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CREEP AND SHRINKAGE ANALYSIS IN REINFORCED CONCRETE STRUCTURES DURING CONSTRUCTION OF HIGH-RISE BUILDING

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ABSTRACT

This thesis presents a simplified assessment of deformations of two reinforced-concrete high-rise buildings during construction, which are constructed at two different construction rates. The primary objective is the calculation of the difference in the vertical shortening of the two buildings, which should be connected thru an expansion joint so that the difference of the height at each floor level is unrecognizable to the user anytime during the service life of the two buildings. Step-by-step procedure, including construction schedule and assessment of structural stiffness, is performed in order to implement real situation and obtain adequate result. Therefore, characteristic values of most expected loads were used, such as self-weight of structure and heavy-weight equipment, in calculations of axial shortening. Time-dependent properties of concrete, such as Young’s modulus of elasticity and compressive strength, as well as creep coefficient and shrinkage strain were evaluated by the Eurocode 2 model. All expected issues can be classified as load-bearing structure problems, problems related to detailing of non-load-bearing structures within one building and between buildings, separated by expansion joint.

Abstract **I**

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1. Introduction

1.1 Motivation

In the modern world of construction industry, market economy challenges construction companies to build better and quicker, nevertheless state standards require compliance. Various types of structures can be built in terms of national's code, however, there are structures or situations that cannot be prescribed by standards. Detailed analysis of concrete's behaviour allows experienced engineers to design outside the standards. With the deep knowledges of construction material's nature, technology of construction and structural analysis, structural engineer becomes an indispensable professional.

1.2 Situation

Reinforced concrete structures subjected not only to elastic and temperature change deformations, but also to creep and shrinkage, that are significant during the early age of concrete. By this means, reinforced concrete structures undergoes bigger deformations during the construction period, than during the service life of structure. The effects of creep and shrinkage, especially for high-rise structures and heavy-weight structures, during construction, must be evaluated and handled to avoid undesirable results.

In this study, total axial shortening of two buildings, separated by expansion joint, constructed at various construction speed, will be assessed to investigate the total deformation due to constant stress increment, causing bigger elastic and creep strains, shrinkage of concrete members, with respect to curing conditions, geometry and strength class of the concrete members, ambient relative humidity of the environment and the class of used cement.

The following situation is examined:

Customer decided to build a high-rise building of 24 floors with the total height of 72 meters (Block A). Construction speed takes 7 days per floor. When the erection of mentioned above building reached 75% (18 floors already built), customer resolved to build exactly the same building (Block B) adjacent to existing one, that both structures will share the inner space and shall be divided by expansion joint. The deadline for construction of both buildings is due to completion of the Block A, therefore Block B must be built within abnormal time sequence of 42 hours per floor.

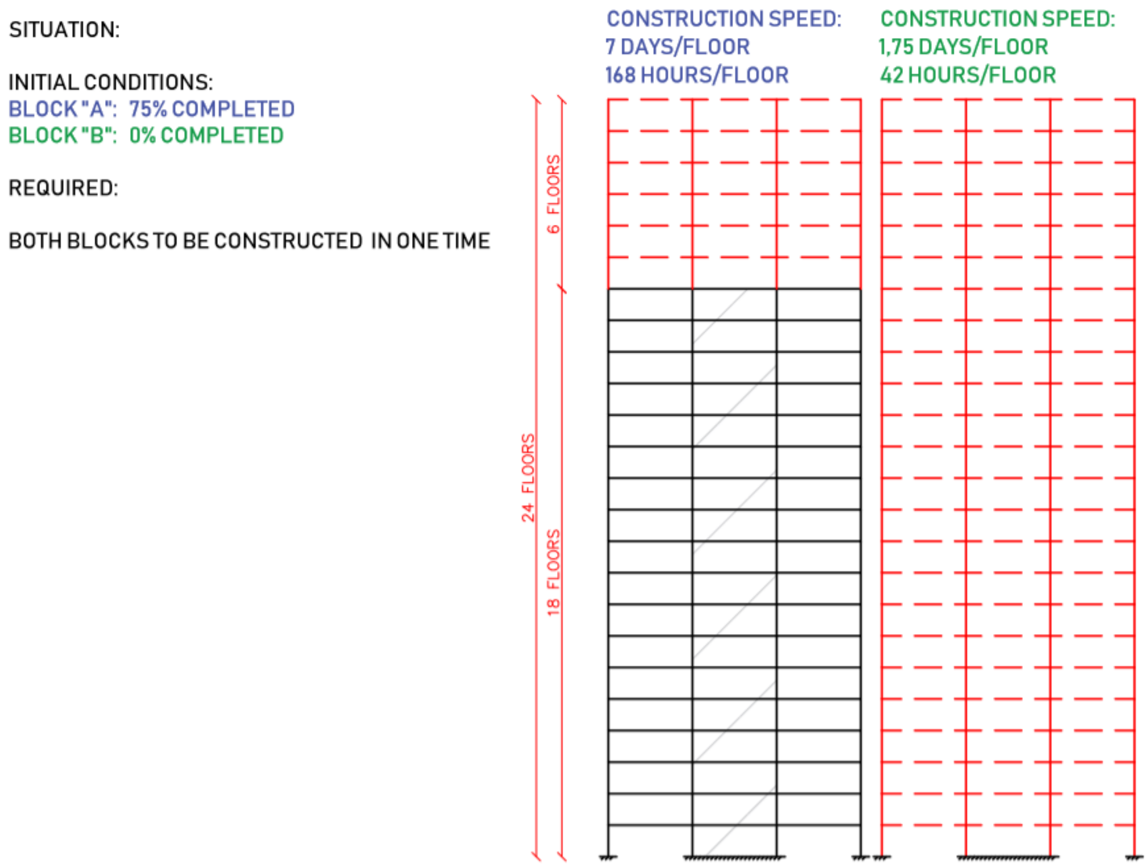


Fig. 1.2.1 Illustration of situation

Both buildings to be made of cast-in-place reinforced concrete.

Since deformation is a time-dependent parameter and creep and shrinkage are huge during the early stage, construction schedule of Block B must be implemented in details. Construction schedule will be discussed individually in details.

1.3 Expected results

Between two buildings:

Early age loading on concrete columns leads to bigger shortening, since mechanical properties of a material are not fully developed and the nature of the concrete subjected to creep and shrinkage. It is expected, that equally loaded columns, at relatively young concrete age and more matured concrete, shall deform differently. Thereby, early load of concrete follows with bigger inelastic deformations.

Within one building:

Due to structural plan, different columns within one building carries different loads, apparently causing various axial compressive stresses in members. With the described above fact, more loaded columns shall deform bigger by virtue of elastic and creep strains.

Usually columns suffer bigger deformations than shear walls. Differential shortening between columns and walls as well expected to take a place.

1.4 Possible problems

Between two buildings:

Differential level of floors between two buildings is not dangerous for structure, but is a problem for serviceability, related to detailing of joint connection between corresponding levels.

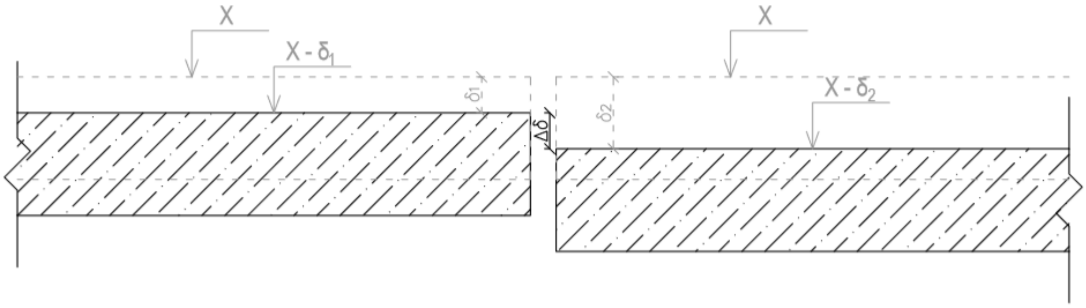


Fig. 1.4.1 Schematic illustration of differential level of floors

Within one building:

Since the building is made of cast-in-place reinforced concrete, differential axial shortening of columns/columns and columns/shear walls causing additional internal forces, such as bending moment and shear force, in horizontal structures. Imposed internal forces affects on limitation of possible cracks width, leading to deterioration of steel reinforcement by corrosion.

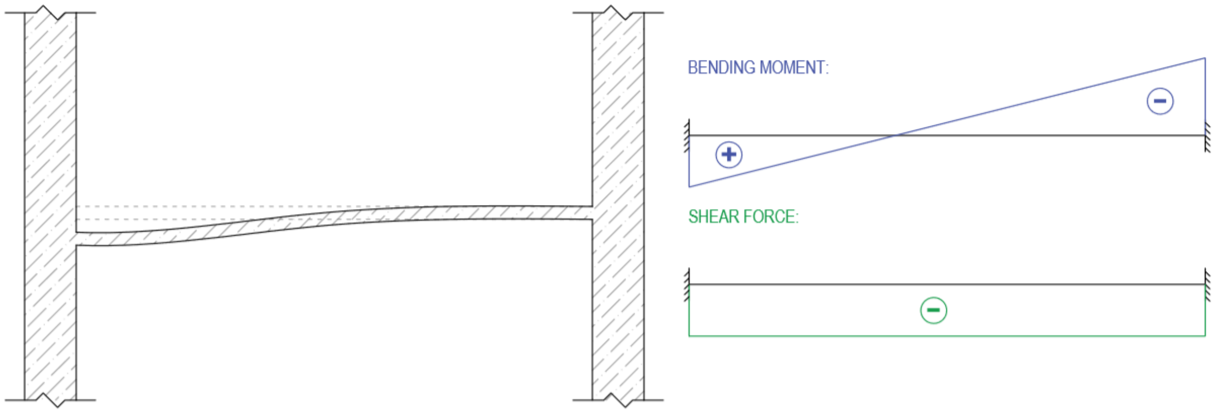


Fig. 1.4.2 Schematic illustration of differential shortening and internal forces

However, crack limitation control is out of scope of this study.

Mentioned above deformations also effects on non-structural elements, such as glazed façade, exterior brickwork and flooring details. Excessive deformations lead to cracking of the non-load-bearing elements.

2. Creep and Shrinkage

2.1 Creep of concrete and modelling of creep

2.1.1 Creep of concrete

Nowadays, determining the value of creep is still a complicated task due to huge set of variables. Many parameters including geometrical shape of specimen, parts of member in contact with air, temperature and relative humidity, curing conditions, age and class of concrete, quality of cement paste, characteristics of used aggregates, mix proportion, value and duration of load, admixtures, degree of hydration and many others affecting on result.

Creep of concrete is a time-dependent parameter, defined as a deformation of a concrete member under the permanent load. This deformation occurs along the axes of applied stress. When the load is applied, concrete member undergoes instantaneous elastic strain and further developing into creep strain, if the applied stress is sustained. Inelastic deformation develops over time even without the increase of load.

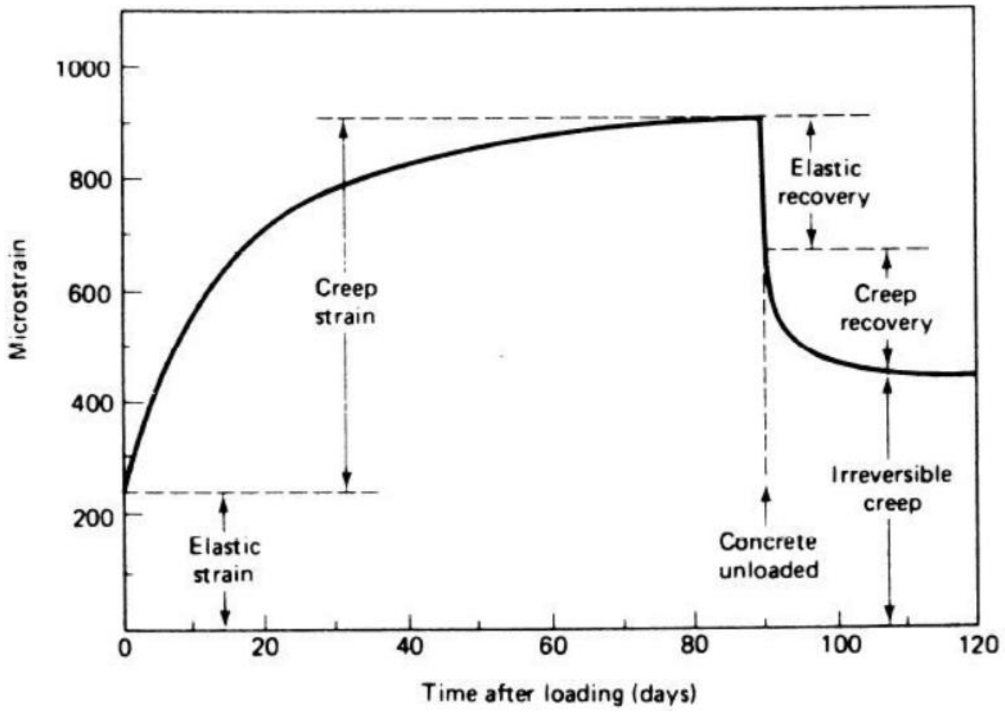


Fig. 2.1.1.1 Illustration of elastic and creep strain in response to loading

In respond to applied load, specimen undergoes with instantaneous elastic strain (Fig. 2.1.1.1), which transfers into creep strain. The rate of creep strain is high at early stage and is then decreases over time. When specimen is unloaded, elastic strain immediately fully recovers, while creep strain recovers slowly over time and not completely. Residual (irreversible) strain is the part of unrecovered creep strain.

Creep of concrete mainly depends on some factors:

1. Age of concrete when load is applied.
   Younger concrete tends to creep more. Generally, even more when degree of hydration (ratio between volume of cement fully reacted with water to total volume of cement in specimen) is low.
2. Type of aggregate
   With higher modulus of elasticity of aggregate the lower is creep. Geometrical proportion of aggregates also influencing on creep.
3. Mix proportion
   Creep is higher with higher water/cement ratio.
4. Concrete class
   With higher compressive strength of concrete, the lower is creep.
5. Relative humidity of an ambient air
   With higher relative humidity, the lower is creep.

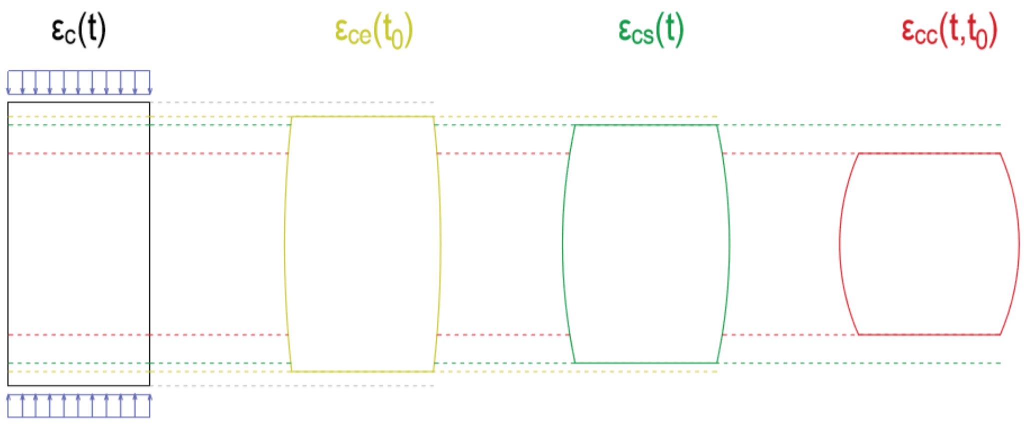


Fig. 2.1.1.2 Components of deformation

The process of solidifying of concrete mix undergoes transition from liquid to solid state. When the construction requires early stage loading of concrete elements, creep is significant and must be properly investigated in order to obtain the most accurate value.

2.1.2 Creep model

Nowadays, wide range of creep models are developed for hardened concrete. Every model code using their own mathematical expressions for creep compliance. Researchers from *Korea*

Advanced Institute of Science and Technology, Daejeon, South Korea investigated different creep models, such as *ACI* model, *fib* model, *KCI* model, *B3* model and *AASHTO* model, on predicting capability. They [4] came up with the following main conclusions:

There is no creep model superior to the rest. One model may underestimate creep especially for extrapolate regions.

Selected creep model cannot perfectly reflect realistic behaviour. Predicting capability of one chosen creep model is ignoring capabilities of others, while combining different creep models (i.e., forecasting), uniting their prediction performance, becomes a powerful tool.

Density forecast combination method (DFCM) is developed to cover weak sides of selected creep model, complementing it with a better performance model. In reality, when great accuracy is necessary, proposed *DFCM* will reflect the most precise, at least until this time, behaviour of creep based on all conditions of construction, concrete composition and environmental circumstances.

Deciding on which creep model should be used in calculations it is necessary to know the range of applicability of selected model. One code may be good for selected time interval, while be bad in another. Due to variety of depended factors and their final result on creep, it is a preference choice on the creep model. Eurocode's range of applicability is suitable for the early load sequence and is chosen for calculations of case study.

Further calculations will include Eurocode 2 creep model.

2.1.3 Eurocode 2 - Creep model

Eurocode 2 (Annex B) [2] creep model:

"Creep coefficient at time t can be calculated from the following expression:

$$\varphi(t, t_0) = \varphi_0 * \beta_c(t, t_0) \tag{2.1}$$

where:

φ_0 is the notional creep coefficient,

$\beta_c(t, t_0)$ is a coefficient, describing development of creep over time after applied load.

Notional creep coefficient φ_0 is obtained from:

$$\varphi(t, t_0) = \varphi_{RH} * \beta(f_{cm}) * \beta(t_0) \tag{2.2}$$

where:

φ_{RH} is a factor of relative humidity subjected to notional creep coefficient and represented as follows:

$$\varphi_{RH} = 1 + \frac{1 - RH/100}{0{,}1 * \sqrt[3]{h_0}} \quad for\ f_{cm} \leq 35\ N/mm^2 \tag{2.3}$$

$$\varphi_{RH} = \left(1 + \frac{1 - \frac{RH}{100}}{0{,}1 * \sqrt[3]{h_0}} * \alpha_1\right) * \alpha_2 \quad for\ f_{cm} > 35\ N/mm^2 \tag{2.4}$$

where:

RH relative humidity of the environment (in %),

h_0 section of element (in mm),

$h_0 = 2A_c/u$, A_c is the cross-sectional are of concrete (mm^2),

u is perimeter of section exposed to drying (mm),

$\beta(f_{cm})$ is a factor of concrete strength and is expressed as follows:

$$\beta(f_{cm}) = \frac{16{,}8}{\sqrt{f_{cm}}} \tag{2.5}$$

where:

f_{cm} is the mean compressive strength of concrete at 28 days,
$\beta(t_0)$ is a factor of concrete age at loading and is expressed as follows:

$$\beta(t_0) = \frac{1}{(0{,}1 + t_0^{0{,}20})} \tag{2.6}$$

$$\beta_c(t, t_0) = (\frac{(t - t_0)}{\beta_H + (t + t_0)})^{0{,}3} \tag{2.7}$$

where:

t is the age of concrete at specific time,

t_0 is the age when load is applied,

β_H is the coefficient related to ambient humidity and is evaluated with following expressions:

$$\beta_H = 1{,}5 * (1 + (0{,}012 * RH)^{18}) * h_0 + 250 \leq 1500\ for\ f_{cm} \leq 35\ N/mm^2 \tag{2.8}$$

$$\beta_H = 1{,}5 * (1 + (0{,}012 * RH)^{18}) * h_0 + 250 * \alpha_3 \leq 1500 * \alpha_3 \quad for\ f_{cm} \leq 35\ N/mm^2 \tag{2.9}$$

α_i are correction factors and are evaluated as follows:

$$\alpha_1 = (\frac{35}{f_{cm}})^{0,7} \tag{2.10}$$

$$\alpha_2 = (\frac{35}{f_{cm}})^{0,2} \tag{2.11}$$

$$\alpha_3 = (\frac{35}{f_{cm}})^{0,5} \tag{2.12}$$

In the case if stress in concrete prevailing value of $0{,}45 f_{ck}(t_o)$ at the age t_0, then non-linear notional creep coefficient should be obtained from expression:

$$\varphi_{nl}(\infty, t_0) = \varphi_0(\infty, t_0) * \exp(1{,}5 * (k_\sigma - 0{,}45)) \tag{2.13}$$

where:

$\varphi_{nl}(\infty, t_0)$ is the non-linear notional creep coefficient, which replaces $\varphi(\infty, t_0)$, k_σ is the stress-strength ratio $\sigma_c / f_{ck}(t_0)$,

where:

σ_c is the compressive stress $f_{ck}(t_0)$ is the characteristic compressive strength at the time of loading."

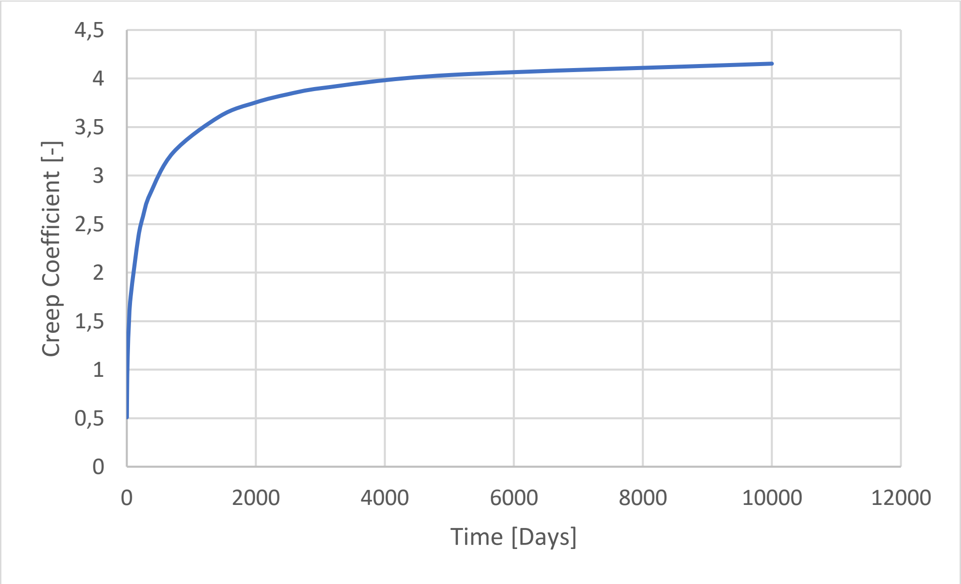


Fig. 2.1.3.1 Eurocode 2 creep model

Fig. 2.1.3.1 shows development of creep coefficient valid for case study parameters.

Simplified form procedure with nomogram (*EN 1992-1-1:2004 (Annex E)*) [2] can be used to estimate creep coefficient. Mentioned nomogram is useless in the case of early loading of concrete.

Therefore, expanded version with described parameters procedure on determining creep coefficient must be used.

2.2 Shrinkage of concrete and modelling of shrinkage

2.2.1 Shrinkage of concrete

Shrinkage of concrete is defined as a contracting of concrete member due to evaporation and absorption of water. Strain, caused by shrinkage, developing over time and, in contradistinction to creep, do not depend on magnitude and duration of load.

Shrinkage of concrete categorized as follows:

1) Plastic shrinkage

The first type of shrinkage that develops, exactly at setting time when concrete is in plastic state, called Plastic Shrinkage. Strain in concrete is mainly caused by two processes: drying of water from surface of concrete and absorption of water by aggregates/admixtures. Situational factors, such as high water to cement ratio, is also responsible for plastic shrinkage since excessive water accumulates on surface of concrete and then leading to cracks when is evaporated. The strain is even higher if exposed surface under direct sunlight. Plastic shrinkage is responsible for microcracking of fresh concrete. In order to mitigate plastic shrinkage, concrete can be covered by waterproof foil or by casting concrete during evening time.

Fig. 2.2.1.1 Picture of cracks, caused by plastic shrinkage

2) Drying shrinkage

Drying shrinkage is a relatively longer process. It occurs firstly when excess of water in relatively big pores evaporates and not significantly affects in change of sizes. Changes in volume generally occurs when water dries from gel pores. Drying shrinkage is bigger with lower gel pore sizes.

3) Autogenous shrinkage

Autogenous shrinkage occurs when there is no moisture movement in cement paste and the temperature in constant. In reality, autogenous shrinkage frequently ignored when water to cement ratio prevailing 0,4, except mass structures, such as bridges, dams, etc.

4) Carbonation shrinkage

Carbonation shrinkage is a long-term process, that happens in contact with atmosphere. Carbon dioxide in air reacts with hydrated cement. Product of chemical reaction is calcium carbonate, and some other cement paste compounds are decomposed. Strain appears when volume of products is less than volume of reagents before reaction. Carbonation shrinkage is beneficial, since it makes outer layer of concrete less permeable and protects steel reinforcement against corrosion. The value of strain is insignificant and can be ignored.

2.2.2 Eurocode 2 - Shrinkage model

Eurocode 2 (Annex B) [2] shrinkage model:

“The total shrinkage strain of concrete, ε_{cs}, is composed of two components:

$$\varepsilon_{cs}(\infty) = \varepsilon_{cd}(\infty) + \varepsilon_{ca}(\infty) \tag{2.2.2.1}$$

where:

$\varepsilon_{cd}(\infty)$ is the drying shrinkage strain,

$\varepsilon_{ca}(\infty)$ is the autogenous shrinkage strain

The final value of the drying shrinkage strain $\varepsilon_{cd}(\infty)$ is:

$$\varepsilon_{cd}(\infty) = k_h * \varepsilon_{cd,0} \tag{2.2.2.2}$$

where:

k_h is a coefficient depending on the notional size of the member h_0 and is expressed as:

$$k_h = \frac{2 * A_c}{u} \tag{2.2.2.3}$$

A_c is the cross-sectional area of concrete specimen,

u is a perimeter of member in contact with atmosphere,

$\varepsilon_{cd,0}$ is the nominal unrestrained drying shrinkage value, which expressed as:

$$\varepsilon_{cd,0} = 0{,}85 * \left((220 + 110 * \alpha_{ds1}) * \exp\left(-\alpha_{ds2} * \frac{f_{cm}}{10}\right) \right) * 10^{-6} * \beta_{RH} \tag{2.2.2.4}$$

$$\beta_{RH}(RH) = 1{,}55 * (1 - (\frac{RH}{100})^3) \tag{2.2.2.5}$$

where:

f_{cm} is the mean compressive cylinder strength of concrete at the age of 28 days

α_{ds1} is a coefficient which depends on the type of cement

$= 3$ for cement Class S

$= 4$ for cement class N

$= 6$ for cement Class R

α_{ds2} is a coefficient which depends on the type of cement

$= 0{,}13$ for cement Class S

$= 0{,}12$ for cement class N

$= 0{,}11$ for cement Class R

RH is the ambient relative humidity (%)."

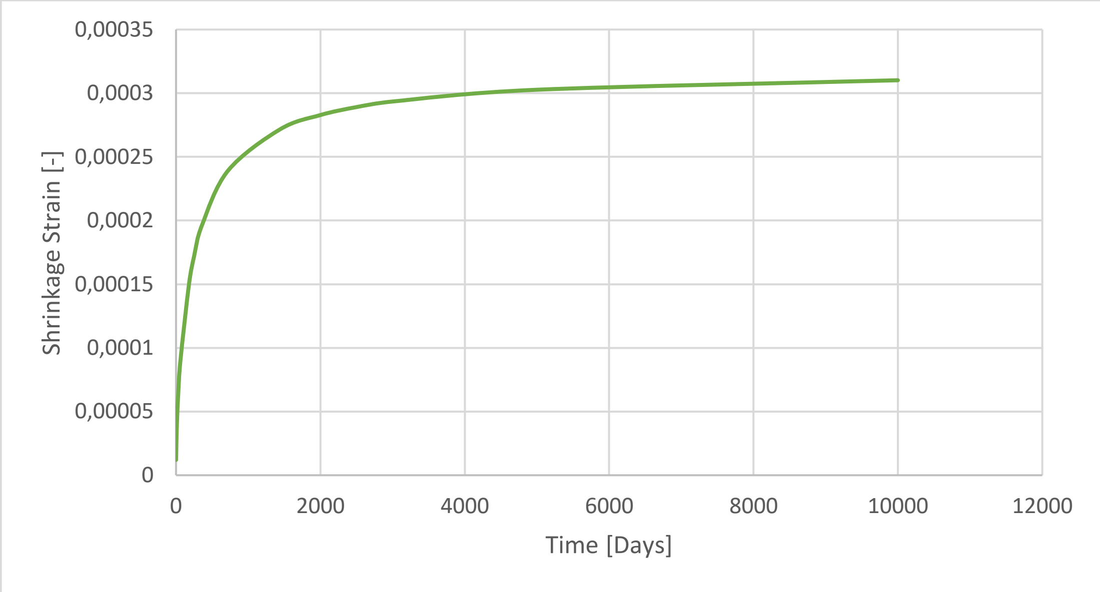


Fig. 2.2.2.1 Eurocode 2 shrinkage model for rapid hardening concrete

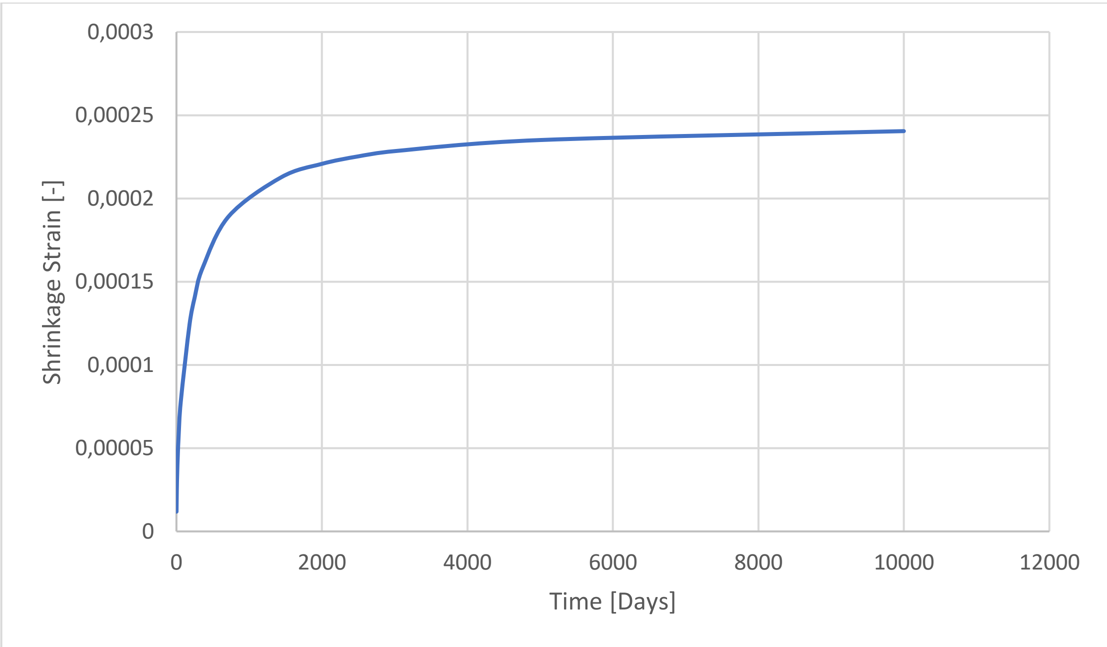


Fig. 2.2.2.2 Eurocode 2 shrinkage model for normal hardening concrete

As it may seem (Fig. 2.2.2.1 and Fig. 2.2.2.2) rate of shrinkage is higher for rapid hardening concrete, than for normal hardening concrete. Both graphs valid for case study.

The following publications, [1, 5, 6, 7, 8, 10, 11], were also found as most useful for developing the computation strategy.

3. Objectives of research

Total deformation of building shall be calculated based on principle of superposition. Exactly on superposition of strains, that causes long-term inelastic deformations. In order to evaluate appropriate values of elastic, creep and shrinkage strains, construction schedule to be created. Construction sequence not only gives us basic understanding of stage progression with actual load increment, but the maturity of concrete elements over time.

In reality, construction of structure undergoes with parallel works related to interior and exterior works. Such assumption is crucial for prediction of actual loading. As it was mentioned above, fresh hardened concrete is sensual to deformations caused by loading. For example, it is expected, that after removal of props, works on interior shall begin. Such stress increment has a direct impact on calculation of deformations. Moreover, distribution and number of props, as well as number of levels, supported by props, must be designed to avoid cracking, overbending or even collapse of structure. Preliminary design on number and distribution of shores to be evaluated in SCIA software. Apparently, each shore has a limit on load-bearing capacity. Being subjected to normal compressive force, design load on shore should not exceed design resistance.

With the help of Finite Element Analysis software, implement construction sequence analysis in a 3D environment. Moreover, it is possible to determine the weakest parts of structure, such as parts of horizontal structures suffering from deepest deflections. In the real construction process, workers could start collecting heavy-weight equipment in one place. The most unfavourable location is a critical region of horizontal structure, where expected the deepest deflection.

Based on obtained results, choose appropriate solution to differential deformation of levels.

Time-varying physical parameters of concrete, such as compressive strength of concrete and Young's modulus of elasticity, to be evaluated by mathematical expressions given by Eurocode 2.

4. Methodology

4.1 Compressive strength development

Varying over time compressive strength of concrete under normal curing conditions can be estimated from Eurocode 2 [2] expression:

$$f_{cm}(t) = \beta_{cc}(t) * f_{cm} \quad (4.1.1)$$

with

$$\beta_{cc}(t) = \exp(s * (1 - (\frac{28}{t})^{\frac{1}{2}})) \quad (4.1.2)$$

where:

$f_{cm}(t)$ is a time-varying compressive strength of concrete

f_{cm} is a mean compressive strength of concrete at 28 days

$\beta_{cc}(t)$ is a coefficient that depends on age of concrete

s is a constant, which depends on type of used cement;

0,2 for rapid hardening cement,

0,25 for normal hardening cement.

4.2 Development of Young's modulus of elasticity

One of the most important time-varying parameters on estimation of deformations is Young's modulus of elasticity. There is a strong dependence of compressive strength with modulus of elasticity. Eurocode 2 [2] propose following estimation model, when data from testing is missing. Mean Young's modulus of elasticity over time can be expressed as:

$$E_{cm}(t) = (\frac{f_{cm}(t)}{f_{cm}})^{0,3} * E_{cm} \quad (4.2.1)$$

where:

$E_{cm}(t)$ is a time-varying mean modulus of elasticity of concrete,

$f_{cm}(t)$ is a time-varying compressive strength of concrete

f_{cm} is a mean compressive strength of concrete at 28 days

E_{cm} is a mean modulus of elasticity of concrete at 28 days

The tangent modulus of elasticity is then calculated as:

$$E_c(t) = E_{cm}(t) * 1,05 \quad (4.2.2)$$

4.3 Assessment of structure stiffness

During the construction process, assessment of structural stiffness is one of the main tasks, especially when construction is accelerated. Mentioned above assessment shall be performed on horizontal structure of *“Block B”* to ensure that slab is rigid enough, i.e. able to carry the loading without undesirable deflection and cracks without props. Simplified assessment is consisted of two check: stress limitation in reinforced concrete slab and limitation of deflection.

Stress limitation expressed as:

$$\sigma_c \leq 0{,}15 * f_{ck}(t) \tag{4.3.1}$$

where:

σ_c is a stress in concrete, caused by loading,

$f_{ck}(t)$ is a characteristic compressive strength at time considered

Deflection limitation expressed as:

$$\omega \leq \frac{l}{500} \tag{4.3.2}$$

where:

ω is a depth of deflection

l is a span of slab

The values of coefficients are more severe than those given by Eurocode 2, since structure is in construction process, i.e. concrete is relatively young and sensitive to load. Such situation is out of norms and regulations, and must be assessed by an experienced engineer. Every single situation is unique due to affecting factors and must be under proper control of specialist. Hence, provided coefficients are not suitable for each case.

4.4 Superposition of elastic, creep and shrinkage strains

The total strain at any time t can be written in the form, by *Ghali et al.* [13]:

$$\varepsilon_T(t) = \varepsilon_{ce}(t_0) + \varepsilon_{cc}(t, t_0) + \varepsilon_{cs}(t, t_0) \tag{4.4.1}$$

where:

$\varepsilon_T(t)$ is a total strain at time considered,

$\varepsilon_{ce}(t_0)$ is an instantaneous elastic strain,

$\varepsilon_{cc}(t, t_0)$ is a creep strain from time of loading to time considered,

$\varepsilon_{cs}(t, t_0)$ is a shrinkage strain from time of setting to time considered.

For an element, loaded with sustained load without stress increment, Equation (4.4.1) can be expressed as (*Bažant, 1982 [12]*):

$$\varepsilon_T(t) = \sigma_c(t_0) * (\frac{1}{E_c(t_0)} + \frac{\varphi(t, t_0)}{E_{c,28}}) + \varepsilon_{cs}(t, t_0) \tag{4.4.2}$$

Since each column of building experiences more than one load increment, except the upmost floor, equation (Eq. 4.4.2) simply transforms into superposition of strains:

$$\varepsilon_T(t) = \sum_{i=1}^{n} (\sigma_c(t_0) * (\frac{1}{E_c(t_0)} + \frac{\varphi(t, t_0)}{E_{c,28}})) + \varepsilon_{cs}(t, t_0) \tag{4.4.3}$$

4.5 Simplified assessment of axial shortening

Total shortening of concrete member is then calculated by:

$$\delta(t, t_0) = \sum_{i=1}^{n} \varepsilon_T(t) * h \tag{4.5.1}$$

where:

$\delta(t, t_0)$ is a total axial shortening of concrete member,

$\varepsilon_T(t)$ is a total strain at time considered,

h is a length of concrete member

Finally, total shortening of building calculated as superposition of deflections.

5. Case study

5.1 Structural system of building and environmental conditions

A high-rise office building with 24 floors and total height of 72 meters is examined. The structural system of both buildings is represented by reinforced concrete core and columns around the perimeter. Both buildings have a symmetrical location of load-bearing elements. Simplification of structural plan is done to concentrate on study problems, i.e. mitigate distraction details.

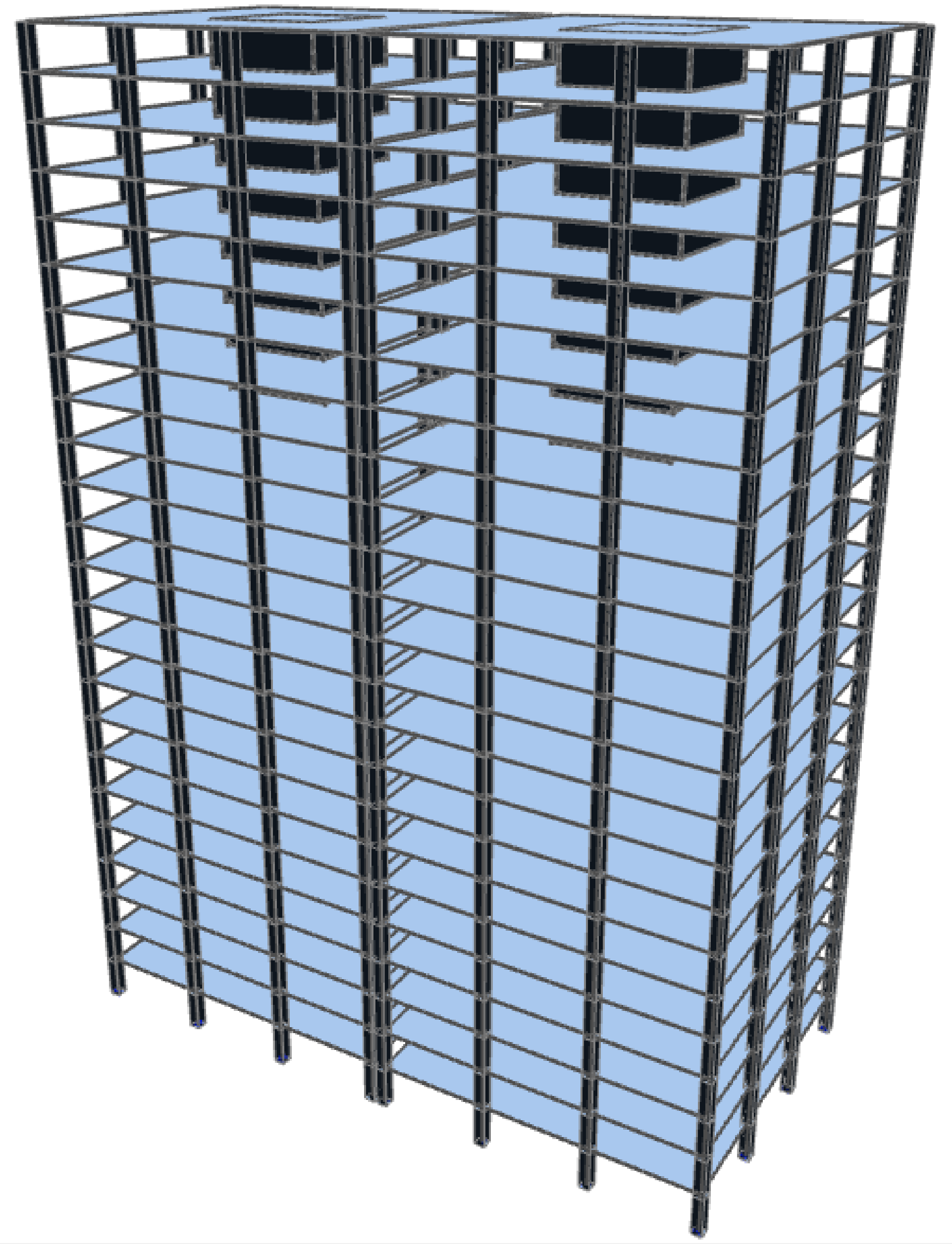


Fig. 5.1.1 3D perspective view of case study building

Vertical elements:

The load bearing vertical elements of superstructure include reinforced concrete columns and reinforced concrete core. The thickness of reinforced concrete wall is *250 mm*. The span between columns in *«x»* and *«y»* direction is *8 meters*. The cross-section of each reinforced concrete column is $800x800\ mm$. Both walls and columns are made of concrete $C35/45\ -XC1$ and steel reinforcement *B500B*. The minimum and nominal concrete cover of all vertical elements is *10 mm* and *15 mm* respectively.

Horizontal elements:

Monolithic reinforced concrete slab is designed for entire building. It has a height of *200 mm*. Slab is in a shape of rectangle and has a span of *8 meters* between supports. Slab is made of concrete $C35/45\ -XC1$ and reinforcement *B500B*. The minimum and nominal concrete cover is *10 mm* and *15 mm* respectively.

Foundation:

Study case do not consider any type of foundation and soil classification. Rigid supports under vertical load-bearing elements are considered.

The main and only difference is in type of used cement. For *"Block A" – 42.5N*, for *"Block B" – 42.5R.*

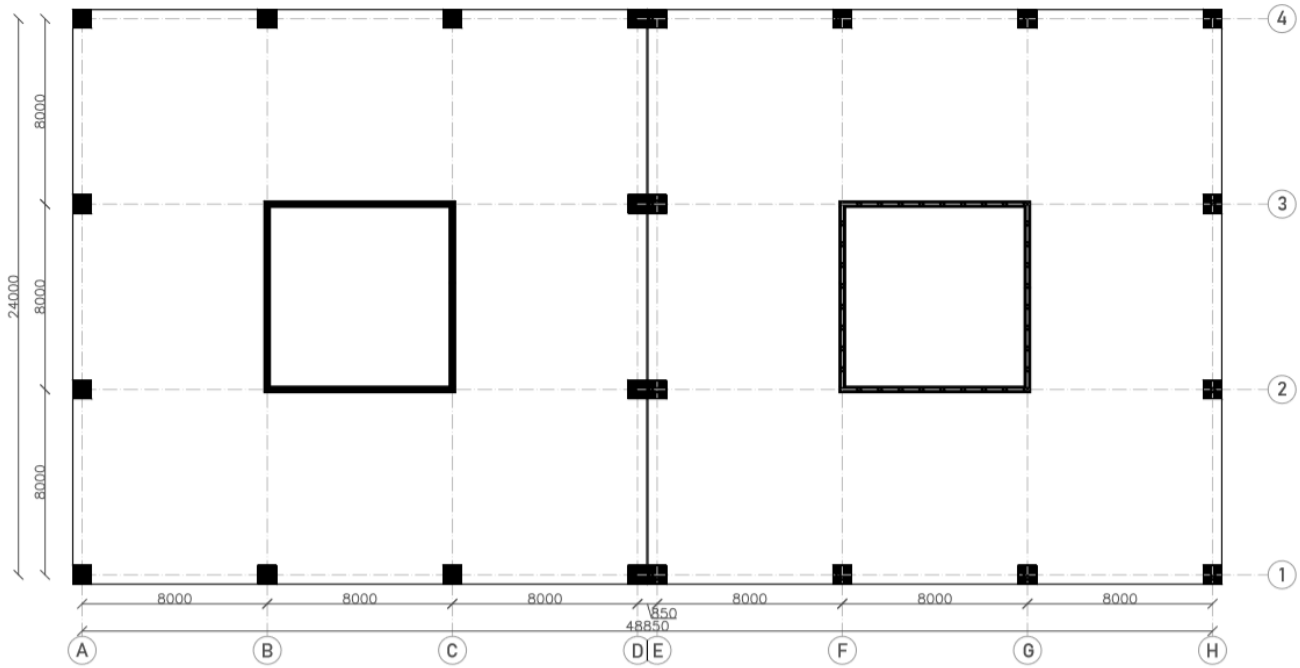


Fig. 5.1.2 Structural system of case study building

Environmental conditions:

Relative humidity of ambient air plays a crucial role for accurate estimation of creep and shrinkage. Since relative humidity differs with geographical location and depends on time of day, value is taken as an annual average for Prague, Czech Republic and assigned as *80%*.

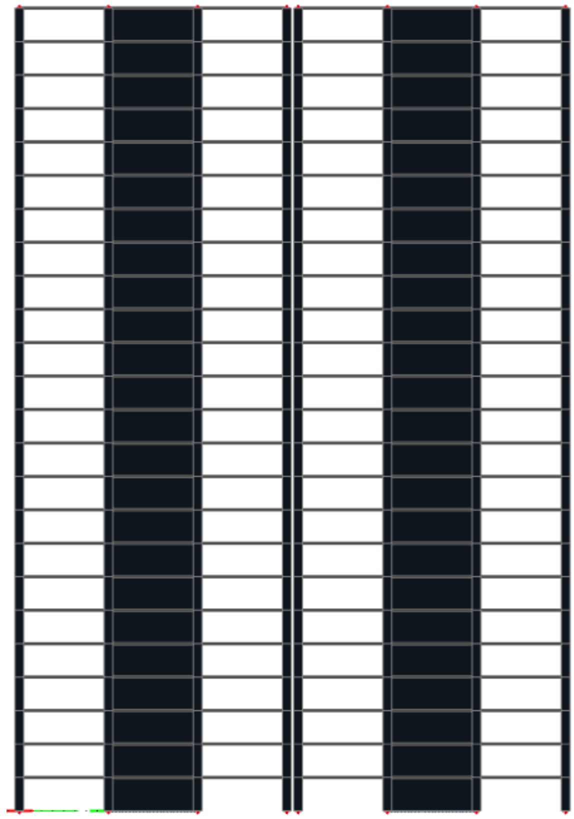


Fig. 5.1.3 Elevation of case study building

5.2 Strength development of concrete

Strength development of concrete for "*Block A*":

Being subjected to normal construction speed, great accuracy of concrete evolution over time is disregarded. Data is plotted using Eurocode 2 model (Eq. 4.1.1 and 4.1.2) with *s* coefficient, which is responsible for cement class, for normal hardening concrete.

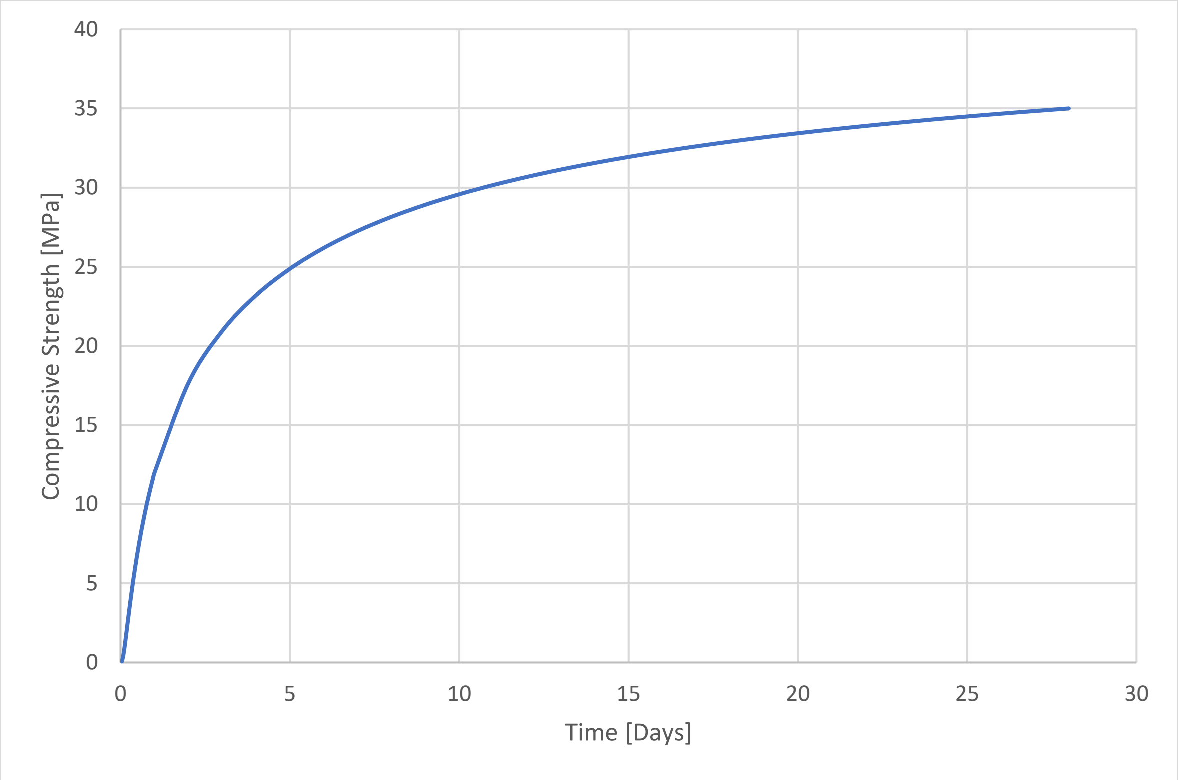


Fig. 5.2.1 Development of compressive strength

Procedure follows with the time-varying Young's modulus of elasticity. With the given expressions (Eq. 4.2.1 and 4.2.2) can be plotted over time as:

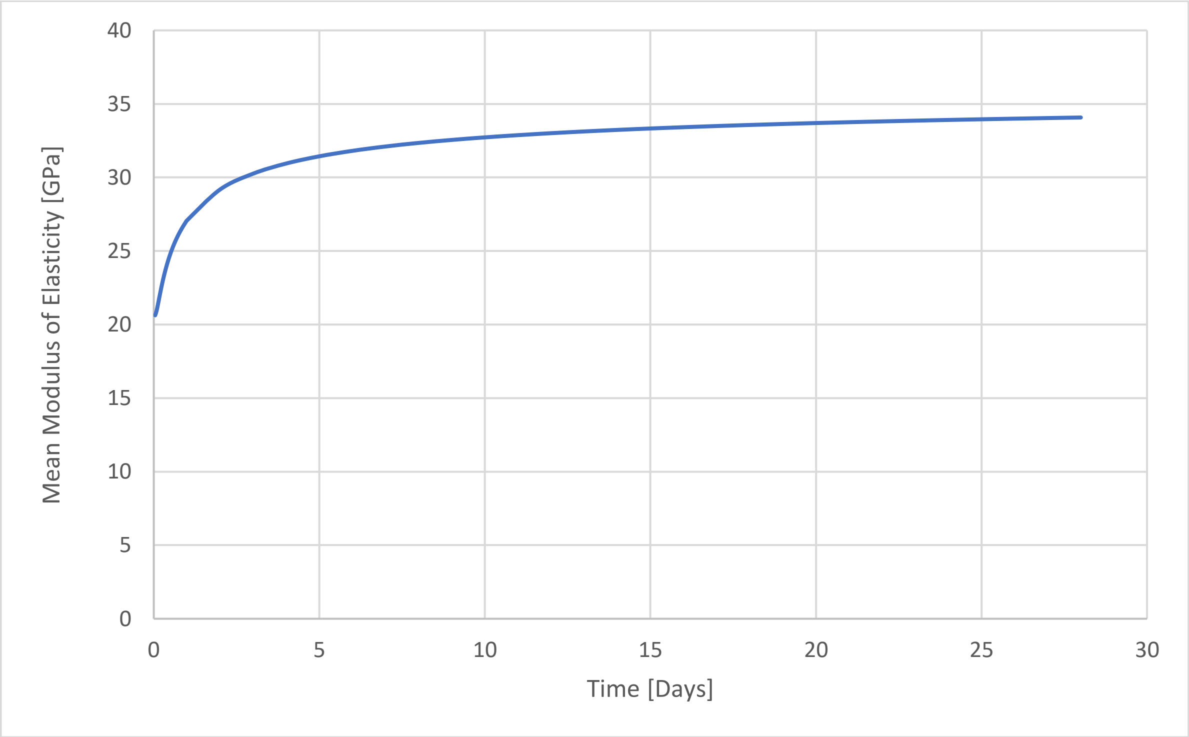


Fig. 5.2.2 Development of modulus of elasticity

Strength development of concrete for "*Block B*":

Development of compressive strength over time for rapid hardening concrete is modified version of what is given by Eurocode 2. The reason of modifying is that proposed model does not represent the real behaviour of hardening in early stages, which is crucial in case of accelerated construction. The modified version is obtained by linear proportional dependence between degree of hydration and compressive strength development [9].

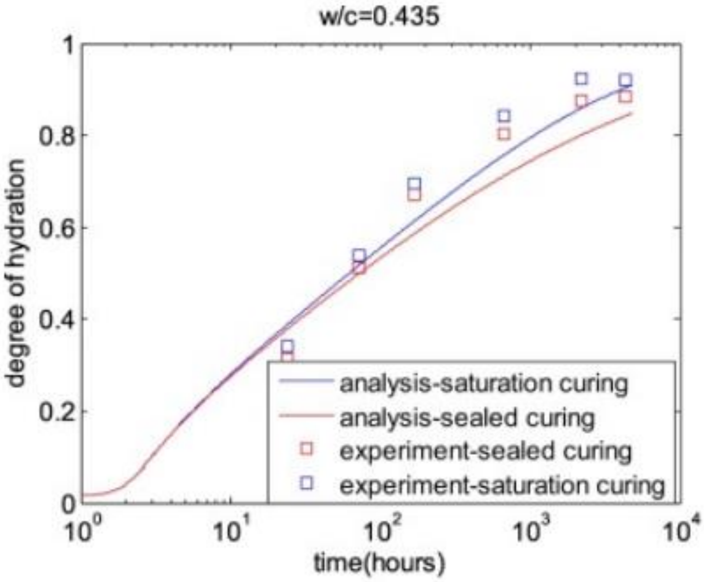


Fig. 5.2.3 Degree of hydration over time

So called "fake experiment" is performed for assessment of compressive strength development. In real situation, responsible engineer must be provided with adequate physical properties of used concrete, obtained by testing.

The following graph (Fig. 5.2.4) represents gradation of concrete, used in calculations for "*Block B*".

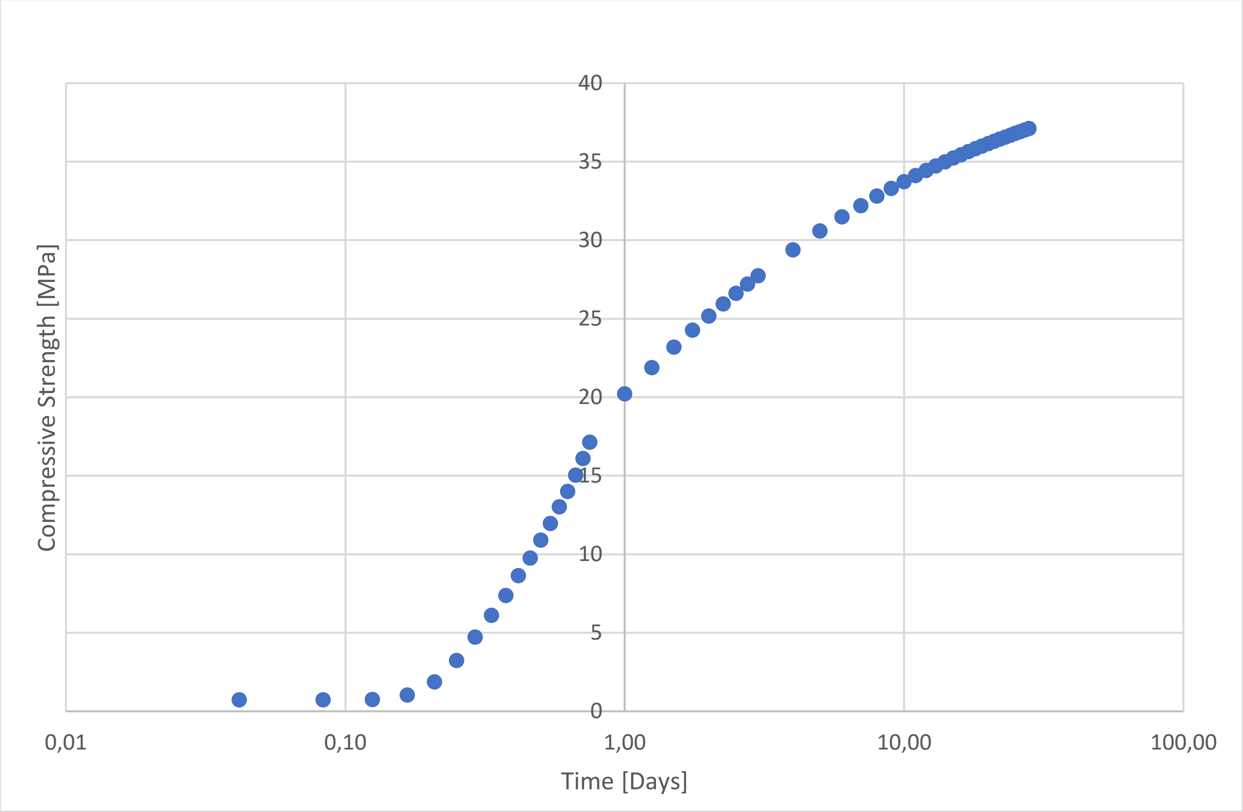


Fig. 5.2.4 Development of compressive strength

Part of graph, until concrete roughly gains 50% of compressive strain, is obtained by following expression:

$$f_{ck}(t) = k(t) * f_{ck} \tag{5.2.1}$$

where:

$f_{ck}(t)$ is a characteristic compressive strength at time considered,

$k(t)$ is a degree of hydration at time considered (-),

f_{ck} is a characteristic compressive strength at *28* days

Another half of graph is obtained by exponential function (Eq. 4.1.1 and 4.1.2), given by Eurocode 2. Both curves are combined with correlation *0,97*.

Procedure follows with the time-varying Young's modulus of elasticity. With the given expressions (Eq. 4.2.1 and 4.2.2) can be plotted over time as:

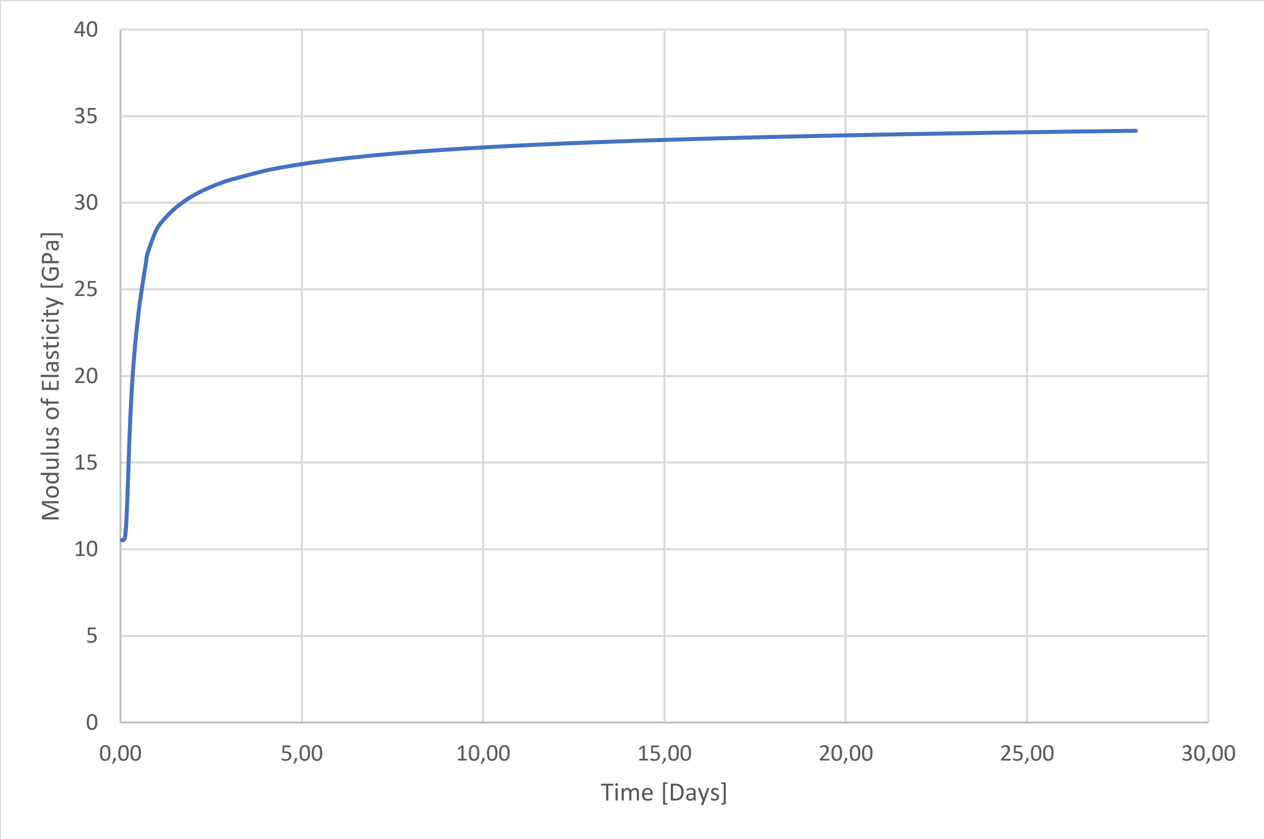


Fig. 5.2.5 Development of modulus of elasticity

5.3 Construction schedule

5.3.1 Block A

Construction schedule for *"Block A"* is assessed as *7 days per floor*. Detailed scheduling not considered and assumed that age of concrete columns on the respective floor *+7 days* with each new level.

5.3.2 Block B

By conditions of assigned task, construction of one floor must be implemented within *42 hours*. However, the assigned time is inconvenient in terms of realistic construction schedule. Therefore, construction speed is reduced to relatively more convenient value, exactly *36 hours per floor*. With the mentioned above speed, construction of *"Block B"* will be finished *6 days* before deadline. The residual time can be beneficial in case of delays on construction site.

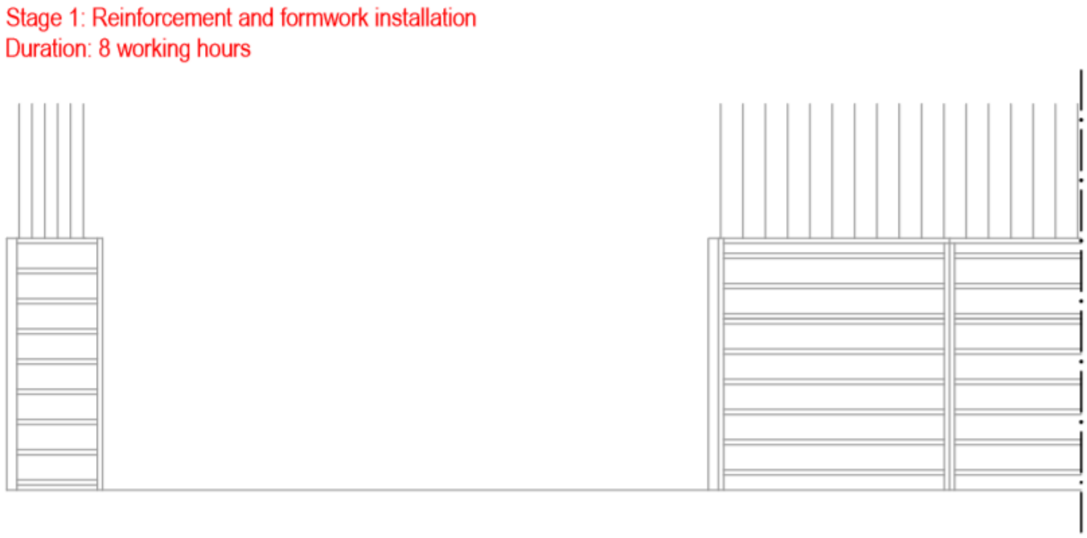
The construction of building will be performed during day and night time. There are two assigned working shifts with start and end time:

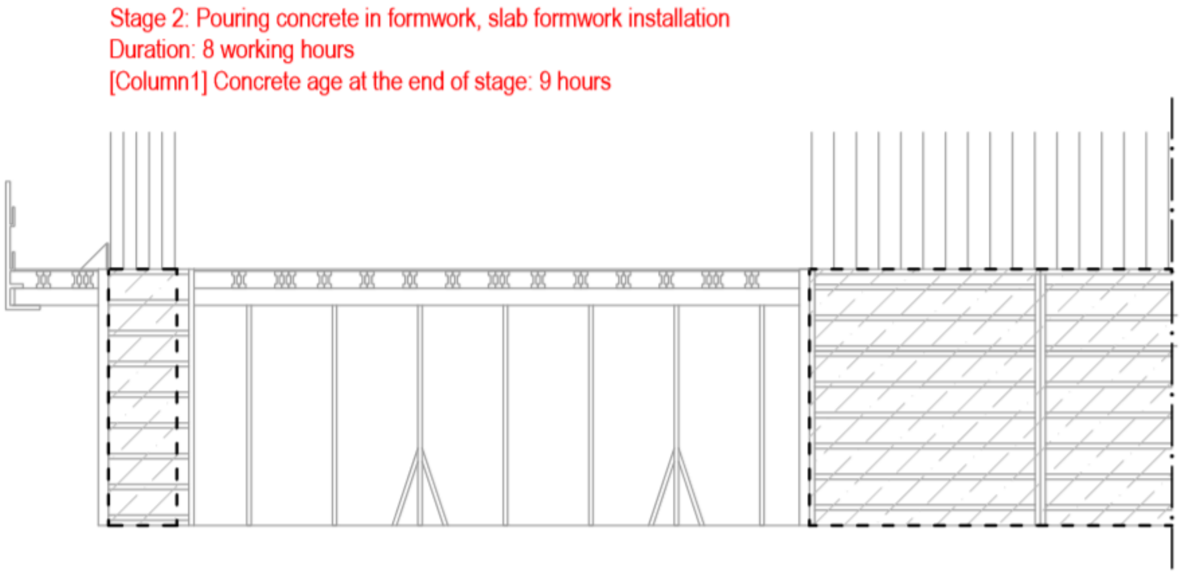
First shift: *09:00 – 18:00 (9 full hours, 8 working hours)*

Second shift: *21:00 – 06:00 (9 full hours, 8 working hours)*

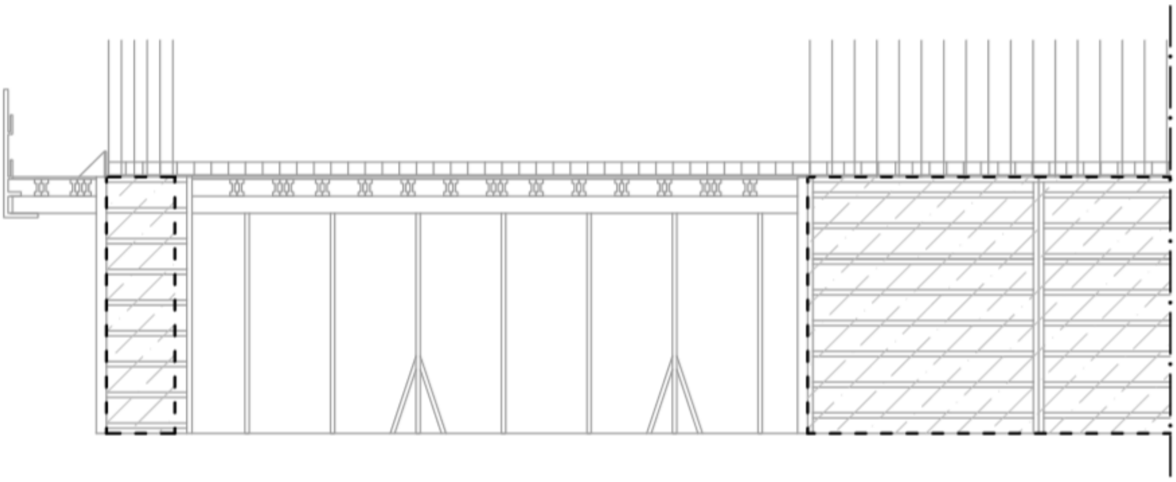
It is necessary to account actual time from cast of concrete, since curing of concrete does not depend on working hours. One construction cycle is done within *3 shifts.*

The following graphics describe construction schedule in stages over time:

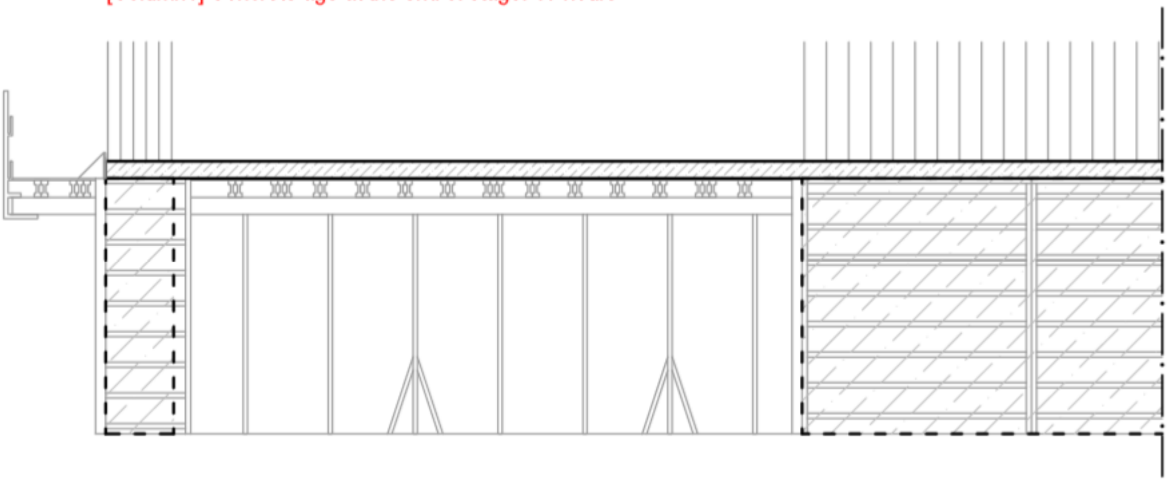




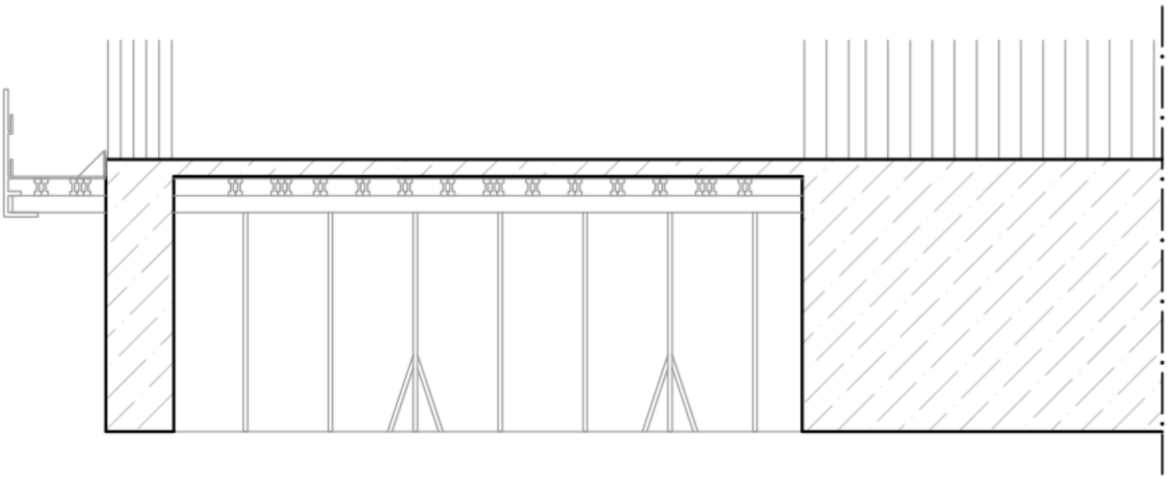
Stage 3: Reinforcement installation for slab
Duration: 4 working hours
[Column1] Concrete age at the end of stage: 17 hours

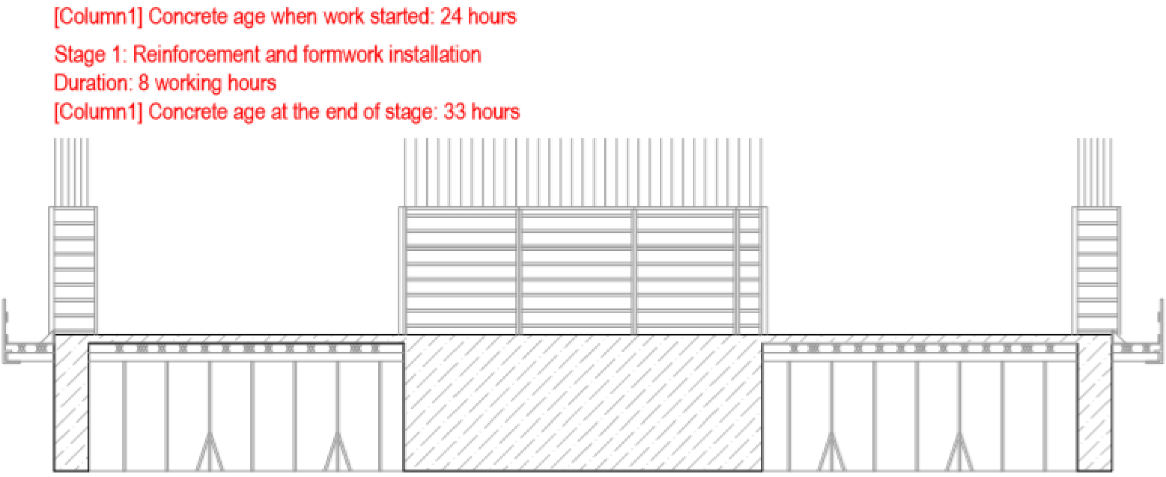


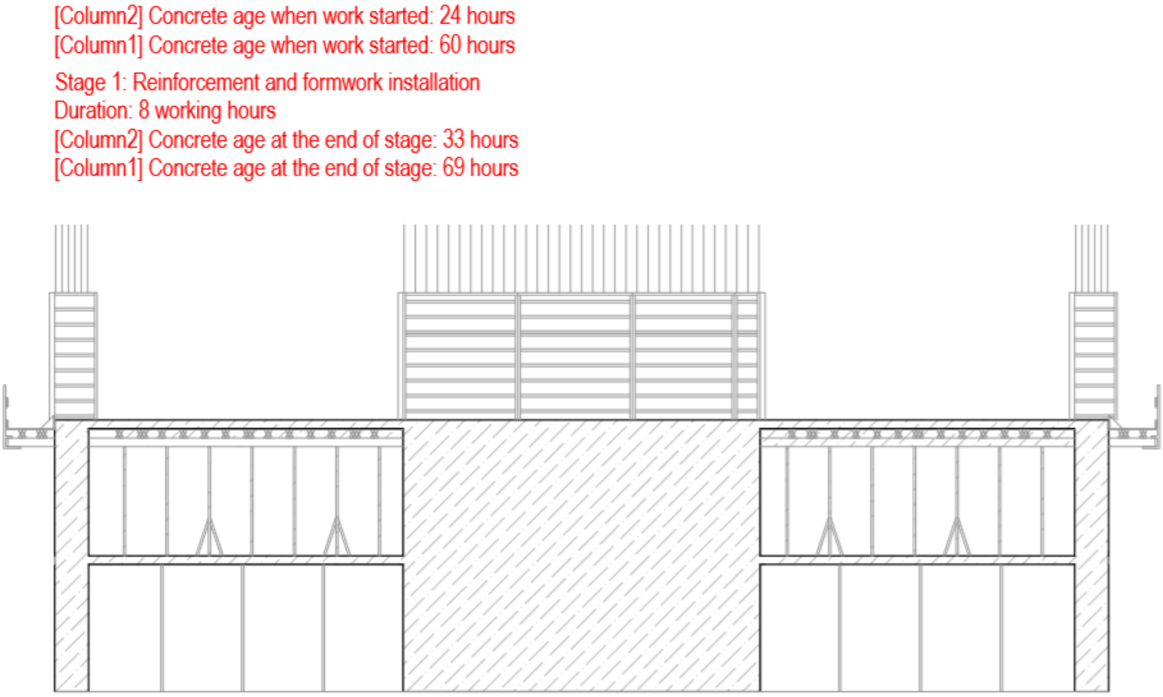
Stage 4: Pouring concrete in slab formwork
Duration: 2 working hours
[Column1] Concrete age at the end of stage: 19 hours



Stage 5: Vertical formwork removal
Duration: 2 working hours
[Column1] Concrete age at the end of stage: 21 hours







Age of concrete columns on the first floor (or respective floor) is *1 day* when works on the second floor (or respective floor +1 floor) begin and is *2,5 days* when works on the third floor (or respective +2 floors) begin. With each new level age of concrete on the first floor (or respective floor) is *+1,5 days.*

5.4 Structure stiffness evaluation

5.4.1 Estimation of required steel reinforcement in slab

Reinforced concrete slab is analysed for required amount of steel reinforcement in SCIA software. Slab is loaded with design values of load and assessed in Ultimate Limit State (ULS).

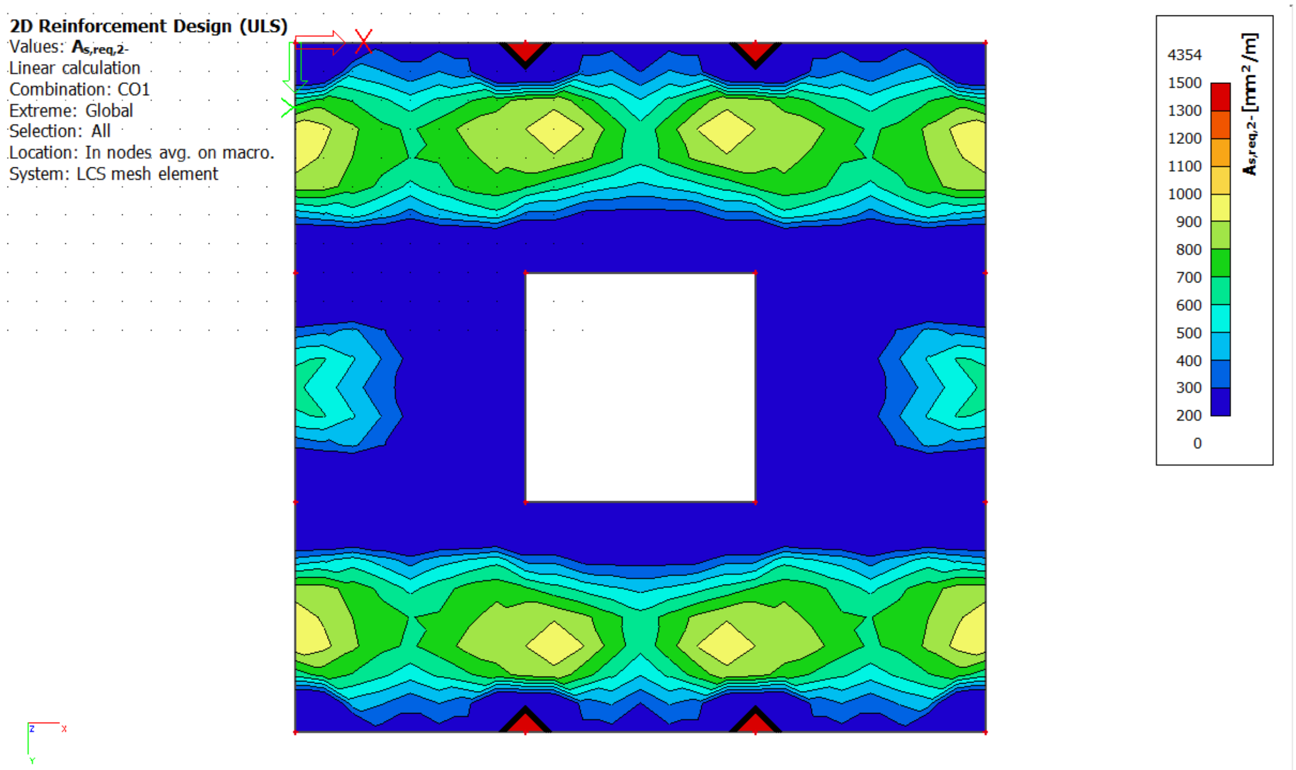


Fig. 5.4.1.1 Amount of minimum required steel reinforcement

Software shows (Fig. 5.4.1.1) distribution of minimum required steel reinforcement for positive bending moment in *"x"* direction. Yellow parts of slab (extreme values) shows a minimum required reinforcement (Fig. 5.4.1.1) $A_{s,req} = 1000\ mm^2/m$. It means, that at least $1000\ mm^2/m$ of steel reinforcement should be provided and shall be distributed in slab.

With the help of table (Tab. 5.4.1.2), design value of steel reinforcement is easily selected.

Tab. 5.4.1.2 Sectional areas per meter width for various bar spacings

Sectional areas per meter width for various bar spacings (Square mm per meter)										
Bar Dia mm	Number of Bars									
	75	100	125	150	175	200	225	250	275	300
8	670	503	402	335	287	251	223	201	183	168
10	1 047	785	628	524	449	393	349	314	286	262
12	1 508	1 131	905	754	646	566	503	452	411	377
16	2 681	2 011	1 608	1 340	1 149	1 005	894	804	731	670
20	4 189	3 142	2 513	2 094	1 795	1 571	1 396	1 257	1 142	1 047
25	6 545	4 909	3 927	3 272	2 805	2 454	2 182	1 964	1 785	1 636
32	10 720	8 042	6 434	5 362	4 596	4 021	3 574	3 217	2 924	2 681
40	16 760	12 570	10 050	8 378	7 181	6 283	5 585	5 026	4 570	4 189

Design value of steel reinforcement:

$A_{s,prov} = 1131\ mm^2/m,\ \emptyset 12\ with\ s = 100\ mm$

5.4.2 Construction stage analysis and distribution of internal forces in slab structure

During construction, bearing capacity of slab is influenced by distribution of internal forces caused by creep and shrinkage.

SCIA software allows to analyse structure stiffness during the construction sequence, when parts of structure, such as columns, walls, supports etc., or loads are added or removed. Step-by-step approach is performed by special option *"Construction Stage"*, which takes into consideration necessary time to complete assigned stage. Enabled option *"Young's modulus function"*, which is respective for material aging/strength development or deterioration over global time. In this case, strength development of concrete and creep effect on effective modulus of elasticity are taken into account.

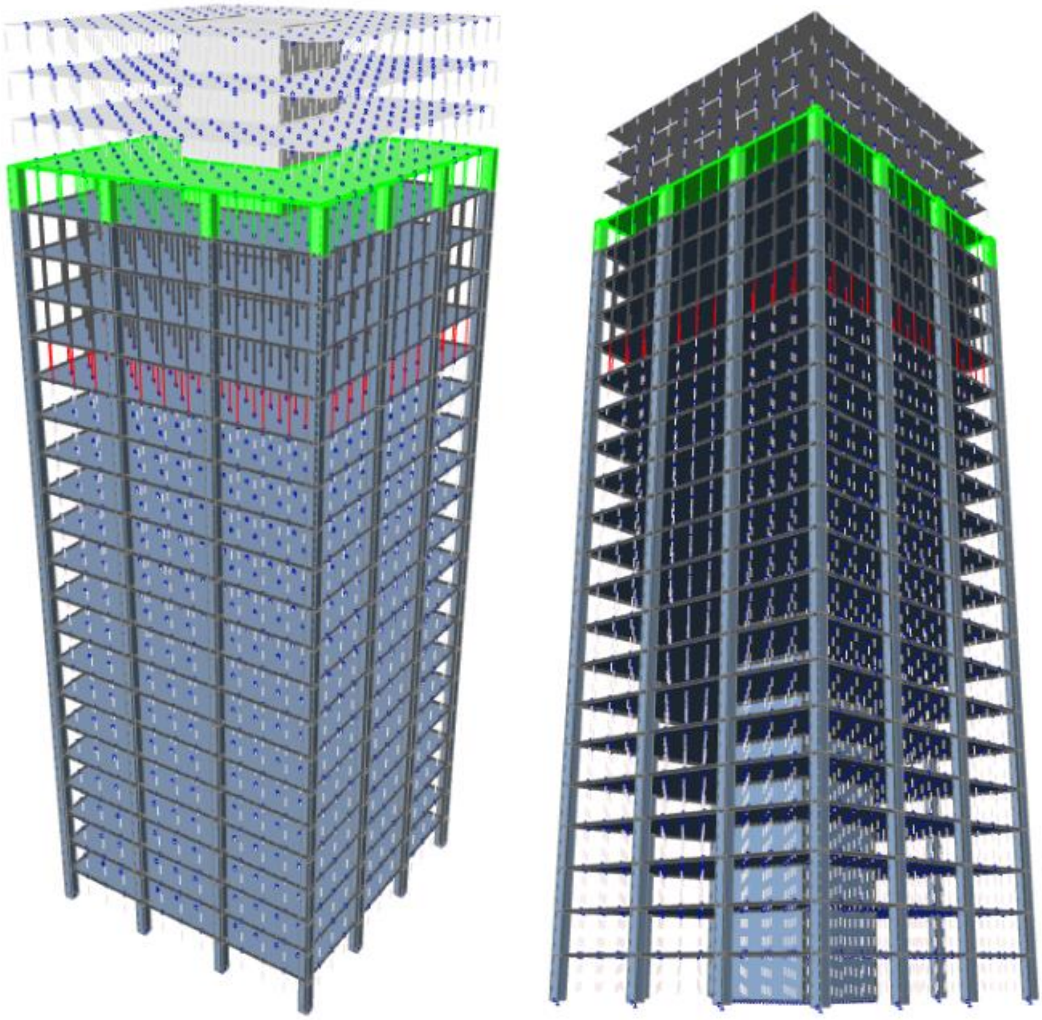


Fig. 5.4.2.1 3D perspective view of construction sequence

Green coloured parts of structure (Fig. 5.4.2.1) shows elements of building installed in respective stage, red coloured – to be removed, white coloured – to be installed in future stages.

Structure is loaded by self-weight and characteristic dead load $14\ kN/m^2$ in upmost installed floor, which is estimated load from heavy-weight equipment and reinforcement. Distribution of positive bending moment (Fig. 5.4.2.2) with the option *"Construction Stage"* and *"Young's modulus function"* in downmost slab without shore support:

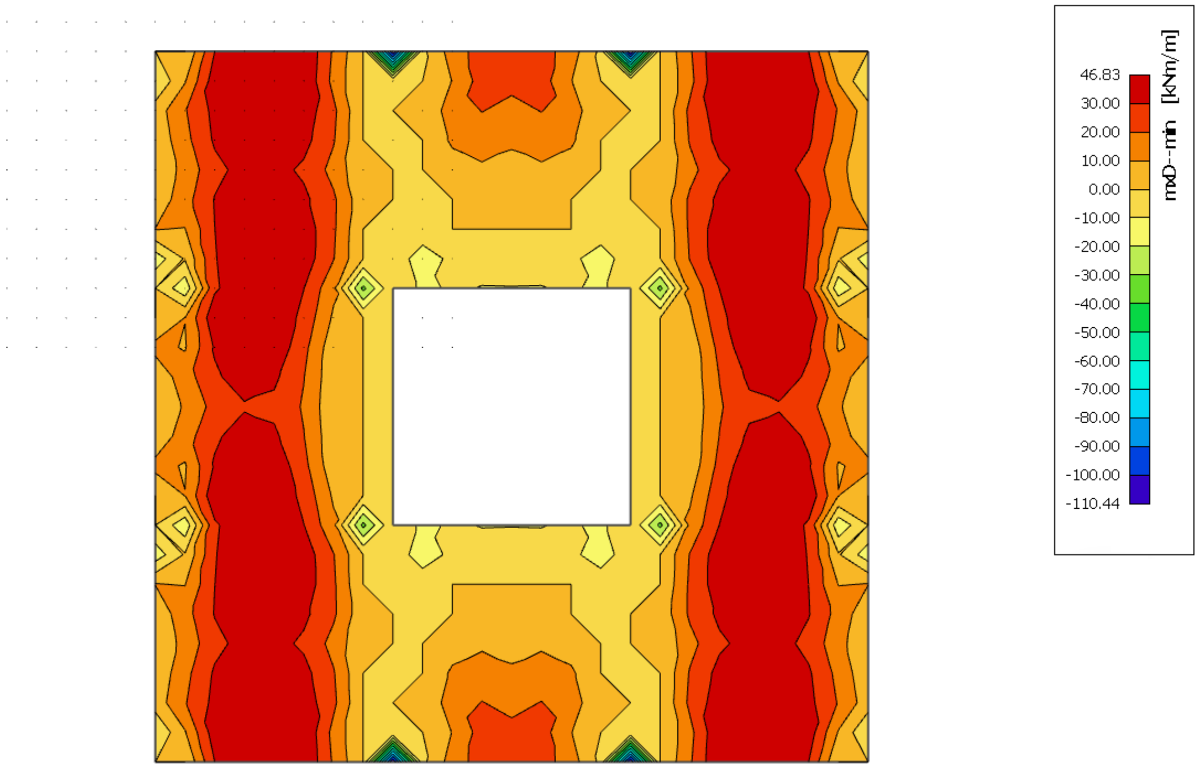


Fig. 5.4.2.2 Top view; Distribution of positive bending moment in *"y"* direction

5.4.3 Design number of support levels

Stress limitation check:

$$\sigma_c = \frac{M_{Ed}}{I} x$$

$$M_{Ed} = 46{,}83\ kNm/m$$

$$I = \frac{1}{12} b{h_s}^3 = \frac{1}{12} * 1000 * 200^3 = 0{,}33 * 10^9 mm^4$$

$$x = \frac{A_{s,prov} f_{yd}}{0{,}8 b f_{cd}} = \frac{1131 * 435}{0{,}8 * 1000 * 23{,}33} = 26{,}4\ mm$$

$$\sigma_c = \frac{M_{Ed}}{I} x = \frac{46{,}83 * 10^6}{0{,}33 * 10^9} * 26{,}4 = 3{,}746\ MPa$$

$$\sigma_c = 3{,}746\ MPa < 0{,}15 * f_{ck}(7\ days) = 0{,}15 * 32{,}22 = 4{,}833\ MPa$$

Deflection limitation check:

$$\omega \leq \frac{l}{500} = \frac{8000}{500} = 16\ mm$$

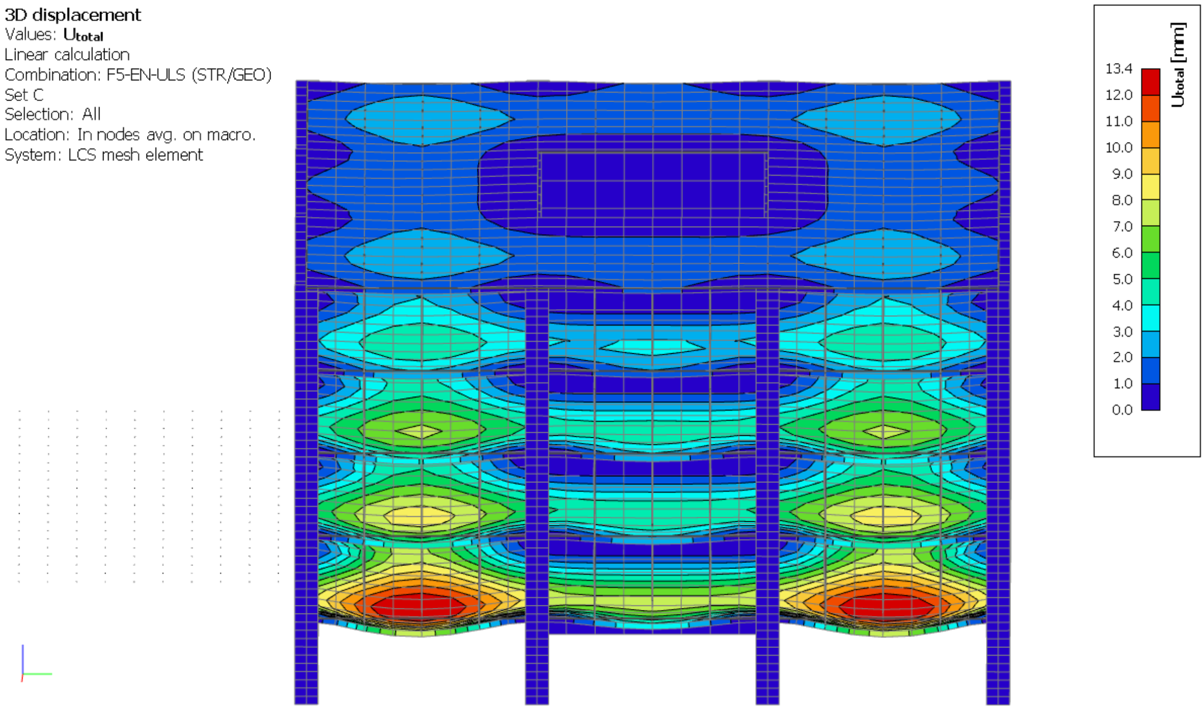


Fig. 5.4.2.3 Deflection of slab

$\omega = 13{,}4\ mm < 16\ mm$

Effect of creep on deflection of slab is automatically taken into consideration as:

$$E_{c,eff}(t) = \frac{E_c}{1 + \varphi(t, t_0)}$$

<u>Bearing resistance of props check:</u>

$N_{Ed} \leq N_{Rd}$

$N_{Rd} = 50\ kN$ for *PERI PEP Ergo Props*

As it seen (Fig. 5.4.2.4), the maximum compressive force is *48 kN*.

$N_{Ed} = 48\ kN < N_{Rd} = 50\ kN$

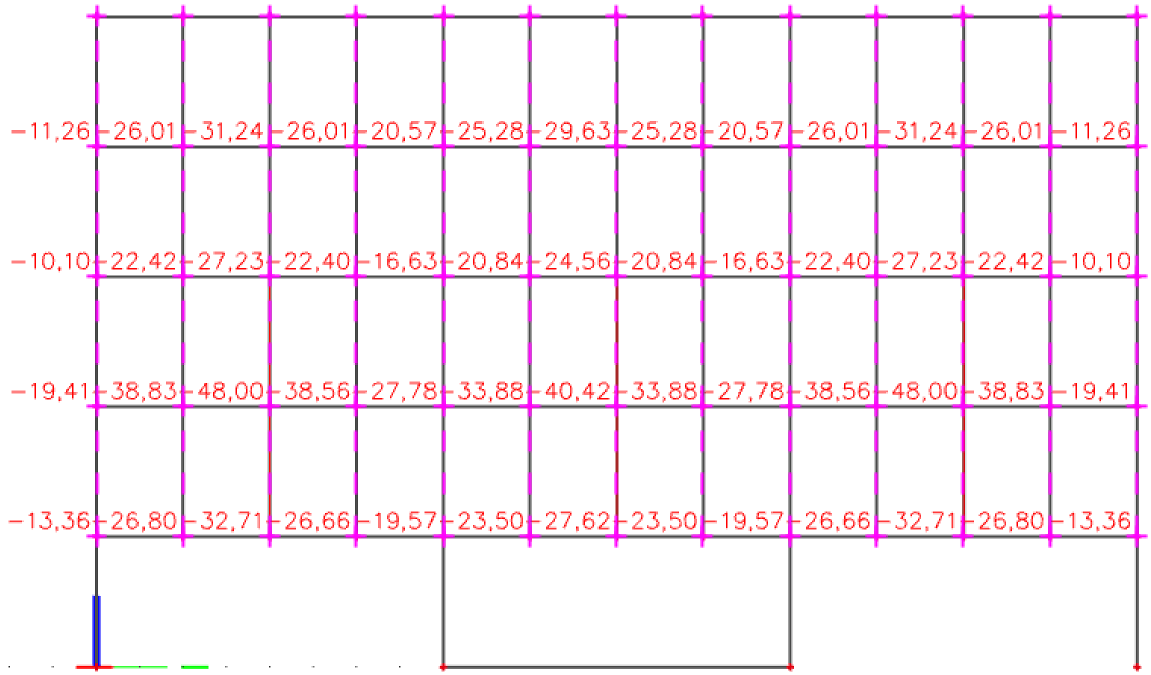


Fig. 5.4.2.4 Elevation; Distribution of normal compressive force in props

<u>Design number of support levels:</u> *4*

5.5 Imposition of actual load due to interior/exterior work

Actual load directly affects on deflections. Assumed, that after removal of props additional load on structure is added. Estimated characteristic values of loads are 1,7 kN/m^2 for dead load and 3 kN/m^2 for live load. Both values imposed in one slab of building.

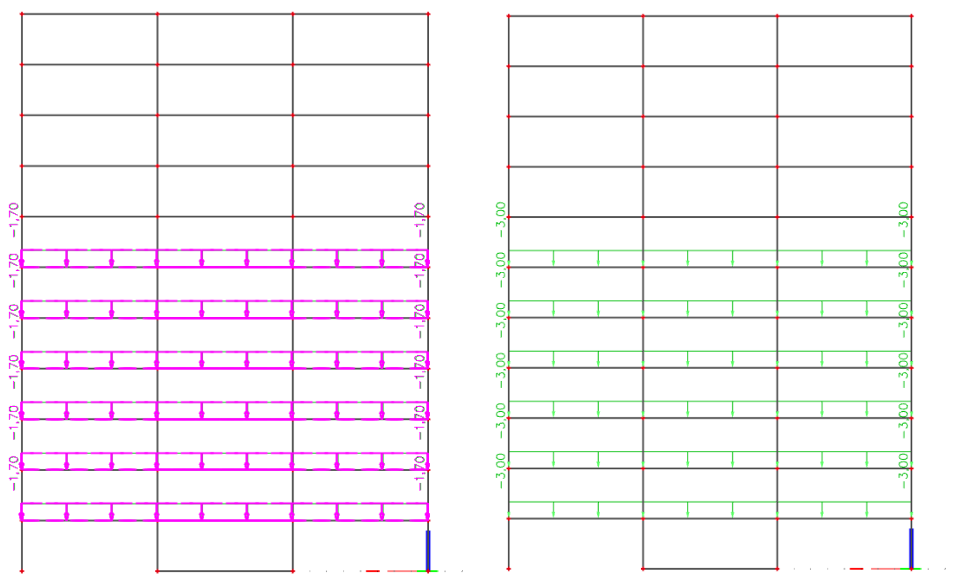


Fig. 5.5.1 Characteristic values of load added below shore supported levels

5.6 Total axial shortening

Case study building consists of *24 floors*, therefore first floor subjected to *24 stages* of loading and it means, that each floor is suffering from $N_{Stage} - 1$ stages of loading from previous floor. To simply describe the procedure for axial shortening calculation, part of it for *"Block B"* is presented:

Tab. 5.6.1 Part of calculation procedure

	Story 22		
	Stage 1	Stage 2	Stage 3
t0	0,0833	1	2,5
t	189	189	189
h	3000	3000	3000
$\varphi(t,t_0)$	2,402236	1,544926	1,303431
Stress Increment	0,3	0,3	0,3
Elastic Strain	2,85E-05	1,05E-05	9,7E-06
Creep Strain	2,12E-05	1,36E-05	1,15E-05
Shrinkage Strain	0,000153		
Total Strain	0,000203	2,42E-05	2,12E-05
Axial shortening	0,609202	0,072506	0,06361
	Story 23		
	Stage 1	Stage 2	
t0	0,0833	1	
t	187,5	187,5	
h	3000	3000	
φ(t,t0)	2,397326	1,541751	
Stress Increment	0,3	0,3	
Elastic Strain	2,85E-05	1,05E-05	
Creep Strain	2,12E-05	1,36E-05	
Shrinkage Strain	0,000153		
Total Strain	0,000203	2,41E-05	
Axial shortening	0,607515	0,072422	

where:

22nd floor is subjected to 3 stages of loading:

1st stage – self-weight and imposed load of 22nd floor

2nd stage – self-weight and imposed load of 23rd floor

3rd stage – self-weight and imposed load of 24th floor

23rd floor is subjected to 2 stages of loading:

1st stage – self-weight and imposed load of 23rd floor

2nd stage – self-weight and imposed load of 24th floor

t_0 is the age of concrete in days when load is imposed and obtained from the construction schedule,

t is the time considered in days, when strain/deformation computed and is $-1{,}5$ days for each following story, with respect to construction sequence,

h is the height of column,

$\varphi(t, t_0)$ is the creep coefficient, obtained from initial time when load is applied to time considered.

Values of stress increment are calculated in FEA software *SCIA*, with the step-by-step *"Construction Stage"* option. Instantaneous elastic strain is calculated as $\varepsilon_{ce} = \sigma_c * (\frac{1}{E_c(t_0)})$, where time-dependent modulus of elasticity is taken with respect to age of concrete at time t_0. Strain, caused by creep, is calculated multiplying stress increment by creep compliance. Shrinkage strain is a load-independent strain, hence calculated from time of casting of concrete t_0 to time considered t. Finally, total strain is calculated (Eq. 4.4.1) at every stage and then multiplied by height of column to get respective axial shortening to each load increment. By summing up axial shortening in one line, value of shortening of column is obtained.

Total axial shortening of building is the sum of shortenings of each column.

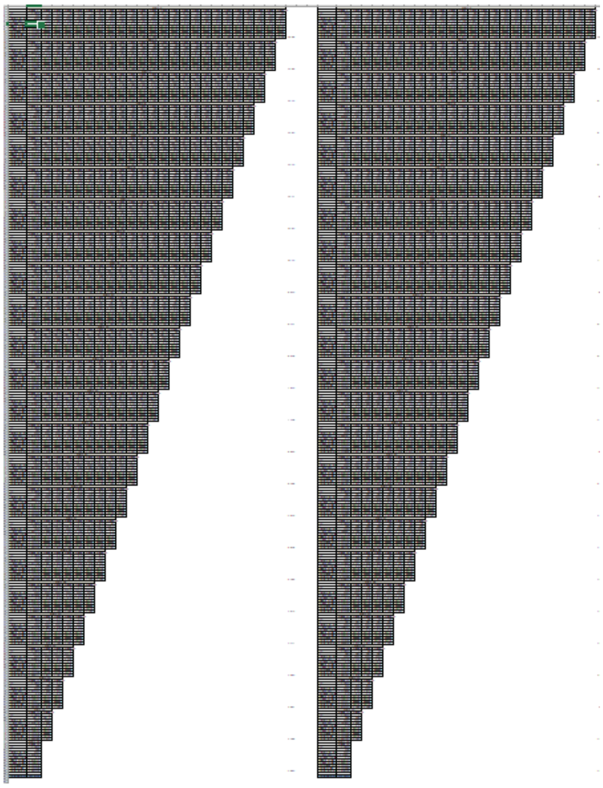


Fig. 5.6.2 Illustrative example of calculations

VBA function is created to get precise values and save time on numerous calculations:

VBA function for creep coefficient:

```
Function CreepEC(t0 As Double, t As Double) As Double
   Dim Alpha1 As Double
   Dim Alpha2 As Double
   Dim Alpha3 As Double
   Dim h0 As Double
   Dim PhiRH As Double
   Dim Betafcm As Double
   Dim Betatt0 As Double
   Dim Phi0tt0 As Double
   Dim Betah As Double
   Dim Betactt0 As Double
   Dim Acc As Double
   Dim u As Double
   Dim RHH As Double
   Dim fcm As Double
   Dim fcm0 As Double
   Dim fck As Double


   Acc = Sheets("Creep model").Range("Acc").Value
   u = Sheets("Creep model").Range("u").Value
   RHH = Sheets("Creep model").Range("RHH").Value
   fcm = Sheets("Creep model").Range("fcm").Value
   fcm0 = Sheets("Creep model").Range("fcm0").Value
   fck = Sheets("Creep model").Range("fck").Value

   Alpha1 = (35 / fcm) ^ 0.7
   Alpha2 = (35 / fcm) ^ 0.2
   Alpha3 = (35 / fcm) ^ 0.5
   h0 = 2 * Acc / u
   PhiRH = (1 + ((1 - RHH) * Alpha1) / (0.1 * (h0 / 0.001) ^ (1 / 3))) * Alpha2
   Betafcm = 16.8 / (fcm ^ (1 / 2))
   Betatt0 = 1 / (0.1 + (t0) ^ 0.2)
   Phi0tt0 = PhiRH * Betafcm * Betatt0
   Betah = WorksheetFunction.Min(1.5 * (1 + (0.012 * RHH / 0.01) ^ 18) * (h0) / 0.001 + 250 * Alpha3, 1500 * Alpha3)
   Betactt0 = ((t - t0) / (Betah + (t - t0))) ^ 0.3

   CreepEC = Phi0tt0 * Betactt0

End Function
```

VBA function for shrinkage strain:

```
Function ShrinkageRapid(ts As Double, t As Double) As Double
   Dim h0 As Double
   Dim Alphads1 As Double
   Dim Alphads2 As Double
   Dim BetaRH As Double
   Dim Epsicd0 As Double
   Dim kh As Double
   Dim Betadstts As Double
   Dim Epsicdt As Double
   Dim Epsicainf As Double
   Dim Betaast As Double
   Dim Epsicat As Double
   Dim Acc As Double
   Dim u As Double
   Dim RHH As Double
   Dim fcm As Double
   Dim fcm0 As Double
   Dim fck As Double

   Acc = Sheets("Creep model").Range("Acc").Value
   u = Sheets("Creep model").Range("u").Value
   RHH = Sheets("Creep model").Range("RHH").Value
   fcm = Sheets("Creep model").Range("fcm").Value
   fcm0 = Sheets("Creep model").Range("fcm0").Value
   fck = Sheets("Creep model").Range("fck").Value
   Alphads1 = Sheets("Creep model").Range("Alphads1").Value
   Alphads2 = Sheets("Creep model").Range("Alphads2").Value

   h0 = 2 * Acc / u
   BetaRH = 1.55 * (1 - (RHH) ^ 3)
   Epsicd0 = 0.85 * (220 + 110 * Alphads1) * Exp(-1 * Alphads2 * fcm / fcm0) * 0.00001 * BetaRH
   kh = (h0 * (0.7 - 0.75)) / (0.5 - 0.3) + 0.825
   Betadstts = (t - ts) / ((t - ts) + 0.04 * (h0 / 0.001) ^ (3 / 2))
   Epsicdt = Betadstts * kh * Epsicd0
   Epsicainf = 2.5 * (fck - 10) * 0.00001
   Betaast = 1 - Exp(-0.2 * (t) ^ (1 / 2))
   Epsicat = Betaast * Epsicainf

   ShrinkageRapid = (Epsicat + Epsicdt) / 10

End Function
```

5.7 Results

Obtained results of total axial deformations of both blocks are plotted in one graph. Graph represents development of shortening over global time. Where global time starts with the beginning of construction of *"Block A"*. Time gap between two blocks is *126 days*. By the geometry of case study building, two types of columns are subjected to different stresses

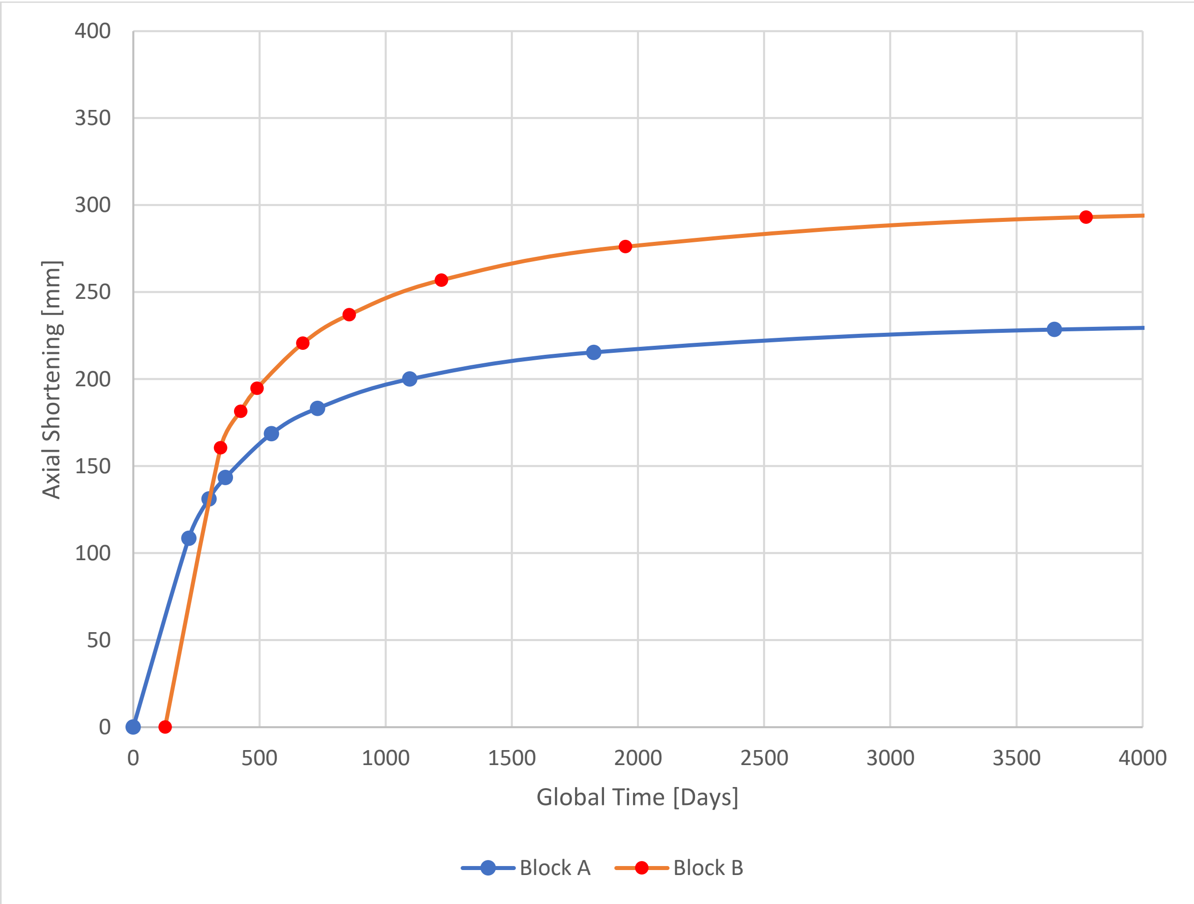


Fig. 5.7.1 Development of axial shortening over global time

Specification of column location in structural plan (Fig. 5.1.2):

Corner columns: A1, A2, D1, D4 and E1, E4, H1, H4

As it was expected, fast construction leading to bigger deformations. At 1 year of deformations, "Block A" reaches 143,38 mm, and "Block B" reaches 194,78 mm. While for 50 years of deformations, "Block A" reaches 240,12 mm, and "Block B" reaches 308,81 mm. Differential shortening of columns is approximately 51 mm at 1 year and 69 mm at 50 years.

Tab. 5.7.2 Axial shortening over time

Time [Days]	Axial Shortening [mm]		Dif. Short.
Local	Block A	Block B	
300	131,16	181,6	50,44
365	143,38	194,78	51,4
547,5	168,65	220,58	51,92
730	183,23	237,04	53,81
1095	199,99	256,91	56,92
1825	215,41	276,13	60,72
3650	228,49	293,13	64,64
9125	237,1	304,69	67,59
18250	240,12	308,81	68,69

Relatively low differential shortening can be explained by fast strength development of concrete, used for construction of *"Block B"*.

Solving the problem of differential shortening, both curves are imposed one to another over local time, so that distinction between them becomes equal to 0. Preferably at time, when deflection reaches adequately deep value in soonest time, so that it's development in long-term shall not significantly affect in differential shortening.

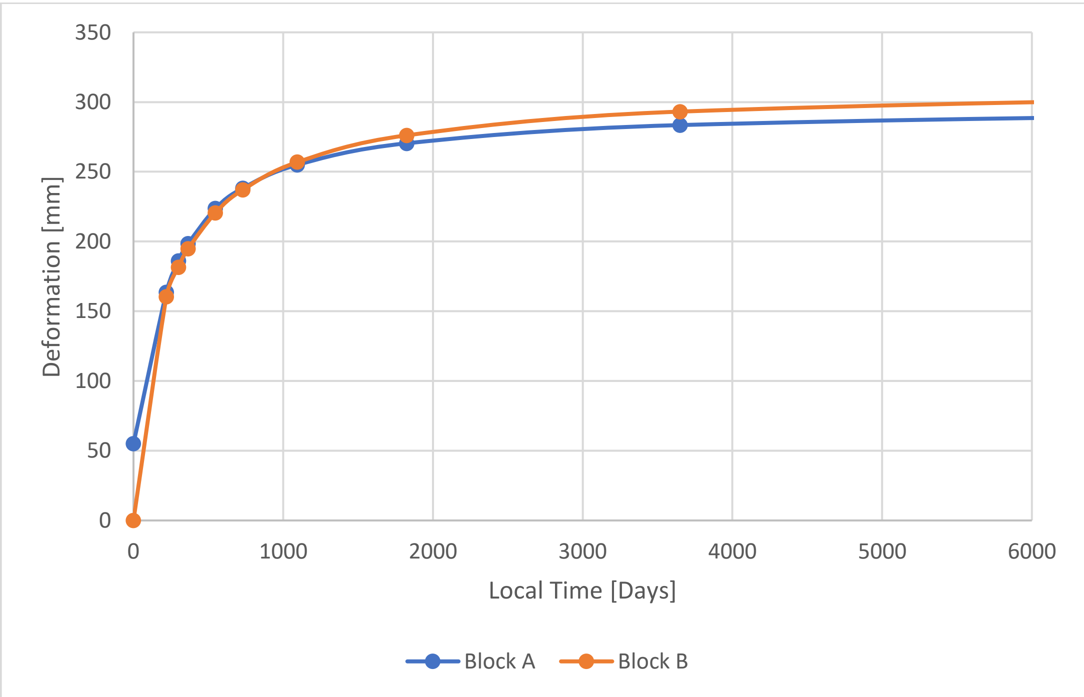


Fig. 5.7.3 Imposition of curves

Selected time is 1 year after construction. During which both blocks shortened in a fast manner.

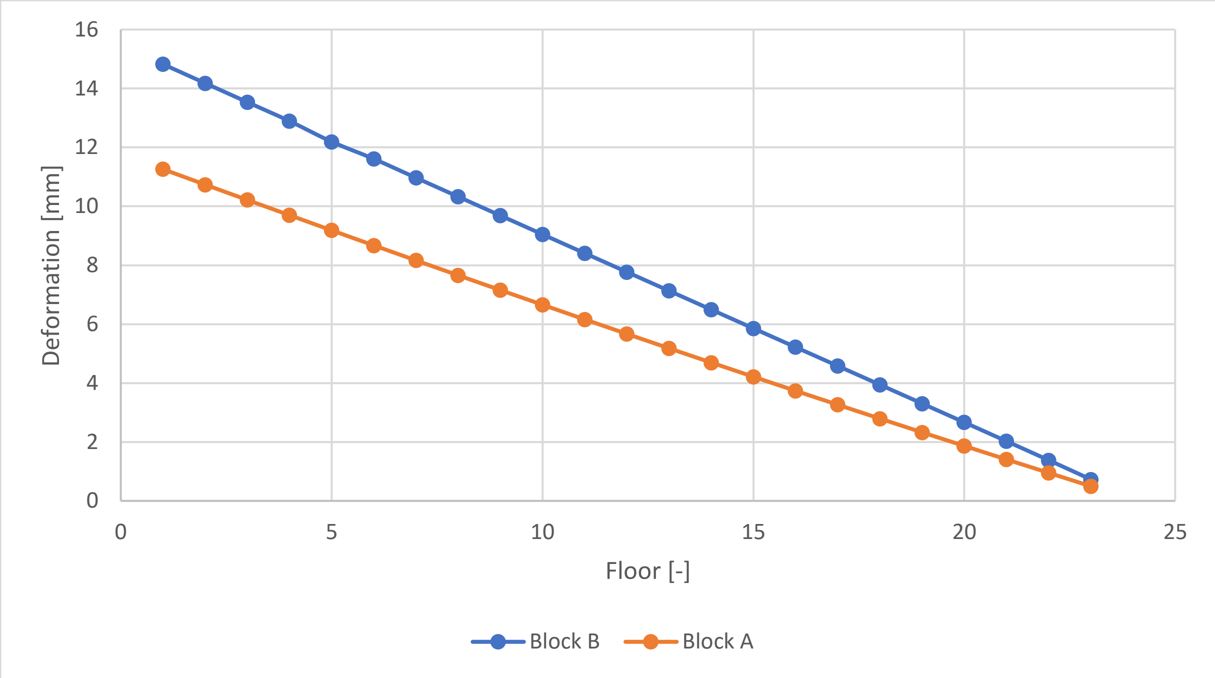


Fig. 5.7.4 Differential deformation of floors at 1 year

Tab. 5.7.5 Differential deformation between corresponding floors

Floor	Block B	Block A	Dif. Deform.
Floor 1	16	12,18	3,82
Floor 2	14,82	11,26	3,56
Floor 3	14,18	10,74	3,44
Floor 4	13,54	10,22	3,32
Floor 5	12,89	9,7	3,19
Floor 6	12,19	9,18	3,01
Floor 7	11,61	8,67	2,94
Floor 8	10,97	8,16	2,8
Floor 9	10,33	7,66	2,67
Floor 10	9,69	7,16	2,53
Floor 11	9,05	6,66	2,39
Floor 12	8,41	6,16	2,25
Floor 13	7,77	5,67	2,1
Floor 14	7,13	5,18	1,95
Floor 15	6,49	4,7	1,8
Floor 16	5,86	4,21	1,64
Floor 17	5,22	3,74	1,48
Floor 18	4,58	3,26	1,32
Floor 19	3,94	2,79	1,15
Floor 20	3,31	2,33	0,98
Floor 21	2,67	1,87	0,8
Floor 22	2,03	1,41	0,62
Floor 23	1,38	0,95	0,43
Floor 24	0,73	0,5	0,23

As it may seem (Fig. 5.7.4), first floor suffers from biggest deformations and, with increase of floor level, shortening decreasing linearly. Compensation on each floor, by adding more concrete, eliminates structure from differential level in each respective floor.

6. Conclusions

Based on calculation procedure the following main conclusions are given below:

- Accelerated construction causes irreversible excessive deflection due to larger creep of concrete at early stages, which cannot be neglected.
- Precision of calculations strongly depends on predicting capability of selected creep and shrinkage models, careful implementation of construction sequence in calculations, accuracy of tests on physical parameters of used material, actual load on structure, etc.
- Obtained results of deformations cannot perfectly reflect material's behaviour, but could be adequately estimated for solving assigned task.
- Differential deformation between structural elements could be compensated by adding more concrete in acceptable range, otherwise creep effect should be reduced by lowering water to cement ratio, increasing strength class of concrete, etc.
- Horizontal structure during accelerated construction must be properly investigated in order to avoid possible cracks, excessive deflections or even collapse.

7. References

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