Master Thesis

Czech
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in Prague

F3

Faculty of Electrical Engineering
Department of Control Engineering

Modelling of Electromechanically Actuated Differential Wet Clutch

Bc. Jan Cabicar

Supervisor: Ing. Michal Sojka, Ph.D.
Field of study: Cybernetics and robotics
Subfield: Cybernetics and robotics
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Acknowledgements

I would like to thank my supervisor, EATON corporation for allowing me to work on their project and my friends and family for their support.

Declaration

I declare that I have completed this thesis independently and I have cited all used sources in accordance with Methodical instruction n. 1/2009 about ethical principles for academic thesis writing.

In Prague, May 24, 2019

Prohlašuji, že jsem předloženou práci vypracoval samostatně a že jsem uvedl veškeré použité informační zdroje v souladu s Metodickým pokynem č. 1/2009 o dodržování etických principů při přípravě vysokoškolských závěrečných prací.

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Abstract

This thesis is focused on modeling of a wet clutch system used for controlling electronic limited slip differential manufactured by EATON corporation. The friction coefficient is important part of the wet clutch model, however its exact model is unknown. Therefore, process for its modeling and identification was designed.

The friction coefficient model introduced describes its dependency on slip speed, pressure, temperature, automatic transmission fluid and friction material properties and is based on Stribeck equation. Identification process based on nonlinear regression and Monte Carlo method using two distinct datasets was designed and tested.

Keywords: wet clutch, limited slip differential, elsd, friction coefficient, Stribeck equation, Monte Carlo, nonlinear regression, EATON

Supervisor: Ing. Michal Sojka, Ph.D.

Abstrakt

Tato práce se zaměřuje na modelování systému kapalinové spojky použité pro ovládání elektronicky ovládaného diferenciálu s omezeným prokluzem vyráběným firmou EATON. Koeficient tření je důležitou součástí systému kapalinové spojky, nicméně jeho přesný model je neznámý. Proto byl navrhnut proces jeho modelování a identifikace.

Představený model koeficientu tření popisuje jeho závislost na rychlosti otáčení, tlaku, teplotě a vlastnostem kapaliny automatické převodovky a třecího materiálu a je odvozen od Stribeckovy rovnice. Proces identifikace založený na nelineární regresi a metodě Monte Carlo využívající dvou odlišných datasetů byl navrhnut a otestován.

Klíčová slova: kapalinová spojka, diferenciál s omezeným prokluzem, elsd, koeficient tření, Stribeckova rovnice, Monte Carlo, nelineární regrese, EATON

Překlad názvu: Modelování elektronicky ovládané diferenciální kapalinové spojky

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MASTER‘S THESIS ASSIGNMENT

I. Personal and study details

Student's name:	**Cabicar Jan**	Personal ID number:	**420034**
Faculty / Institute:	**Faculty of Electrical Engineering**		
Department / Institute:	**Department of Control Engineering**		
Study program:	**Cybernetics and Robotics**		
Branch of study:	**Cybernetics and Robotics**		

II. Master’s thesis details

Master’s thesis title in English:

Modelling of Electromechanically Actuated Differential Wet Clutch

Master’s thesis title in Czech:

Modelování elektronicky ovládané diferenciální kapalinové spojky

Guidelines:

1. Introduction; Make yourself familiar with differentials and their mission in modern vehicles: focus on open, locked and electronically controlled limited slip differentials (eLSD). Also familiarize with function of clutches in differentials, their modeling and control algorithms
2. Clutch Model; Design physical model structure of a wet clutch (in Simulink), determine impact of friction coefficients (sensitivity analysis).
3. Friction model; Propose suitable equation for a friction model. Propose friction model with respect to pressure, temperature and speed dependencies.
4. Parameterization of friction model with respect to experimental data. Use multivariate regression and Monte Carlo analysis.
5. Simulation; Compare proposed solutions (with respect to pressure, temperature and speed dependencies). Choose the best solution with respect to design of a suitable algorithm for wet clutch control.
6. V&V; Compare proposed solutions (simulations) against available experimental data.

Bibliography / sources:

[1] Achtenová, G.: Převodná ústrojí motorových vozidel. ČVUT v Praze. 2015. ISBN 978-80-01-04855-9
[2] Ivanović, V., Deur, J., Herold, Z., Hancock, M., and Assadian, F. Modelling of electromechanically actuated active differential wet-clutch dynamics. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering. 2012, 226(4), p. 433–456.
[3] Ljung, L. System identification (2nd ed.): theory for the user, Prentice Hall PTR, Upper Saddle River, NJ, 1999. 609 pages. ISBN 0-13-656695-2.
[4] Candy, James V. Bayesian Signal Processing : Classical, Modern, and Particle Filtering Methods, John Wiley & Sons, Incorporated, 2016. 637 pages. ISBN 9781119125495.

Name and workplace of master’s thesis supervisor:

Ing. Michal Sojka, Ph.D., Embedded Systems, CIIRC

Name and workplace of second master’s thesis supervisor or consultant:

Date of master’s thesis assignment: **09.01.2019** Deadline for master's thesis submission: **24.05.2019**

Assignment valid until:
by the end of summer semester 2019/2020

Ing. Michal Sojka, Ph.D. Supervisor’s signature	prof. Ing. Michael Šebek, DrSc. Head of department’s signature	prof. Ing. Pavel Ripka, CSc. Dean’s signature

III. Assignment receipt

The student acknowledges that the master’s thesis is an individual work. The student must produce his thesis without the assistance of others, with the exception of provided consultations. Within the master’s thesis, the author must state the names of consultants and include a list of references.

.

Date of assignment receipt

Student’s signature

Symbols table

h	...	fluid film thickness
t	...	time
$\phi(h)$	...	Patir and Cheng's flow factor
$\xi(h)$	...	film hydrodynamic pressure
$\delta(h)$	...	permeability parameter of friction material
$g(h)$	...	surface roughness parameter of friction material and separator plate
A_{red}	...	contact area over the surface area of the friction plate (area excluding grooves)
γ	...	scaler
F_{app}	...	force applied to driving part of the clutch
F_a	...	asperity force
r_o	...	outer radius of the friction disc
r_i	...	inner radius of the friction disc
P_a	...	mean asperity pressure
E	...	Youngus modulus of the friction material
A_R	...	real contact area
A_N	...	friction lining surface area
N	...	asperity density
β	...	asperity tip radius
σ	...	surface RMS roughness
K_{perm}	...	friction lining permeability
d	...	friction lining thickness
η	...	Beavars and Joseph slip factor
χ	...	Beavars and Joseph slip coefficient
μ	...	ATF viscosity
T_{clu}	...	clutch torque
T_h	...	hydrodynamic torque
T_a	...	asperity torque
N_f	...	number of friction surfaces
θ_0	...	space between two grooves
ω	...	slip speed
ϕ_f	...	Patir and Cheng's pressure flow factor
ϕ_{fs}	...	Patir and Cheng's shear stress factor for smooth surface
μ	...	ATF viscosity
$\alpha_{0,1}$	...	ATF viscosity model parameters
μ_f	...	friction coefficient
θ_{sump}	...	sump temperature
θ_{pack}	...	temperature caused by friction on clutch plates
ρ	...	density of the separator plate
V	...	volume of the separator plate
c_p	...	density of the separator plate
N_{sp}	...	number of separator plates
H_{tf}	...	heat transfer coefficient

Table 1: Symbols table

Symbols table

θ_0	...	angle between grooves of the friction disc
μ_f	...	friction coefficient
μ_C	...	Coulomb friction
μ_S	...	static friction
ω_S	...	slip speed coefficient
i	...	empirical constant
k_v	...	viscous friction coefficient
F_N	...	normal load
P	...	pressure
θ	...	temperature
$a_{0...24}$	...	polynomial parameters
$b_{1...4}$	...	heat transfer coefficient model's parameters

Table 2: Symbols table

List of abbreviations

ATF	...	automatic transmission fluid
RMSE	...	root mean square error

Table 3: List of abbreviations

Chapter 1

Introduction

There is always a strive to make cars as safe and reliable as possible. Control design of key parts of the vehicle in order to provide the best possible response to the driver's commands is one way to achieve this. One of the key parts of the vehicle which can be used for this is a differential. A differential is a device which can distribute and transfer torque between wheels, which influences the vehicle's response during turns. The differential also plays a major role in other situations, e.g. when one of the wheels is in the air or on a slippery surface.

The focus of this thesis is on modeling of a wet clutch system used for controlling electronic limited slip differential made by company EATON. A model based on scientific literature is introduced. Special attention is given to the friction coefficient which is not modeled sufficiently in the surveyed literature and is shown to be important by the means of sensitivity analysis.

The friction coefficient is dependent on multiple factors such as slip speed, pressure, temperature, friction material and automatic transmission fluid, while only its dependency on slip speed is described. Hence a modeling and identification process of the friction coefficient that will account for the influence of all the factors is introduced.

Chapter 2
Differentials

Differential is a set of gearwheels that transfers and splits driving torque between wheels (Figure 2.1). Typical application of this device is during turning, when it can make the outer wheel, which needs to bridge longer distance, rotate faster than the inner wheel. It can also be used to prevent under/over steering and thus help the car drive along the desired path.

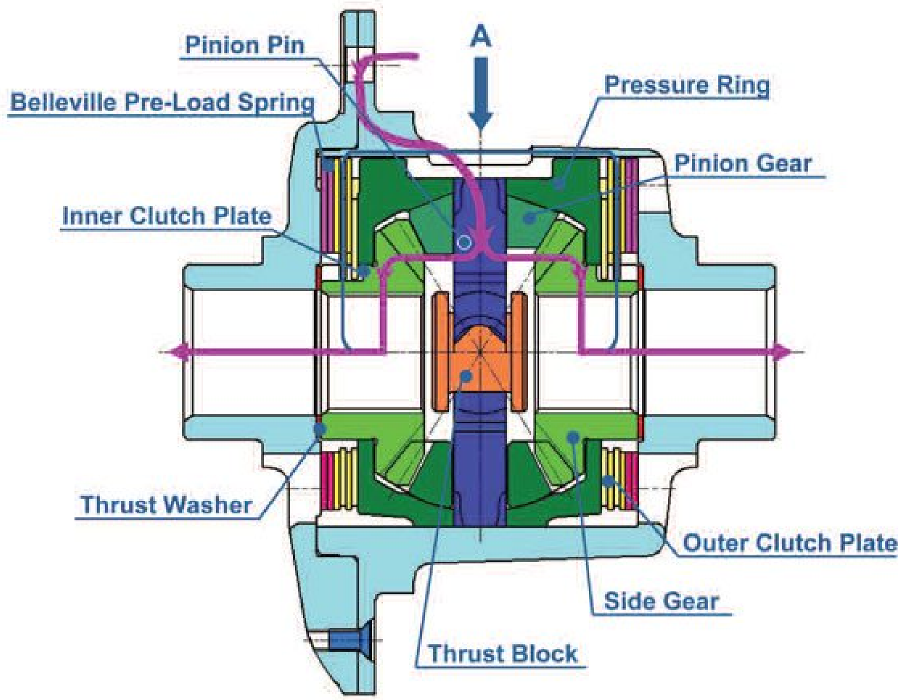


Figure 2.1: Example of a differential scheme [20]

There are three main types of differentials:

- Open differentials
- Locked differentials
- Combination of both

Different types of mechanisms are used to control locking of a differential - mechanical, electronic, hydraulic etc.

In this section, all the basic types of differentials are presented and a control mechanism used in this project is described.

2.1 Open differential

Simplified open differential is shown in Figure 2.2. The input shaft brings the torque from the motor and makes the outer ring with the housing of the differential turn. From inside the housing there are two spider gears attached to it. Both output shafts for wheels have a gear attached to their ends.

If there is same resistance against both wheels, the spider gears are moving at the same speed without spinning and torque is split equally. However if there is a higher resistance against one wheel and less resistance against the other, the spider gears will start to turn and thus will transfer more torque to the wheel with less resistance.

This is especially useful while turning at low speeds - the outer wheel requires more torque to turn than the inner wheel. The problem with this differential is that when one side is blocked (tire having a bigger obstacle in the way) or the other side is having no resistance (wheel in the air or on a slippery road), all the torque will go to the side where there is no resistance while its needed on the other side (e.g. to get over the obstacle).

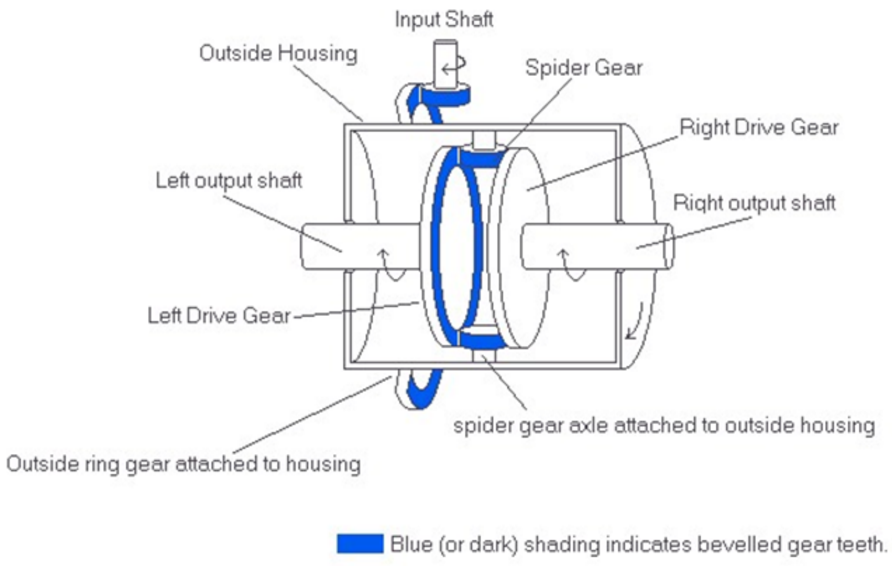


Figure 2.2: Open differential [1]

2.2 Locking differential

This type of differential has a locking system (Figure 2.3), which means it can stop the gears inside the differential from transfering torque between the wheels. Thus it can behave like a solid axle and output the same torque to both sides, which solves the problem from the previous subsection[1]. The locking can be achieved manually or automatically using various mechanisms.

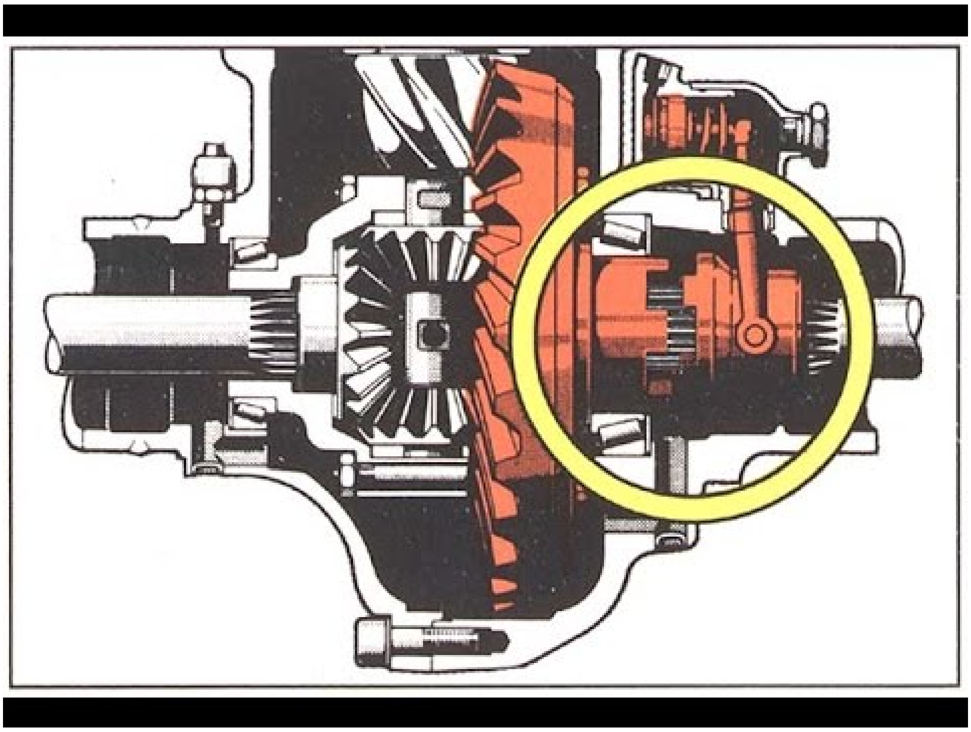


Figure 2.3: Locking differential [26]

2.3 Limited slip differential

One particular type of locking differential is a limited slip differential. This device has a locking mechanism that can make the differential transition fluently between the open state and the locked state. There is plenty of different implementations (viscous coupling, clutches etc.), however the main focus here is on the electronic limited slip differential (Figure 2.4).

2.3.1 Electronic limited slip differential

This differential has an electronically controlled clutch or two clutches that lock or partially lock the differential in certain situations. Thus it controls redistribution of torque which provides lots of possibilities for controlling the whole vehicle. The particular differential presented is made by EATON.

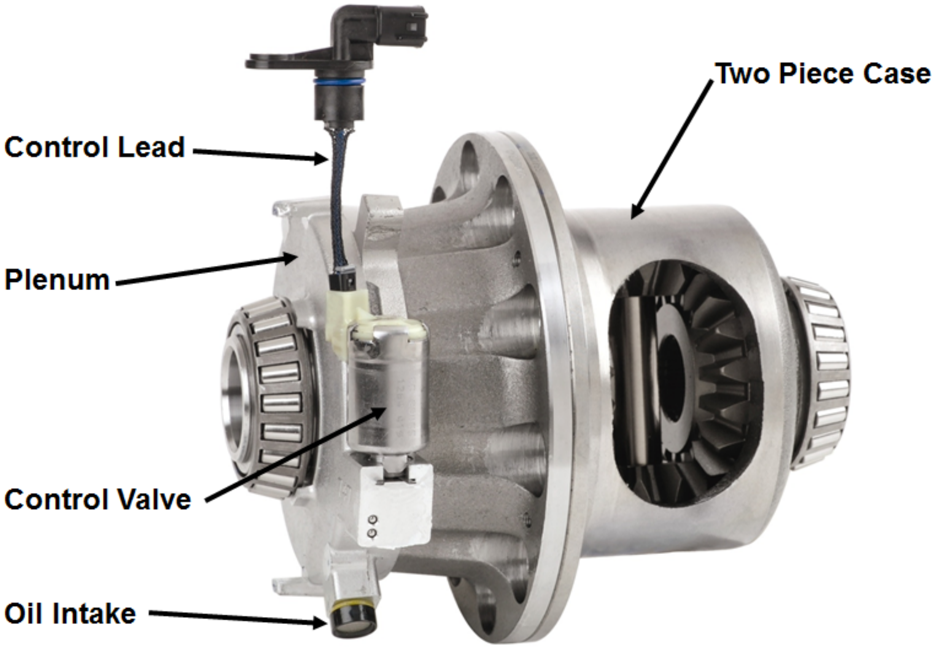


Figure 2.4: Electronic limited slip differential ©Eaton

The wet clutch used as a locking mechanism is mounted only to one side of the differential. When torque load difference between the wheels is bigger than set limit, the clutch starts to influence the amount of torque transfer. The clutch can act like the open differential or it can block the torque transfer completely and thus lock the differential. It can also function in any intermediate step between completely locked or open state and transfer only certain amount of torque.

Main advantage of this device is in enabling high level control to provide desired response of the vehicle to drivers intention perfectly at all times, while taking care of stability of the vehicle (rollover prevention).

The desired response of the vehicle is predicted from car's current dynamics e.g. speed and steering wheel angle. If the yaw rate of the vehicle, which describes its movement to the sides, is below the desired one, the differential is open and if it is the other way around, the differential is locked.[2] Following the desired yaw rate of the vehicle is one of the most efficient ways of ensuring the safety of the driver.

Chapter 3

Wet clutch modeling

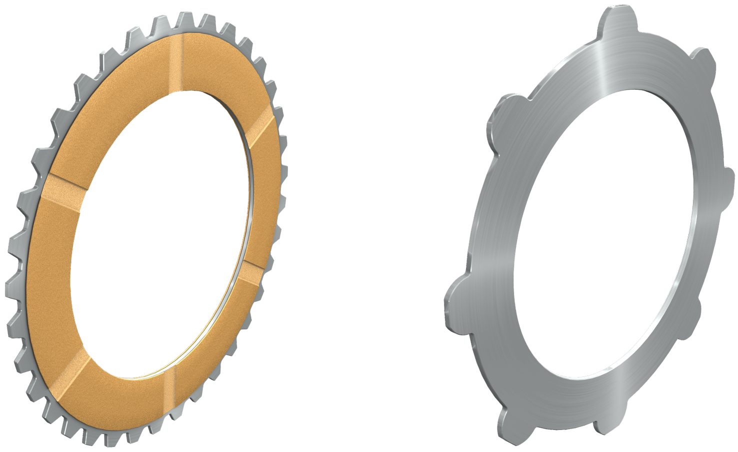
Wet clutch consists of a package with multiple discs - driving ones and a driven ones (Figure 3.2). For the illustration purposes only two discs are now considered.

The driving disc is rotating and can be pushed against the driven disc and when they get in contact the torque of the driving disc is transferred to the driven disc. This is called the engagement process. The driving disc can be also pulled away from the driven disc in order to stop transferring torque, which is called the disengagement process.

One of the discs is a separator plate made of steel (Figure 3.1b) and the other disc is a core plate coated with friction lining (Figure 3.1a), which is usually some paper based permeable friction material. The friction disc may or may not have grooves (Figure 3.4) however in case of this project, the friction discs have grooves.

The schematic on Figure 3.2 can be extended to multi-disc clutch, which is actually the one used in this project.

The package with separator plates and friction discs is partially submerged in the ATF (automatic transmission fluid). This fluid creates a film during engagement between the friction discs and separator plates - it fills the asperities between the disc and the plate.



(a) : Friction disc [19]   **(b) :** Separator plate [19]

Figure 3.1: Clutch plates

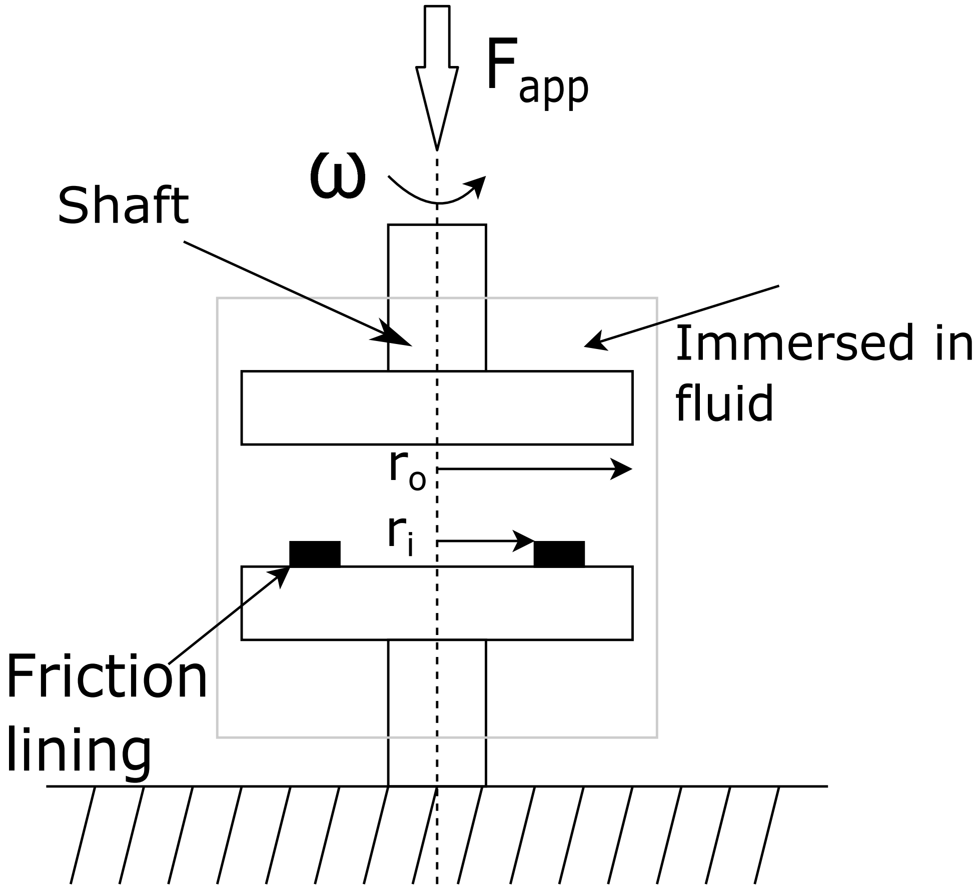


Figure 3.2: Scheme of wet clutch, taken from [4]

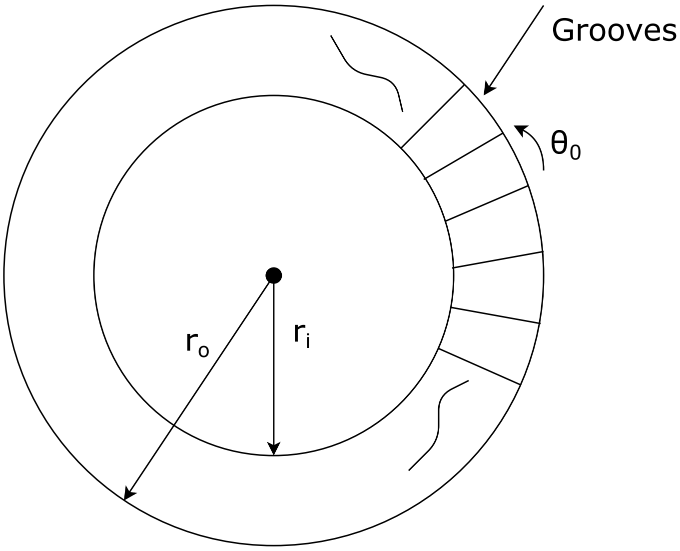


Figure 3.4: Grooves on friction discs, taken from [4]

3.1 Engagement process

Torque T_{clu} transferred through clutch has two components, hydrodynamic (viscous friction) torque T_h and asperity (contact) torque T_a.

$$T_{clu} = T_h + T_a \tag{3.1}$$

The hydrodynamic torque is most effective in the pre-engagement phase because it is transferred solely through the ATF. It has only a minor role during engagement.

The asperity torque is the torque transferred due to the contact of separator plates and friction discs through asperity contacts.

When the force is applied to the shaft in order to push the driving disc onto the driven one, the process of engagement is then following[4]:

1. Hydrodynamic lubrication:
   - At the initial engagement the film thickness is large and torque is transferred only through fluid.
   - As the discs and the separator plates get pressed together, fluid discharges from inbetween and the film thickness is decreasing.
2. Partial lubrication
   - When the discs and separator plates get pressed together to the point when film thickness reaches the heights of the asperity contacts, the asperity torque starts to contribute to the total torque.
3. Mechanical contact
   - When the whole pack of the separator plates and friction discs starts rotating at the same speed (relative speed = 0), the hydrodynamic torque disappears and only the asperity torque contributes to the total torque.

There is heat generated during engagement due to friction between friction discs and separator plates. The separator plates get the most heat since friction discs have low conductivity. This leads to non-uniform temperature distribution. The temperature at the friction interface is important for determining torque response due to its influence on ATF viscosity (increase in temperature causes decrease in ATF viscosity)[5].

Depending on the friction coefficient, there may be a rooster tail towards the end of engagement, which is usually undesirable. This depends on the properties of the friction material and the ATF [5](see section 4). The disengagement process generally works in the same manner as the engagement process backwards, however it is not considered in this thesis.

3.2 Modeling

Figure 3.5 shows the block diagram of model of the wet clutch. Inputs to the model are force applied to the driving part of the clutch, angular (slip) speed difference between driving part and the driven part and axle sump temperature. Output of the model is the torque transferred through the clutch.

The model was provided by EATON, and was partially based on the scientific literature, which is explained further in this section. However thermal, viscosity and friction coefficient models were developed directly by EATON. In case of thermal and viscosity models, which cannot be disclosed in this thesis, sufficient substitutes are introduced based off scientific literature. New approach to the friction coefficient modeling is introduced in this thesis (see chapter 4).

The mathematical model of torque response for the wet friction clutch engagement was introduced by Berger et al. [3] which was expanded upon by Yang et al. [5] and Deur et al. [4]. The model introduced in this project is mainly taken from [5] and nomenclature from this article is also used here. Some parts of the model however are changed due to insufficient modeling of the phenomenon in question, particularly thermal effects and considering grooves in the clutch.

Central part of the model is description of the fluid film thickness. In [5], the fluid film thickness is dimensionless, which is not the case in the model presented here.

Fluid film thickness development over time is described by approximate[1] Reynolds equation:

$$\frac{dh}{dt} = \frac{\phi(h)\xi(h)\delta(h)}{g(h)A_{red}} \cdot \gamma h^3 \tag{3.2}$$

where:

- $\phi(h)$ is the Patir and Cheng's pressure flow factor, which modifies Reynolds equation so that it is applicable to any general roughness surfaces. This flow factor partially expresses the average lubricant flow[6]. This factor is realized with a look-up table with data provided by EATON.

[1]The exact model is too complex[5]

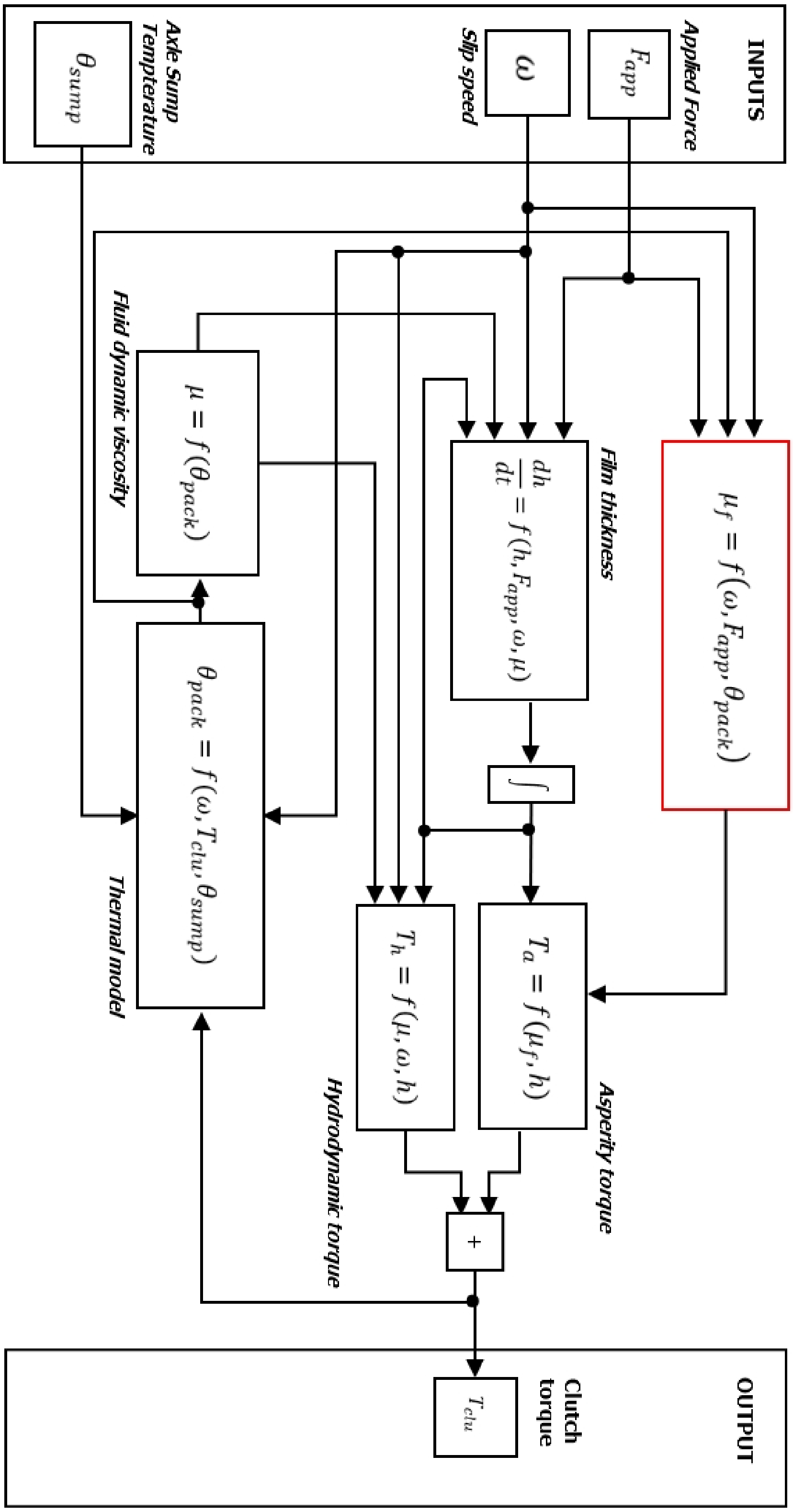


Figure 3.5: Wet clutch model diagram

- $\xi(h)$ is the film hydrodynamic force:

$$\begin{aligned}
\xi(h) &= F_{app} - F_a && (3.3)\\
F_a &= A_N P_a && (3.4)\\
P_a &= E\frac{A_R}{A_N} && (3.5)\\
A_R &= A_N \pi N \beta \sigma F_e(h/\sigma) && (3.6)\\
F_e &= \frac{1}{\sqrt{2\pi}}\int_x^{\infty}(s-x)e^{\frac{-s^2}{2}}ds && (3.7)\\
A_N &= \pi(r_o^2 - r_i^2) && (3.8)
\end{aligned}$$

  where F_a is force in the asperity contacts. It is calculated from asperity pressure on the contact area. Asperity pressure is obtained from fluid film thickness and geometric and material properties of asperity contacts of the mating surfaces - density of asperity contacts on the surface N, asperity contacts' tip radius β, surface RMS roughness σ and stiffness of the friction material described by its Young modulus E.

- $\delta(h)$ is the permeability parameter:

$$\begin{aligned}
\delta(h) &= \frac{h^3(1+3\eta)+12K_{perm}d}{h^3} && (3.9)\\
\eta &= \frac{1}{1+\frac{\chi h}{\sqrt{K_{perm}}}} && (3.10)
\end{aligned}$$

  where η is Beavers and Joseph slip factor and χ is Beavers and Joseph slip coefficient, which define boundary condition on interface of the fluid and permeable material, in this case ATF and friction lining.

  In [5], this equation contains term describing ratio of ATF viscosity and effective ATF viscosity in the friction lining. It was found that they are effectively the same and therefore this term was omitted in this model.

- $g(h)$ is surface roughness parameter of the friction material and separator surface:

$$g(h) = \frac{1}{2}[1 + erf(\frac{h}{\sqrt{2}\sigma})] \qquad (3.11)$$

- γ is a scaler:

$$\begin{aligned}
\gamma &= \frac{-1}{6\mu A_N Q} && (3.12)\\
Q &= r_o^2 + r_i^2 + \frac{r_o^2 - r_i^2}{ln(\frac{r_i}{r_o})} && (3.13)
\end{aligned}$$

which covers viscosity, geometry of the friction disc and loading effects. This form of a scaler was specifically introduced for a clutch made by EATON.

The output of the model is the clutch torque:

$$T_{clu} = T_h + T_a \tag{3.14}$$

where:

- T_h is hydrodynamic torque:

$$T_h = N_f \omega \frac{(\phi_f + \phi_{fs})}{h} \mu \frac{r_o^4 - r_i^4}{2} \pi A_{red} \tag{3.15}$$

  where A_{red} is fraction of non-grooved surface included in [4] to account for grooves; ϕ_f and ϕ_{fs} are Patir and Cheng's flow factors describing mean hydrodynamic shear stress of the asperity contacts[6]. The flow factors are realized with look-up table with data provided by EATON.

- T_a is asperity torque:

$$T_a = N_f \mu_f P_a \frac{r_o^3 - r_i^3}{3} 2\pi \tag{3.16}$$

The ATF viscosity is highly dependent on temperature and the relation is described using following empirical relation[4]:

$$\mu(\theta_{pack}) = \alpha_0 \theta_{pack}^{-\alpha_1} \tag{3.17}$$

The thermal effects are described by thermal model presented by Ivanović et al. [7]:

$$\frac{d\theta_{pack}}{dt} = \frac{1}{\rho V c_p} [\frac{\omega T_{clu}}{N_{sp}} - H_{tf} A_N (\theta_{pack} - \theta_{sump})] \tag{3.18}$$

where H_{tf} is heat transfer coefficient.

3.3 Sensitivity analysis

The friction coefficient is suspected to be important for this model. This is implied from the equation 3.16, because friction coefficient is a multiplicative factor for asperity torque. To support this claim, sensitivity analysis was performed [15]. Sensitivity of the output (clutch torque T_{clu}) of the system to the changes in inputs or its parameters (friction coefficient μ_f) can be quantified using sensitivity function. Sensitivity function is defined as follows:

$$S(t) = \frac{\partial T_{clu}}{\partial \mu_f} \tag{3.19}$$

The output of the system is called sensitive to parameter if small changes in the parameter produce significant changes in the output[16]. Due to the complexity of the model, derivation cannot be computed directly and finite difference approach has to be used.

Model described in this chapter was used with constant inputs of $\omega = 60rpm$, $\theta_{sump} = 60°C$ and $P_{app} = 500kPa$. The friction coefficient was set to $\mu_f = 0.09$, so the perturbation of 1% is $\Delta\mu_f = 0.0009$.

The finite difference approach formula is following:

$$\frac{\partial T_{clu}}{\partial \mu_f} \approx \frac{T_{clu}(t, \mu_f + \Delta\mu_f) - T_{clu}(t, \mu_f - \Delta\mu_f)}{2\Delta\mu_f} \tag{3.20}$$

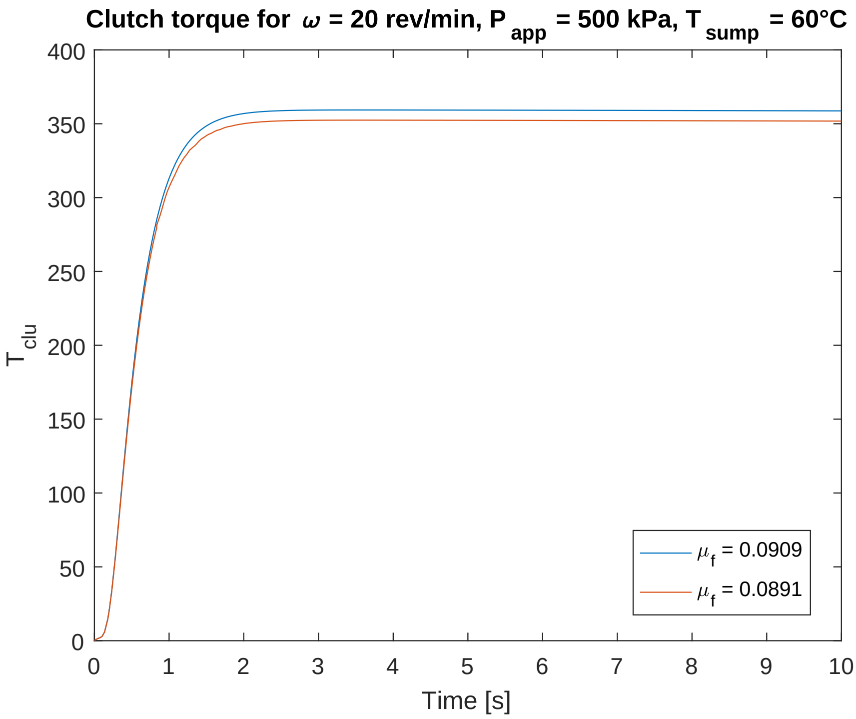


Figure 3.6: Clutch torque response

The simulation results can be found on Figure 3.6. As can be seen from the finite difference graph (Figure 3.7), for steady state the derivation is significant so the clutch torque is sensitive to small changes in friction coefficient. Thus our suspicion is confirmed.

Output of the wet clutch model is sensitive to changes in other parameters too, however the equations describing them are already known which is not the case with the friction coefficient and thus efforts should be made towards correct modeling and identification of friction coefficient, as it might improve the overall wet clutch model accuracy considerably.

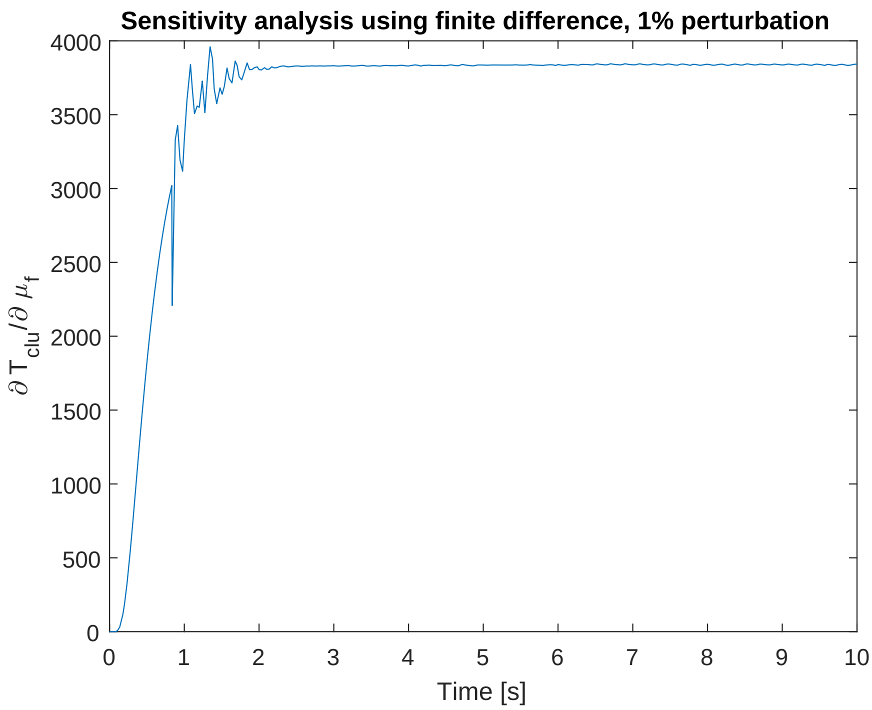


Figure 3.7: Finite difference graph

Chapter 4

Friction coefficient

The friction coefficient is a measure of force generated by relative motion of two surfaces sliding against each other. There are many different types of friction but the main focus here is on the lubricated friction where there is a lubricant between the two moving surfaces. The friction coefficient is positive because if the normal load in the contact increases the friction increases too[22]. However, there is some research (unrelated to this project) that demonstrated negative friction coefficient arising on chemically modified graphite at nanoscale[23].

The clutch engagement is a mechanical lubricated contact and as such, the friction coefficient has a strong influence on its performance and behaviour[8]. Generally speaking, friction is influenced by sliding speed, pressure, temperature, type of ATF and friction material and their properties[5]. Geometry of the contact surfaces in this case is already taken into account (see section 3.2, hydrodynamic and asperity torque). Type of ATF and friction material used can be considered a categorical variable and thus the model is created only for one particular type.

The friction coefficient dependency on sliding speed is described by Stribeck equation(Figure 4.1):

$$\mu_f = \mu_C + (\mu_S - \mu_C)e^{-|\frac{\omega}{\omega_S}|^i} + k_V\omega \tag{4.1}$$

where:

- μ_S is static friction. Static friction force is a force needed to make the two bodies of mass in contact with each other moving. It is the value of friction coefficient when the sliding speed is zero. This parameter is therefore by definition positive:

$$\mu_S > 0 \tag{4.2}$$

- μ_C is Coulomb or sliding force friction, which describes friction acting in dry contact of two bodies sliding on each other. The Coulomb (sliding) friction force is described as follows:

$$F_C = \mu_C F_N \tag{4.3}$$

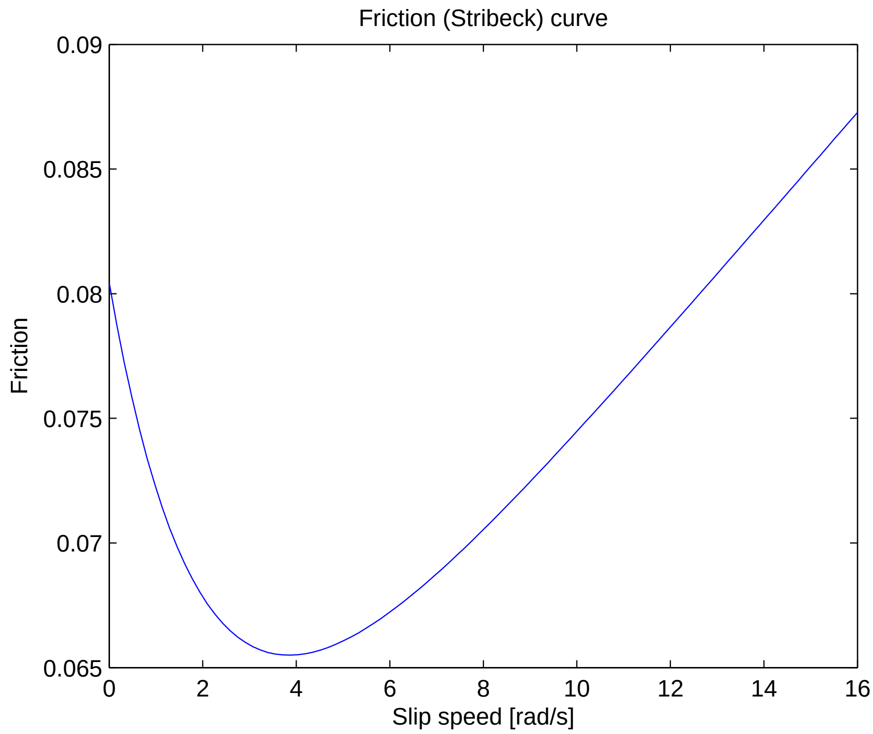


Figure 4.1: Friction dependency on slip speed

where F_N is normal load in the contact [8]. Usually, Coulomb friction is less than the static friction (the friction curve is ascending):

$$0 < \mu_C < \mu_S \tag{4.4}$$

However if certain type of ATF is used, this condition doesn't have to apply [4][5][18] and Stribeck friction can be curved differently. Practically it means that for an object to start moving in this fluid is easier than to remain in motion. Thus based on data (maximal achievable friction), Coulomb friction has to be smaller than some offset (the friction curve is descending):

$$0 < \mu_C < \mu_{offset} \tag{4.5}$$

As mentioned in 3.1, the friction curve affects the torque response. If the curve is descending, the torque may have a rooster tail towards the end of engagement. If the curve is ascending, the torque response is more desirable and smooth [5].

- ω_S is sliding speed coefficient (Stribeck velocity). This parameter is usually small and positive. In [11], it is defined as breakaway velocity multiplied by $\sqrt{2}$. In [12] this parameter is set as follows:

$$0.001 < \omega_S < 0.05 \tag{4.6}$$

- i is empirical parameter[12], also positive:

$$i > 0 \tag{4.7}$$

- k_V is viscous friction coefficient. Its value determines, if the friction curve is ascending or descending for higher speeds, which depends on the ATF type[5][4]. In this project, value range was chosen based on how the parameter corresponds to data:

$$-0.01 < k_V < 0.01 \tag{4.8}$$

4.1 Modeling

As can be seen from previous section, only sliding speed dependency of the friction coefficient is known. To model pressure and temperature dependency, following assumption is used:

Every parameter of Stribeck equation is pressure and temperature dependent.

which can be written as follows:

$$\mu_f(\omega, P, \theta) = \mu_C(P,\theta) + (\mu_S(P,\theta) - \mu_C(P,\theta))e^{-|\frac{\omega}{\omega_S(P,\theta)}|^{i(P,\theta)}} + k_V(P,\theta)\omega \tag{4.9}$$

These dependencies are approximated with 2nd order polynomials:

$$\begin{aligned}
\mu_C &= a_0 + a_1P + a_2P^2 + a_3\theta + a_4\theta^2 && (4.10)\\
\mu_S &= a_5 + a_6P + a_7P^2 + a_8\theta + a_9\theta^2 && (4.11)\\
\omega_S &= a_{10} + a_{11}P + a_{12}P^2 + a_{13}\theta + a_{14}\theta^2 && (4.12)\\
i &= a_{15} + a_{16}P + a_{17}P^2 + a_{18}\theta + a_{19}\theta^2 && (4.13)\\
k_V &= a_{20} + a_{21}P + a_{22}P^2 + a_{23}\theta + a_{24}\theta^2 && (4.14)
\end{aligned}$$

As can be seen, the polynomials used are of second order and do not include terms with both temperature and pressure. This is because the number of variables increases computational complexity of the model and therefore higher order and interdependent terms are omitted. Another reason for not including the interdependent terms is that the cross-dependency influence is considered to be small. Also since the tests conducted to collect data from the wet clutch were done by a third party who did not consider the influence, we are not able to account for them.

The i parameter is responsible for the slope of the Stribeck curve after breakaway point. Mostly it's been defined as empirical parameter relating mainly to used material[9][18], however some research suggests that for a system with effective boundary lubricant it can be large[14] and the slope is then almost vertical. However according to sources cited, its value is usually set to $i = 1$ or $i = 2$. The Stribeck curve with different i parameters is visualized on Figure 4.2.

Therefore, this parameter seems to be dependent mostly (if not only) on friction material and thus it can be approximated as a constant instead of being dependent on pressure and temperature.

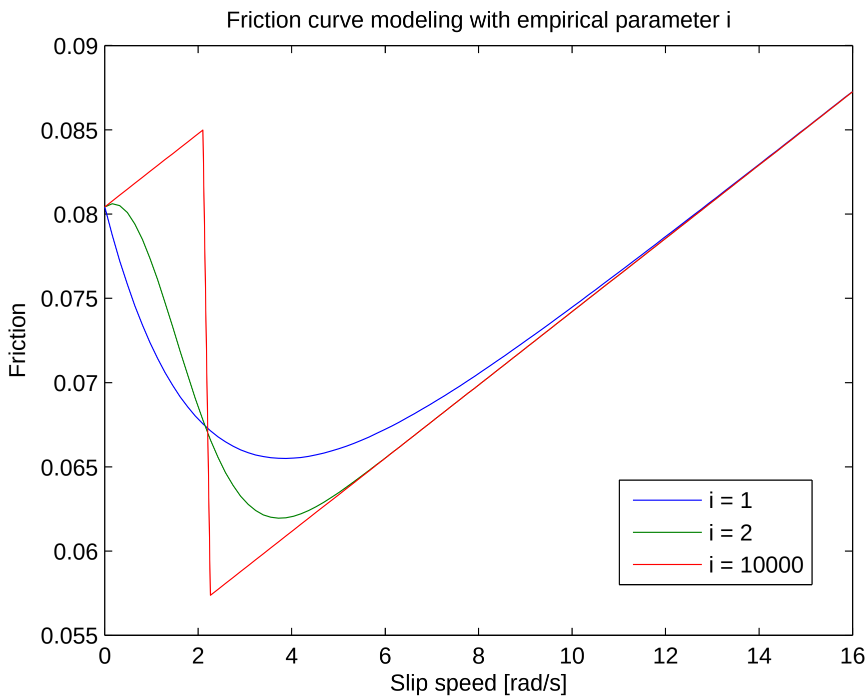


Figure 4.2: Friction curves graph with different i

Chapter 5

Identification

There are three models, that need to be identified:

- viscosity and thermal model (see section 3.2) because the clutch model provided by EATON was equipped with production version of these models, which cannot be disclosed in this thesis.
- friction model

As a method of identification, nonlinear regression was chosen. There are two available datasets used for friction model identification - verification and validation data. The verification dataset is used for regression while validation dataset is used for refinement of the parameters with a Monte Carlo method.

5.1 Data

Tests conducted on a wet clutch are following:

- SAE#2 testing bed[13], which measured friction coefficient as well as other parameters needed for identification (pressure, plate temperature, slip speed) - verification dataset
- real car experiments where friction coefficient isn't measured, however resulting clutch torque is as well as all the other parameters - validation dataset

The verification dataset was measured for multiple different values of slip speed and sump temperature. Slip speed and sump temperature were made constant, then the pressure was changed in steps multiple times and plate temperature and friction coefficient were measured. Then the slip speed was changed to another value and the whole process was repeated. This whole process of changing slip speed and pressure was then repeated with different sump temperatures. This dataset is used for nominal values identification with nonlinear regression.

The validation data was also measured for multiple different values of slip speed and sump temperature in the same manner as the verification data. System validation consisted of multiple test runs, each with 4 different torque requests. The response of the system (the clutch torque) and input variables (pressure, slip speed, sump temperature) were measured. The collected data was then filtered with median filter and then adjusted by a bias, so that all data points were above zero. This was done due to the noise from the sensor, which was sometimes causing the torque measurement to be negative. This dataset is used for validation and refinement of nominal values with Monte Carlo method.

The thermal and viscosity models were developed by EATON and could not be disclosed. Therefore the data collected from these models was fitted to known equations 3.17 and 3.18. The datasets for viscosity and thermal model identification were created from validation data using slip speed, applied pressure and sump temperature as inputs for production models used by EATON. The response of the viscosity and thermal models to these inputs was used as a dependent variable for regression analysis - in case of thermal model, plate temperature, which was used as input for viscosity model, which produces ATF viscosity as an output.

5.2 Regression

Regression is set of statistical processes used for estimating relationships among variables[21]. In this particular case, nonlinear or linear least squares method is used to estimate parameters of models. This method is based on minimizing of squared error between measured and estimated variables:

$$min \sum_{j=1}^{m} (y_j - f(x_j, \mathbf{b}))^2 \tag{5.1}$$

where m is number of measured datapoints, y_j is dependent variable, x_j is independent variable and $\mathbf{b}$ is vector of parameters used to put the measured variables in a relation. The linear version of this method has closed form solution, for nonlinear version the result can be achieved through numerous iterative methods.

5.2.1 Friction coefficient identification

The verification dataset contains measurements of slip speed, pressure, temperature of the plates and friction coefficient which are used for identification of nominal values. Thus, nonlinear least squares method is used with equations 4.9 - 4.14 (with the exception of i, which is considered a constant) as follows:

$$min \sum_{j=1}^{m} [(\mu_C(P_j,\theta_j) + (\mu_S(P_j,\theta_j) - \mu_C(P_j,\theta_j))e^{-|\frac{\omega_j}{\omega_S(P_j,\theta_j)}|^i} + k_V(P_j,\theta_j)\omega_j) - \mu_{f_j}]^2 \tag{5.2}$$

where μ_f is measured friction coefficient.

Because the friction coefficient is equal to static friction coefficient when the slip speed is zero (see definition in section 4), the friction coefficient values for low slip speed are very close to true values of static friction coefficient. Therefore, static friction coefficient polynomial is fit to a friction coefficient measured for the lowest slip speed with MATLAB function $nlinfit()$ and following cost function:

$$min \sum_{j=1}^{m} [\mu_S(P_j, \theta_j) - \mu_{f_j}]^2 \tag{5.3}$$

Then the rest of the parameters of Stribeck equation (or more precisely their polynomials) is fit to the data using MATLAB function $fmincon()$ with inequalities defined in chapter 4. The function conducts a minimization on sum of squared differences between the result of Stribeck equation and measured values of friction coefficient while accounting for the defined constraints and the static friction coefficient fit.

As the inequalities are not defined for the parameters of the polynomials, but for the coefficients of the Stribeck equation, their application is more complex. With the knowledge of possible pressure ($P_{min} = 0$, $P_{max} = 20MPa$) and temperature ($\theta_{min} = 0°C$, $\theta_{max} = 230°C$) range (inferred from verification data), for second order polynomials the inequalities have to be checked for:

- all endpoints of defined intervals, i.e. [P_{max}, θ_{max}], [P_{max}, θ_{min}], [P_{min}, θ_{max}], [P_{min}, θ_{min}]
- extreme of the polynomial, if it is in the defined interval of pressure and temperature; e.g. for

$$\omega_S = a_{10} + a_{11}P + a_{12}P^2 + a_{13}\theta + a_{14}\theta^2 \tag{5.4}$$

  the extreme is $[\frac{-a_{11}}{2a_{12}}, \frac{-a_{13}}{2a_{14}}]$, and if following is correct:

$$P_{min} < \frac{-a_{11}}{2a_{12}} < P_{max} \tag{5.5}$$

$$\theta_{min} < \frac{-a_{13}}{2a_{14}} < \theta_{max} \tag{5.6}$$

  then the extreme has to be checked as follows:

$$0.001 < a_{10} + a_{11}(\frac{-a_{11}}{2a_{12}}) + a_{12}(\frac{-a_{11}}{2a_{12}})^2 + a_{13}(\frac{-a_{13}}{2a_{14}}) + a_{14}(\frac{-a_{13}}{2a_{14}})^2 < 0.05 \tag{5.7}$$

- extreme in both endpoints of defined interval of pressure/temperature, if it is in the corresponding interval of pressure or temperature; e.g. for P_{max}:

$$\omega_S = a_{10} + a_{11}P_{max} + a_{12}P_{max}^2 + a_{13}\theta + a_{14}\theta^2 \tag{5.8}$$

Therefore the extreme of the polynomial is $\frac{-a_{13}}{2a_{14}}$; if it belongs to the interval $[\theta_{min},\theta_{max}]$, following inequality has to be checked:

$$0.001 < a_{10}+a_{11}P_{max}+a_{12}P_{max}^2+a_{13}(\frac{-a_{13}}{2a_{14}})+a_{14}(\frac{-a_{13}}{2a_{14}})^2 < 0.05 \quad (5.9)$$

To evaluate the results, root mean square error (RMSE) is used:

$$RMSE = \sqrt{\frac{1}{m}\sum_{j=1}^{m}(\mu_{f_j} - \hat{\mu}_{f_j})^2} \quad (5.10)$$

where m is number of measured datapoints, μ_{f_j} is measured friction coefficient and $\hat{\mu}_{f_j}$ is estimated friction coefficient.

5.2.2 Viscosity and thermal model

The only unknown variable in the thermal model is the heat transfer coefficient and therefore the equation 3.18 is transformed as follows:

$$H_{tf} = \frac{\rho V c_p}{A_N(\theta_{pack} - \theta_{sump})}(\frac{\omega T_{clu}}{N_{sp}\rho V c_p} - \frac{d\theta_{pack}}{dt}) \quad (5.11)$$

When the measured data and known parameters are used to calculate the heat transfer coefficient, together with other inputs it is used to fit the following regression curve which was obtained through parameter optimization in [7]:

$$H_{tf} = b_1 + b_2\omega + b_3F_{app} + b_4\omega F_{app} \quad (5.12)$$

As regression method, linear least squares were used:

$$\mathbf{H_{tf}} = \begin{pmatrix}\mathbf{1} & \omega & \mathbf{F_{app}} & \omega\mathbf{F_{app}}\end{pmatrix}\begin{pmatrix}b_1\\ b_2\\ b_3\\ b_4\end{pmatrix} = \mathbf{X}b \quad (5.13)$$

$$b = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{H}_{tf} \quad (5.14)$$

The viscosity model represented by empirical relation in equation 3.17 was fit to data using nonlinear least squares method with MATLAB function $nlinfit()$.

As evaluation metric, root mean square error was used.

5.3 Monte Carlo

The "Monte Carlo" in the name of any method implies involvement of a stochastic element, usually repeated random sampling from a given probability distribution. They are used because it allows for exploring behaviour of complex systems in response to random inputs which can help with simplifying a problem where deterministic solution is too complex or non-viable.

In case of this project, initial estimates of parameters of the polynomials in the friction model are obtained through regression. However these estimates turned out to be not much accurate when the model is used with validation data. Also this dataset is not suitable for regression as it does not contain measured friction coefficient. Therefore method for using this dataset needed to be introduced.

The method is based on substituting parameters of the polynomials in the friction model with random samples from interval centred around nominal values of the parameters. The sampling is done uniformly and the endpoints of intervals are chosen as a percentage (<100%) of the nominal value. The intervals chosen are ±5% and ±20%:

$$\mathbf{a} \sim U(\mathbf{a_{nominal}} - 0.05\mathbf{a_{nominal}}, \mathbf{a_{nominal}} + 0.05\mathbf{a_{nominal}}) \quad (5.15)$$

$$\mathbf{a} \sim U(\mathbf{a_{nominal}} - 0.2\mathbf{a_{nominal}}, \mathbf{a_{nominal}} + 0.2\mathbf{a_{nominal}}) \quad (5.16)$$

where $\mathbf{a_{nominal}}$ is vector of polynomial coefficient values of the friction model.

The output of the whole model relies on combinations of all these parameters. An exhaustive deterministic search for the true estimates would therefore be quite difficult and computationally expensive. However if random element is utilized in the search there is a chance that the estimates can be found more easily as the interval of each parameter is explored with enough samples. The mean of those samples that give better results than nominal values (if any) could be close to true estimate of the parameters.

This idea is based on law of large numbers, which states that if there are independent and identically distributed samples from a probability distribution, then the sample average converges to true mean of said distribution with growing number of samples.

Samples generation in Matlab is done by using $rand()$ function, which generates samples uniformly in the interval (0,1). The samples are then calculated into some desired interval (lb,ub) as follows:

$$p = lb + (ub - lb)rand() \quad (5.17)$$

where ub is upper boundary of the interval, lb is lower boundary of the interval and p is generated sample. The generated sample is also checked for if the resulting friction is positive in the possible pressure and temperature interval. That is because some combinations of the parameters may actually yield negative friction which is by definition positive.

There were 300000 samples generated which were then simulated and the outputs were then compared with validation data using root mean square error. The simulations were computed in parallel using MATLAB Parallel computing toolbox and Simulink on a computer with 16 core processor.

Chapter 6

Results

6.1 Thermal and viscosity model

The viscosity model was fit to the data with $RMSE = 0.0021$. On Figure 6.1 there is comparison of the measured viscosity and output of the identified model:

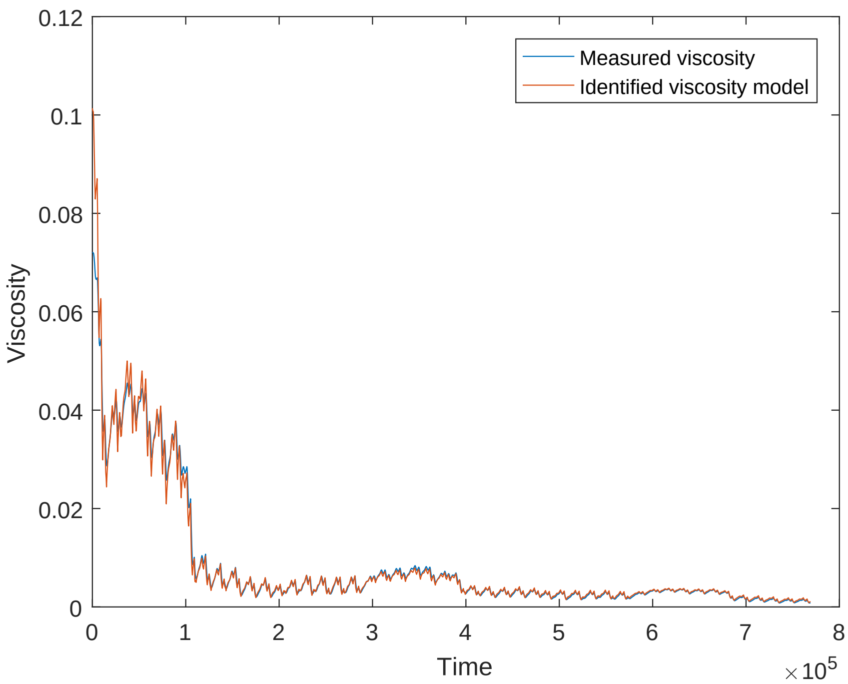


Figure 6.1: Viscosity model fit

The values of identified parameters can be found in the table 6.1:

Parameter	Value
α_0	31.67
α_1	1.89

Table 6.1: Table of viscosity model parameters

The heat transfer coefficient was identified and when used in the thermal model, the output with respect to original EATON model has $RMSE = 6.1204$. On Figure 6.2 the comparison is shown:

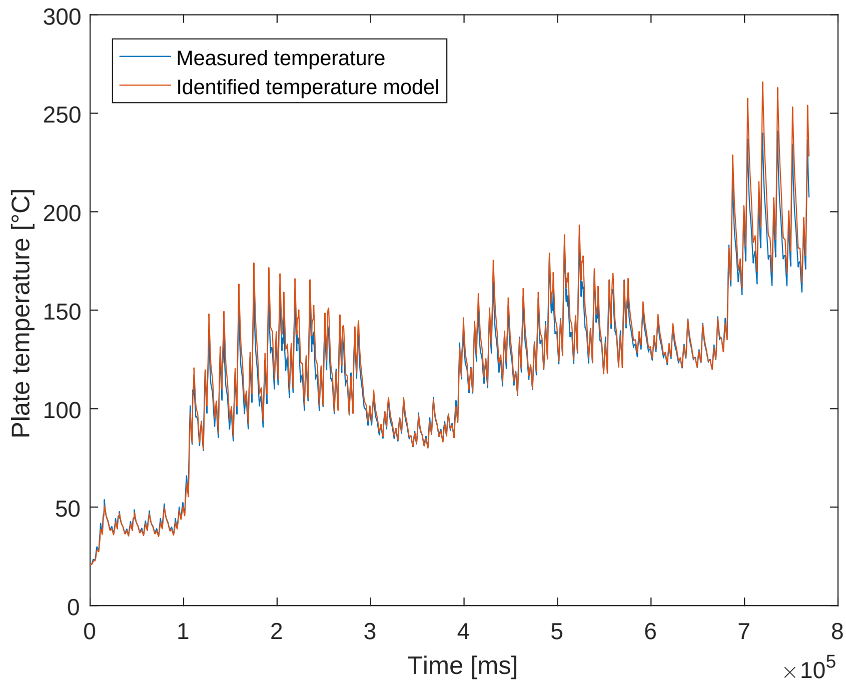


Figure 6.2: Temperature model fit

The identified parameters of heat transfer coefficient can be found in the table 6.2:

Parameter	Value
b_1	149.40
b_2	13.72
b_3	-0.0036
b_4	-0.0099

Table 6.2: Table of heat transfer coefficient model parameters

Identification of these two models was done because the models used in the wet clutch model by EATON cannot be disclosed for this thesis. However these results show that the models are at least based in the equations presented here. This is also supported by the fact that the whole model output with production models in comparison to the output with models presented here has $RMSE = 18.19$. This identification process could be easily used if the data for thermal and viscosity model were to be collected.

6.2 Friction model

6.2.1 Nonlinear regression

Since the properties of ATF used in the differential are unknown, the condition (equations 4.4 and 4.5) for Coulomb friction parameter (4.3) has to be discovered empirically.

When applying nonlinear regression to the data with condition 4.4 defined, the solver reports not being able to find a solution. The RMSE of best parameters found with respect to identification data is $RMSE = 0.0269$.

When condition 4.5 is applied, solver reports solution to be a possible local minimum, however the result cannot be certain because of first order optimality measure not being less than optimization settings for the measure[27]. The result satisfies given constraints and the solver stopped because the step size (relative change in the parameters[17]) was too small[24]. The RMSE of best parameters found was $RMSE =$ 0.0108 with respect to identification data. Therefore it is reasonable to assume the condition 4.5. This is also supported by measured clutch torque from the car because it doesn't exhibit rooster tail towards the end of engagement (as mentioned in section 3.1), which can be seen on Figure 6.10. As can be seen from graphs below, the identified parameters comply to their constraints and the model fits the data well.

First, static friction $\mu_S(P, \theta_{pack})$ was fit to the lowest speed data. The graph is shown on Figure 6.3. As can be seen, its value drops with rising temperature.

The Coulomb friction $\mu_C(P, \theta_{pack})$, sliding speed coefficient $\omega_S(P, \theta_{pack})$, empirical parameter i and viscous friction coefficient $k_V(P, \theta_{pack})$ were then fit to the data. The graphs of μ_C, ω_S and k_V dependency on pressure and temperature can be seen on Figures 6.4, 6.5 and 6.6. The graph for Coulomb friction μ_C also shows that its value drops with rising temperature, similarly to static friction. This could be explained by the fact that with growing temperature, viscosity of the ATF drops (see equation 3.17). Viscosity is a measure of fluids resistance to deformation - if it is high, it is harder for an object to move in it and the friction is high. If it is low, objects can move more easily in the fluid and the friction is low[28]. That corresponds to the trend observed in graphs for parameters μ_C and μ_S.

The empirical parameter i was identified as $i = 3.11$. The rest of the parameters can be seen in the table 6.3.

The resulting friction coefficient dependency on pressure and slip speed for temperature of $\theta_{pack} = 20$, $\theta_{pack} = 60$ and $\theta_{pack} = 100$ can be seen on Figures 6.7, 6.8 and 6.9.

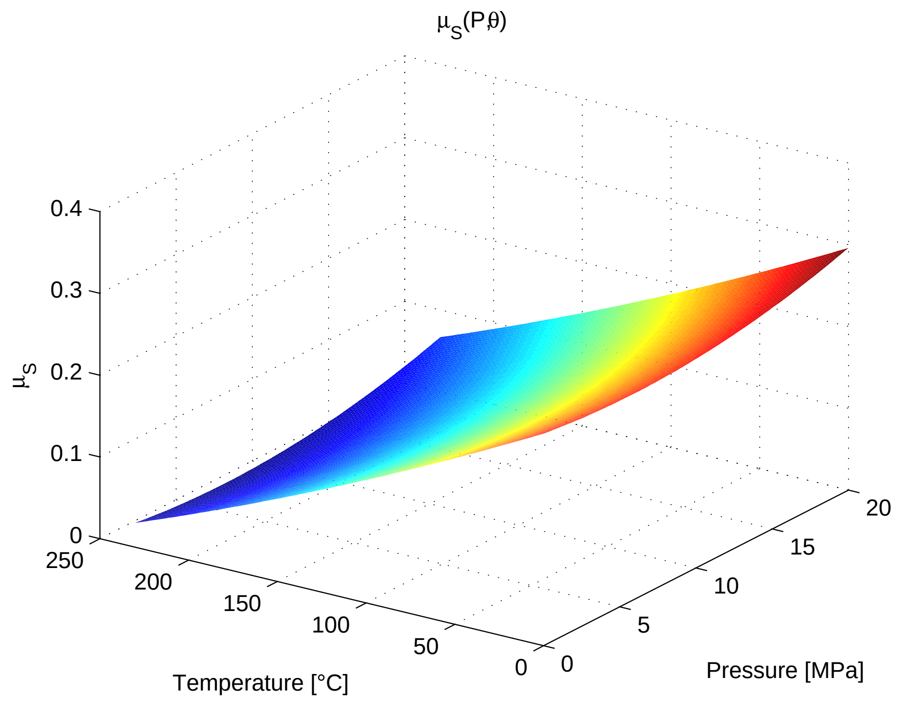


Figure 6.3: μ_S fit

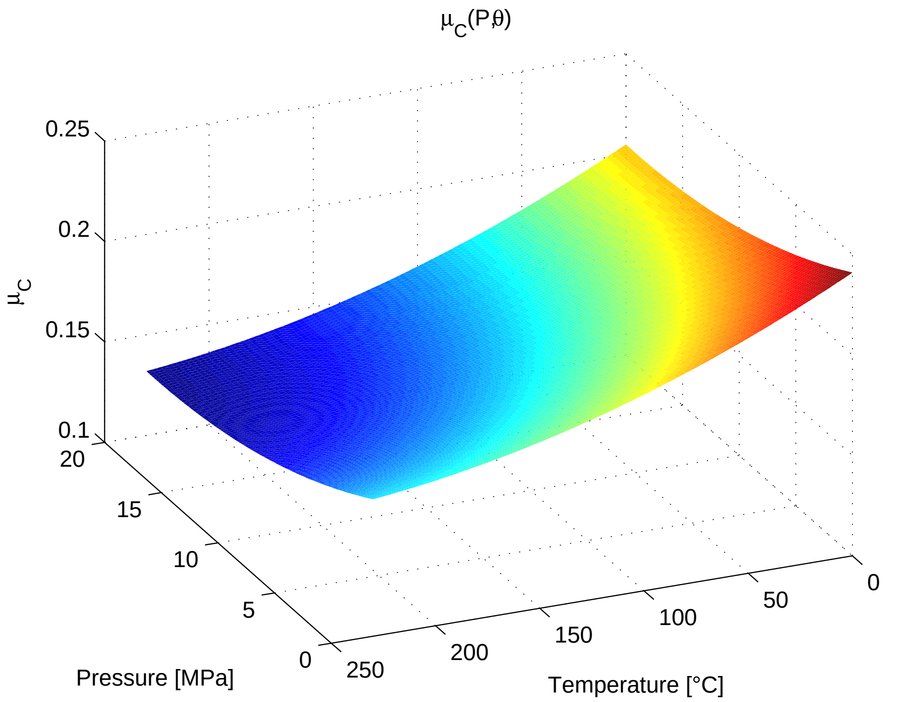


Figure 6.4: μ_C fit

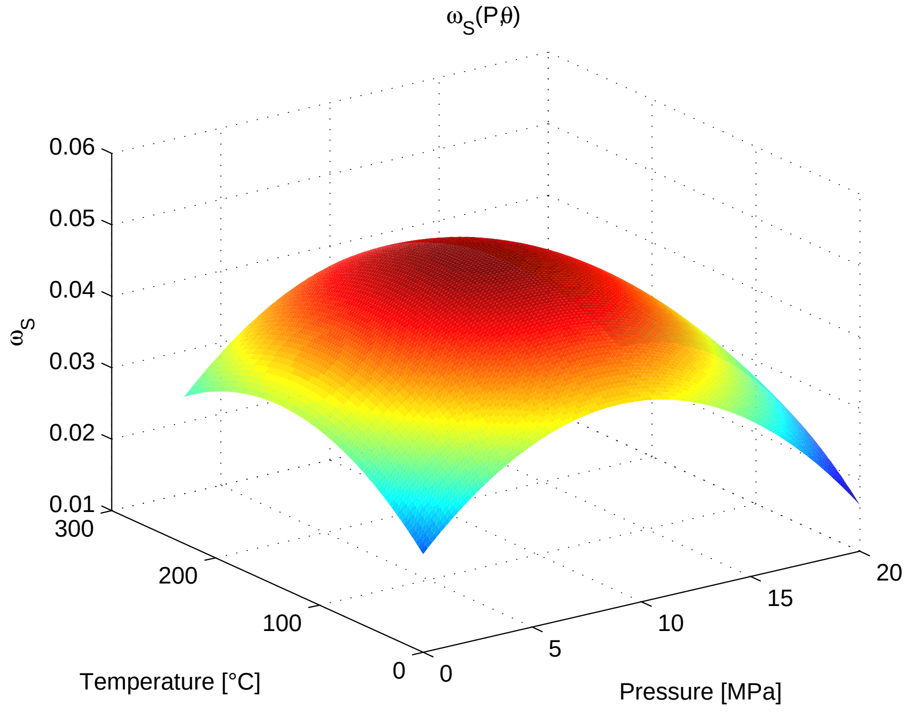


Figure 6.5: ω_S fit

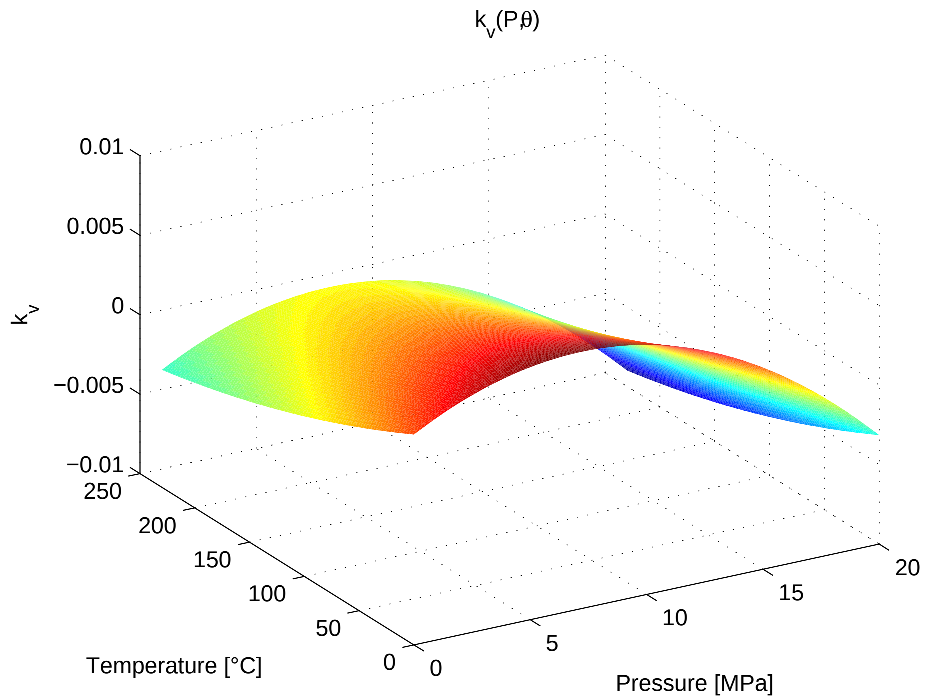


Figure 6.6: k_V fit

Parameter	Polynomial Parameter	Value
μ_C	a_0	0.24
	a_1	-0.0037
	a_2	9.63e-05
	a_3	-5.24e-04
	a_4	9.02e-07
μ_S	a_5	0.26
	a_6	-0.0026
	a_7	2.22e-04
	a_8	-0.0012
	a_9	7.08e-07
ω_S	a_{10}	0.024
	a_{11}	0.0032
	a_{12}	-1.80e-04
	a_{13}	1.67e-04
	a_{14}	-5.98e-07
i		3.11
k_V	a_{15}	0.0032
	a_{16}	8.13e-04
	a_{17}	-5.66e-05
	a_{18}	-3.65e-05
	a_{19}	4.81e-08

Table 6.3: Table of friction model parameters

The identified model was then used in simulation with the validation data from real car measurements. The root mean square error was $RMSE = 360.57$ and the graph[1] can be seen on Figure 6.10.

Upon inspection of the comparison of the simulated and measured clutch torque it can be noticed that the amplitude of the simulated clutch torque is close to the measured one mostly for the lower torque steps. The higher torque steps are not as close, which may be due to a modeling error. The dynamics of each torque step do not match up as well. Because the dynamics are largely determined by the properties of the friction material and the ATF is suspected that there isn't sufficient modeling of these properties.

[1]The graph contains only part of validation data for confidentiality and visualization reasons.

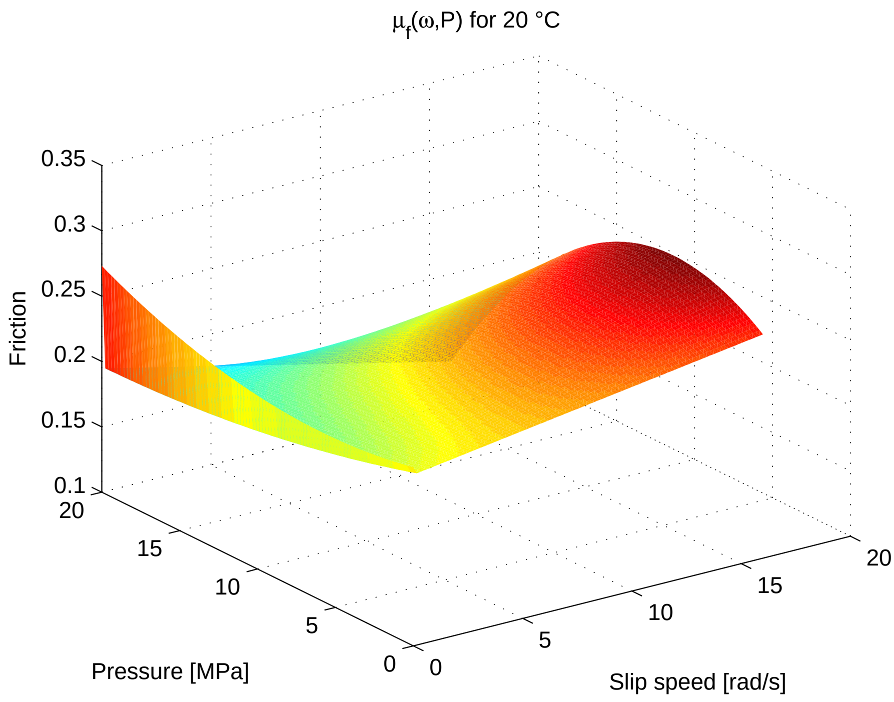


Figure 6.7: Friction coefficient for $20°C$

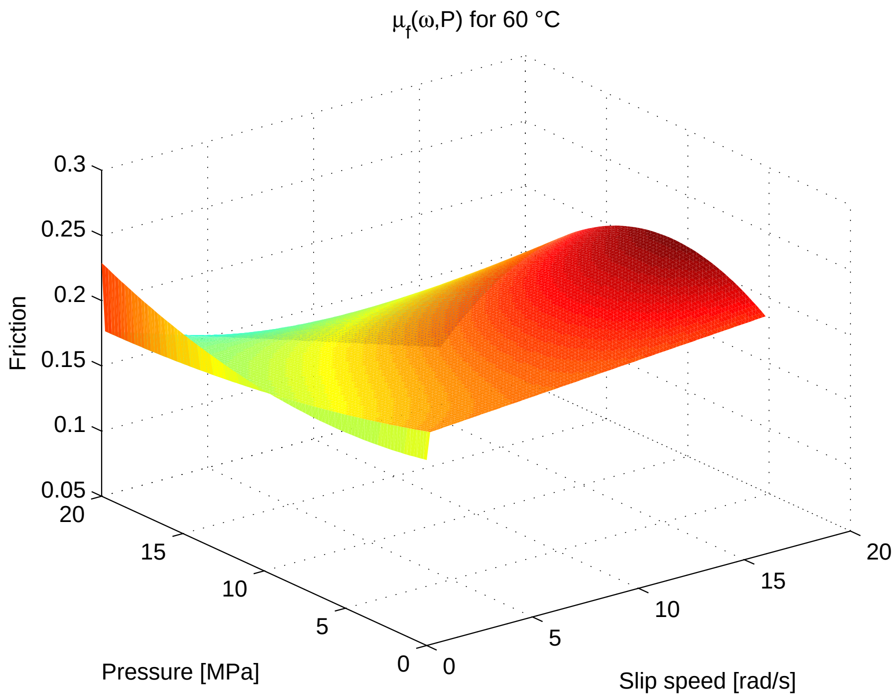


Figure 6.8: Friction coefficient for $60°C$

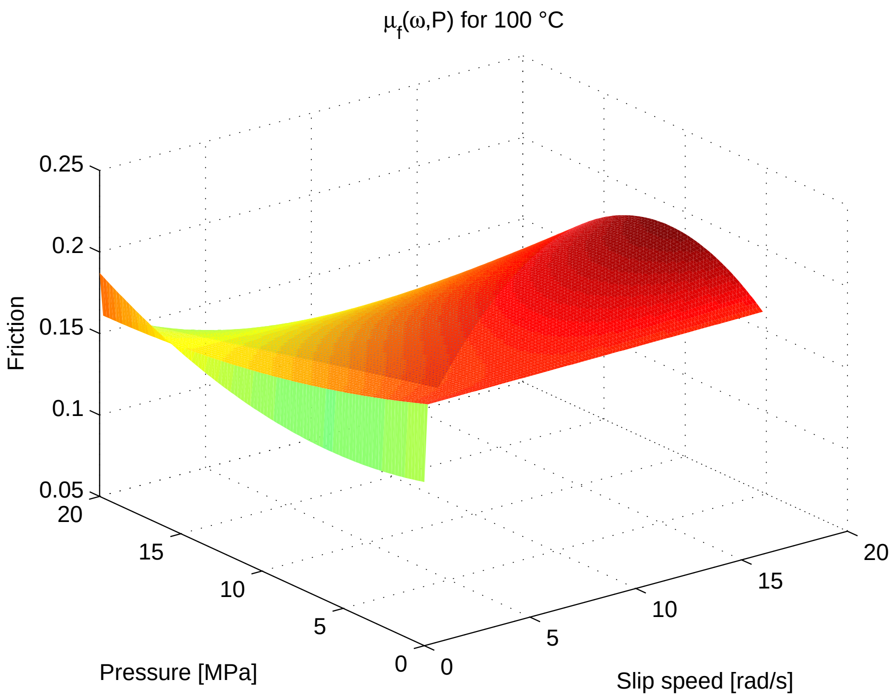


Figure 6.9: Friction coefficient for $100°C$

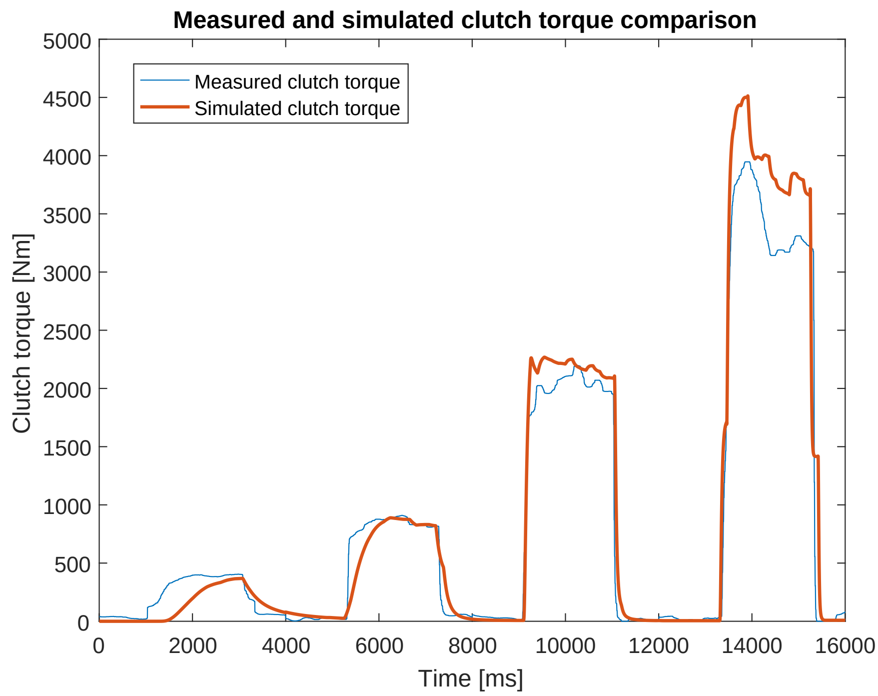


Figure 6.10: Measured and simulated clutch torque comparison

6.2.2 Monte Carlo

For simulating the model in Simulink, Rapid accelerator was used. The total time of one full simulation can be up to 20 seconds with sampling rate of 0.001 ms. The computation was done in parallel on a batch of 80 samples which made the simulations considerably faster and it prevented the computer from overloading its RAM, which can cancel the simulation without a possibility of data retrieval. However simulation of all the samples can take up to two weeks of running on a computer with 16 core processor and 32 GB of RAM.

There were 300 000 samples generated in both 5% and 20% intervals centered around the nominal values. For 5% interval only half of the simulations were done because the results were sufficient for further analysis. The samples for 20% interval were all simulated.

For the 5% interval the best result was $RMSE = 359.77$ and for the 20% interval the best result was $RMSE = 356.45$. Given the fact that the nominal values had $RMSE = 360.57$, improvement can be observed but it is not of great significance. However if the samples of all the simulations that resulted in better RMSE than nominal values are examined, we can discover trends in some of the parameters. If there would be a similar trend with the same parameters in samples from both 5% and 20% their actual value could be further deduced, which might improve the overall accuracy of the model.

On Figures 6.11, 6.12 and 6.13 it can be seen that the samples from 5% interval around the nominal values of parameters a_0, a_3 and a_4 visually exhibit a significant trend. The mean of the generated samples which supports this observation is also displayed.

Other parameters don't exhibit such a significant trend. When the samples from 20% interval are examined, the results are quite similar. In case of parameters a_3 and a_4, the trend is similar as in the results from 5% interval simulations. However in the case of parameter a_0 the samples are concentrated around a value inside the 20% interval. This could mean that the true value of the parameter had been discovered. The graphs can be seen on Figures 6.14, 6.15 and 6.16.

When the mean of these samples is used in simulation where the rest of the parameters had their nominal values, the resulting clutch torque has $RMSE = 354.09$ with respect to measured clutch torque. There is a minor improvement to the results from the Monte Carlo simulations. Also, the trend seen in samples for parameters a_3 and a_4 can be explored with the method of interval halving.

This method is designed to find a value of a parameter in given interval based on some evaluation metric. The middle of the interval is taken as reference point and the evaluation metric is calculated for this point. Then the interval is split into two halves and the evaluation metric is calculated for middle points of both of these new intervals. The interval whose middle point has better result is taken as the new starting interval and the whole process is repeated until the desired value of the parameter is found with defined accuracy. The algorithm used is described in [25].

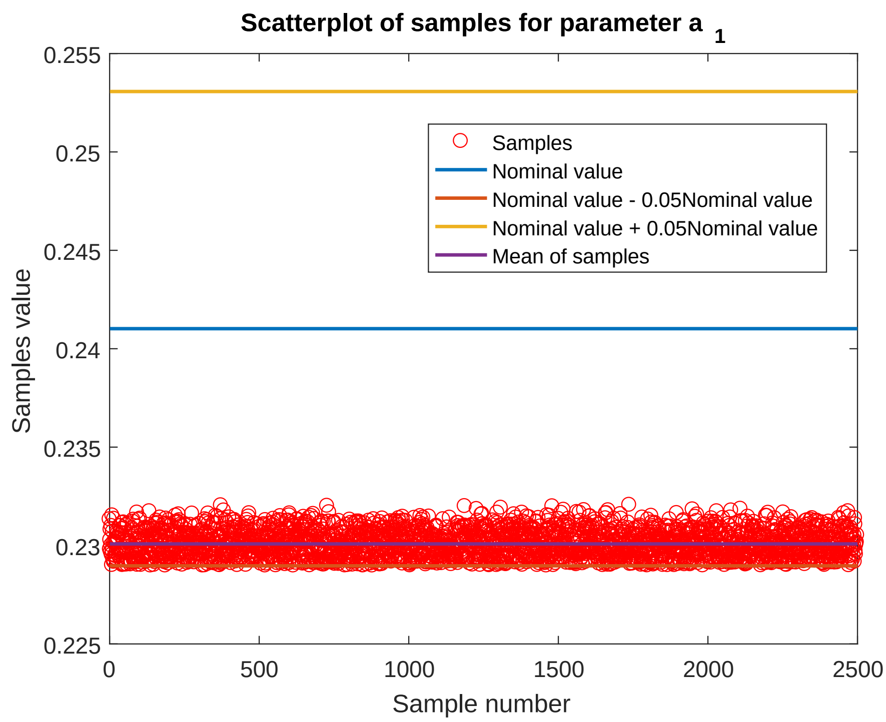


Figure 6.11: Scatterplot of samples within 5% interval that result in better RMSE than nominal values for parameter a_0

This method was applied first on parameter a_3 and then a_4, with the starting interval defined as $< a_{nominal}, a_{nominal} + 0.4a_{nominal} >$. Given the fact that the parameter a_4 shows weaker trend than parameter a_3 (the samples are scattered in both directions around the nominal value where the mean of all samples shows a small trend towards upper boundary of the interval) its value is not expected to move much. The results can be found in Table 6.4.

Parameter	Mean Value	Interval halving
a_0	0.2236	-
a_3	-4.57e-04	-4.04e-04
a_4	9.34e-07	9.11e-07

Table 6.4: Table of parameters improved by monte carlo method

The simulation with these improved parameters yielded $RMSE = 352.14$ of the simulated clutch torque with respect to measured clutch torque. The simulation result can be seen on Figure 6.17.

Even though the overall model didn't improve much and the execution of this method is not easy, it may still prove useful. Originally the model was identified with more loosely defined constraints on the parameters of the Stribeck equation. This was based on article [8] which concerned parameters ω_S which was defined as a positive empirical constant and k_V which also seemed to be defined as only positive.

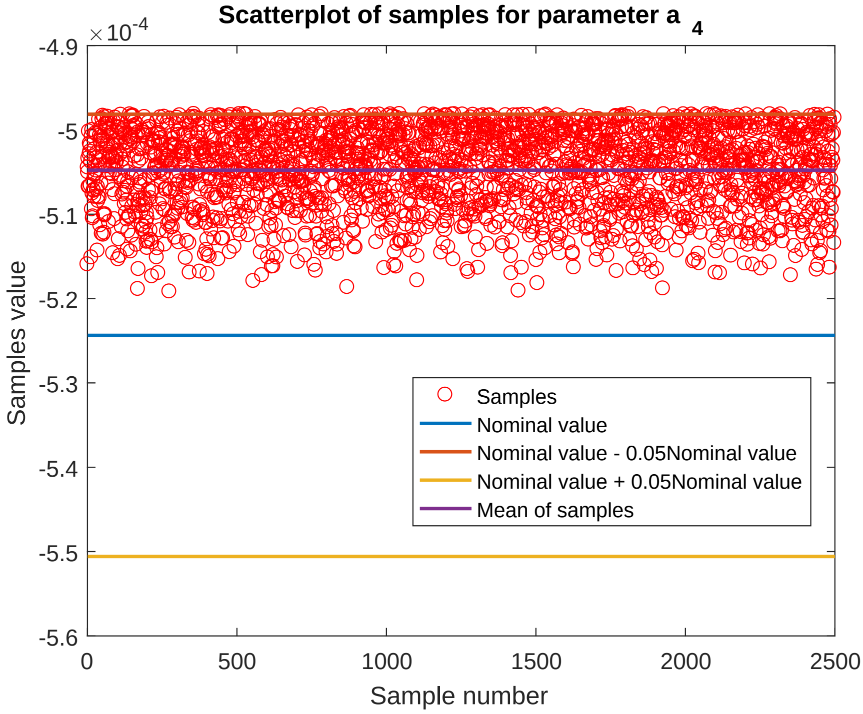


Figure 6.12: Scatterplot of samples within 5% interval that result in better RMSE than nominal values for parameter a_3

During the early stages of the project, an experiment based on these assumptions similar to Monte Carlo method presented here was conducted. The experiment set each parameter with either $+100\%$ or -100% of its nominal value and simulated every combination. Given 20 parameters, there was 2^{20} possible combinations to be examined. The RMSE improved significantly by 16% during the experiment.

However then the constraints on the Stribeck equation parameters were defined more accurately based on articles [5], [4] and [12]. This resulted in significantly better model presented in section 6.2.1 and thus the process described above was applied.

This may serve as a proof that the identification method presented in this thesis might help in detecting problems such as when the apriori setting of the process is not correct.

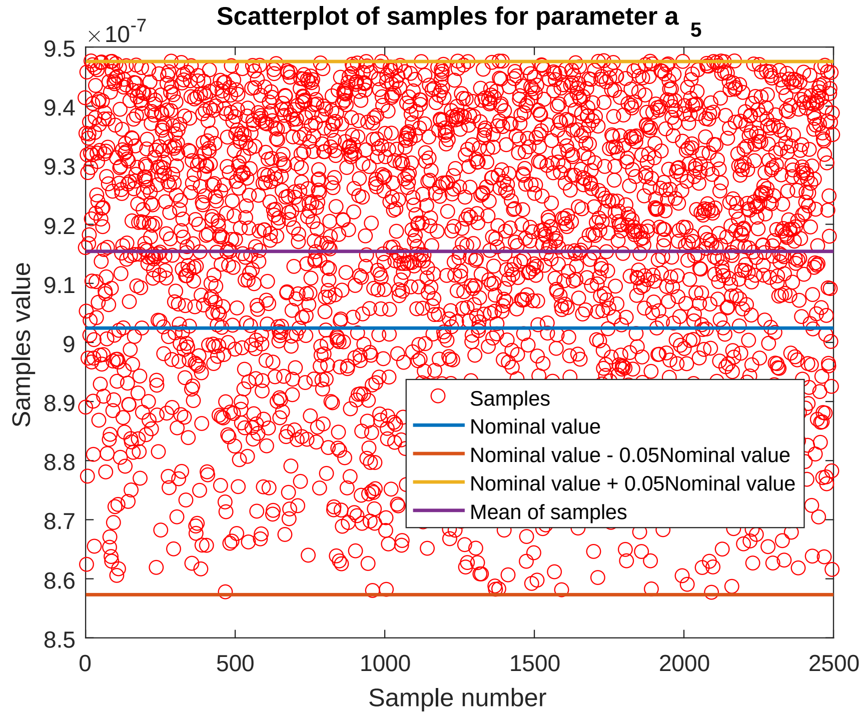


Figure 6.13: Scatterplot of samples within 5% interval that result in better RMSE than nominal values for parameter a_4

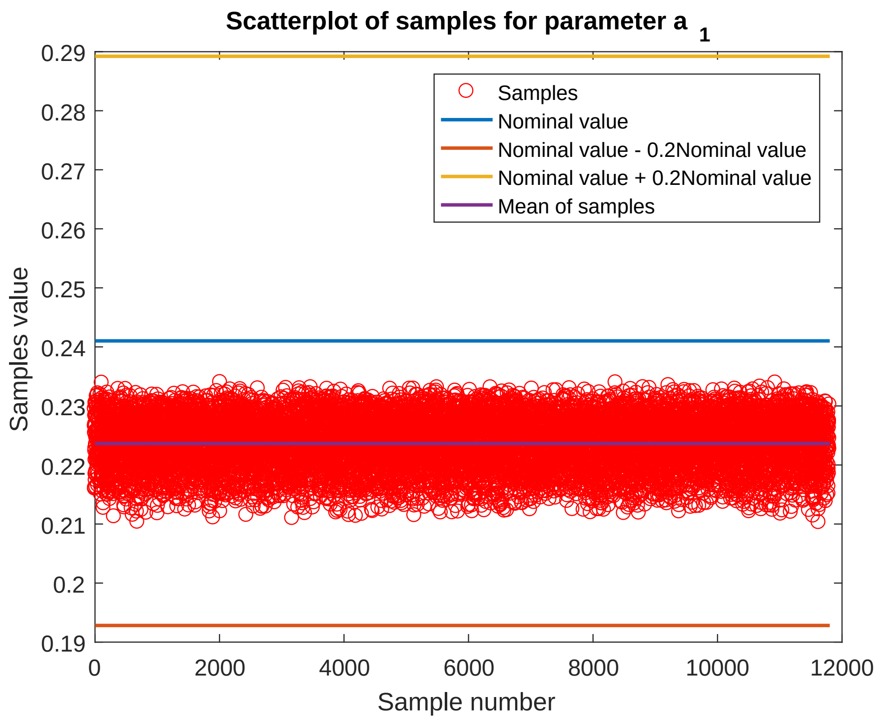


Figure 6.14: Scatterplot of samples within 20% interval that result in better RMSE than nominal values for parameter a_0

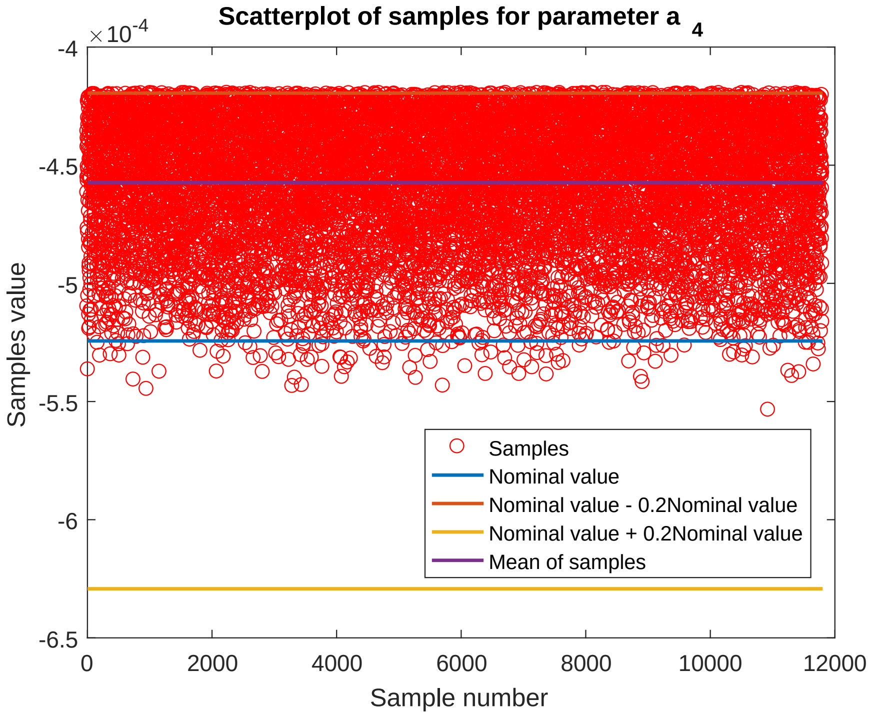


Figure 6.15: Scatterplot of samples within 20% interval that result in better RMSE than nominal values for parameter a_3

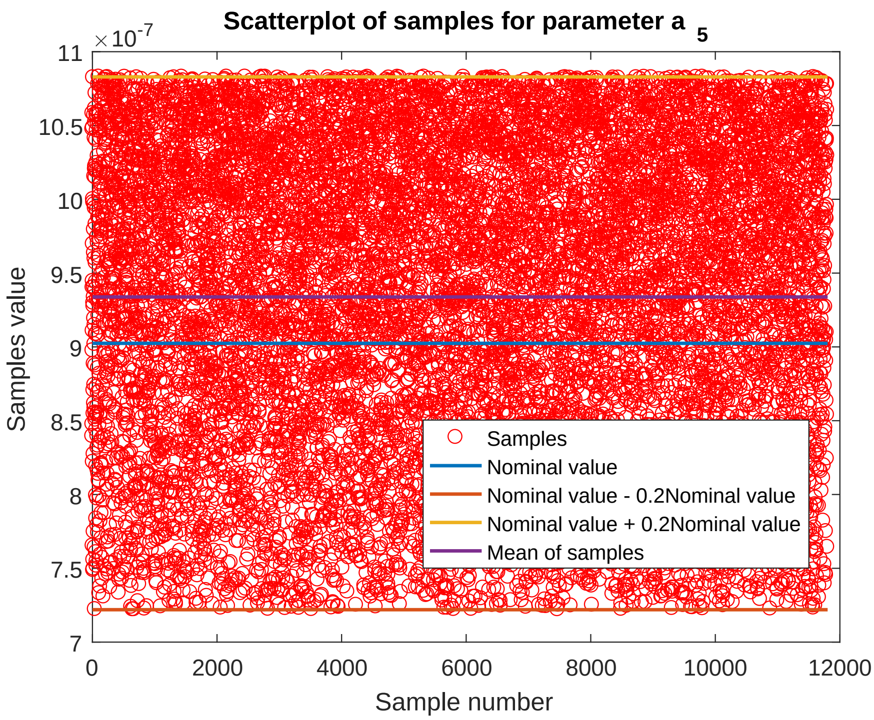


Figure 6.16: Scatterplot of samples within 20% interval that result in better RMSE than nominal values for parameter a_4

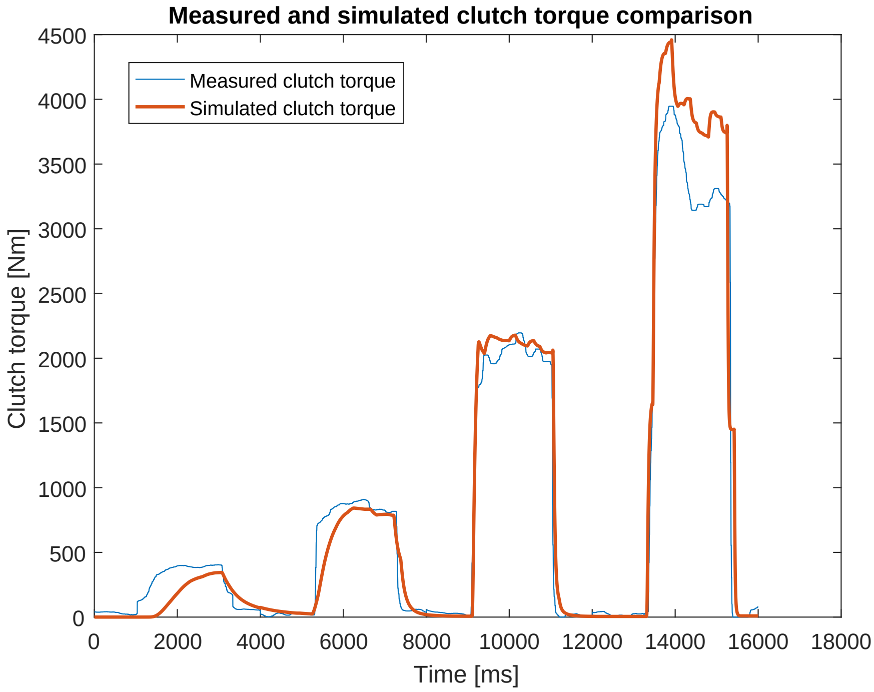


Figure 6.17: Measured clutch torque compared to output of simulation with parameters gained from Monte Carlo

Chapter 7

Conclusion

A model of a wet clutch used in an electronic limited slip differential made by company EATON was introduced. The model description was put together based on research papers which however provided insufficient modeling of the friction coefficient.

The wet clutch model was shown to be sensitive even to small changes (1% perturbation) in friction coefficient by performing sensitivity analysis. It is known that the friction coefficient is dependent on pressure, temperature, slip speed, properties of automatic transmission fluid contained in the clutch case and properties of the friction material used. This dependency was modeled and it was based on Stribeck equation which models dependency of the friction coefficient on slip speed. The model introduced considers parameters of Stribeck equation to be dependent on pressure and temperature which was modeled with polynomials. The model is designed for one type of automatic transmission fluid and friction material.

The model was then identified with nonlinear regression on a dataset collected from a test bed designed for clutches according to SAE#2 standard. The identification was successfully done with root mean square error $RMSE = 0.0269$. However this identified model's accuracy on a second dataset consisting of measurements of clutch torque in a real car's differential was much lower with $RMSE = 360.57$.

Another method based on random sampling was used to try further refine the parameters. Each polynomial parameter was substituted with number of random samples uniformly drawn from interval with boundaries $\pm 5\%$ and $\pm 20\%$ around nominal values obtained from nonlinear regression on testbed dataset. The model with each of these samples was then simulated. The mean of samples which yielded better results than the nominal values was then used as a better estimate of the true values of the parameters. The model accuracy was slightly improved to $RMSE = 352.14$.

This approach also significantly improved model's accuracy during early stages of this project when it improved identification of the model that was based on not well defined apriori information for nonlinear regression.

The results lead us to conclusion, that there is probably some unmodeled phenomena or an error in the wet clutch model. The data provided by EATON was not collected under our supervision and are not exhaustive enough. However if the wet clutch model that was obtained in this thesis is compared to EATON's production wet clutch model significant improvement can be observed (Figure 7.1). The production model has $RMSE = 468.59$ which means almost 25% improvement was achieved.

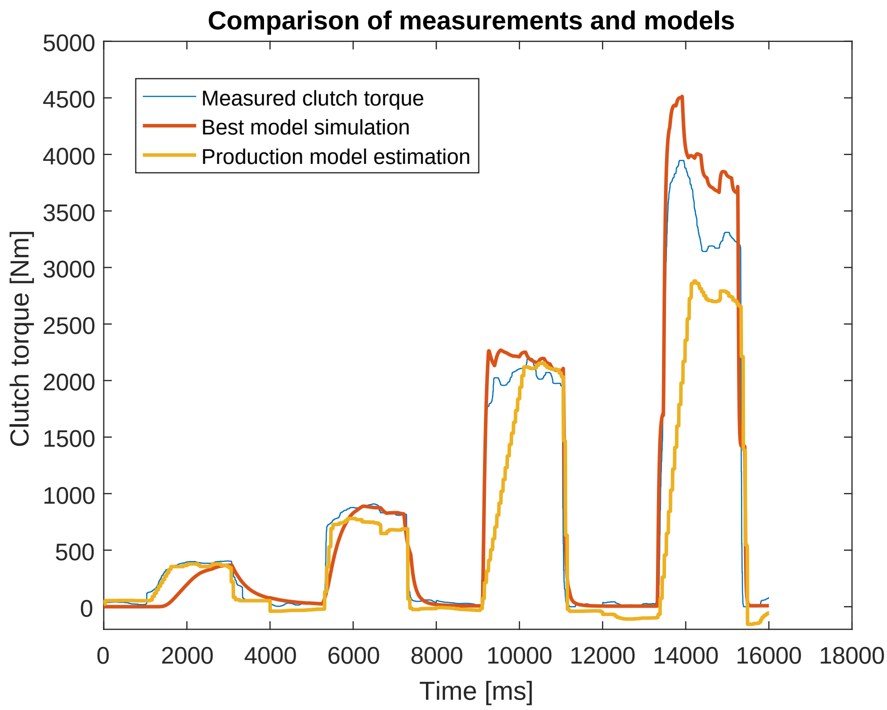


Figure 7.1: Measured clutch torque compared to output of simulation with parameters gained from Monte Carlo and to production model estimation

In conclusion, friction coefficient model for wet clutch model based on Stribeck equation was introduced and identification method for this model was designed. The wet clutch model will continue to be further improved as it is an important part of control design for the electronic limited slip differential, which ensures safety and reliability of the car response. Possible improvement can be achieved by eliminating modeling errors and conducting new tests to collect more exhaustive data.

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Appendix A

CD content

CD attached to this thesis contains:

- folder *Nonlinear_regression*, where:
  - folder *Viscosity_and_Thermal_model* contains files:
    - *thermal_data.mat* with data for thermal model identification
    - *viscosity_data.mat* with data for viscosity model identification
    - *identification.m* which is a Matlab script for running nonlinear regression on the data in the folder. To run either viscosity or thermal model identification uncomment corresponding sections.
  - folder *Friction_coefficient* contains files:
    - *friction_and_inputs.mat* with data for friction coefficient identification
    - *ineqs_2_constant_i.m* which is a Matlab script containing constraints definitions for nonlinear regression
    - *nonlinear_regression.m* which is a Matlab script for running nonlinear regression on the data in the folder.
- folder *Simulink_model*, which contains files:
  - *friction_model_params.mat* with friction model parameters
  - *flow_factor.mat* with ϕ_f look up table data
  - *sheer_stress.mat* with ϕ_{fs} look up table data
  - *film_thickness_correction.mat* with $\phi(h)$ look up table data
  - *clt_data.mat* with validation data
  - *WetClutchModel.slx* which is a Simulink model of the wet clutch
  - *load_wcmodel.m* which is a Matlab script for loading data and parameters for the Simulink model
- folder *Monte_Carlo*, which contains files:
  - *friction_model_params.mat* with friction model parameters
  - *flow_factor.mat* with ϕ_f look up table data

- *sheer_stress.mat* with ϕ_{fs} look up table data
- $film\_thickness\_correction.mat$ with $\phi(h)$ look up table data
- *clt_data.mat* with validation data
- $WetClutchModelParallelSim.slx$ which is a Simulink model of the wet clutch prepared for parallel simulations
- *load_wcmodel_parallel.m* which is a Matlab script for loading data and parameters for the parallel Simulink model
- *to_sim_2nd_order_20_percent.mat* which contains random samples for parallel simulations in the 20% interval around the nominal values of the friction model
- *s_start.mat* with variable containing number of starting parallel simulation, implicitly set to 0
- *parallel_sim.m* which is a Matlab script that runs parallel simulations of the wet clutch model with different friction model samples.
- *run_parallel_sim.m* which is a Matlab script that runs file *parallel_sim.m* in a try-catch construct.