CZECH TECHNICAL UNIVERSITY IN PRAGUE

FACULTY OF CIVIL ENGINEERING

Department of Geomatics

BACHELOR THESIS

Evaluation of Measuring at Testing Levelling Area of Praha-Podbaba from Epoch 1997-2017

Vyhodnocení měření na zkušebním nivelačním okruhu v Praze-Podbabě z etapy 1997-2017

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Branch of Study: Geodesy and Cartography

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22. 02. 19

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STATEMENT

I declare that I have worked out this thesis independently with the help of the supervisor, Zdeněk Vyskočil, Ph.D., assuming that the results of the thesis can also be used at the discretion of the supervisor of the thesis as its co-author. I also agree with the potential publication of the results of the thesis or of its substantial part, provided I will be listed as the co-author.

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ABSTRACT

This bachelor thesis deals with evaluation of levelling measurement accuracy at the circuit in Praha-Podbaba from epoch 1997-2017 and creating a useful software for processing the work in the computing environment MATLAB R2018a. First, the data is being digitalised, then adjusted with the help of a method based on least squares. Later follows spectral analysis for determining periodicity of height changes. At the end the program based on all the scripts written for the computations was created with the help of creating standalone applications environment MATLAB GUIDE.

KEY WORDS

Levelling, adjustment, MATLAB, levelling circuit, levelling base, height differences, levelling analysis, spectral analysis, GUI, MATLAB GUIDE

ABSTRAKT

Tato bakalářská práce se zabývá vyhodnocením přesnosti nivelačního měření na nivelačním okruhu Praha-Podbaba z etapy 1997-2017 a vytvořením užitečného software pro zpracování ve výpočetním prostředí Matlab R2018a. Zaprve se data převedla do digitální podoby, potom byla vyrovnaná pomocí metody, základem které je metoda nejmenších čtverců. Dále následuje spektrální analyza pro určování peridiodicity výškových změn. Na konci se pomocí skriptů, napsaných pro výpočty, udělal program v prostředí pro vytvoření aplikací Matlab GUIDE.

KLÍČOVÁ SLOVA

Nivelace, vyrovnání, Matlab, nivelační okruh, nivelační základna, převýšení, analýza nivelace, spektrální analýza, GUI, Matlab GUIDE

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INTRODUCTION

This bachelor thesis is aiming to process observations from the levelling circuit set in Praha-Podbaba (Prague, the Czech Republic) and to create a software, that would allow easier evaluation during the whole work.

The software would be a better way of representing the data and would help to evaluate the circuit and the base partially. Nowadays, manual processing is being replaced with computer one, because it enables to evaluate data faster and to avoid random mistakes. If any mistakes are made during the creation of the software, they could be easily eliminated by correcting the code and the results would overwrite themselves without repeating steps, that follow the one with the mistake.

The measurement at the levelling circuit of Praha-Podbaba is being taken since 1952, the observations at the levelling base from epoch 1952-1997 and the whole circuit have been evaluated in thesis of Jan Řezníček in 1997/98.

According to an agreement with Land Survey Office (ZÚ) it was decided to evaluate the later period (1997-2017) in this bachelor thesis. The data was provided by the Office.

The results after the processing should be compared to earlier ones in order to find correlations and trends. The results should also represent accuracy of taken measurements.

It was chosen to write the paper in English language, so the thesis could be international, and could be used for further works outside the Czech Republic. Therefore, notes, discriptions and quoting are made in the international way as well (f.ex. bibliography is done according to the APA norms).

1. LEVELLING CIRCUIT IN PRAHA-PODBABA

1.1 History of the circuit

Before creating the circuit in Praha-Podbaba, there were two other levelling sets: Praha-Zabraslavi, built in 1939, that was replaced with Praha-Slivenec in 1941.
The second one had an inconvenient position, because the measurements were done in an open area, and, therefore, were constantly influenced by wind, and the circuit was not suitable for testing purposes: unstable benchmarks, small height differences.
The testing levelling area in Praha-Podbaba was created in the year 1952. All the previous inconveniences were eliminated, so the circuit could be good for training gangs and testing equipment.

The circuit and its history are described in [7].

1.2 Purpose of the levelling base

Nowadays the created in 1952 levelling circuit is more important than at the start.

Over the period of 80 years, measurement at the area was taken with the help of different types of instruments, starting with the ones using a spirit level vital, then compensator instruments and more modern types – compensator instruments with automatic registration. Not only the instruments were changed with the time, but levelling staffs as well. Their length was verified using different comparative meters. Measurements from the period of this bachelor thesis were made with compensator instruments and with the ones with automatic registration.

1.3 Circuit Description

Varied terrain (plane and hilly) enables better levelling gang trainings, it is also suitable for instrument testing.
The circuit itself (*appendix 1*) was starting with a point 1.1, which was replaced with 1.2 in 2016, going to point 2.1 by a pavement, then to point 6 by an asphalt road, after that to point 11 by a fieldway, and by an asphalt road again to its ending at the point 1.1(1.2).

1.4 Changes during the evaluated period

Points 13a, 13b and 13c were destroyed after the agreement between ZÚ and land owners in year 2017. They were not measured in autumn 2016 because there was no permission. However, in 2017 new levelling points 13d and 13e were established and measured. They are not evaluated in this thesis, because they don't have a long-term repetitive measurement which can be evaluated. That is why data from autumn 2016 and both observations of 2017 are represented only till the point 13.

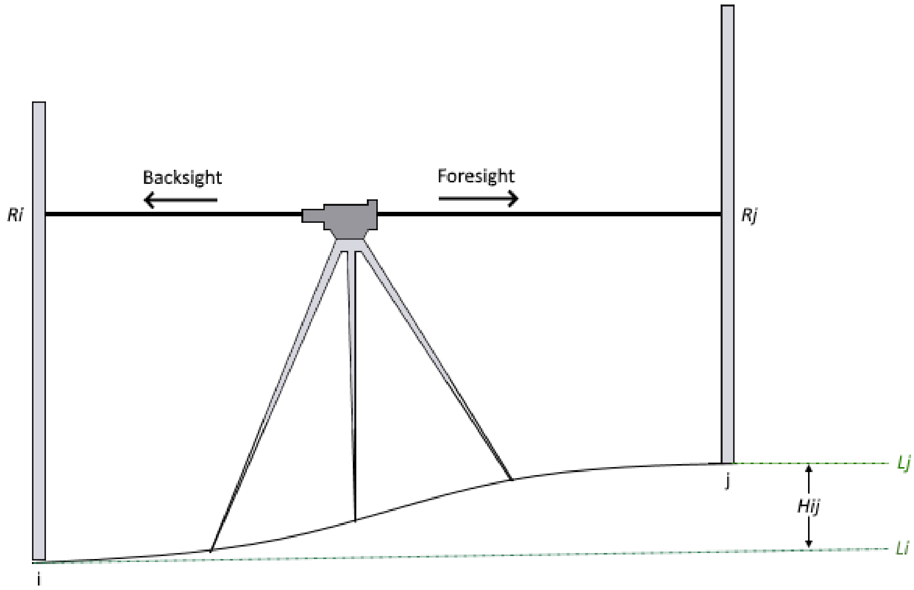
Levelling point 6 of the circuit in Praha-Podbaba was destroyed in the year 2016, there was no other point even in the year 2017.

As it has already been stated in 2.3 while describing the circuit, point 1.1 had been replaced with 1.2 in 2016.

1.5 Measuring Method

Geodetic levelling as the most precise type of measuring differences of heights between two given points has already been known back in the ancient days. Its meaning and applications are described in details in [3]. All in all, instruments and procedure of taking measurements experienced huge changes over the years, however, the basic principle has remained the almost the same since the middle of the nineteenth century.

The applied method is called high-precision levelling and it represents geometric type of levelling(Figure.2.1). Geometric levelling appears to be more precise with usage of lighter monumentation and cheaper instruments than trigonometric.



Pic.2.1 Geometric Levelling

The method is easy to understand: height differences are determined as differences of readings on two staffs, which are positioned on both points at the same time. Optical levels are used to do readings. An optical level is a levelling instrument, which consists of a telescope used for reading values on a vertical staff with the help of cross hairs, having a very sensitive spirit level. Those instruments were used in the area of Praha-Podbaba till the year 1964. Optical level instruments were used together with a newer, compensator-type instruments over a period of almost 25 years, in 1964-1988. Then they were completely replaced with the newer type, which is used in observations nowadays as well. A modern version of instruments with a built-in compensator is applied in the levelling circuit since year 1993. It automatically reads the value on the levelling staff. Accuracy of those three types of levelling instruments is compared in [7].

In the high-precision levelling the processing of the measured data and the way of an observation allows to reach the required accuracy, although it is about 0,1-0,3 mm, as mentioned in [6]. It is also achieved with the use of a special type of levelling staff, which are solid, have two scales and tested in a laboratory before an observation for the measuring crew to know its deviations.

It is also required due to its importance to pay attention to convergence of the measured surface area during an observation.

1.6 Applied Height System – Baltic Sea Vertical Datum/ Baltic Sea Height System 1957

Nowadays Baltic Sea is used as vertical datum in the Czech Republic. According to [8] it was first established and the network was adjusted in year 1977 for the Soviet Union countries. It is suitable for use in a few countries of the former USSR, the Slovak Republic and the mentioned earlier, Czech Republic. As it is said in [9], in 2017-2018 Baltic countries switched to European Vertical Reference System (EVRS).

Its origin is in Kronstadt, near Saint Petersburg where the average water level was observed since year 1707 and the mark, that was put in there, is equivalent to the water level in the period between 1825 and 1839, as it is mentioned in [10]. It is the only origin that's why the whole vertical datum depends on the value of the Kronstadt mark and it is a free height system that uses normal heights. This vertical datum used in geodesy (geodetic and engineering surveying) and topographic mapping.

As it was mentioned earlier, the adjustment of year 1977 was used in the countries of former USSR. However, it was not adopted in the Czech and Slovak Republics, although they were included. That is why there is still in use the 1957 adjustment as it is said in [11].

According to [12] till the year 2000, despite the change of the height system, there still was a different vertical datum in some parts of the Czech Republic (for example, Prague), which was commonly used till the end of the World War II. It is called Jadran in Czech language and known as Metres Above the Adriatic in English. It was defined in year 1875. This vertical datum uses normal orthometric heights and its origin is in the Port of Trieste. There's an approximately 35-42cm difference between those two systems, Baltic Sea has smaller values (for example, Lišov is 388,6 mm higher in the Metres Above the Adriatic).

It could be more useful for the Czech Republic to switch to EVRS in the future, so all the members of the European Union would use the same height system. It would eliminate the computations needed to switch to another vertical system, which would be easier for travelers for example, and would increase integration between the members of the EU. Using the same height system would also simplify planning, processing and executing projects, that involve different countries.

2.LEVELLING ERRORS

Every measurement has to be done to some limit of precision, and when the limit is crossed it means, that at some point there was an influence that led to it. Those influences are sources of levelling errors.

There are two main ways to divide levelling errors into groups. The first one is by their cause: natural, instrumental, manipulation of equipment and reading and booking. The second one creates three groups by how they influence the output: gross, random and systematic errors.

2.1 Types of Errors

2.1.1 Gross Errors

Those errors can be caused by any of the reasons and have any size, however they mostly appear to be so called careless errors: misreading staff due to different markings, booking in wrong column, etc.

1. **Staff Not Vertical**
2. **Disturbing Level**
3 **Errors in Staff Graduation**

2.1.2 Random Errors

Random errors are to be a result of an observation, of the reading process itself, they are not caused by circumstances or conditions.
Their qualities cannot be predicted: random errors are not easily detected as gross ones, but they have a very specific statistical behaviour and can be solved with the help of a proper (fitting to the given situation) statistical method.

Examples of gross errors:

1. **Error in Aiming**
2. **Bubble Not Central**

When reading is taken or wrong sight adjusting by a compensator

3. **Difference Between Backsight and Foresight**

2.1.3 Systematic Errors

Those are the errors that can be mathematically modelled and eliminated by using a correct computation or procedure of taking measurements. They usually represent a change in the whole observation, that is why they are easily removed, usually before or after the observation. Systematic errors can be at least minimised by careful observations, approaching special models and special checks.

1. **Defective or Insensitive Bubble**
2. **Curvature of the Earth**

Is eliminated by its compensation with double measurement (backsight and foresight) or by a computation. Curvature of the Earth could be eliminated together with the refraction.

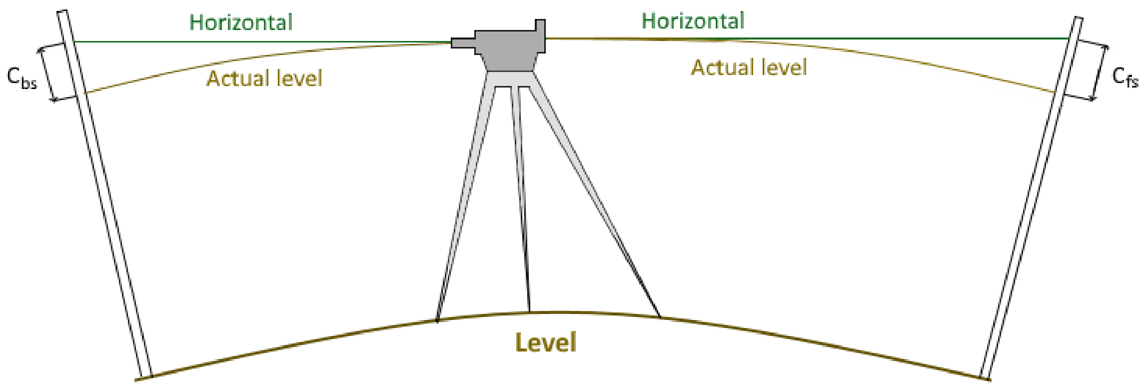


Figure.3.1 Curvature Error Compensation

3. **Atmospheric Refraction**

Atmospheric refraction exists even without any other atmospheric anomalies, because line of sight has to go through atmosphere, which leads to the error. It correlates with temperature gradient at about 50cm above the surface.

Is usually eliminated by setting a minimal sight reading (helps to avoid chaotic refraction caused by ground heating, too) or by a computation after an observation.

4. **Parallax**

Parallax is caused by physical specifics of the observer's eye and malfocusing. It should be cleared every time, when the person taking the measurements is changed. It is eliminated by proper focusing: adjusting the crosshairs and then focusing the object.

5. **Influence of the Sun and the Moon**

This error is eliminated by means of astronomical correction.

6. **Movements of the Earth Crust**

This error can be eliminated by using a method of an interrupted observation.

7. **Differences of Height of Levelling Points**

With the help of evaluation of the measurement those changes can be detected and if having a linear trend predicted for the future.

8. **Change of the Instrument Height**

Eliminated by measuring a few times at the same point by a special method BFFB, which means backsight-foresight-foresight-backsight.

9. **Not Precise Staff**

Staff tests are done quite often which helps to detect the deviation in the staff and write it down, so it could be used while doing computations and be evaluated.

10 **Vertical Deformations of the Earth Crust**

This error is eliminated by dividing an observation into parts, because deformations can last for several weeks.

11 **Influence of the Earth Gravitation**

The Earth's density is not even, which causes magnetic anomalies, they are represented by gravitation deviations.

The error can be eliminated by applying corrections.

12 **Applying Wrong/Incorrect Temperature or/and Pressure During an Observation**

This error can be eliminated mathematically after the measurement.

13 **Change of Benchmark Heights**

Not declared manual changes of benchmark position are prohibited by the law. However, in case of need, a person might get a permission to remove it.

A fee for this kind of work is calculated with the consideration of the work, that was done to the benchmark earlier. All the prices can be found on pages of the Land Survey Office.

2.2 Error Sources

Levelling errors can be caused by many factors, which could be divided into three major groups: natural, instrument and human errors. Each of those types is described in [15].

2.2.1 Instrument Errors

This type of errors is caused by bad service, application or construction of an instrument. Major part of them is eliminated/minimised by following the rules of levelling: levelling the instrument the right way, keeping the same distances of foresight/backsight shots, double centering, etc.

Any instrument should be also adjusted from time to time in order to determine and reduce such errors as: sighting errors, instrument maladjustment, bad storage consequences. It is also advised not to start an observation right after the instrument is levelled, but let it adjust to the conditions.

2.2.2 Human Errors

These errors are caused by the surveyor’s specifics: physical specifics, habits, etc.

Human errors can be eliminated by following levelling rules for the levelling process itself.

2.2.3 Natural Errors

Natural errors are the ones that are caused by environmental circumstances and conditions of an observation.

They can be minimised by the ppm correction, which is defined by temperature and pressure. Curvature of the Earth and refraction are natural errors, too. They are eliminated mathematically by using the constants. Sunny weather can be a cause of a measurement error as well, influencing the instrument. Sun can also cause distortion, which increases near the surface. That is why cloudy, cool weather is believed to be the best for observations.

2.3 Meaning of Repeated Measurements

Every measurement has some errors that influence the result. As it follows from the error sources, those errors cannot be eliminated completely. However, it is possible to minimise their influence, predict their behaviour and to eliminate some of them after taking the measurements themselves.

One of the methods is statistical evaluation of the measurement, as it was mentioned earlier in *3.1.2*. In order to improve the power of any statistical test and to find the error distribution and any outliers, that could be caused by gross errors or some different conditions in that particular observation, the observation is usually taken several times.

Repeated observations is a powerful instrument, that allows not just to improve the results of statistical tests, but to rise confidence in experimental data as well. This method helps to estimate the uncertainty in the measurements better. Precision of the results depend on the number of taken measurements, N, and on their spread. With more observations done, standard uncertainty will decrease:

$$u_{\bar{x}} = \frac{s}{\sqrt{N}} \tag{3.1}$$

Method of repeated measurements provides the best result for the determination of terrestrial displacement (vertical for levelling). Of course, drawing a direct comparison between measurements taken along the same line (levelling circuit) would be the most accurate way to compute the displacements.

As it is done in this thesis, an adjusted polynomial is chosen as an average for evaluating the measurement.

3.PROCESSING THE MEASURED DATA

As it has already been mentioned in *2.4*, in autumn of 2016 points 13a, 13b, 13c were not measured due to prohibition of entering the site. Later those points were cancelled and in the year 2017 points 13d and 13e were created. However, one-year observation cannot provide enough of data for evaluation of the measurement and point/benchmark changes, which are based on repeated measurements (described in *3.3*). That is why, if years 2016 or 2017 are chosen, the last available point is 13.

3.1 Set of the Levelling Base 6b-11

At the beginning, point 11 was the final point of the levelling base. In year 1962, 13c was chosen as a new final point of the base in Praha-Podbaba. However, the final height difference since then is being calculated to point 11 as well. Since 11 and 13c were chosen as the major final points in [7], it was decided to perform the same computations in order to compare the results.

3.1.1 Levelling Base over the Period

Firstly, the observation data was digitalised and represented in a form of a plot (*appendices 2-4*). The data was depicted both separately (*appendices 3-4*) and together (*appendix 2*) for the spring and autumn values in order to draw a rough comparison between two seasons over the whole period. Nevertheless, both of the graphs have the same trend and they don't have any significant differences that could be possibly occurred by some while observing errors or unexpected changes of the bench marks position.

After processing and visualising the results of measurements in the area with the final point 11, linear regression was chosen as the most fitting type due to a constant gradual decrease of the last benchmark of the levelling base. However, it's not the only type of dependence, because the point number 6 causes a non-linear trend of the base with its levelling changes.

3.1.2 Regression Model

Regression line was constructed with the help of a normal linear regression model, that was described in [1] and [2], where

$e_i \sim N(\mu, \ ^2)$

Where μ is the mean or the expectation of the distribution, in this case it's equal to 0, so

$e_i \sim N(0, \ ^2)$

It is a normal distribution of deviations e_i

The method used here is called least-squares estimation, where the model is expressed by

$$y = X \cdot \beta + 1e \tag{4.1}$$

Where Nx1 vector of observations of the dependent variable is denoted by

$y = (y_1 \, y_2 \dots y_n)^T$, which is represented by reduced values of elevation differences of the levelling base (in [m], so the original data could be used).

$$X_{(n,k+1)} = \begin{pmatrix} 1 & 1997 & 1997^{2} & \dots & 1997^{k} \\ 1 & 1997.5 & 1997.5^2 & \dots & 1997.5^k \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 2017.5 & 2017.5^2 & \dots & 2017.5^k \end{pmatrix}$$

X is a matrix of explanatory variables, which represents n number of changes. Values here are expressed in years and their halves, where a full year, for example 1997, responses to a spring measurement and a year and a half, for example 1997.5, to an autumn one. This helps corresponds with the period between measurements – half a year.

Matrix X has k number of columns, where k is a polynomial degree, in this case it will be 1, because the function's trend is a straight line. So, in this case matrix X size is (42,1), since the observed period is 20 years.

$\beta = (\beta_0 \;\; \beta_1 \;\; \beta_2 \dots \beta_k)^T$ is a vector of unknown regression coefficients, also known as the regression line's slope;

$e = (e_1 \;\; e_2 \;\; e_3 \dots e_n)$ is a vector of adjusted errors or residuals from the original measuring, which represents the line's intercept;

The ordinary least squares estimator $\hat{\beta}$ has a multivariate normal distribution with covariance matrix

$var[\hat{\beta}|X] = \ ^2 \cdot (X^T \cdot X)^{-1}$

This is a method of adjustment for indirect measurements, where there is an extra number of measurements. The problem was solved with the help of a method described in [4]. With this method's help the real errors nor the slope can be computed, there's no such a vector that could possibly lead to $e = 0$. In this case only regression coefficients of b can be evaluated by approximation, where

$$\hat{y} = X \cdot b \tag{4.2}$$

$\hat{y} = (\hat{y}_1\ \hat{y}_2\ \hat{y}_3 \dots \hat{y}_n)^T$ are approximated values of a responsive variable. An equation of deviations represented by vector v, where $v^T = (v_1\ v_2\ v_3 \dots v_n)$ here will be

$$v = \hat{y} - y = X \cdot \hat{b} + l' \tag{4.3}$$

Where l' are reduced values of l represented by

$$l' = l^0 - l = X \cdot b_0 - y \tag{4.4}$$

Unknown adjusted values of b are equal to the sum of approximate values and their increments:

$$b = b_0 + \hat{b} \tag{4.5}$$

Vector $\hat{b}$ is the ordinary least squares estimator, that minimises the squared residuals sum. It is defined from normal equations:

$$X^T \cdot X \cdot \hat{b} + X^T \cdot l' = 0 \tag{4.6}$$

$$\hat{b} = -(X^T \cdot X)^{-1} \cdot X^T \cdot l' \tag{4.7}$$

Unknown values could be then computed with the help of equation (5)

3.1.3 The Standard Deviation

Mostly errors of the b are not known, and they describe the accuracy of the measurement, that is why a standard deviation is being calculated. Those methods are represented in [5]. In this case we assume, that each measurement (and as it follows – their means) has the same weight, because they were taken with the same accuracy requirements, therefore we use the following equation

$$m_0 = \sqrt{\frac{v^T \cdot v}{n - k - 1}} \tag{4.8}$$

Matrix Q is a covariance matrix is used in statistics to represent covariance between two elements of a random vector. It is an orthogonal matrix, which consists of sub matrices with diagonal components in the coefficient on the normal equation. It can be also computed before an actual measuring, which would optimise the whole process of the observation.

$$Q = (X^T \cdot X)^{-1} \tag{4.9}$$

With the help of equations *(4.8)* and *(4.9)* regression parameters can be computed by the following equation

$$m_{b_i} = m_0 \cdot \sqrt{Q_{ii}} \tag{4.10}$$

3.1.4 Statistics

A t-*test* was chosen as the most suitable for the observation to test the β significance due to [6], because we are using only a sample mean.

$H_0 : \beta_i = 0$ and $H_1 : \beta_i \neq 0$

T-statistic is the name of the given(null) hypothesis, which can be computed using the following equation

$$\frac{|b_i|}{m_{b_i}} \geq t_{\alpha/2}(n-k-1) \tag{4.11}$$

The calculations were done at the significance level of $\alpha = 0.05 = 5\%$. Number of b depends on the polynomial degree, which is 1 in this case (because point 11 has a linear trend), therefore there will be b_0 and b_1.

Statistics Results

$N = 42, k = 1$

i	0	1
b_i	-80.5	-3.87^{-4}
m_{b_i}	0.228	1.14^{-4}
$\lvert b_i\rvert / m_{b_i}$	353.0	3.4
t	2.0211	

Tab.4.1 T-statistics for 6b-11

According to the results, it is clearly seen, that both of the b parameters have much higher values than t, consequently, at the significance level of 5% they both don't meet the requirements of H_o, because they are far from being equal to 0. From this, it follows that none of the values of the linear regression can or should be removed from the computations and, therefore, none of them is an outlier. Mean change of the levelling base's size is represented with the help of b_1, which is a significant result, that allows to predict future changes under the same circumstances.

Aside from the *table 4.1*, the results are also displayed in the *appendix 7*, that consists the first part of a protocol, which is one of the program output forms.

3.1.5 Standard Error of the Levelling Base Height Differences

This error is mostly dependent on the residual of levelling staffs. However, the value of the error can be also influenced by the accuracy of the used method (0.1-0.3 mm for one setup of levelling in this case) and of the whole set.

The standard error of the levelling base height differences was computed for both spring and autumn measurements are equal to the regression parameter, which was computed right after constructing the regression line *(4.10)*. It is denoted by $m_{\Delta H}$ and represented by

$$m_{\Delta H} = \sqrt{\frac{[vv]}{n(n-1)}} \tag{4.12}$$

Where n is the number of the survey crews participated in the observation.

3.2 Set of the Levelling Base 6b-13c

3.2.1 Levelling Base over the Period

The over time changes of the levelling base 6b-13c are represented in the same way and with the help of the same methods as the levelling base 6b-11. The results can be found in the *appendices 8* and *9* for separated values of spring and autumn and in the *appendix 6*, where both groups of values are depicted in the same plot. However, in this case there's no visible linear trend to be found due to non-linear changes of the bench mark 6 (which has already been mentioned in *4.1.1*) and the levelling point 13c at the same time.

3.2.2 Constructing the Fitting Curve

To describe and depict the height differences of the given levelling base were used polynomical functions of different degrees, and the fuction of the 5^{th} degree was considered as the best one for the representation due to the residual sum of squares, which can be found in the protocol. Descriprtion of how the protocol is created can be found later in the chapter 7, Program Manual. However, for the two parts of the levelling base evaluated in the thesis there are examples of the first part of the protocol in the appendices part: for 6b-13 it would be *appendix 11*. The plot of both height differences and fitting curve can be found in the *appendix* number *10*. While finding the best fitting curve was used a tool: a function in Matlab computing the curve according to the given by a user degree. Nevertheless, it is important that in this case the curve cannot help us to predict any future changes of the levelling base, because it was constructed by methods of interpolation *to fit in*, and it is not a one-way trend, that almost doesn't change over the time, since the changes are not stable.

3.2.3 Statistics Results

The same t-*test* was used to compute the significance of b_i, the number of which will be 5 in this case (b_0 up to b_5). The results are given in the table below.

$N = 42, k = 5$

i	0	1	2	3	4	5
b_i	-7.14^7	1.82^5	-185.0	9.41^{-2}	-2.39^{-5}	2.43^{-9}
m_{b_i}	8.477^3	6.79	2.1^{-3}	2.73^{-6}	5.18^{-10}	1.4^{-13}
$\lvert b_i\rvert / m_{b_i}$	8.422^3	2.68^4	8.81^4	3.45^4	4.62^4	1.73^4
t	2.0639					

Tab.4.2 T-Statistics for 6b-13c

4.SPECTRAL ANALYSIS

Spectral analysis (also called Fourier analysis due to use of the Fourier transform) is one of the most widely used methods for data analysis. It is mainly used for signals or time series. The analysis represents a random process as a combination of periodical components: sin and cosine waves, which have different frequencies. As a result, spectral analysis helps to find those frequencies and to calculate the intensity or power of the signal at various frequencies. This is often called a spectrum of a random process. There're several methods for testing periodicity of time series, the most often used are described below and compared for the application on the given data.

4.1 Periodogram

Given a signal of N length, one could fit $N/2$ sinusoids, but those must be true sinusoids of the signal. It is where periodogram is used. A periodogram is a tool where a plot the frequency of the signal and its intensity is plotted. It helps to test periodicity of the given time series. This is

$$I(\lambda) = \frac{1}{2\pi N}\left|\sum_{t=1}^{N} X_t e^{-it\lambda}\right|^2 \tag{5.1}$$

The equation given above might be also computed as a combination of sin and cosine waves with the help of the following equation

$$I(\lambda) = \frac{1}{4\pi} * \left(A^2(\lambda) + B^2(\lambda)\right) \tag{5.2}$$

Where

$$A^2(\lambda) = \sqrt{\frac{2}{N}} * \sum_{t=1}^{N} X_t * \cos(t * \lambda) \tag{5.3}$$

And

$$B^2(\lambda) = \sqrt{\frac{2}{N}} * \sum_{t=1}^{N} X_t * \sin(t * \lambda) \tag{5.4}$$

It should be noted, that in the case of using this method the energy of the periodic signal in such hence and infinite. That is why the method should be applied carefully and a statistical testing should follow it, so at least the null that signal is white noise is checked.

Fisher's g *test* is one of the best methods for statistical calculations of periodogram reliability as it is noted in [13], it helps to decide if the computed peak at a specific frequency is not random.

Sampling error of periodogram might occur in our case, since the computations are made only for a single time series. And *Fisher's* g *test* is considered to be conservative in nature nowadays. That is why it is better to use power spectrum analysis. However, one of the main conditions for using another method is actual testing of both and comparing how each of the results fits in the original data. After reconstructing sinusoids, it was decided, that power spectrum analysis is a better method for the spectral analysis of the observations.

4.2 Fast Fourier Transform

A fast Fourier transform (FFT) is widely known as computationally effective algorithm of discrete Fourier transform (DFT), which is actually an equivalent of the continuous Fourier transform (DTFT) for signals, which are known only at N instants separated T times by samples, which creates a finite sequence of data. FFT is an algorithm that is almost always used to compute DFT in practice.

FFT is not a new method, it was invented by Carl Friedrich Gauss, a German mathematician, in 1805. Original purpose of FFT was assisting in computing orbits as a faster method for DFT. Through the history, different algorithms for FFT were invented independently by scientists. However, it became widely known because of the Cooley and Tukey algorithm, which was also useful in a new era of computations with the help of digital computers.

In FFT we have the-N-long DFT, in [14] is defined as

$$X(k) = \sum_{n=0}^{N-1} \left(x(n) * e^{-\left(j\frac{2\pi nk}{N}\right)} \right) \tag{5.5}$$

Where

$X(k)$...is complex-valued and $k \geq 0$

$X(n)$...is complex-valued and $n \leq N - 1$

The algorithm requirements are

1) in order to compute each $X(k)$ there should be N complex multiplies;
2) to compute all N frequency samples directly, N^2complex multiples and $N(N-1)$ complex additions are required.

Therefore, precomputation of the DFT coefficients is in the computation, so the cost would not be even higher

$$W_N^{nk} \doteq e^{-\left(j\frac{2\pi nk}{N}\right)} \tag{5.6}$$

All in all, the idea of FFT is to take an N-long DFT and to factor it into several shorter DFTs, compute the outputs, which will be then used multiple times in order to calculate the final result.

A power of two, where $N = 2^M$, is one of the most commonly used select of FFT. It is a part of a very well organised FFT algorithm called power-of-two. That type is a very efficient algorithm, which is gained by a multiple reuse of shorter DFT's results and by simplicity of length-2 DFTs, since no multiplications are required there.

5. PROGRAM MANUAL

5.1 Main Information

As a part of a project a program for various computations in the testing levelling circuit in Praha-Podbaba in the period over 20 years (1997-2017) was created.

It allows the user to make wider calculations and represent the base or the circuit not only as one value, but divide it into parts as well and evaluate the parts.

MATLAB (matrix laboratory) was used for creating the computation scripts, GUI and to compile the program. MATLAB is a product developed by Mathworks and a proprietary programming language, which is also interfacing various programs written in other languages. The language was based on three commonly used programming languages: C++, C and Java. For example, in GUI creation it partially uses C for the program compilation. MATLAB is described in [16].

5.1.1 Program Installation

A MATLAB standalone application is the output of the done work. In order to execute the computations without buying a license, MATLAB provides two variants, how to install the program:

1. Download MATLAB Runtime manually, which is a standalone set of shared libraries
2. Insert MATLAB Runtime into the application

With adding the libraries directly into the application, its size increases from about 7MB to 800MB, but this results in easier installation process.

To install the app properly the user should follow the next steps:

1. Open the executive file
2. Start the installation
3. Install both the application and MATLAB Runtime
4. Insert the “data” folder into the same folder as the installed executive is in (if there are several .exe files, this folder should be in each of them!)
5. Run the application

5.2 Main Page

Figure.6.1 Application Main Page

This is a start page of the application. The full name of the application is the same as the bachelor thesis's. However, it was decided to put Praha-Podbaba 1997-2017 in the top, because it describes two of the main properties of the application: its area and the period.

A new logo was created to make the application more independent. It is a simple picture of crosshairs. However, due to Mathworks policy, the logo may be used only for the program itself, but not for the single windows.

The popupmenu under the string "Choose an Option" allows the user to open one of the following windows

1. Plotting the Levelling Base – just *Levelling Base* in the list.
2. Plotting the Circuit and Height Profiles– just *Circuit* in the list
3. Adjustment of the Levelling Base and Accuracy – just *Adjustment* in the list
4. *Spectral Analysis* (the name is the same in the list)

5.3 Plotting the Levelling Base

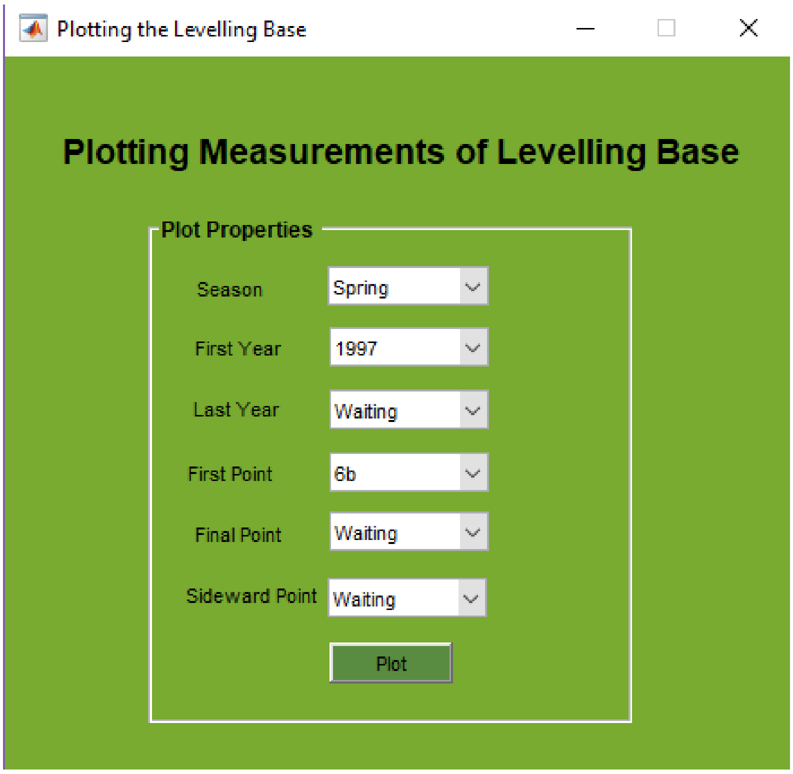


Figure.6.2 Plotting the Levelling Base Window

Plotting the Levelling Base window was created for separated graphical representation of the levelling base in Praha-Podbaba in 1997-2017.

It allows to plot a graph for spring, autumn or both seasons at the same time in order to show an overtime change. The period can be chosen by the user by changing first and last year values in popupmenues.

The First Year and the First Point popupmenues are filled at the same moment the application is opened. However, the Last Year's, the Final Point's and the Sideward Point's values have only a "Waiting" string. It was made to protect the application from falling: overtime change might be represented from past to the future, points start with the first levelling base point. So, once those values are chosen, the Last Year and the Final Point popupmenues are filled with lists starting with the chosen value. If the final point has any sideward points measured, the corresponding popupmenu will get them, too.

There are several things to remember:

1. Starting with autumn 2016 points 13a/b/c were removed (more details in 2.4), so once those values are chosen, the user is not allowed to choose an observation further than to point 13, because the data does not represent a repeated measurement yet (only 3 observations taken).
2. Although the application allows to plot different observations, it is important to know, that this page works only with height differences in order to avoid the influence of different types of benchmarks.

Examples of plot outputs, that were used earlier for main computations, are shown in *appendices 2* and *4*.

5.4 Plotting the Circuit

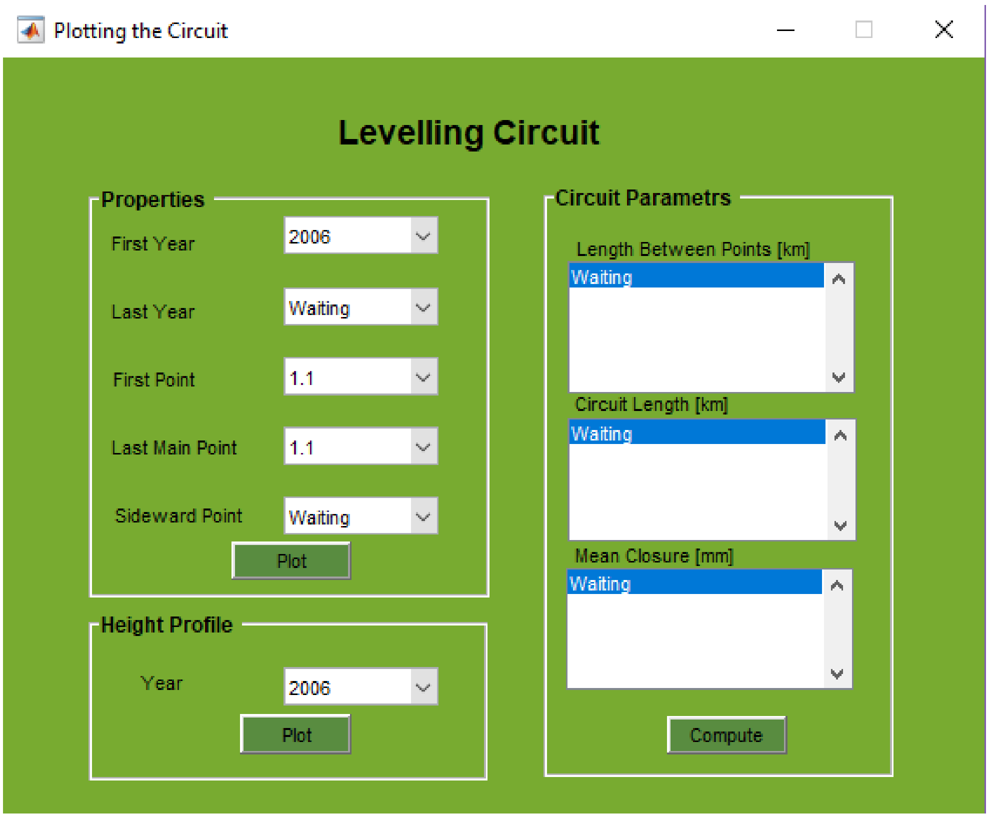


Figure.6.3 Plotting the Circuit Window

The Plotting the Circuit window was created to plot the circuit and height profile of the circuit.

The first Plot button was made the same way as the Plot button in the Plotting Levelling Base window: it shows an overtime change for the chosen last point.

The window also shows some properties of the measurement. Such as: length between the chosen by the user points, length of the whole circuit (depends only on the period) and mean closure, which depends on the points.

For example, if the user chooses not 1.1 as a final point of interest, mean closure is going to show big values.

All the mentioned properties are represented for each year. Listboxes are going to show several numbers following each other, each of them corresponds with a year in an ascending way.

Height Profile part allows to plot a height profile which is close to the classical representation of height profiles in geodesy. Regardless the fact that point 10 is supposed to be the most reliable one, the circuit's height profile computes the lengths with the help of point 1.1 (1.2 for later observations), which was taken from the official database on [17].

In the Height Profile window, if point 13 is chosen as the last main point, a sideward point is chosen as well (13a,b or c), and the first/last year is 2017, the program automatically corrects the sideward point list to new built in 2017 13d,e. Errors and circuit closure are computed the same way.

5.5 Adjustment of the Levelling Base and Accuracy

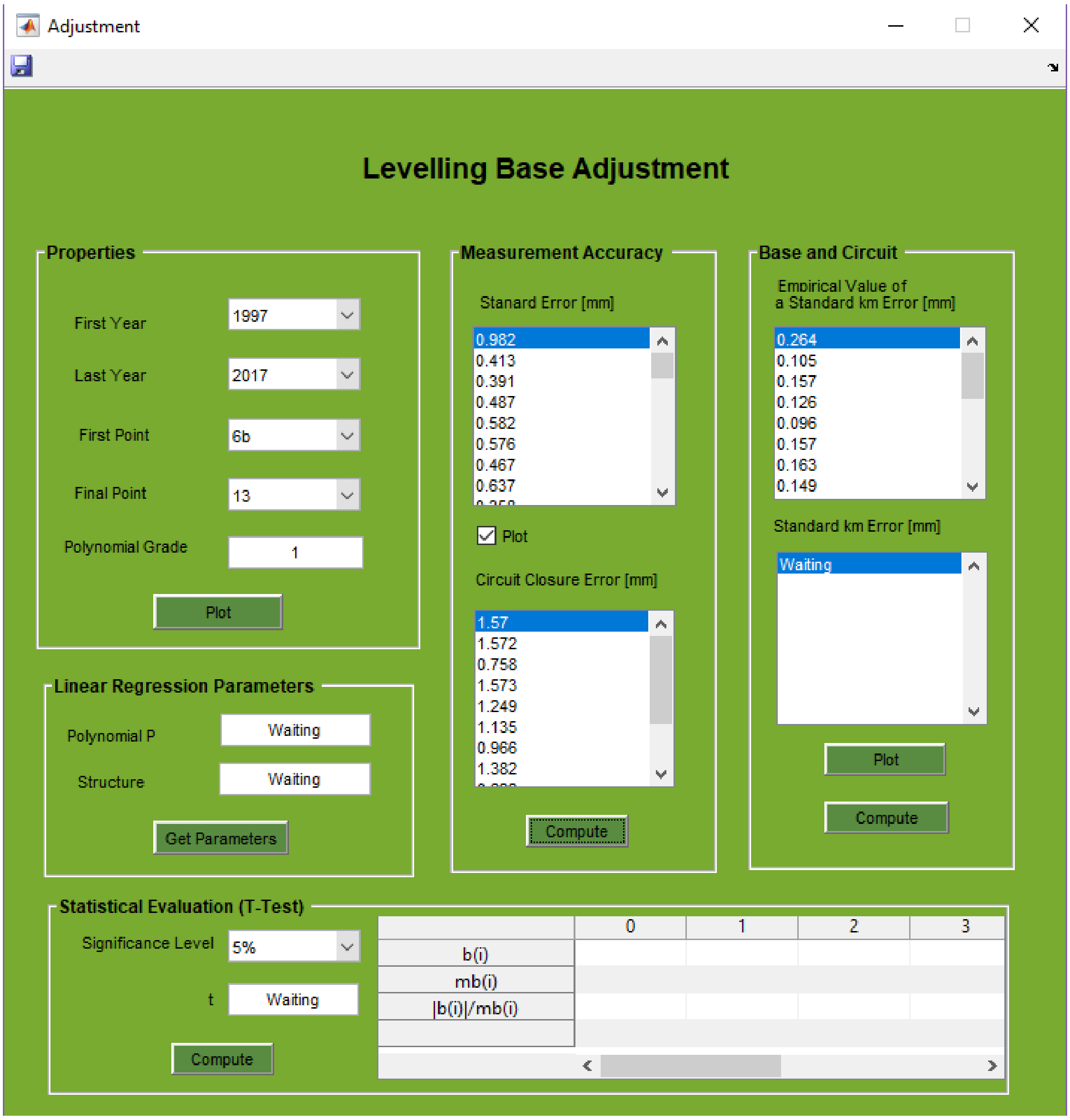


Figure.6.4 Levelling Base Adjustment Window

This window is titled Levelling Base Adjustment. Nevertheless, it allows to compute and plot accuracy of observations as well.

Despite working with the classical final points of the levelling base in Praha-Podbaba the user could choose any point to start and to finish the plotting and computations for the base. This should allow to check trends in different parts of the base in order to find abrupt errors that could occur only a few times or even ones as the consequence of a measurement error, for example.

Pressing the Plot button in Properties section automatically computes standard errors and circuit closure error in the measurement section regardless of the chosen start/final point. It also plots the chosen part of the observation and the constructed with the help of functions *polyfit* and *polyval* polynomial of the chosen grade.

Circuit closure error (m_φ) is immediately computed for the period of 2006-2017, if the chosen period is till the year 2006. If the last year is equal to 2006, the values will be computed for 2006-2017, if the last year is within the boundaries, but the first year is smaller than 2006, the values will be computed for the period of 2006-last year.

Those data are later used in Base and Circuit section. However, if the chosen period does not correspond with the period when the Circuit was observed (2006-2017), the user will get an error message, that the chosen period does not fit any observation and an informing one, that empirical value of a standard km error is automatically computed, once the Compute button is pressed in the Measurement Accuracy section.

Linear Regression Parameters section was created specifically for the situation, when polynomial grade is equal to 1 in order to show the main parameters of the linear regression constructed for the measurement.

Statistical part was fully created to correspond with a table form, as it is represented in *4.1.4* and *4.2.3*. Only t-value is shown separately to make it easier to find for the user.

There is a function save in the Toolbar right at the top of the window. This function creates a protocol for the chosen period. To avoid problems with switching to a different format (cell array with a string to just a string) it was decided to create a protocol for all the computations, except for the statistics, because it usually helps to evaluate the certainty of the results and is not as sufficient for common results as the other parts. In case of the repetitive computations executed for the same period, the protocol will rewrite itself. All the protocols could be found in the folder with the application, where they are automatically saved by the system.

5.6 Spectral Analysis

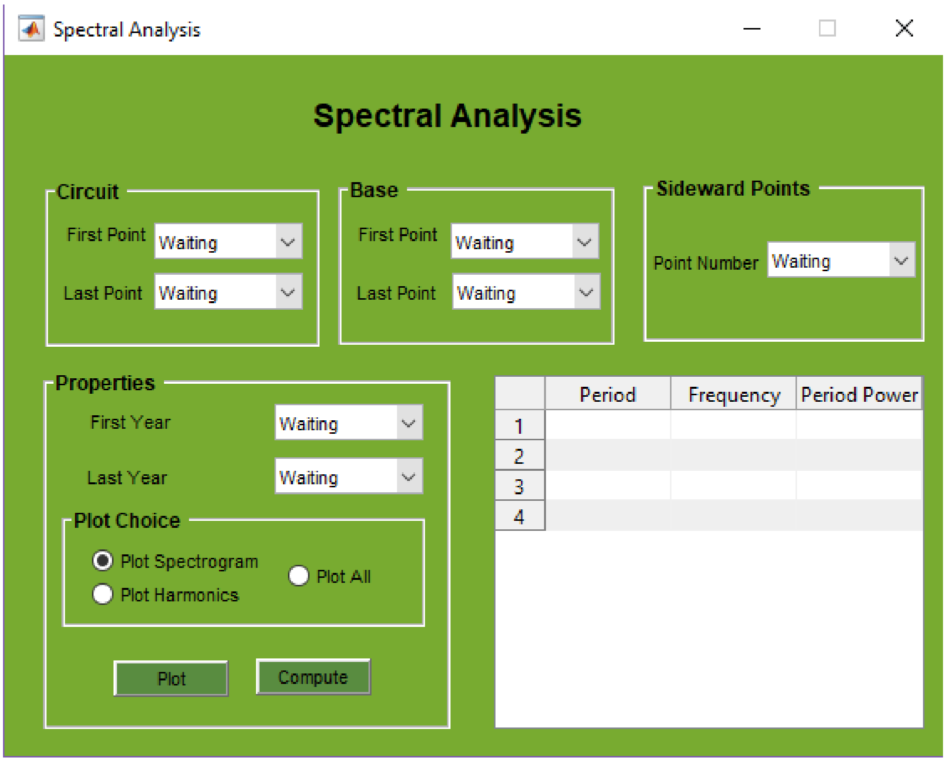


Figure.6.5 Spectral Analysis Window

The Spectral Analysis window allows the user to perform FFT on the levelling circuit or levelling base for a certain period of time chosen by the user as well.

Function *FFT* in MATLAB is used for the computations. Its power (Period Power in the table) must be evaluated manually by the user, since fast Fourier transform does not always show the real period, but a sinusoid that fits in.

In the Plot Choice button group, the user is allowed to plot only the spectrogram (FFT real output), all the harmonics (constructed sinusoids for the data) and all, which means plotting the following parts of FFT

1. The input data itself
2. Power of FFT
3. Real and imaginary parts of FFT
4. Angle of the constructed sinusoid
5. Difference between the input and the output function
6. FFT harmonics

CONCLUSION

One of the aims of this thesis was creating a useful application, that would help to evaluate precision of the data without manual calculations.

During the computations it was noticed, that the standard error didn't have any trend, but there were years, when its value was higher than usual. For example, year 2007. However, all those great differences are mentioned in conclusions of protocols from the provided.

It was also discovered, that trends of the levelling base didn't change since the last evaluation: point 11 is still decreasing and for 6b-13c polynomial of the 5^{th} grade was considered the best one due to the residual sum of squares (RSS).

With the help of the Adjustment and the Levelling Base parts of the program point 10 was proven to be the most stable, that represented the point and then calculated the most fitting polynomial function. There is a slight difference between RSS of the polynomial of the first and second grades, which leads to the conclusion, that point 10 despite of its stability has a trend of changes – it slowly decreases.

During the thesis processing I extended my knowledge in MATLAB. Mostly in GUI and creating standalone applications for users, that don't have this computational software.

Spectral analysis was processed in chapter 5. The results may vary from the ones that were provided earlier, because the duration of the observations is less, and the used method is different from the one, that was used previously.

Due to the changes in points, stated in 2.4, number of samples for the data processing was less, which influenced the results' uncertainty. Nevertheless, the program was made the way, that the data could be added to it, but the user should maintain the rules, that are mentioned in chapter 6. Hopefully, the program is going to be useful for the Land Survey Office, and will be developed further.

LIST OF FIGURES

LIST OF TABLES

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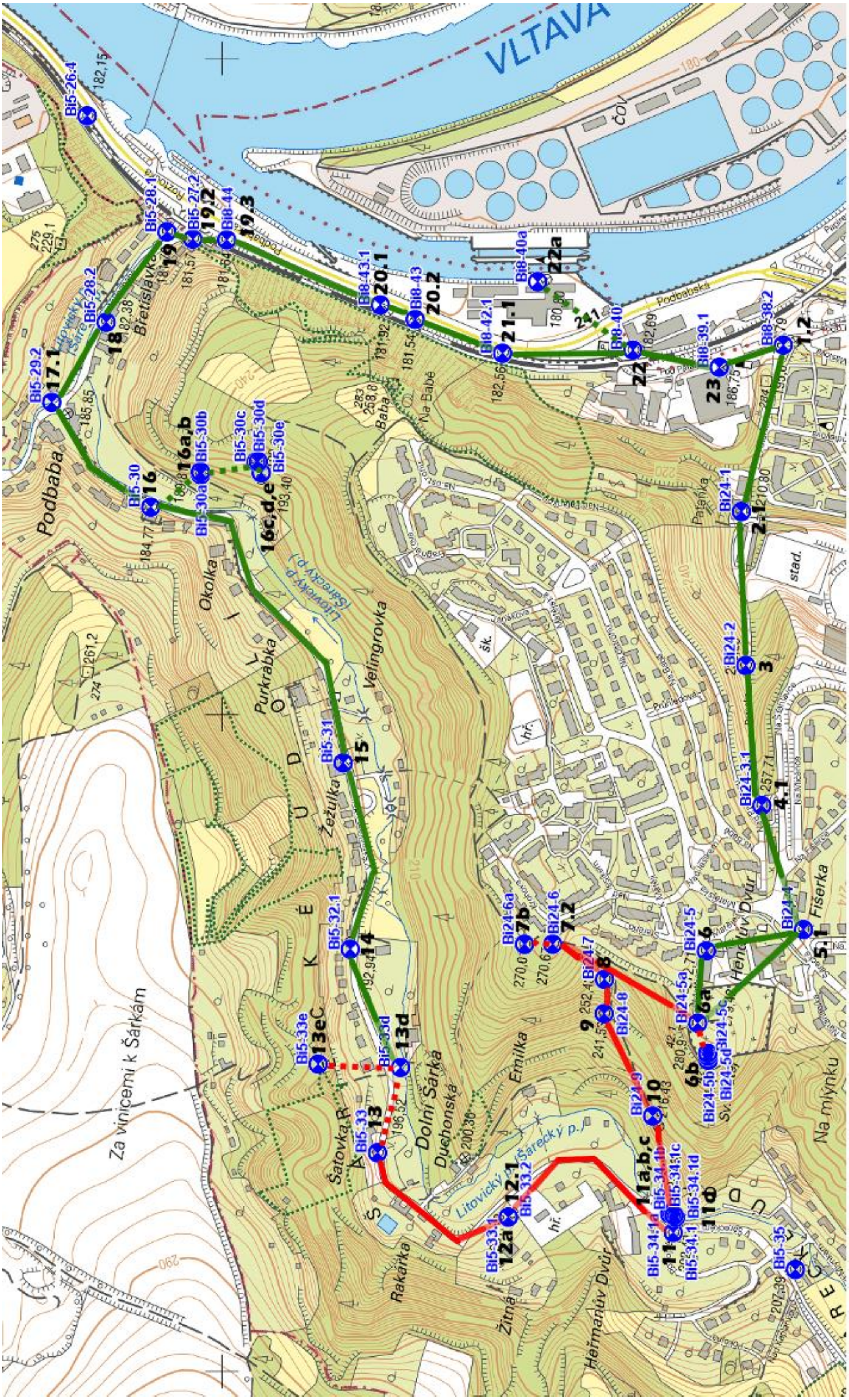
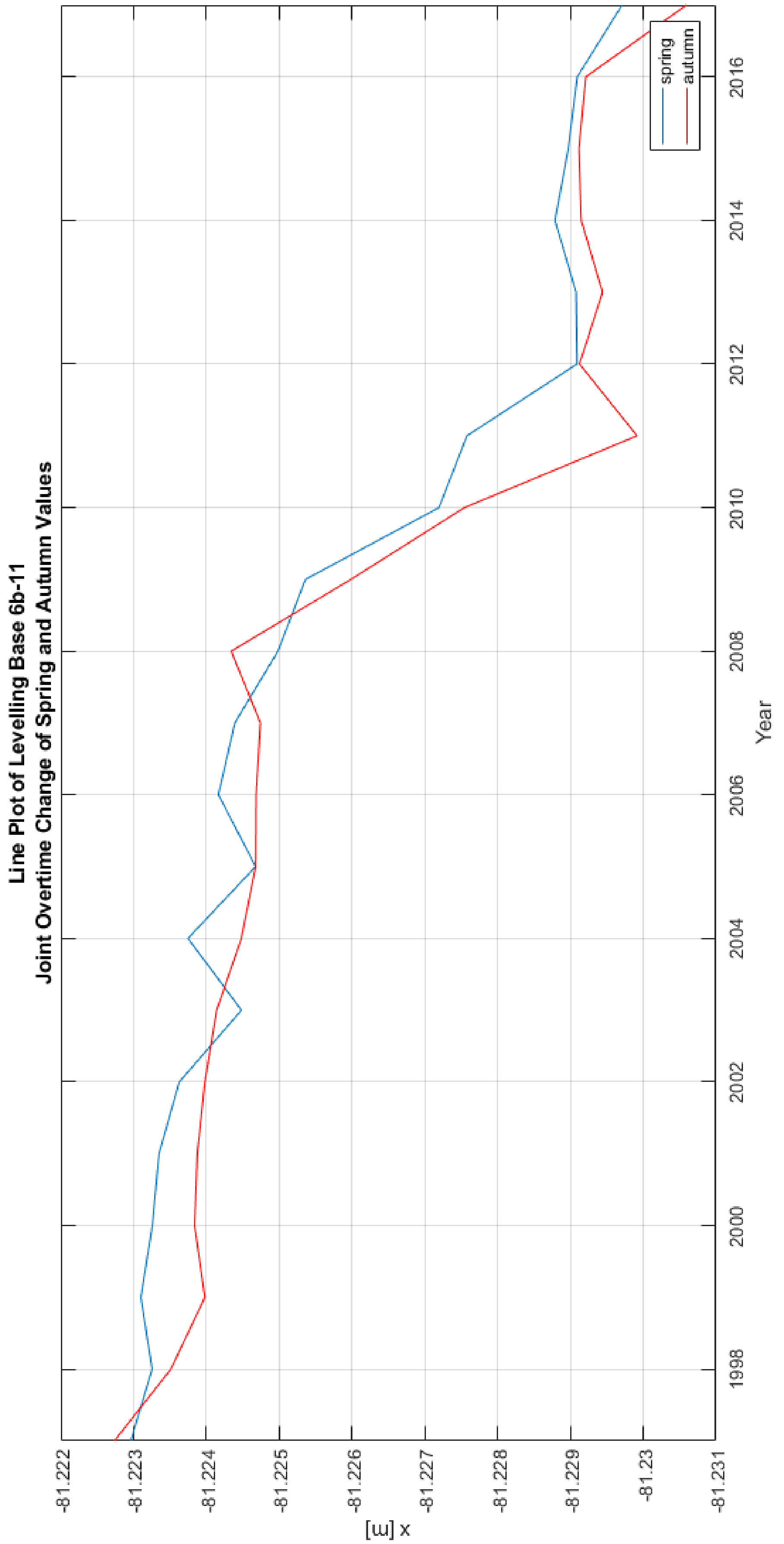
LIST OF APPENDICES

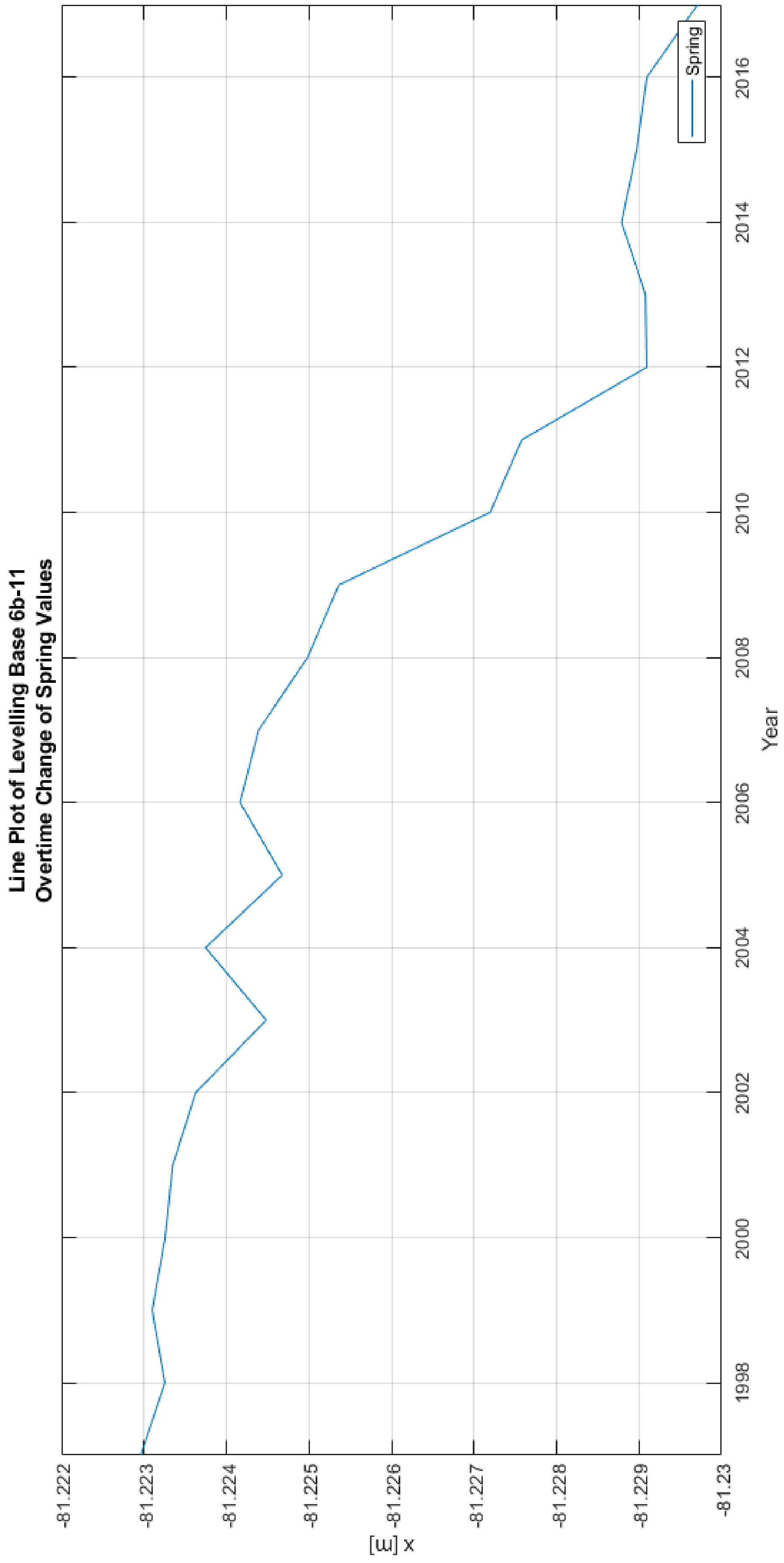
Appendix 1 – Testing Levelling Circuit in Praha-Podbaba



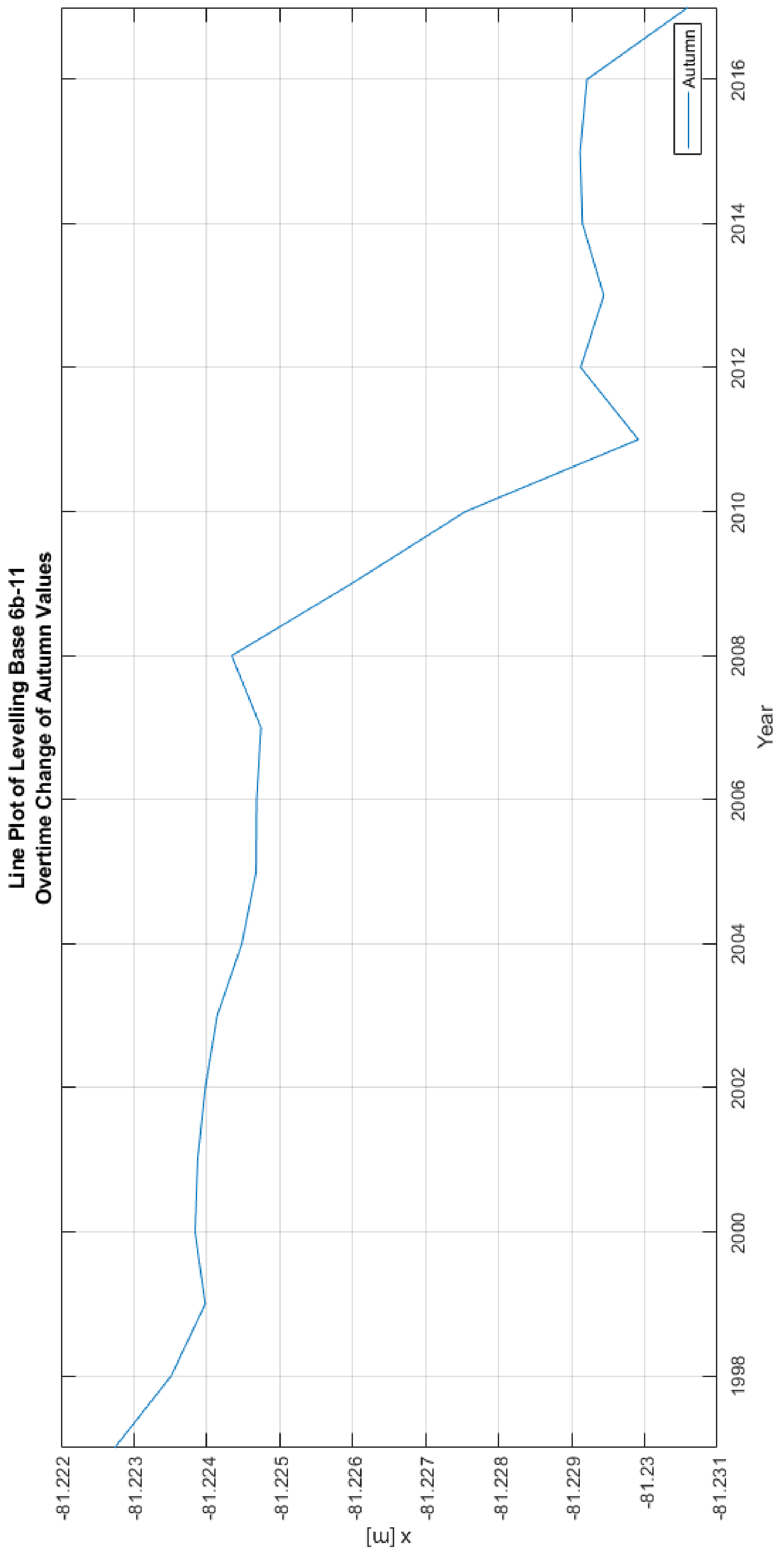
Appendix 2 – Levelling Base 6b-11. Spring and Autumn Values



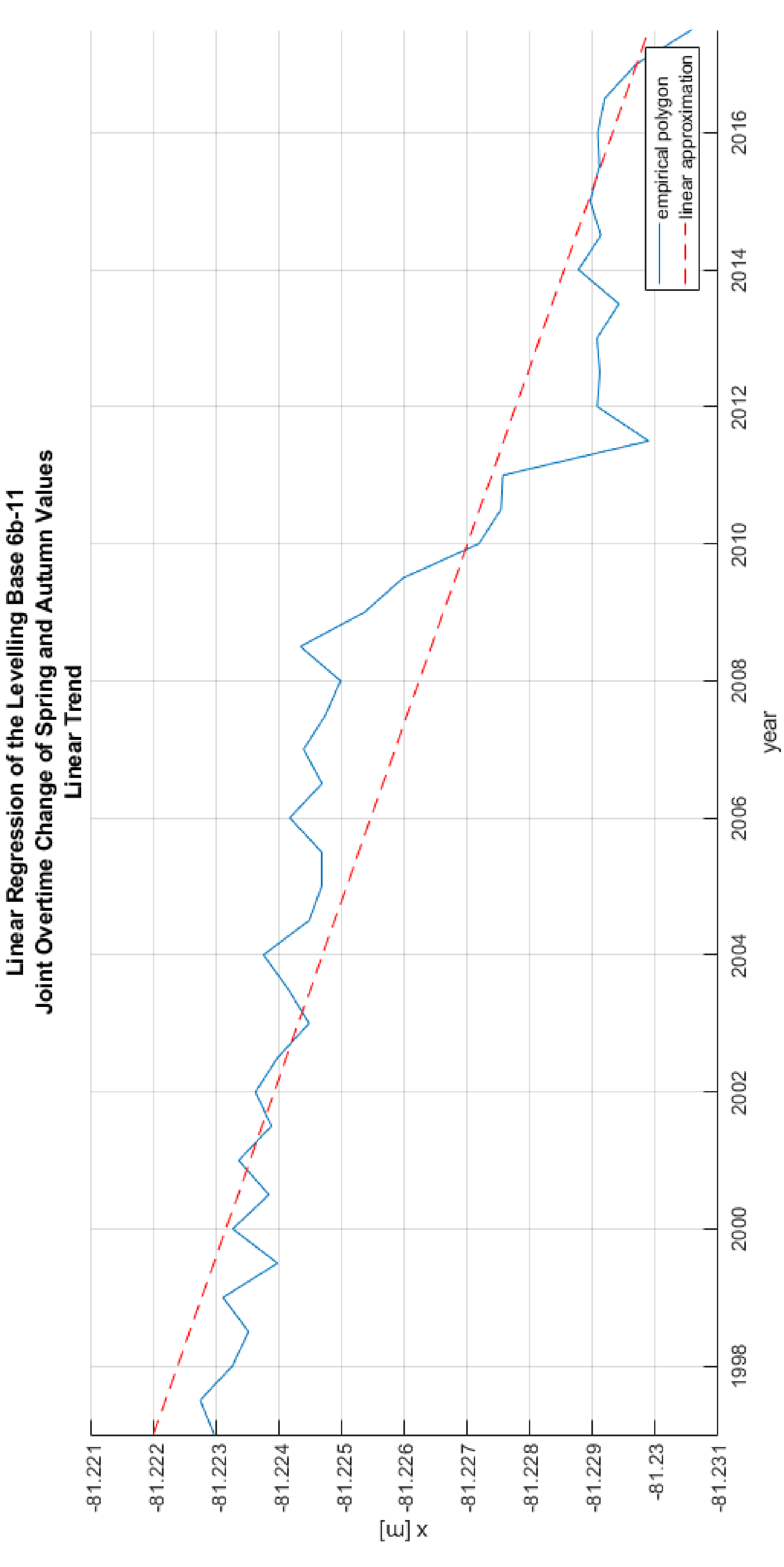
Appendix 3 - Levelling Base 6b-11. Spring Values



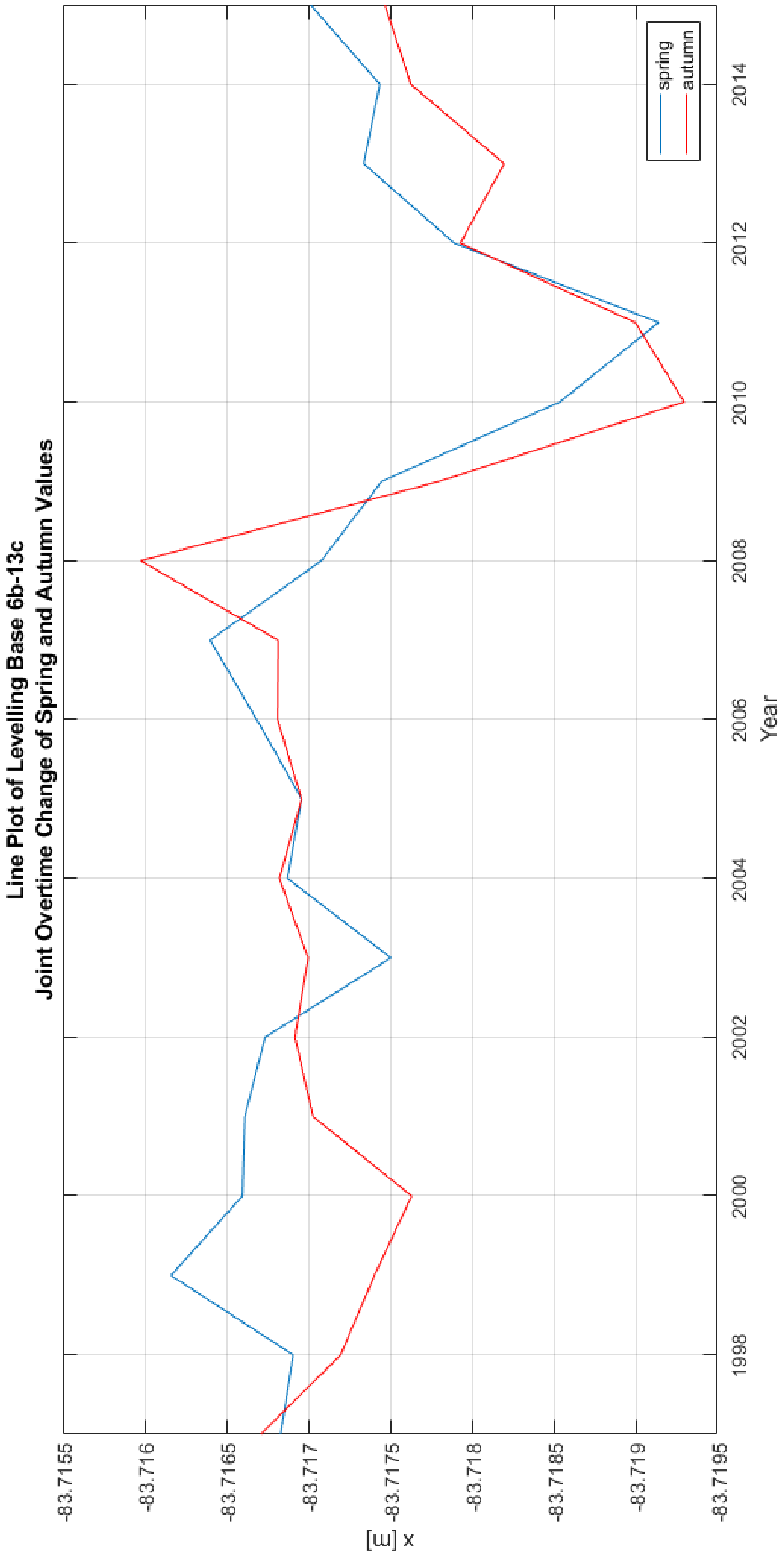
Appendix 4 - Levelling Base 6b-11. Autumn Values



Appendix 5 - Levelling Base 6b-11. Linear Trend



Appendix 6 - Levelling Base 6b-13c. Spring and Autumn Values



Appendix 7 – Protocol 6b-11

Adjustment Protocol 6b-11

Period 1997-2017

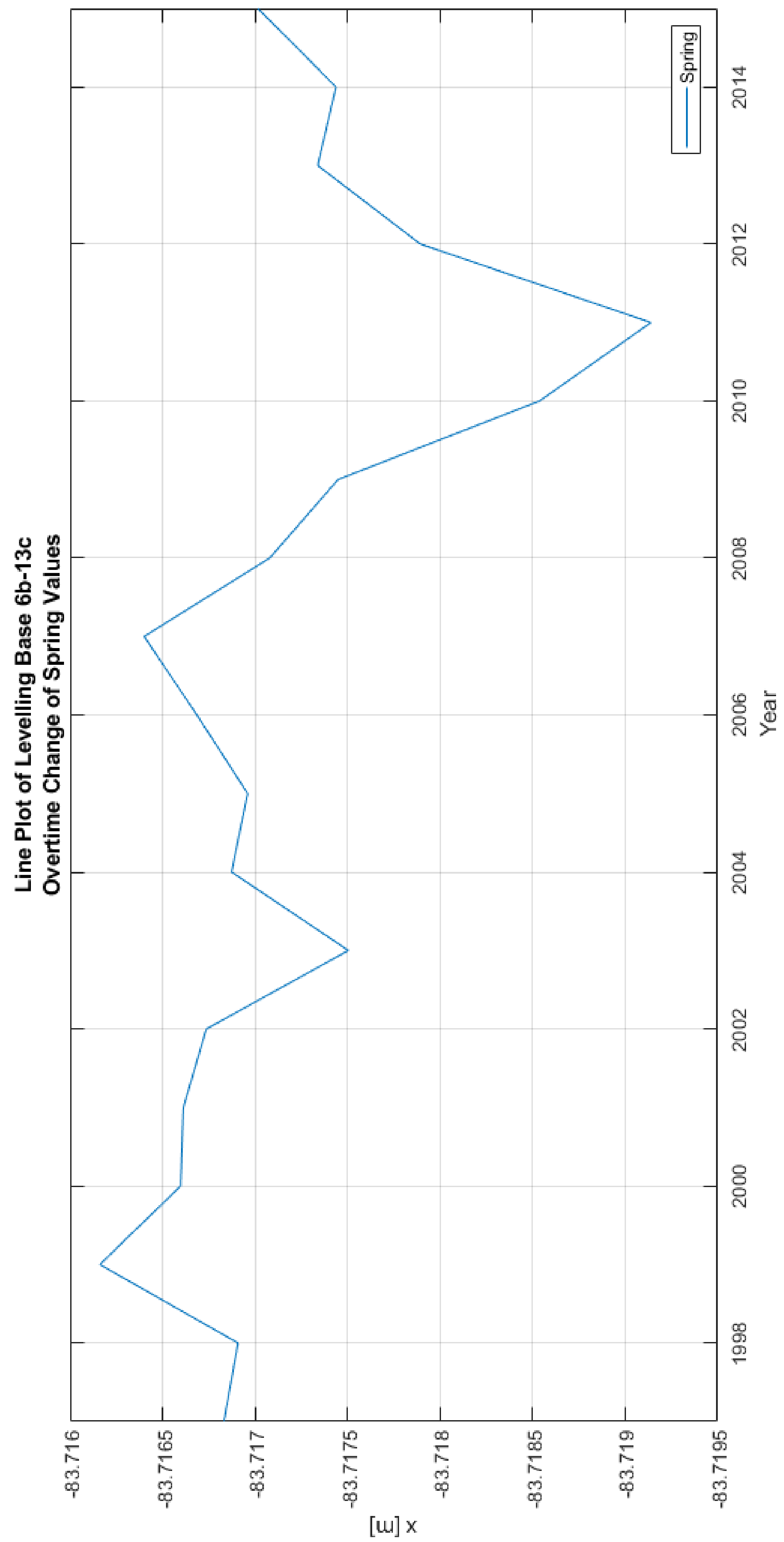
Polynomial Grade 1

Data

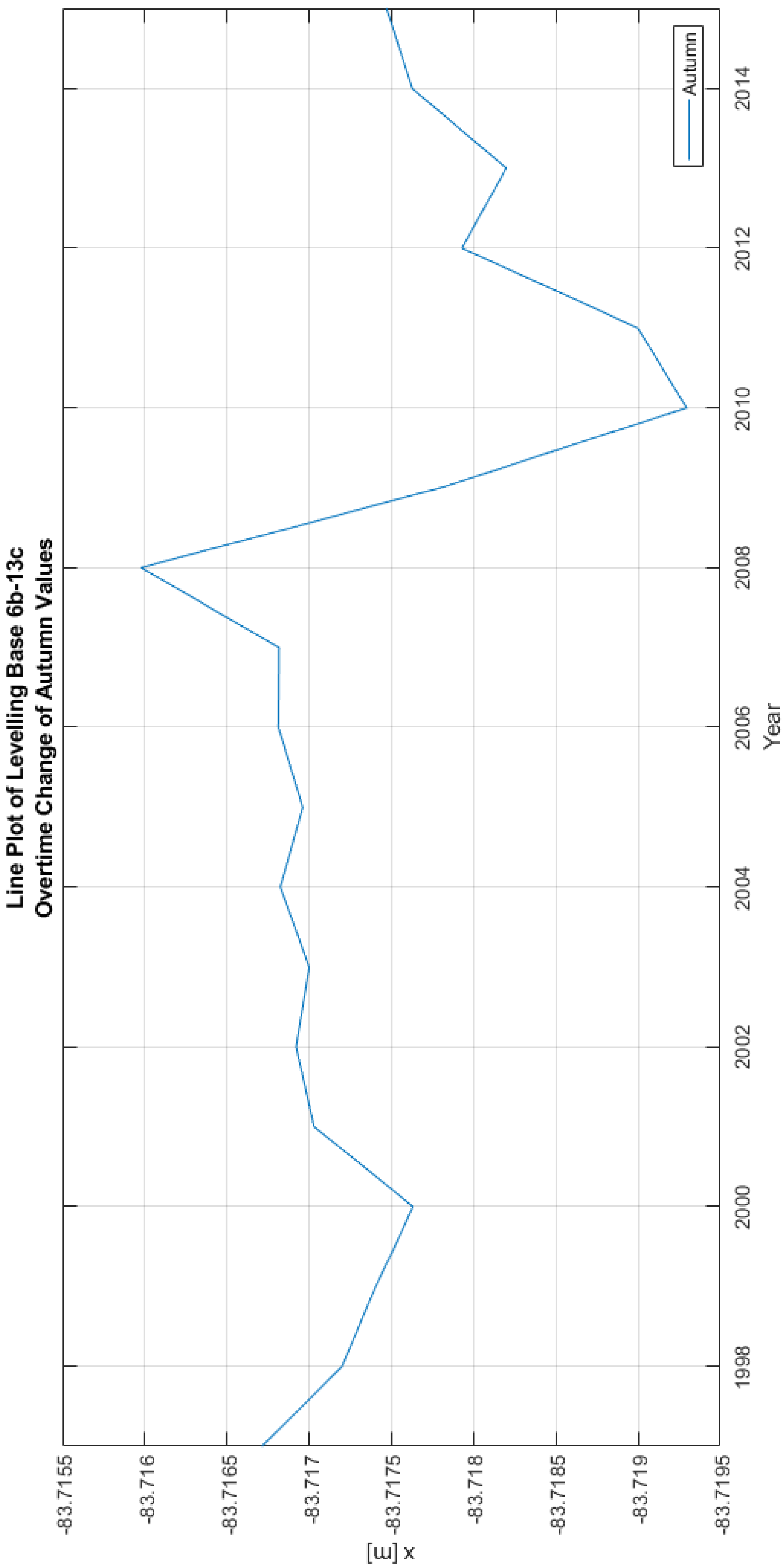
Original	Adjusted
-81.222962	-81.222735
-81.223253	-81.223511
-81.223100	-81.223979
-81.223257	-81.223838
-81.223349	-81.223874
-81.223627	-81.223979
-81.224481	-81.224142
-81.223747	-81.224480
-81.224676	-81.224676
-81.224165	-81.224683
-81.224390	-81.224743
-81.224983	-81.224340
-81.225361	-81.225985
-81.227195	-81.227544
-81.227578	-81.229915
-81.229091	-81.229125
-81.229078	-81.229441
-81.228789	-81.229149
-81.228971	-81.229118
-81.229094	-81.229212
-81.229710	-81.230595

Residual Sum of Squares 3.2392e-05

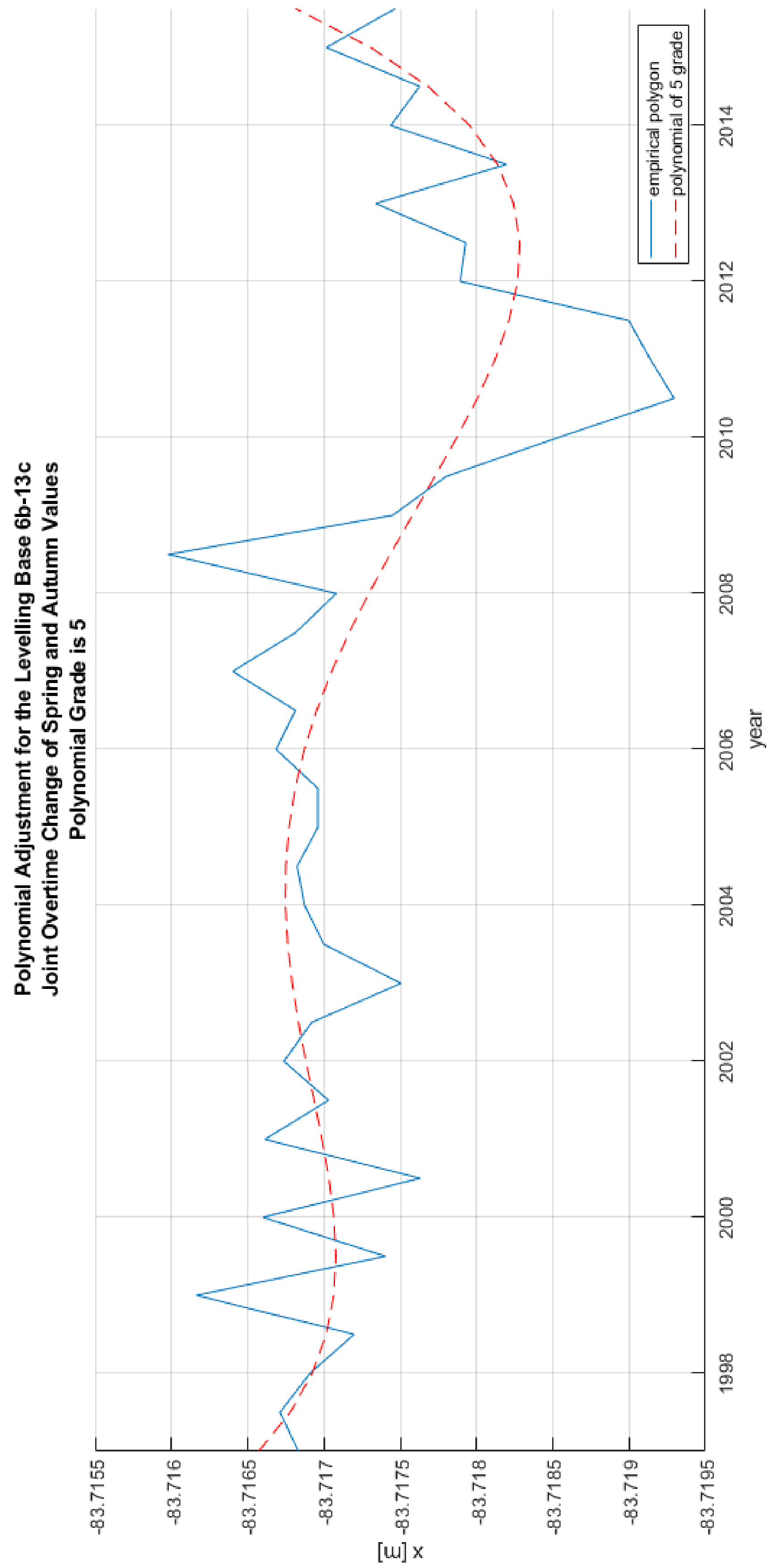
Appendix 8 – Levelling Base 6b-13c. Spring Values



Appendix 9 – Levelling Base 6b-11. Autumn Values



Appendix 10 – Levelling Base 6b-13c. Polynomial of 5th Grade



Appendix 11 – Protocol 6b-13c

Adjustment Protocol 6b-13c

Period 1997-2015

Polynomial Grade 5

Data

Original	Adjusted
-83.716832	-83.716712
-83.716908	-83.717199
-83.716162	-83.717404
-83.716598	-83.717633
-83.716613	-83.717030
-83.716737	-83.716920
-83.717505	-83.717000
-83.716873	-83.716824
-83.716960	-83.716960
-83.716686	-83.716812
-83.716401	-83.716816
-83.717080	-83.715976
-83.717449	-83.717801
-83.718538	-83.719296
-83.719140	-83.718998
-83.717893	-83.717929
-83.717339	-83.718198
-83.717437	-83.717627
-83.717015	-83.717469

Residual Sum of Squares 1.0858e-05