting gas is a function of the specific heat k, pressure ratio p1/p2, the temperature of the nozzle inlet T_1 and the gas constant R.

$$V_2 = \sqrt{\frac{2k}{k-1} RT_1 [1-(\frac{p_2}{p_1})^{\frac{k-1}{k}}]} \tag{15}$$

$$V_2 = \sqrt{\frac{2k}{k-1}\frac{R'T_1}{\mathfrak{m}}[1-(\frac{p_2}{p_1})^{\frac{k-1}{k}}]} \tag{16}$$

The maximum value of the exhaust velocity is obtained in the vacuum of space with an infinite nozzle expansion

$$(V_2)_{max} = \sqrt{2kRT_1/(k-1)} \tag{17}$$

2.6 Nozzle expansion area ratio

The most common nozzle in use today is the de Laval nozzle. It consists of a converging section, the section of minimum area called the throat of the nozzle and the diverging section.

The ratio of the nozzle exit area A_2 to the throat area A_t is called the nozzle expansion ratio. It represents an important nozzle parameter.

$$\epsilon = A_2/A_t \tag{18}$$

Rocket Engine	Manufacturer	Nozzle Expansion Ratio
Merlin	SpaceX, USA	16
RD-180	NPO Energomash, Russia	37
SSME	Rocketdyne, USA	69
Raptor	SpaceX, USA	40
Vikas-2	ISRO, India	13.9

Table 2: The nozzle expansion ratio of a few rocket engines

2.7 Nozzle operation

Expansion is the process that converts the heat and pressure of combustion into kinetic energy. The flow expands against the walls of the nozzle to create a force that pushes the vehicle forward. The behavior of the expansion process is mainly dictated by the exhaust pressure and the ambient pressure. In an ideal nozzle, the exit pressure of the nozzle p_2 will be equal to the ambient pressure of the atmosphere p_3. Unfortunately, the nozzle cannot always be ideally expanded. This is because the atmospheric pressure decreases with increase in altitude. Hence nozzles are designed to be optimum only at one altitude but with minimum losses at lower or higher altitudes. These losses result from the fact that the exhaust pressure will either be higher (At high altitudes) or lower (At low altitudes) than the atmospheric pressure.

2.7.1 Over Expansion

In an over-expanded nozzle, the ambient pressure is greater than the exhaust pressure. A_t higher external pressures, flow separation begins to take place inside the divergent portion of the nozzle. The diameter of the exiting supersonic jet will be smaller than the nozzle exit diameter. The separation point of the flow and the nozzle wall travels upstream with increasing external pressure. The separation of flow reduces the efficiency of the nozzle.

2.7.2 Under Expansion

In an under-expanded nozzle, the gases discharge at an exit pressure higher than the ambient pressure. This takes place when the exit area A_2 is too small for optimum expansion. Gas expansion is therefore incomplete within the nozzle, and further expansion will take place outside the nozzle exit since the nozzle exit pressure is higher than the atmospheric pressure. This reduces the efficiency of the rocket engine because the external expansion does not exert any force on the nozzle wall. This energy can therefore not be converted into thrust and is lost. Ideally, the nozzle should have been longer to capture this expansion and convert it into thrust.

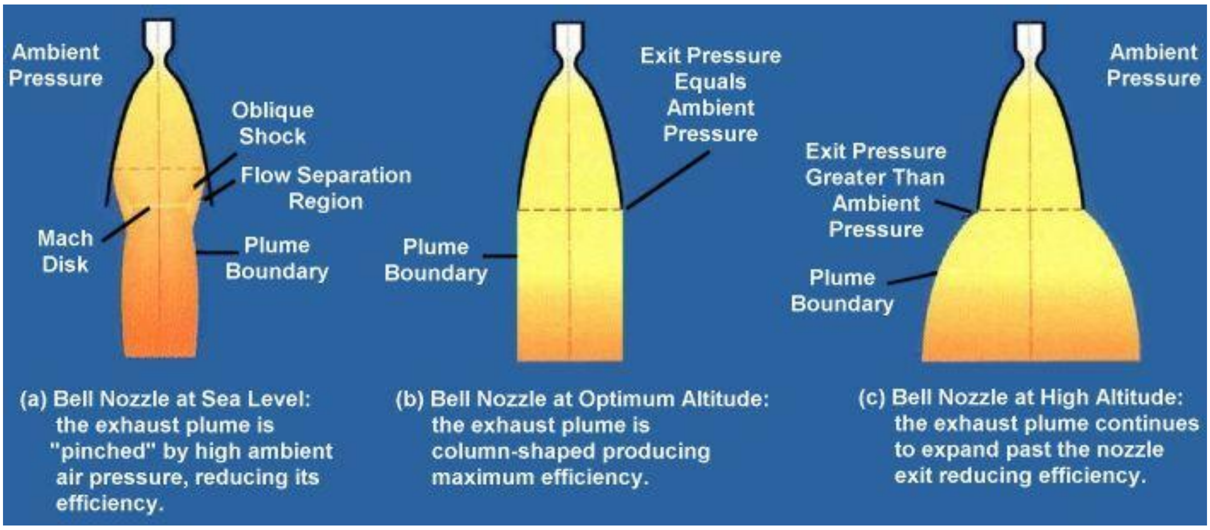

Figure 5 Nozzle operation [6]

2.8 Nozzle Configuration

There are various nozzle configurations available today. Nozzles typically have a converging section, a throat, and a diverging section. The converging section of the nozzle between the chamber and the throat does not significantly affect nozzle performance. The subsonic flow can easily be turned with any given radius, cone angle, and nozzle inlet shape. The throat contour is also not very critical to performance and any smooth shape is acceptable. The difference in nozzle configurations arises in the diverging supersonic-flow section. In general, the internal wall surfaces should be smooth and without any holes, sharp edges or protrusions.

The two most widely used nozzles are taken into consideration for this thesis.

2.8.1 Conical Nozzle

In early rocket engine applications, the conical nozzle was used almost extensively. It is easy to fabricate and is still used in many rocket engines today. A theoretical correction factor λ must be applied to the nozzle exit momentum in any ideal rocket propulsion system using a conical nozzle. This factor is the ratio between the momentum of the gases exhausting with a finite nozzle angle 2α and the momentum of an ideal nozzle with all gases flowing in the axial direction. For ideal rockets λ = 1.0 .

$$\lambda = \frac{1}{2}(1 + \cos \alpha) \qquad (19)$$

Small nozzle divergence angles may allow most of the momentum to remain axial and thus produce high specific impulses, but they result in long nozzles introducing performance penalties and increasing the vehicle mass. Larger divergence angles give shorter, lightweight designs but their performance becomes unacceptably low.

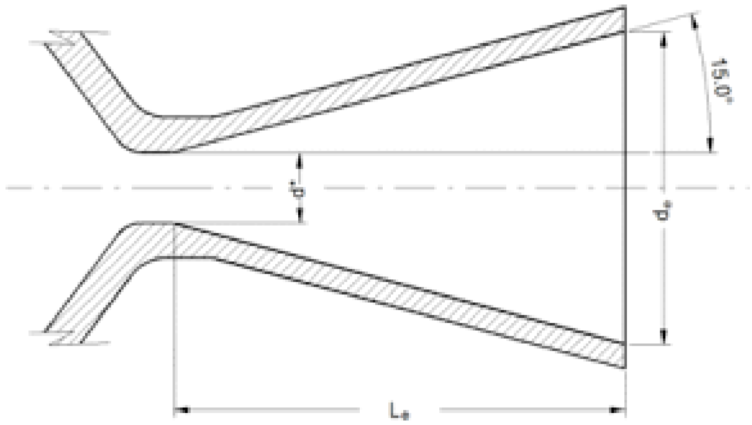

Figure 6 Cross-section of a conical nozzle [7]

2.8.2 Bell-Shaped nozzle

Bell-shaped or contour nozzles are the most common nozzles in use today. They have a high angle expansion section (20° to 50°) immediately downstream of the nozzle throat followed by a gradual reversal of nozzle contour slope so that at the nozzle exit the divergence angle is small, usually less than a 10° half angle. Gas expansion in supersonic bell nozzles is more efficient than in simple straight cones of similar area ratio and length because wall contours can be designed to minimize losses.

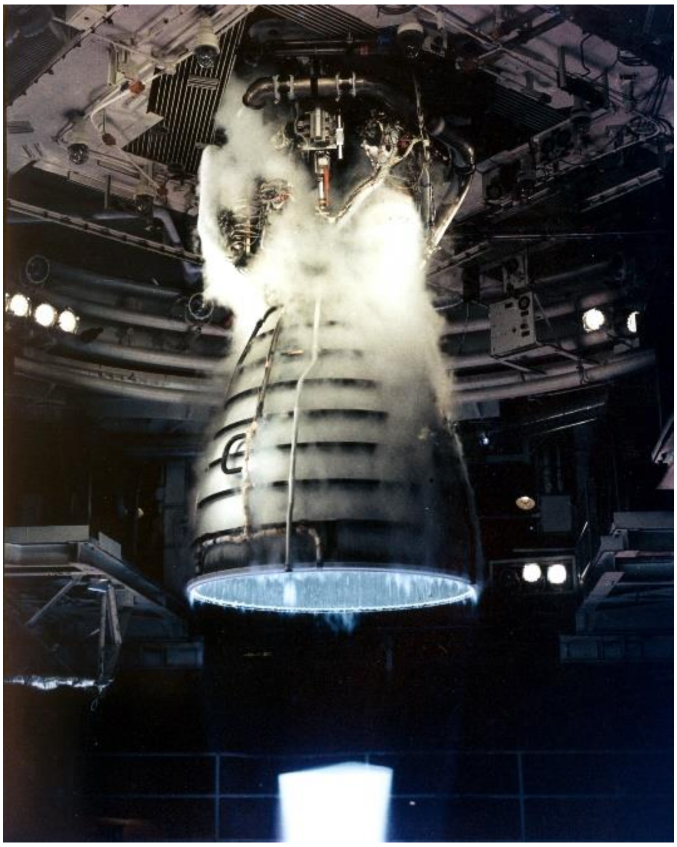

Figure 7 A Contour nozzle under operation[8]

2.9 Boundary Layers

Nozzles always develop thin viscous boundary layers adjacent to the inner walls where gas velocities are much lower than in the free-stream region. Immediately next to the wall, the flow velocity is zero. Beyond the wall, the boundary layer may be considered as being built up of successive thin layers of increasing velocity until the free-stream is reached. The low-velocity flow region next to the wall is always subsonic and laminar while the high-velocity region of the boundary layer is supersonic. The temperature at some locations in the boundary layer can be higher than the free-stream temperature because of the conversion of kinetic energy into thermal energy that occurs when the velocity is slowed down and by heat created by viscous effects. The flow layer right next to the wall is cooler because of heat transfer to the wall. The boundary layers affect the rocket performance, particularly in applications with very small nozzles and with relatively long nozzles with high nozzle area ratios, where a comparatively high proportion of the total mass flow (2 to 25%) can be in the lower-velocity region of the boundary layer.

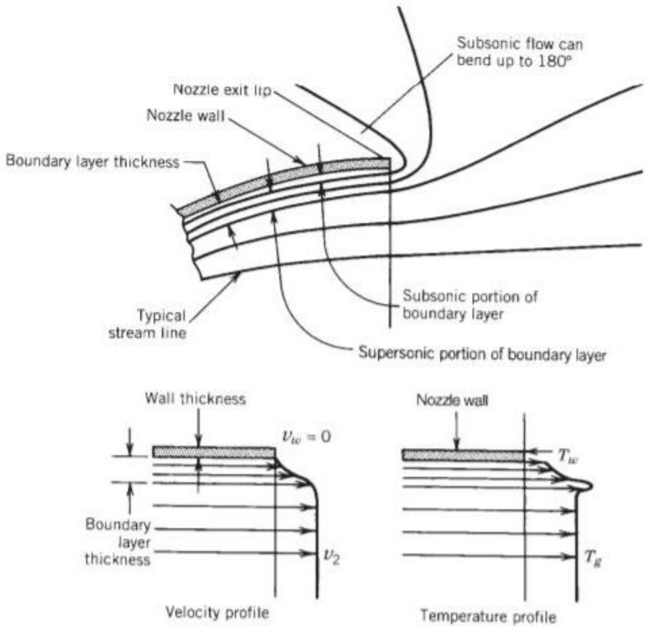

Figure 8 Boundary layers of a nozzle [5]

3 Practical Part

This section of the thesis includes the mathematical calculation required for the design of the nozzle as well as the simulations performed to analyze the performance of the nozzles.

3.1 Description of the rocket engine

The rocket used for this thesis utilizes hybrid rocket propulsion. The combustion chamber has a diameter of 25mm and a length of 200mm. It is made of stainless steel. It has an entry valve for the oxidizer and an injector. The fuel grain sits in the combustion chamber beside the ignitor. The nozzle is attached to the combustion chamber externally with the help of four M3 screws.

Figure 9 Rocket engine in the test setup

Figure 10 Top view of the hybrid rocket engine

The rocket engine uses solid fuel and liquid oxidizer. The oxidizer used is gaseous oxygen O_2. The solid fuel used is Polyoxymethylene also called acetal and Delrin. It has a density of 1.41 g/cm^3. It is a synthetic polymer and is used primarily in parts requiring high stiffness, low friction, and high dimensional stability. It is in the shape of a hollow cylinder with external diameter 25mm and internal diameter 10mm. The rocket has an oxidizer to fuel (O/F) ratio of 5:1.

3.2 Initial Input

Gaseq is a chemical equilibrium program for windows. It was used to calculate the adiabatic temperature and composition at constant pressure. It took into consideration the reactants in the reaction, the pressure p_1 as well as the oxidizer to fuel ratio of the rocket engine. The calculation for determining pressure p_1 has been included in the next section.

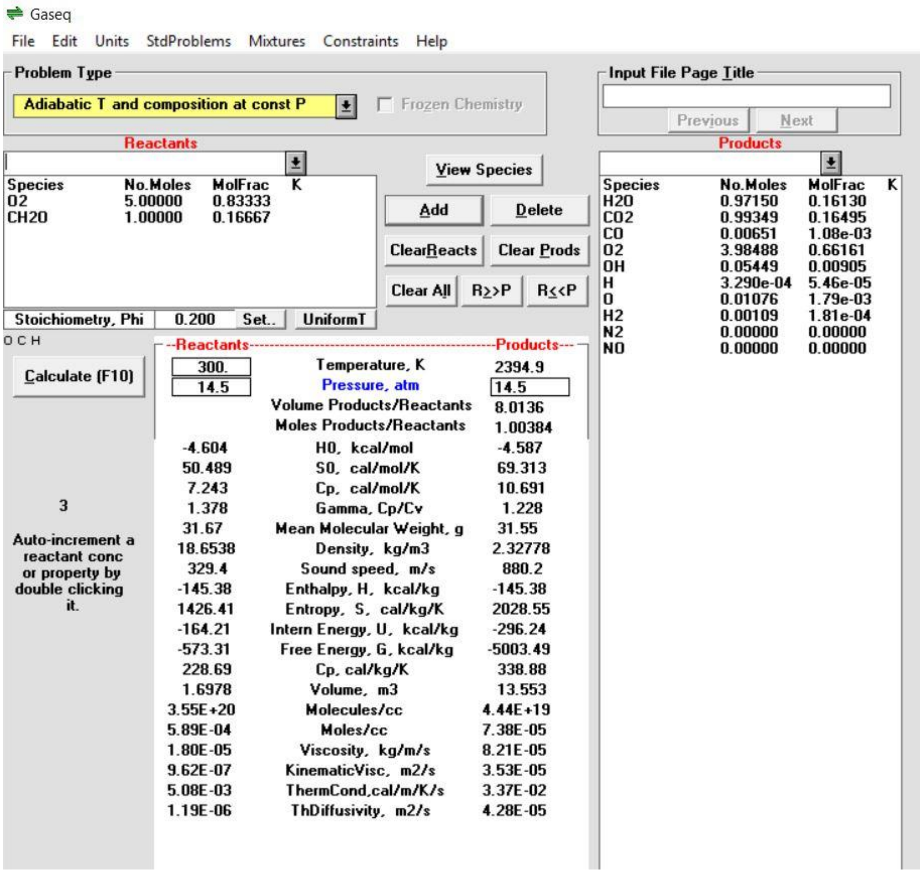

Figure 11 Gaseq program output

The significant quantities it generates are given in the table below:

Quantity	Magnitude	Unit
Temperature T_1	2394.9	K
Specific Heat, k	1.228	-
Mean Molecular Weight, ɱ	31.55	g
Specific heat at constant pressure, C_p	1417.87	J/Kg/K
Thermal Conductivity	0.141	J/m/K/s

Table 3: Output values from Gaseq

3.3 Calculation

This section includes the calculation required to determine parameters such as the nozzle throat area A_t, combustion pressure p_1 and the mass flow rate $\dot{m}$.

We have with us the following quantities:

Combustion temperature T_1 = 2394.9 K

Ambient Pressure $p_3 = p_2$ = 101325 Pa

Specific heat k = 1.228

Thrust F = 60 N

The Gas constant R is calculated from the Universal Gas Constant R' and Molecular weight ɱ.

$$R = \frac{R'}{ɱ}$$

$$= \frac{8314.3}{31.55}$$

$$= 263.53 \text{ J/Kg-K}$$

The total mass flow rate is the sum of the mass flow rate of the oxidizer and mass flow rate of the fuel.

$$\dot{m} = \dot{m}_f + \dot{m}_o$$

The mass flow rate of the fuel is calculated from the equation:

$\dot{m}_f = 2\pi r l \rho Ɽ$ (20)

Where,

r, the average radius of the fuel grain = 8.75mm

l, the length of the fuel grain = 200mm

ρ, the density of the fuel = 1.41 g/cm^3

Ɽ, the regression rate of the fuel = 0.4 mm/s

$\dot{m}_f = 2\times\pi\times8.75\times200\times1.41\times0.4$

$= 6.2$ g/s

The mass flow rate of the oxidizer is determined by the O/F ratio.

$\dot{m}_o = 5 \times 6.2$

$= 30$ g/s

The total mass flow rate $\dot{m} = \dot{m}_f + \dot{m}_o$

$= 6.2 + 30$

$= 37.2$ g/s

$= 0.0372$ Kg/s

The exhaust velocity is calculated from Eq (5).

$F = \dot{m}c$

$C = \frac{F}{\dot{m}}$

$= \frac{60}{0.0372}$

$= 1612.9$ m/s

The exit pressure p_2 is computed from Eq (15).

$$V_2 = \sqrt{\frac{2k}{k-1} R T_1 [1 - (\frac{p_2}{p_1})^{\frac{k-1}{k}}]}$$

$$1612.9 = \sqrt{\frac{2\times1.228}{1.228-1}\times 263.53 \times 2394.9[1-(\frac{p_2}{p_1})^{\frac{1.228-1}{1.228}}]}$$

$$1612.9 = \sqrt{6{,}798{,}466.5[1-(\frac{p_2}{p_1})^{0.186}]}$$

$$2{,}601{,}446.41 = 6{,}798{,}466.5[1-(\frac{p_2}{p_1})^{0.186}]$$

$$0.385 = 1 - (\frac{p_2}{p_1})^{0.186}$$

$$(\frac{p_2}{p_1})^{0.187} = 0.615$$

$$\frac{p_2}{p_1} = 0.075$$

$$p_1 = \frac{101{,}325}{0.075}$$

$$= 1{,}351{,}000 \text{ Pa}$$

The Mach number has been computed using an isentropic flow relations calculator [9]. It considers the specific heat of the gas k and the pressure ratio $\frac{p_2}{p_1}$. The exit Mach number M_2 = 2.37.

$$\text{Nozzle throat area } A_t = \frac{\dot{m}}{p_1}\sqrt{\frac{RT_1}{k[\frac{2}{k+1}]^{(k+1)/(k-1)}}}$$

$$= \frac{0.0372}{1{,}351{,}000}\sqrt{\frac{263.53\times2394.9}{1.228[\frac{2}{1.228+1}]^{(1.228+1)/(1.228-1)}}}$$

$$= 2.75\times10^{-8}\sqrt{\frac{513{,}947.88}{[0.89]^{9.7}}}$$

$$= 2.75\times10^{-8}\sqrt{\frac{513{,}947.88}{0.323}}$$

$$= 2.75\times10^{-8}\sqrt{1{,}591{,}169.9}$$

$$= 3{,}482.35\times10^{-8} \text{ m}^2$$

$$= 34.82 \text{ mm}^2$$

$$\text{Throat Diameter } d_t = \sqrt{\frac{34.82\times4}{\pi}}$$

$$= \sqrt{44.33} = 6.66\text{mm}$$

The Nozzle area expansion ratio is determined from Eq (13).

$$\epsilon = \frac{A_2}{A_t} = \frac{1}{M_2}\sqrt{\left\{\frac{1+\left[\frac{k-1}{2}\right]{M_2}^2}{\left[\frac{k+1}{2}\right]}\right\}^{(k+1)/(k-1)}}$$

$$= \frac{1}{2.33}\sqrt{\left\{\frac{1+\left[\frac{1.228-1}{2}\right]2.33^2}{\left[\frac{1.228+1}{2}\right]}\right\}^{(1.228+1)/(1.228-1)}}$$

$$= 0.43\sqrt{\left\{\frac{1.62}{1.114}\right\}^{9.77}}$$

$$=0.43\sqrt{38.8}$$

$$= 2.65$$

Exit area $A_2 = \epsilon \times A_t$

$= 2.65 \times 34.82$

$= 92.27\ mm^2$

Exit Diameter $d_2 = \sqrt{\frac{92.27\times4}{\pi}}$

$= \sqrt{117.48} = 10.84mm$

The same set of calculations are performed for the nozzles optimized for 50 and 55N. The parameters for all three nozzles are summarized in Table (4).

Thrust [N]	Length [mm]	Pressure Ratio $\frac{p_2}{p_1}$	Pressure p_1 [Pa]	Area Expansion Ratio ϵ	Area of Throat A_t [mm^2]	Exit Area A_2 [mm^2]	Exit Velocity [m/s]	Mach Number M_2
50	13.33	0.182	556730.8	1.53	83.44	127.7	1344.1	1.81
55	16.26	0.118	858686.4	1.98	54.1	107.1	1478.5	2.067
60	18.73	0.075	1351000	2.65	34.82	92.27	1612.9	2.37

Table 4: Parameters of the 50 and 55N rated nozzles

3.4 Design

3.4.1 Conical nozzle

The conical nozzles were designed in the Spaceclaim tool on Ansys workbench. It considers the throat, inlet and outlet diameters as well as the half divergence angle α =15°.

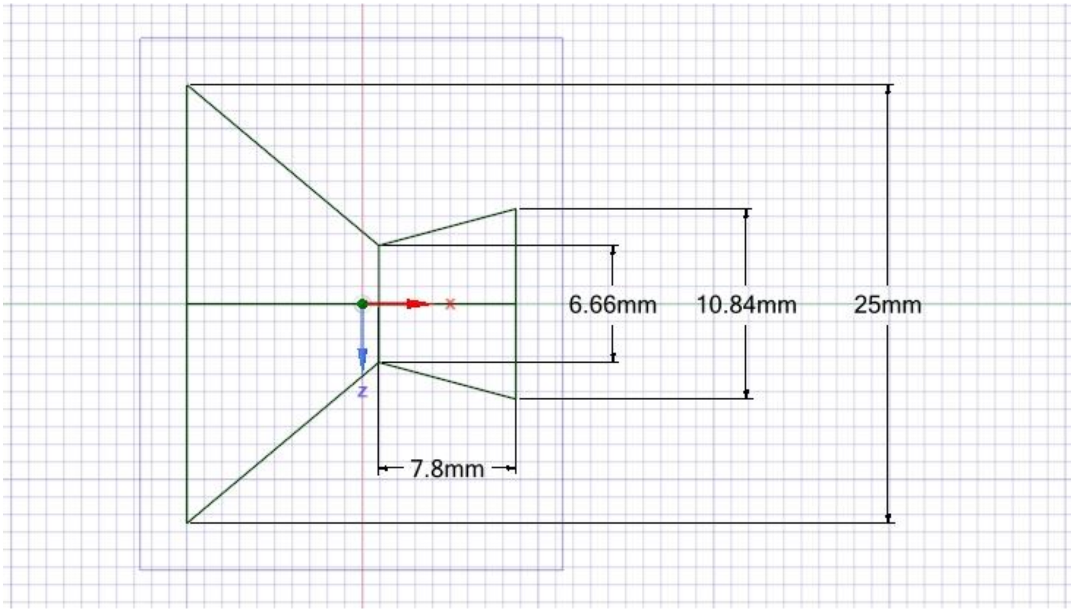

Figure 12 Design of the conical nozzle for 60N

3.4.2 Contour nozzle

A MATLAB program was used to determine the contour of the diverging section of the bell-shaped nozzle [10]. It utilizes the Method of Characteristics (MOC) to determine the shape of the nozzle. It registers the coordinates of the contour into an excel file. The file is exported to Spaceclaim and the rest of the nozzle is drawn using the poly-line tool.

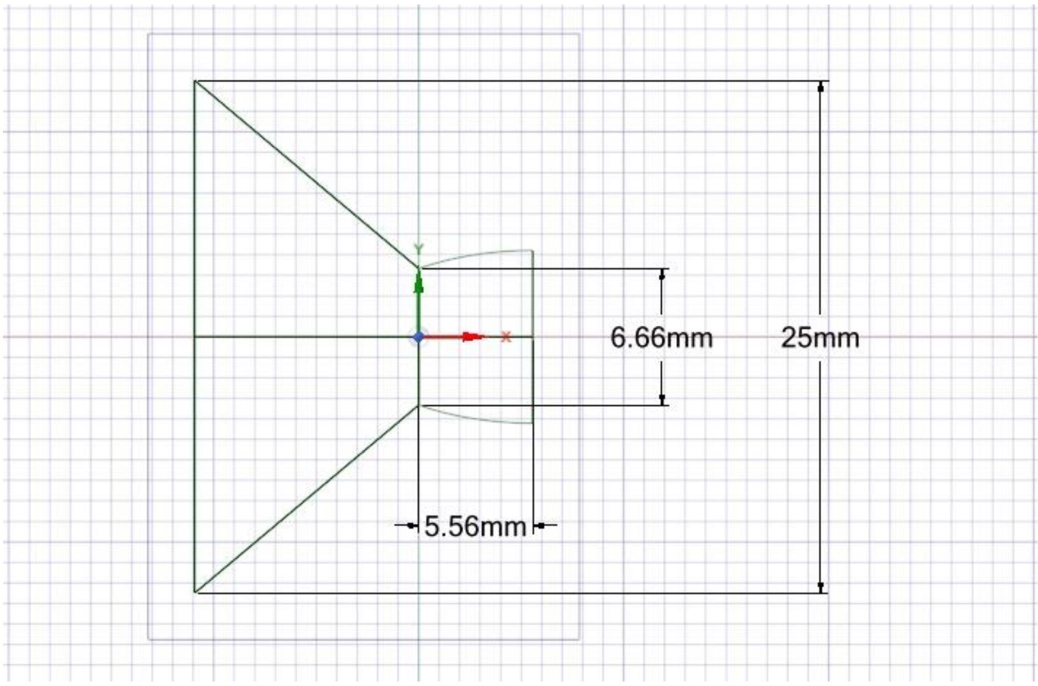

Figure 13 Design of the contour nozzle for 60N

3.5 Simulation

The nozzles along with their surrounding are first imported into the meshing tool. It is meshed with special emphasis given to the areas around the throat of the nozzle and the exhaust of the nozzle.

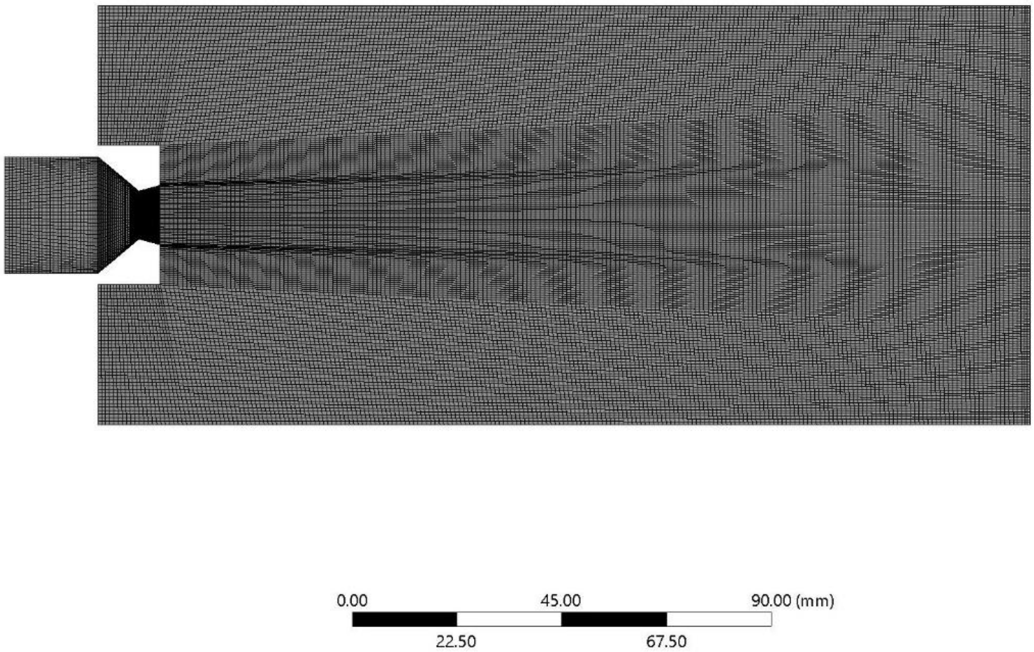

Figure 14 Mesh of the conical nozzle and the surrounding for 50N

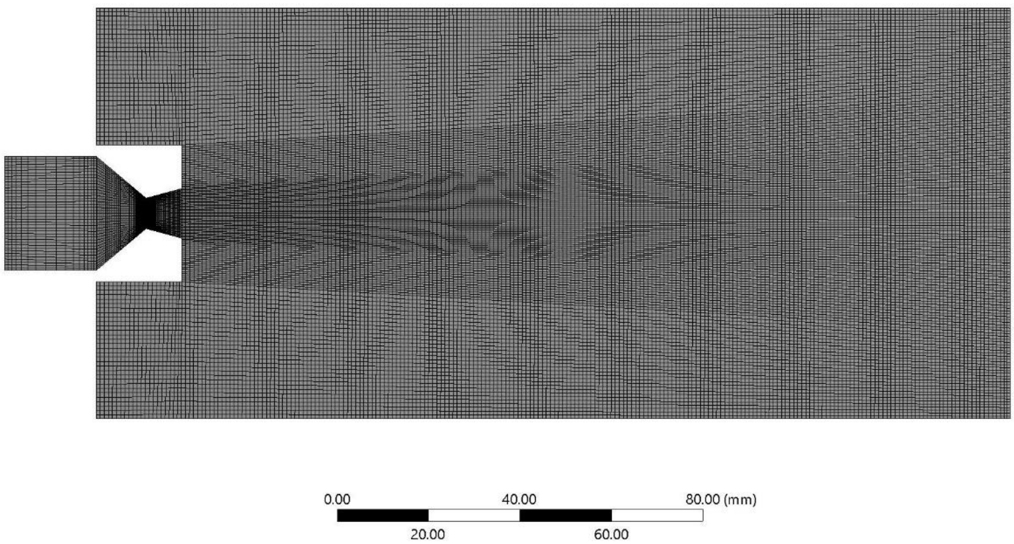

Figure 15 Mesh of the conical nozzle and the surrounding for 60N

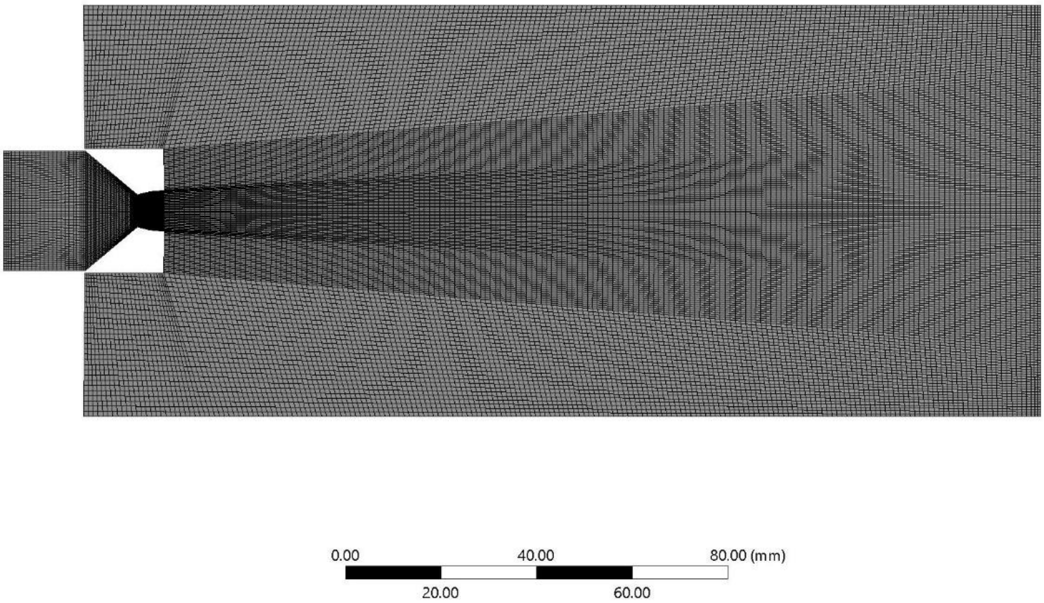

Figure 16 Mesh of the contour nozzle and the surrounding for 60N

After meshing, the nozzles are analyzed in Ansys Fluent. The nozzles are simulated in the conditions at sea level.

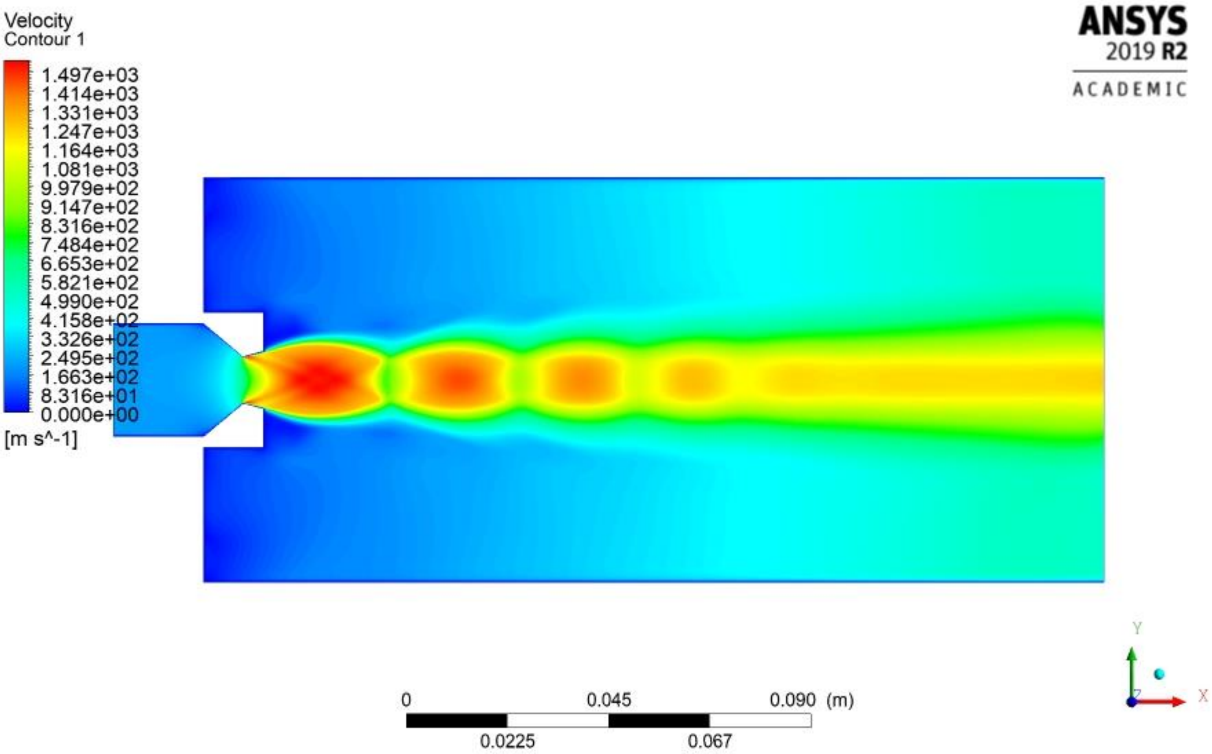

Figure 17 Simulation of the conical nozzle for 50N

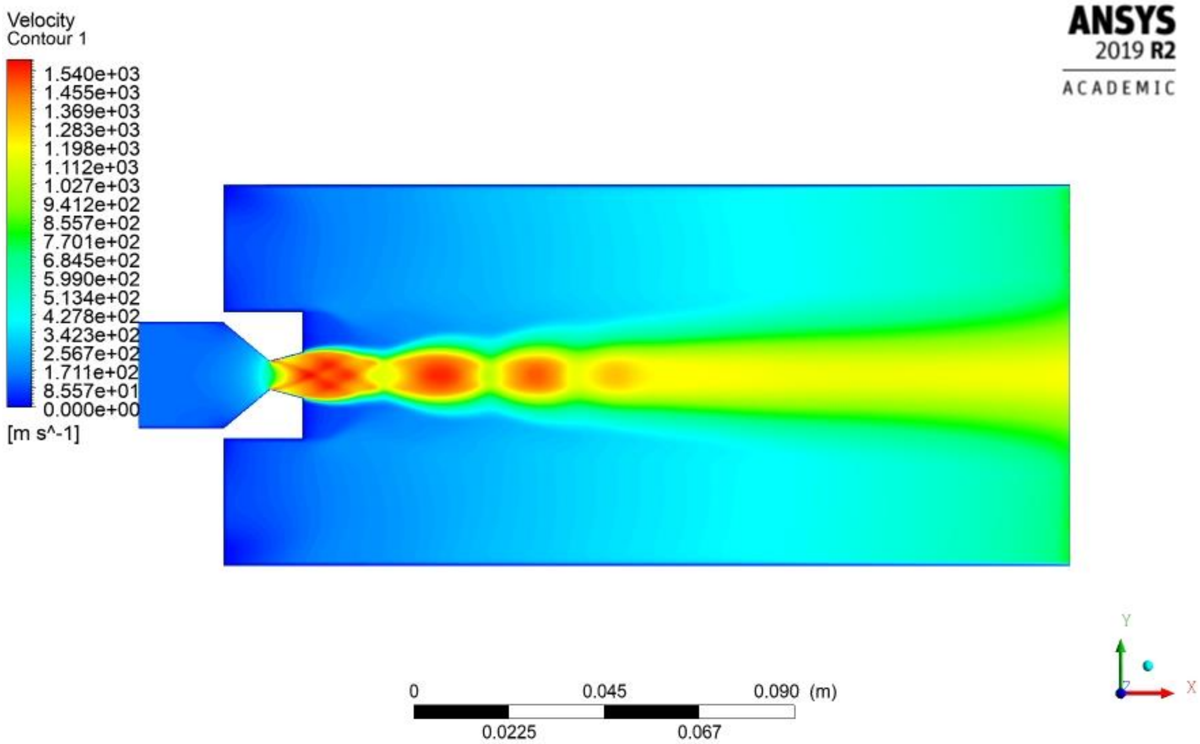

Figure 18 Simulation of the conical nozzle for 60N

The Mach diamonds can be observed beyond the throat of the nozzle. The exit velocity of the conical nozzle rated for 60N is 1584 m/s. The correction factor λ is computed from Eq (19). The divergence angle α of the conical nozzle is 15°.

$$\lambda = \frac{1}{2}(1 + \cos\alpha)$$

$$= \frac{1}{2}\times(1 + 0.96)$$

$$= 1.96/2$$

$$= 0.98$$

Hence, the effective exhaust velocity c = 1552.3 m/s

The specific impulse $I_{sp} = \frac{1568}{9.81}$

$$= 158.24 \text{ s}$$

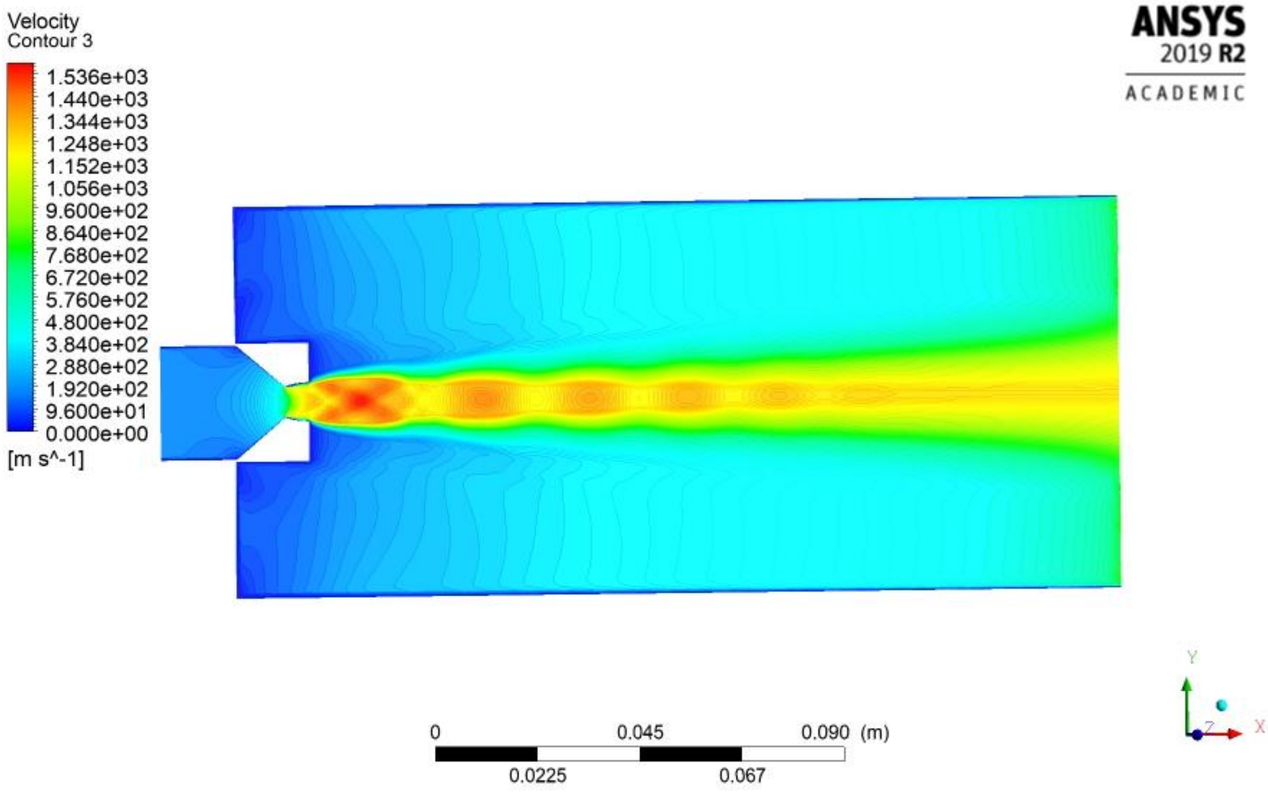

Figure 19 Simulation of the contour nozzle for 60N

The exit velocity of the contour nozzle is 1600 m/s. Since the contour nozzle expels all the propellant in the axial direction, it doesn't require a correction factor.

Specific impulse $I_{sp} = \frac{1600}{9.81}$

$= 163.27$ s

3.6 Manufacturing

The three sets of conical nozzles were manufactured in a CTU lab. The alloy used for fabricating it was Stainless Steel 1.4305. A cylinder of diameter 55mm was used as the blank for milling. In addition to the nozzle itself, the mount consists of four M3 screw holes to attach the nozzle to the body of the rocket engine. A small protrusion of height 2mm is made to better fit the nozzle into the combustion chamber and to avoid loses.

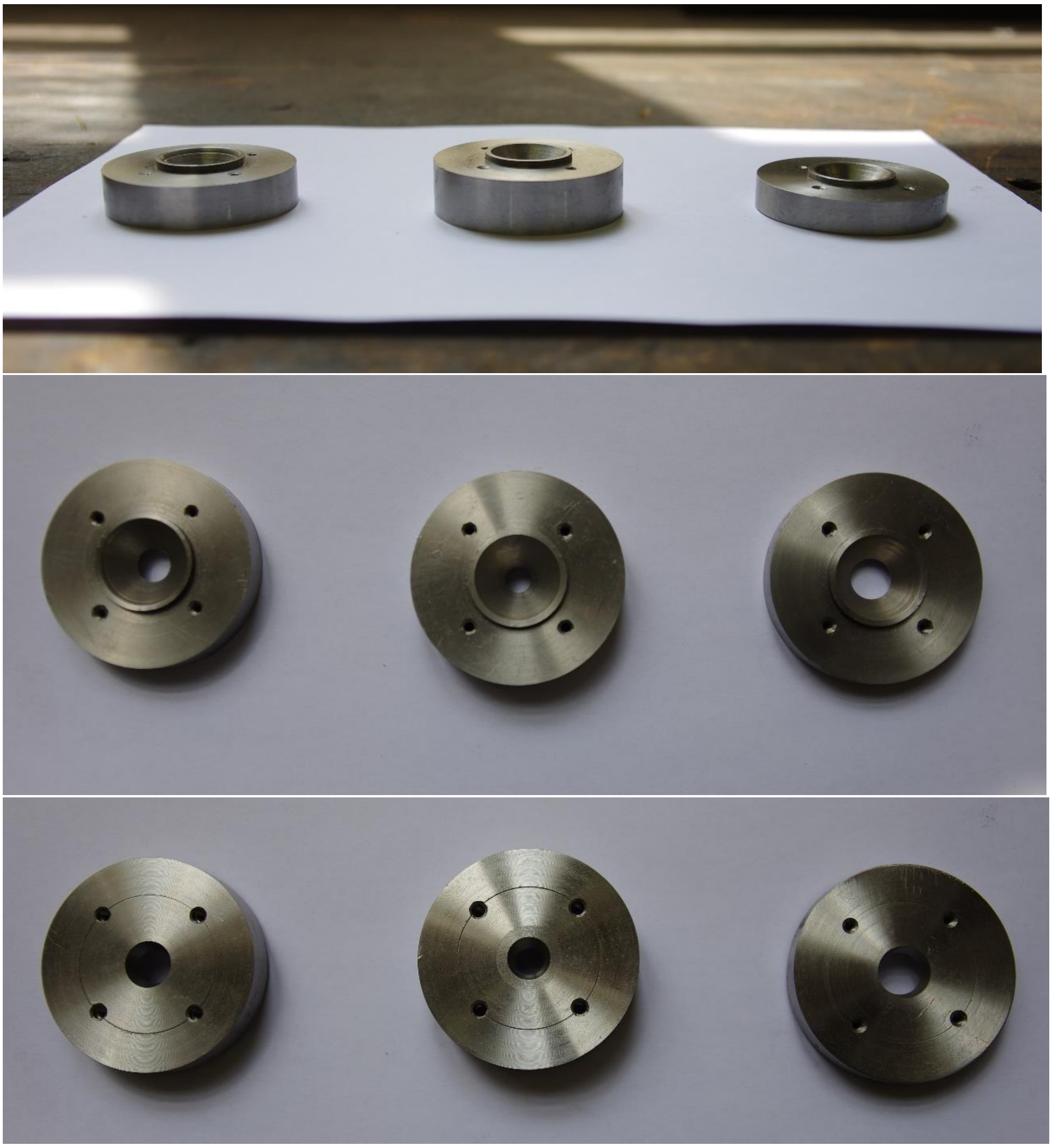

Figure 20 The picture on the top shows the side view of the nozzles. The middle picture shows the inside of the nozzle and the bottom picture shows the outside of the nozzle. The first nozzle from the left is rated for 55N, the nozzle in the middle is rated for 60N and the right-most nozzle is rated for 50N.

3.7 Summary Table

The following table holds the parameters of the nozzle as well as of the rocket engine with both nozzles. The parameters of the three conical nozzles were summarized previously in Table (4).

Nozzle Type	Length [mm]	Thrust[N]	Specific Impulse [s]	Exit Velocity [m/s]
Conical Nozzle	18.73	60	158.24	1552.3
Contour Nozzle	16.49	60	163.27	1600

Table 5: Comparison of nozzle and rocket parameters

4 Conclusion

When comparing nozzles of the same geometry but rated for different levels of thrust, the general trend is that a higher amount of thrust leads to a smaller throat area A_t. However, the length of the nozzle, area expansion ratio ϵ and pressure in the combustion chamber p_2 increases with higher amounts of thrust.

When comparing nozzles with different geometries, we find that the length of the Contour nozzle is 12% lesser when compared to the Conical nozzle for the same thrust. While reducing the vehicle length, the shorter length also implies that the Contour nozzle will weigh lesser than the Conical nozzle. Thus, increasing the performance as well as the payload fraction (ratio of the weight of the payload to the liftoff weight) of the rocket.

Furthermore, the specific impulse of the contour nozzle is 3.2% higher than that of the conical nozzle. This is because conical nozzles allow some of the exiting combustion gases to be directed at an angle from the centerline of the nozzle which is inefficient. The contour nozzle directs all the combustion gases axially, increasing the performance by a small amount. This difference in performance is more significant for bigger rocket engines.

Nomenclature

F = Thrust [N]

$\dot{m}$ = Mass flow rate [Kg/s]

v = Velocity [m/s]

p = Pressure [Pa]

A = Area [mm^2]

c = Exhaust velocity [m/s]

I_{sp} = Specific Impulse [s]

g = Acceleration due to gravity [m/s^2]

k = Specific heat [-]

C_v = Specific heat at constant volume [J/gK]

C_p = Specific heat at constant pressure [J/gK]

R = Gas constant [J/Kg-K]

R' = Universal gas constant [J/Kg-mol-K]

ɱ = Molecular mass [g]

T = Temperature [K]

a = Acoustic Velocity [m/s]

M = Mach number [-]

ϵ = Nozzle area expansion ratio [-]

r = Radius [mm]

l = Length [mm]

ρ = Density [Kg/cm^3]

ɽ = Regression rate [mm/s]

d = Diameter [mm]

α = Divergence angle [°]

Subscripts

t = Throat of the nozzle

1 = Combustion Chamber/ Beginning of the nozzle

2 = End of the nozzle

3 = Beyond the nozzle

o = Oxidizer

f = Fuel

Abbreviations

CFD = Computational Fluid Dynamics

O/F = Oxidizer to fuel ratio

CTU = Czech Technical University in Prague

List of pictures:

List of literature:

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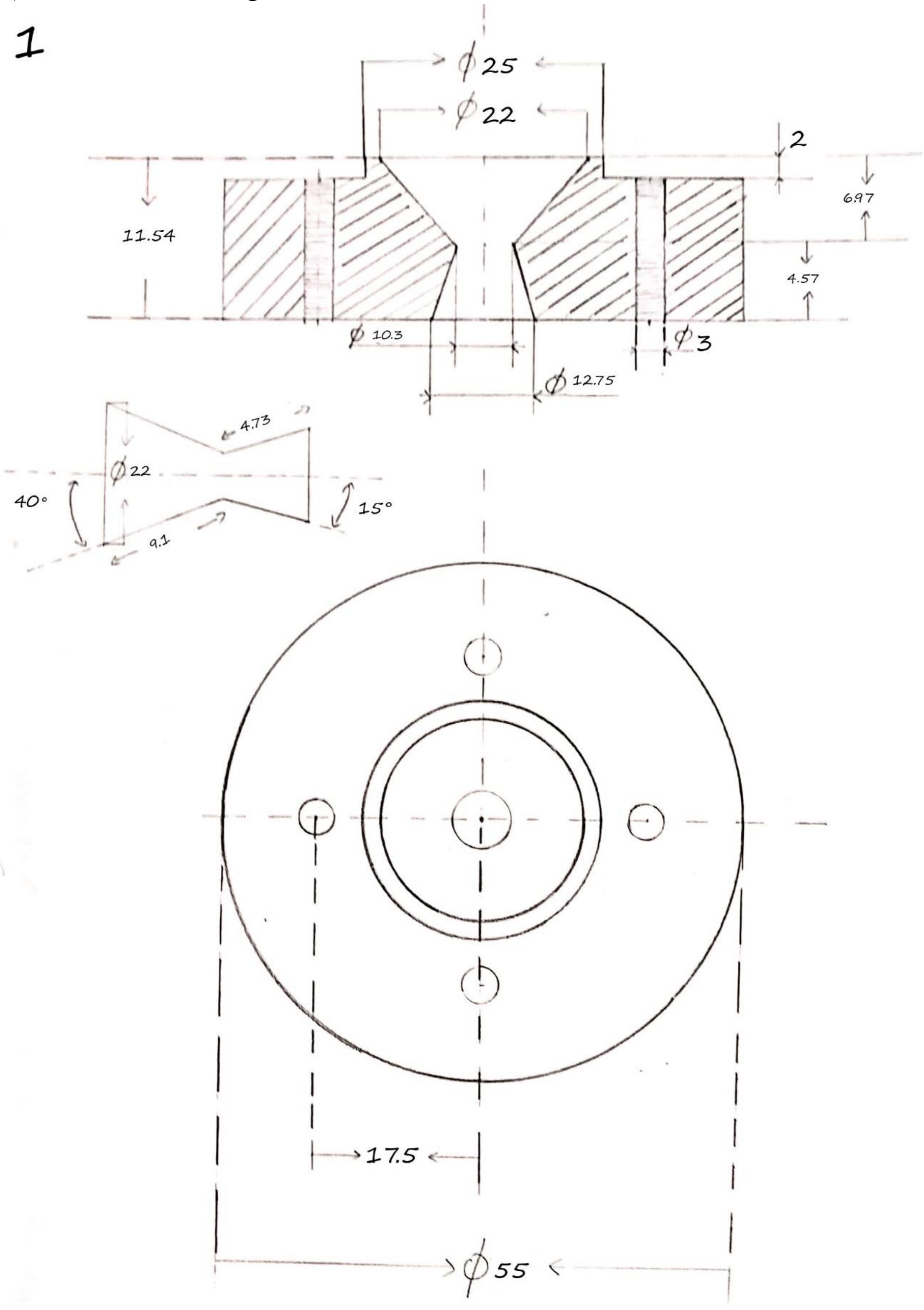
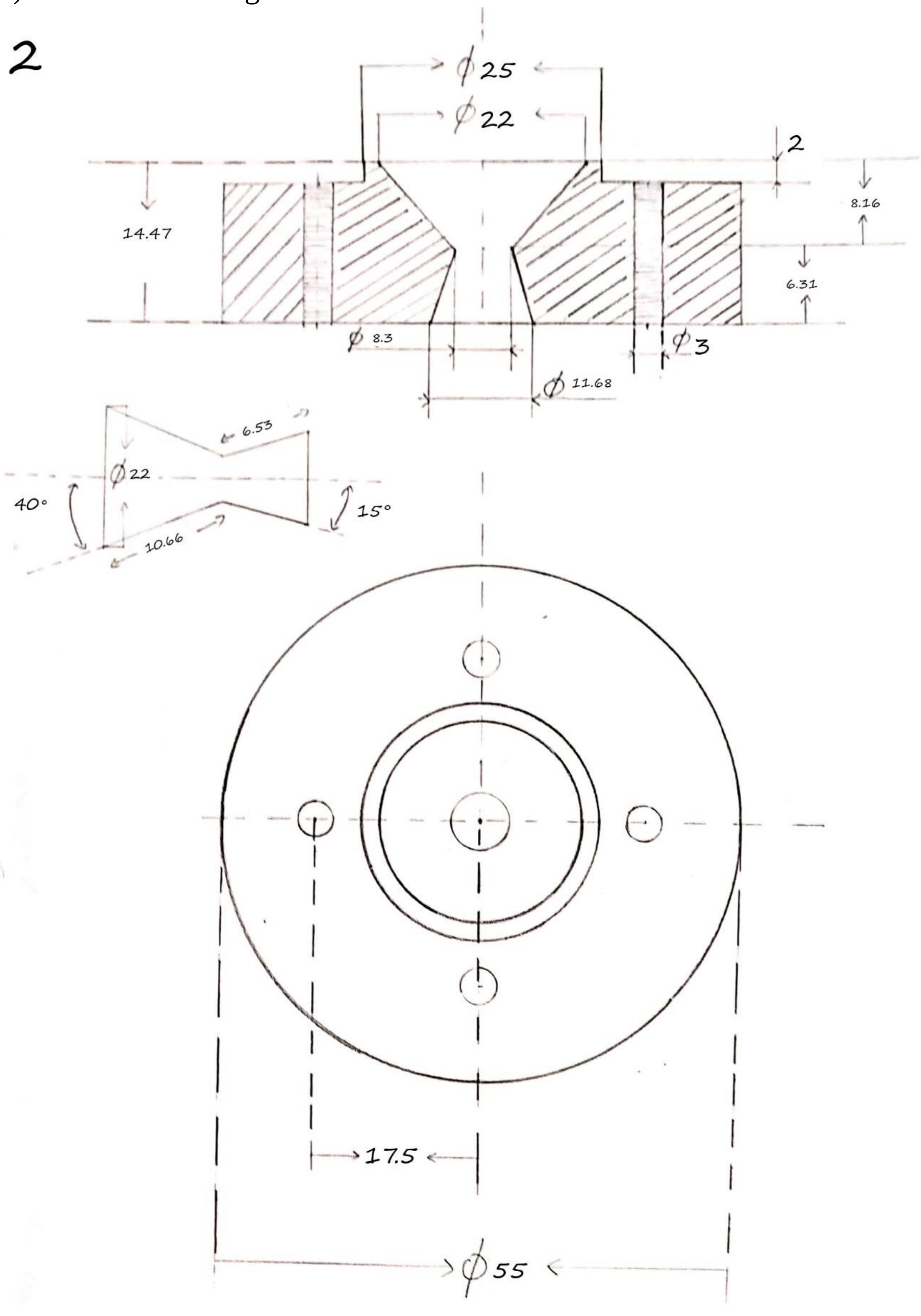
Appendix

1) Technical drawing of the conical nozzle for 50N



2) Technical drawing of the conical nozzle for 55N



3) Technical drawing of the conical nozzle for 60N