Odvození aerodynamických matic (špatně – pro potřebu srovnání)

Rozepsání rovnic aerodynamických sil

Při změně definice efektivních úhlů podle Roddena a Rose, tedy na:

$$\bar{\Theta} = \Theta + \frac{\dot{z}}{v_\infty} = \Theta + \frac{a\dot{\Theta}}{v_\infty}$$

$$\bar{\Psi} = \Psi - \frac{\dot{y}}{v_\infty} = \Psi - \frac{a\dot{\Psi}}{v_\infty}$$

lze vztahy pro aerodynamické síly generované vrtulí rozepsat do tvaru:

$$P_Y = q_\infty F_P \left[c_{\mathrm{y}\Theta}\left(\Theta + \frac{a\dot{\Theta}}{v_\infty}\right) + c_{\mathrm{y}\Psi}\left(\Psi - \frac{a\dot{\Psi}}{v_\infty}\right) + c_{\mathrm{yq}}\left(\frac{\dot{\Theta}R}{v_\infty}\right) + c_{\mathrm{yr}}\left(\frac{\dot{\Psi}R}{v_\infty}\right)\right]$$

$$P_Z = q_\infty F_P \left[c_{\mathrm{z}\Theta}\left(\Theta + \frac{a\dot{\Theta}}{v_\infty}\right) + c_{\mathrm{z}\Psi}\left(\Psi - \frac{a\dot{\Psi}}{v_\infty}\right) + c_{\mathrm{zq}}\left(\frac{\dot{\Theta}R}{v_\infty}\right) + c_{\mathrm{zr}}\left(\frac{\dot{\Psi}R}{v_\infty}\right)\right]$$

$$M_{Y,P} = q_\infty F_P D_P \left[c_{\mathrm{m}\Theta}\left(\Theta + \frac{a\dot{\Theta}}{v_\infty}\right) + c_{\mathrm{m}\Psi}\left(\Psi - \frac{a\dot{\Psi}}{v_\infty}\right) + c_{\mathrm{mq}}\left(\frac{\dot{\Theta}R}{v_\infty}\right) + c_{\mathrm{mr}}\left(\frac{\dot{\Psi}R}{v_\infty}\right)\right]$$

$$M_{Z,P} = q_\infty F_P D_P \left[c_{\mathrm{n}\Theta}\left(\Theta + \frac{a\dot{\Theta}}{v_\infty}\right) + c_{\mathrm{n}\Psi}\left(\Psi - \frac{a\dot{\Psi}}{v_\infty}\right) + c_{\mathrm{nq}}\left(\frac{\dot{\Theta}R}{v_\infty}\right) + c_{\mathrm{nr}}\left(\frac{\dot{\Psi}R}{v_\infty}\right)\right]$$

Pravá strana pohybových rovnic je dána:

$$M_Y = M_{Y,P} - aP_Z$$

$$M_Z = M_{Z,P} + aP_Y$$

A tedy po dosazení, roznásobení a vytknutí:

$$\begin{aligned} M_Y = q_\infty F_P D_P \Bigg[& (c_{\mathrm{m}\Theta})\Theta + \left(\frac{a}{v_\infty} c_{\mathrm{m}\Theta}\right)\dot{\Theta} + (c_{\mathrm{m}\Psi})\Psi - \left(\frac{a}{v_\infty} c_{\mathrm{m}\Psi}\right)\dot{\Psi} + \left(\frac{R}{v_\infty} c_{\mathrm{mq}}\right)\dot{\Theta} \\ & + \left(\frac{R}{v_\infty} c_{\mathrm{mr}}\right)\dot{\Psi} - \left(\frac{a}{D_P} c_{\mathrm{z}\Theta}\right)\Theta - \left(\frac{a^2}{D_P v_\infty} c_{\mathrm{z}\Theta}\right)\dot{\Theta} - \left(\frac{a}{D_P} c_{\mathrm{z}\Psi}\right)\Psi + \left(\frac{a^2}{D_P v_\infty} c_{\mathrm{z}\Psi}\right)\dot{\Psi} \\ & - \left(\frac{aR}{D_P v_\infty} c_{\mathrm{zq}}\right)\dot{\Theta} - \left(\frac{aR}{D_P v_\infty} c_{\mathrm{zr}}\right)\dot{\Psi} \Bigg] \end{aligned}$$

$$M_Z = q_\infty F_P D_P \left[(c_{\mathrm{n\Theta}})\Theta + \left(\frac{a}{v_\infty} c_{\mathrm{n\Theta}}\right)\dot{\Theta} + (c_{\mathrm{n\Psi}})\Psi - \left(\frac{a}{v_\infty} c_{\mathrm{n\Psi}}\right)\dot{\Psi} + \left(\frac{R}{v_\infty} c_{\mathrm{nq}}\right)\dot{\Theta} + \left(\frac{R}{v_\infty} c_{\mathrm{nr}}\right)\dot{\Psi} \right.$$
$$+ \left(\frac{a}{D_P} c_{\mathrm{y\Theta}}\right)\Theta + \left(\frac{a^2}{D_P v_\infty} c_{\mathrm{y\Theta}}\right)\dot{\Theta} + \left(\frac{a}{D_P} c_{\mathrm{y\Psi}}\right)\Psi - \left(\frac{a^2}{D_P v_\infty} c_{\mathrm{y\Psi}}\right)\dot{\Psi}$$
$$\left. + \left(\frac{aR}{D_P v_\infty} c_{\mathrm{yq}}\right)\dot{\Theta} + \left(\frac{aR}{D_P v_\infty} c_{\mathrm{yr}}\right)\dot{\Psi} \right]$$

Nyní po separaci na aerodynamickou tuhost a aerodynamické tlumení a dalším vytknutí:

$$M_y = M_Y^K + M_Y^D$$

$$M_Z = M_Z^K + M_Z^D$$

$$M_Y^K = q_\infty F_P D_P \left[\left(c_{\mathrm{m\Theta}} - \frac{a}{D_P} c_{\mathrm{z\Theta}}\right)\Theta + \left(c_{\mathrm{m\Psi}} - \frac{a}{D_P} c_{\mathrm{z\Psi}}\right)\Psi \right]$$

$$M_Y^D = q_\infty F_P \frac{{D_P}^2}{v_\infty} \left[\left(\frac{a}{D_P} c_{\mathrm{m\Theta}} + \frac{1}{2} c_{\mathrm{mq}} - \frac{a^2}{{D_P}^2} c_{\mathrm{z\Theta}} - \frac{a}{2D_P} c_{\mathrm{zq}}\right)\dot{\Theta} \right.$$
$$\left. + \left(-\frac{a}{D_P} c_{\mathrm{m\Psi}} + \frac{1}{2} c_{\mathrm{mr}} + \frac{a^2}{{D_P}^2} c_{\mathrm{z\Psi}} - \frac{a}{2D_P} c_{\mathrm{zr}}\right)\dot{\Psi} \right]$$

$$M_Z^K = q_\infty F_P D_P \left[\left(c_{\mathrm{n\Theta}} + \frac{a}{D_P} c_{\mathrm{y\Theta}}\right)\Theta + \left(c_{\mathrm{n\Psi}} + \frac{a}{D_P} c_{\mathrm{y\Psi}}\right)\Psi \right]$$

$$M_Z^D = q_\infty F_P \frac{{D_P}^2}{v_\infty} \left[\left(\frac{a}{D_P} c_{\mathrm{n\Theta}} + \frac{1}{2} c_{\mathrm{nq}} + \frac{a^2}{{D_P}^2} c_{\mathrm{y\Theta}} + \frac{a}{2D_P} c_{\mathrm{yq}}\right)\dot{\Theta} \right.$$
$$\left. + \left(-\frac{a}{D_P} c_{\mathrm{n\Psi}} + \frac{1}{2} c_{\mathrm{nr}} - \frac{a^2}{{D_P}^2} c_{\mathrm{y\Psi}} + \frac{a}{2D_P} c_{\mathrm{yr}}\right)\dot{\Psi} \right]$$

Což lze přepsat do maticové podoby:

$$\begin{bmatrix} M_y \\ M_Z \end{bmatrix} = q_\infty F_P D_P \begin{bmatrix} m_{Y1}^K & m_{Y2}^K \\ m_{Z1}^K & m_{Z2}^K \end{bmatrix} \begin{bmatrix} \Theta \\ \Psi \end{bmatrix} + q_\infty F_P \frac{{D_P}^2}{v_\infty} \begin{bmatrix} m_{Y1}^D & m_{Y2}^D \\ m_{Z1}^D & m_{Z2}^D \end{bmatrix} \begin{bmatrix} \dot{\Theta} \\ \dot{\Psi} \end{bmatrix}$$

Kde jsou matice:

$$\begin{bmatrix} m_{Y1}^K & m_{Y2}^K \\ m_{Z1}^K & m_{Z2}^K \end{bmatrix} = \boldsymbol{K}^{\boldsymbol{A}*} = matice\ aerodynamick\acute{e}\ tuhosti\ p\check{r}ed\ \acute{u}pravou$$

$$\begin{bmatrix} m_{Y1}^{D} & m_{Y2}^{D} \\ m_{Z1}^{D} & m_{Z2}^{D} \end{bmatrix} = \boldsymbol{D}^{\boldsymbol{A}*} = matice\ aerodynamického\ tlumení\ před\ úpravou$$

Matice aerodynamické tuhosti

Po vyjádření jednotlivých členů matice aerodynamické tuhosti před úpravou získá tvar:

$$\boldsymbol{K}^{\boldsymbol{A}*} = \begin{bmatrix} c_{\mathrm{m}\Theta} - \dfrac{a}{D_P} c_{\mathrm{z}\Theta} & c_{\mathrm{m}\Psi} - \dfrac{a}{D_P} c_{\mathrm{z}\Psi} \\ c_{\mathrm{n}\Theta} + \dfrac{a}{D_P} c_{\mathrm{y}\Theta} & c_{\mathrm{n}\Psi} + \dfrac{a}{D_P} c_{\mathrm{y}\Psi} \end{bmatrix}$$

S využitím symetrických vlastností vrtule:

$$c_{\mathrm{y}\Theta} = c_{\mathrm{z}\Psi} \qquad c_{\mathrm{m}\Theta} = c_{\mathrm{n}\Psi}$$

$$c_{\mathrm{yq}} = c_{\mathrm{zr}} \qquad c_{\mathrm{mq}} = c_{\mathrm{nr}}$$

$$c_{\mathrm{y}\Psi} = -c_{\mathrm{z}\Theta} \qquad c_{\mathrm{m}\Psi} = -c_{\mathrm{n}\Theta}$$

$$c_{\mathrm{yr}} = -c_{\mathrm{zq}} \qquad c_{\mathrm{mr}} = -c_{\mathrm{nq}}$$

Lze matici upravit:

$$\boldsymbol{K}^{\boldsymbol{A}*} = \begin{bmatrix} c_{\mathrm{m}\Theta} - \dfrac{a}{D_P} c_{\mathrm{z}\Theta} & -c_{\mathrm{n}\Theta} - \dfrac{a}{D_P} c_{\mathrm{y}\Theta} \\ c_{\mathrm{n}\Theta} + \dfrac{a}{D_P} c_{\mathrm{y}\Theta} & c_{\mathrm{m}\Theta} - \dfrac{a}{D_P} c_{\mathrm{z}\Theta} \end{bmatrix}$$

A po převedení na levou stranu pohybových rovnic *whirl flutteru* získá konečný tvar:

$$\boldsymbol{K}^{\boldsymbol{A}} = \begin{bmatrix} \dfrac{a}{D_P} c_{\mathrm{z}\Theta} - c_{\mathrm{m}\Theta} & c_{\mathrm{n}\Theta} + \dfrac{a}{D_P} c_{\mathrm{y}\Theta} \\ -c_{\mathrm{n}\Theta} - \dfrac{a}{D_P} c_{\mathrm{y}\Theta} & \dfrac{a}{D_P} c_{\mathrm{z}\Theta} - c_{\mathrm{m}\Theta} \end{bmatrix}$$

Na levé straně pohybových rovnic bude tedy aerodynamická tuhost reprezentována členem:

$$q_{\infty} F_P D_P \boldsymbol{K}^{\boldsymbol{A}} = q_{\infty} F_P D_P \begin{bmatrix} \dfrac{a}{D_P} c_{\mathrm{z}\Theta} - c_{\mathrm{m}\Theta} & c_{\mathrm{n}\Theta} + \dfrac{a}{D_P} c_{\mathrm{y}\Theta} \\ -c_{\mathrm{n}\Theta} - \dfrac{a}{D_P} c_{\mathrm{y}\Theta} & \dfrac{a}{D_P} c_{\mathrm{z}\Theta} - c_{\mathrm{m}\Theta} \end{bmatrix}$$

Matice aerodynamického tlumení

Stejným způsobem jako v předchozím případě lze získat i výslednou matici aerodynamického tlumení:

$$\boldsymbol{D}^{\boldsymbol{A}*} = \begin{bmatrix} \frac{a}{D_P} c_{\mathrm{m\Theta}} + \frac{1}{2} c_{\mathrm{mq}} - \frac{a^2}{{D_P}^2} c_{\mathrm{z\Theta}} - \frac{a}{2D_P} c_{\mathrm{zq}} & -\frac{a}{D_P} c_{\mathrm{m\Psi}} + \frac{1}{2} c_{\mathrm{mr}} + \frac{a^2}{{D_P}^2} c_{\mathrm{z\Psi}} - \frac{a}{2D_P} c_{\mathrm{zr}} \\ \frac{a}{D_P} c_{\mathrm{n\Theta}} + \frac{1}{2} c_{\mathrm{nq}} + \frac{a^2}{{D_P}^2} c_{\mathrm{y\Theta}} + \frac{a}{2D_P} c_{\mathrm{yq}} & -\frac{a}{D_P} c_{\mathrm{n\Psi}} + \frac{1}{2} c_{\mathrm{nr}} - \frac{a^2}{{D_P}^2} c_{\mathrm{y\Psi}} + \frac{a}{2D_P} c_{\mathrm{yr}} \end{bmatrix}$$

S využitím symetrických vlastností vrtule:

$$c_{\mathrm{y\Theta}} = c_{\mathrm{z\Psi}} \qquad c_{\mathrm{m\Theta}} = c_{\mathrm{n\Psi}}$$

$$c_{\mathrm{yq}} = c_{\mathrm{zr}} \qquad c_{\mathrm{mq}} = c_{\mathrm{nr}}$$

$$c_{\mathrm{y\Psi}} = -c_{\mathrm{z\Theta}} \qquad c_{\mathrm{m\Psi}} = -c_{\mathrm{n\Theta}}$$

$$c_{\mathrm{yr}} = -c_{\mathrm{zq}} \qquad c_{\mathrm{mr}} = -c_{\mathrm{nq}}$$

Lze matici upravit:

$$\boldsymbol{D}^{\boldsymbol{A}*} = \begin{bmatrix} \frac{a}{D_P} c_{\mathrm{m\Theta}} + \frac{1}{2} c_{\mathrm{mq}} - \frac{a^2}{{D_P}^2} c_{\mathrm{z\Theta}} - \frac{a}{2D_P} c_{\mathrm{zq}} & \frac{a}{D_P} c_{\mathrm{n\Theta}} - \frac{1}{2} c_{\mathrm{nq}} + \frac{a^2}{{D_P}^2} c_{\mathrm{y\Theta}} - \frac{a}{2D_P} c_{\mathrm{yq}} \\ \frac{a}{D_P} c_{\mathrm{n\Theta}} + \frac{1}{2} c_{\mathrm{nq}} + \frac{a^2}{{D_P}^2} c_{\mathrm{y\Theta}} + \frac{a}{2D_P} c_{\mathrm{yq}} & -\frac{a}{D_P} c_{\mathrm{m\Theta}} + \frac{1}{2} c_{\mathrm{mq}} + \frac{a^2}{{D_P}^2} c_{\mathrm{z\Theta}} - \frac{a}{2D_P} c_{\mathrm{zq}} \end{bmatrix}$$

A po převedení na levou stranu pohybových rovnic *whirl flutteru* získá konečný tvar:

$$\boldsymbol{D}^{\boldsymbol{A}} = \begin{bmatrix} -\frac{a}{D_P} c_{\mathrm{m\Theta}} - \frac{1}{2} c_{\mathrm{mq}} + \frac{a^2}{{D_P}^2} c_{\mathrm{z\Theta}} + \frac{a}{2D_P} c_{\mathrm{zq}} & -\frac{a}{D_P} c_{\mathrm{n\Theta}} + \frac{1}{2} c_{\mathrm{nq}} - \frac{a^2}{{D_P}^2} c_{\mathrm{y\Theta}} + \frac{a}{2D_P} c_{\mathrm{yq}} \\ -\frac{a}{D_P} c_{\mathrm{n\Theta}} - \frac{1}{2} c_{\mathrm{nq}} - \frac{a^2}{{D_P}^2} c_{\mathrm{y\Theta}} - \frac{a}{2D_P} c_{\mathrm{yq}} & +\frac{a}{D_P} c_{\mathrm{m\Theta}} - \frac{1}{2} c_{\mathrm{mq}} - \frac{a^2}{{D_P}^2} c_{\mathrm{z\Theta}} + \frac{a}{2D_P} c_{\mathrm{zq}} \end{bmatrix}$$

Na levé straně pohybových rovnic bude tedy aerodynamické tlumení reprezentováno členem:

$$q_\infty F_P \frac{{D_P}^2}{v_\infty} \boldsymbol{D}^{\boldsymbol{A}} =$$

$$= q_\infty F_P \frac{{D_P}^2}{v_\infty} \begin{bmatrix} -\frac{a}{D_P} c_{\mathrm{m\Theta}} - \frac{1}{2} c_{\mathrm{mq}} + \frac{a^2}{{D_P}^2} c_{\mathrm{z\Theta}} + \frac{a}{2D_P} c_{\mathrm{zq}} & -\frac{a}{D_P} c_{\mathrm{n\Theta}} + \frac{1}{2} c_{\mathrm{nq}} - \frac{a^2}{{D_P}^2} c_{\mathrm{y\Theta}} + \frac{a}{2D_P} c_{\mathrm{yq}} \\ -\frac{a}{D_P} c_{\mathrm{n\Theta}} - \frac{1}{2} c_{\mathrm{nq}} - \frac{a^2}{{D_P}^2} c_{\mathrm{y\Theta}} - \frac{a}{2D_P} c_{\mathrm{yq}} & +\frac{a}{D_P} c_{\mathrm{m\Theta}} - \frac{1}{2} c_{\mathrm{mq}} - \frac{a^2}{{D_P}^2} c_{\mathrm{z\Theta}} + \frac{a}{2D_P} c_{\mathrm{zq}} \end{bmatrix}$$